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Certain Results OF $(LCS)_n$-Manifolds Endowed with E-Bochner Curvature Tensor

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ABSTRACT: In this paper, we study geometry of $(LCS)_n$-manifold focusing on some conditions of E-Bochner curvature tensor. First, we describe an E-Bochner pseudo-symmetric $(LCS)_n$-manifold is never reduces to E-Bochner semi-symmetric manifold under the condition $((\alpha^2 - \rho) \neq 0)$. Next, we characterize certain results of $(LCS)_n$-manifold satisfying $B^e(U,V)\xi = 0$, $B^e(\xi,V) \cdot B^e = 0$ and $B^e(\xi,V) \cdot S = 0$.

Key Words: E-Bochner curvature tensor, $(LCS)_n$-manifold, Scalar curvature, ξ-Sectional curvature, η-Einstein manifold.

Contents

1. Introduction

In 1989, Matsumoto [7], Mihai and Rosca [9] have introduced and studied the structure of Lorentzian para Sasakian manifolds (briefly, LP-Sasakian manifolds). Since then, many geometers have weakened the structure of LP-Sasakian manifolds with different extent. For instance, by giving a global approach based on the existence of several examples, Shaikh [14] firstly investigated Lorentzian concircular structure manifolds (briefly, $(LCS)_n$-manifolds) and proved that an $(LCS)_n$-manifold is a space of constant curvature $(\alpha^2 - \rho)$ [15]. In addition to this, Shaiakh and Ahmad [16] proved that an $(LCS)_n$-manifold is always remains invariant under a D-homothetic transformation, which does not holds for an LP-Sasakian manifold. Moreover Shaikh and Baishya [17,18] have studied the applications of $(LCS)_n$-manifolds to the general theory of relativity and cosmology. The structure of $(LCS)_n$-manifolds have been weakened by many geometers viz., Hui [5], Hui and Atceken [6], Prakasha [12], Shaikh et al. [19], Shukla and Shukla [20], Venkatesha et al. [21], Venkatesha and Naveen Kumar [22] etc. Some related developments can be found in [10,11,13].

On the other hand, in 1949, Bochner studied Weyl conformal curvature tensor as a Kahler analogue which is popularly known as the Bochner curvature tensor [2]. Later, the geometric meaning of the Bochner curvature tensor was given by Blair [1]. Then by considering the Boothby-Wang's fibration [3], authors Matsumoto and Chuman have introduced the structure of C-Bochner curvature tensor [8] from

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the Bochner curvature tensor given by

$$\begin{aligned} B(U,V)W &= R(U,V)W + \frac{1}{n+3}[S(U,W)V - S(V,W)U \\ &+ g(U,W)QV - g(V,W)QU + S(\phi U,W)V \\ &- S(\phi V,W)U + g(\phi U,W)Q\phi V - g(\phi V,W)Q\phi U \\ &+ 2S(\phi U,V)\phi W + 2g(\phi U,\phi V)Q\phi W \\ &- S(U,W)\eta(V)\xi + S(V,W)\eta(U)\xi \\ &- \eta(U)\eta(W)QV + \eta(V)\eta(W)QU] \\ &- \frac{p+n-1}{n+3}[g(\phi U,V)W - g(\phi V,W)\phi U \\ &+ 2g(\phi U,V)\phi W] - \frac{p-4}{n+3}[g(U,W)V \\ &- g(V,W)U] + \frac{p}{n+3}[g(U,W)\eta(V)\xi \\ &+ \eta(U)\eta(W)V - g(V,W)\eta(U)\xi - \eta(V)\eta(W)U], \end{aligned} \tag{1.1}$$

where S is the Ricci tensor, Q is the Ricci operator defined by $g(QU,V) = S(U,V)$, $p = \frac{r+n-1}{n+1}$ and r being the scalar curvature of the manifold.

As a generalization of C-Bochner curvature tensor, in 1991 Endo [4] defined the structure of E-Bochner curvature tensor as:

$$\begin{aligned} B^e(U,V)W &= B(U,V)W - \eta(U)B(\xi,V)W \\ &- \eta(V)B(U,\xi)W - \eta(W)B(U,V)\xi, \end{aligned} \tag{1.2}$$

for all U,V,W belongs to TM^n, where B is the C-Bochner curvature tensor. Again he shown that a K-contact manifold with vanishing E-Bochner curvature tensor is always be a Sasakian manifold.

The present paper is organized as follows: In Section 2, we recall basic formulas and results of $(LCS)_n$-manifold which is essential throughout the paper. In Section 3, we study E-Bochner pseudo-symmetric $(LCS)_n$-manifold. Here we prove that either ξ-sectional curvature is a differentiable function L_{B^e} or the manifold turns into η-Einstein and the E-Bochner pseudo-symmetric $(LCS)_n$-manifold is never reduces to E-Bochner semi-symmetric manifold. In Section 4, we consider $(LCS)_n$-manifold such that $B^e(U,V)\xi = 0$. In this case the manifold becomes η-Einstein and hence scalar curvature and ξ-sectional curvature are linearly related to each other. Also the manifold admits an η-parallel Ricci tensor provided scalar curvature or ξ-sectional curvature are constant. In fact, Section 5 is devoted to the study of $(LCS)_n$-manifold satisfying $B^e(\xi,X)\cdot B^e = 0$. We show that either the scalar curvature and ξ-sectional curvature are linearly related to each other or the manifold reduces to special type of η-Einstein and also the manifold admits an η-parallel Ricci tensor. Finally, in Section 6 we obtained the Ricci tensor and Ricci operator of an $(LCS)_n$-manifold satisfying $B^e(\xi,X)\cdot S = 0$.

2. Preliminaries

Let M^n be an Lorentzian manifold with unit timelike concircular vector field ξ, we have

$$g(\xi,\xi) = -1, \quad g(V,\xi) = \eta(V), \tag{2.1}$$

from which it follows that:

$$(\nabla_U\eta)(V) = \alpha[g(U,V) + \eta(U)\eta(V)], \quad (\alpha \neq 0), \tag{2.2}$$

where $U,V \in TM^n$, ∇ represent the covariant differential operator corresponding to Lorentzian metric g and α is a non-zero scalar function satisfying

$$\nabla_V\alpha = (V\alpha) = d\alpha(V) = \rho\eta(V), \tag{2.3}$$

where ρ being certain scalar function given by $\rho = -(\xi\alpha)$. Next if we take $\phi V = \frac{1}{\alpha}\nabla_V \xi$, then it follows from (2.2) and (2.3) that

$$\phi V = V + \eta(V)\xi, \tag{2.4}$$

from which it can be seen that ϕ is a symmetric $(1,1)$ tensor. Thus the Lorentzian manifold M^n together with the unit timelike concircular vector field ξ, associated 1-form η and $(1,1)$ tensor field ϕ is called $(LCS)_n$-manifold [14]. Especially, if we take $\alpha = 1$ in $(LCS)_n$-manifold, then we obtain the Lorentzian para-Sasakian structure given by Matsumoto [7]. In an $(LCS)_n$-manifold, the following relations hold [14,15]:

$$\eta(\xi) = -1, \quad \phi\xi = 0, \quad \eta(\phi U) = 0, \tag{2.5}$$
$$g(\phi U, \phi V) = g(U,V) + \eta(U)\eta(V), \tag{2.6}$$
$$R(U,V)W = (\alpha^2 - \rho)[g(V,W)U - g(U,W)V], \tag{2.7}$$
$$(\nabla_U \phi)(V) = \alpha[g(U,V)\xi + 2\eta(U)\eta(V)\xi + \eta(V)U], \tag{2.8}$$
$$S(U,\xi) = (n-1)(\alpha^2 - \rho)\eta(U), \tag{2.9}$$
$$S(\phi U, \phi V) = S(U,V) + (n-1)(\alpha^2 - \rho)\eta(U)\eta(V), \tag{2.10}$$
$$Q\xi = (n-1)(\alpha^2 - \rho)\xi. \tag{2.11}$$

for any vector fields U, V and W, where R and S denotes respectively the Riemannian curvature tensor and the Ricci tensor of the $(LCS)_n$-manifold.

Also in an $(LCS)_n$-manifold, E-Bochner curvature tensor satisfies the following relations:

$$\begin{aligned} B^e(U,V)\xi &= \frac{4(\alpha^2-\rho)+2p-4}{n+3}[3\eta(V)U - 3\eta(U)V] \\ &\quad + \frac{6}{n+3}[\eta(U)QV - \eta(V)QU] \\ &\quad - \frac{p+n-1}{n+3}[2g(U,V)\xi + 2\eta(U)\eta(V)\xi], \end{aligned} \tag{2.12}$$

$$\begin{aligned} B^e(\xi,U)V &= \frac{4(\alpha^2-\rho)+2p-4}{n+3}[2g(U,V)\xi - 3\eta(V)U \\ &\quad -\eta(U)\eta(V)\xi] + \frac{6}{n+3}[\eta(V)QU - S(U,V)\xi] \\ &= -B^e(U,\xi)V, \end{aligned} \tag{2.13}$$

$$\begin{aligned} B^e(\xi,U)\xi &= \frac{4(\alpha^2-\rho)+2p-4}{n+3}[3\eta(U)\xi + 3U] \\ &\quad - \frac{6}{n+3}[(n-1)(\alpha^2-\rho)\eta(U)\xi + QU] \\ &= -B^e(U,\xi)\xi, \end{aligned} \tag{2.14}$$

$$B^e(\xi,\xi)U = 0. \tag{2.15}$$

Definition 2.1. *The ξ-sectional curvature of an $(LCS)_n$-manifold for a unit vector field V orthogonal to ξ is given by $K(\xi,V) = g(R(\xi,V)\xi,V)$.*

Hence from (2.7), we obtain

$$K(\xi,U) = (\alpha^2 - \rho).$$

Throughout this paper we have assumed that an $(LCS)_n$-manifold always admits a non-vanishing ξ-sectional curvature $((\alpha^2-\rho) \neq 0)$.

3. E-Bochner pseudo-symmetric $(LCS)_n$-manifold

Definition 3.1. *An n-dimensional Riemannian manifold M^n is said to be E-Bochner pseudo-symmetric if*

$$R \cdot B^e = L_{B^e} Q(g, B^e), \tag{3.1}$$

holds on the set $X_{B^e} = \{x \in M^n : B^e \neq 0\}$ *at* x, *where* L_{B^e} *is some differentiable function on* X_{B^e} *and* B^e *is the E-Bochner curvature tensor.*

In particular, if we take $L_{B^e} = 0$, then E-Bochner pseudo-symmetric manifold is reduces to E-Bochner semi-symmetric manifold.

Theorem 3.2. *An* $(LCS)_n$*-manifold is E-Bochner pseudo-symmetric, then either* ξ*-sectional curvature is a differentiable function* L_{B^e} *or the manifold reduces to* η*-Einstein.*

Proof. Let us consider an E-Bochner pseudo-symmetric $(LCS)_n$-manifold, then it follows from (3.1) that

$$\begin{aligned}(R(X,\xi)\cdot B^e)(U,V)W &= L_{B^e}[((X\wedge\xi)(B^e(U,V)W) \\ &\quad -B^e((X\wedge\xi)U,V)W \\ &\quad -B^e(U,(X\wedge\xi))W \\ &\quad -B^e(U,V)(X\wedge\xi)].\end{aligned} \tag{3.2}$$

Now the left hand side of equation (3.2) gives that

$$\begin{aligned}&(\alpha^2-\rho)[\eta(B^e(U,V)W)X - g(X,B^e(U,V)W)\xi \\ &-\eta(U)B^e(X,V)W - \eta(W)B^e(U,V)X \\ &+g(X,U)B^e(\xi,V)W + g(X,W)B^e(U,V)\xi \\ &-\eta(V)B^e(U,X)W + g(X,V)B^e(U,\xi)W].\end{aligned} \tag{3.3}$$

Similarly right hand side of (3.2) turns into

$$\begin{aligned}&L_{B^e}[\eta(B^e(U,V)W)X - g(X,B^e(U,V)W)\xi \\ &-\eta(U)B^e(X,V)W - \eta(W)B^e(U,V)X \\ &+g(X,U)B^e(\xi,V)W + g(X,W)B^e(U,V)\xi \\ &-\eta(V)B^e(U,X)W + g(X,V)B^e(U,\xi)W].\end{aligned} \tag{3.4}$$

By virtue of (3.3) and (3.4) in (3.2) implies that

$$\begin{aligned}0 &= ((\alpha^2-\rho) - L_{B^e})[\eta(B^e(U,V)W)X - g(X,B^e(U,V)W)\xi \\ &\quad -\eta(U)B^e(X,V)W + g(X,U)B^e(\xi,V)W \\ &\quad -\eta(W)B^e(U,V)X + g(X,W)B^e(U,V)\xi \\ &\quad -\eta(V)B^e(U,X)W + g(X,V)B^e(U,\xi)W],\end{aligned} \tag{3.5}$$

which gives either $(\alpha^2-\rho) = L_{B^e}$ or

$$\begin{aligned}&\eta(B^e(U,V)W)X - g(X,B^e(U,V)W)\xi \\ &-\eta(U)B^e(X,V)W + g(X,U)B^e(\xi,V)W \\ &-\eta(W)B^e(U,V)X + g(X,W)B^e(U,V)\xi \\ &-\eta(V)B^e(U,X)W + g(X,V)B^e(U,\xi)W = 0.\end{aligned} \tag{3.6}$$

Substituting $V=\xi$ into (3.6) and then using (2.12)-(2.15), we obtain

$$\begin{aligned}B^e(U,X)W &= \frac{4(\alpha^2-\rho)+2p-4}{n+3}[\eta(W)g(X,U)\xi + 3g(X,W)U \\ &\quad +\eta(U)g(X,W)\xi - 2g(U,W)X + \eta(U)\eta(W)X \\ &\quad +2\eta(U)\eta(W)\eta(X)\xi] + \frac{6}{n+3}[S(U,W)X \\ &\quad -g(X,W)QU + \eta(U)S(X,W)\xi - \eta(U)\eta(W)QX \\ &\quad -\eta(U)g(X,W)\xi) + (n-1)(\alpha^2-\rho)(\eta(U)\eta(W)X].\end{aligned} \tag{3.7}$$

Finally contracting above equation along the vector field U yields that

$$\begin{aligned} S(X,W) &= Mg(X,W) + N\eta(X)\eta(W), \\ \text{where } M &= \frac{(n+1)(21-17n)(\alpha^2-\rho) - 2(3n-1)r + n^3 + 3n - 12}{(n+1)(n+7)}, \\ N &= \frac{(14-9n-n^2)(n+1)(\alpha^2-\rho) + n^3 + 7r + n^3 + 2n - 11}{(n+1)(n+7)}. \end{aligned}$$

□

Since $L_{B^e} = (\alpha^2 - \rho)$ and noticing the assumption that the manifold is of non-vanishing ξ-sectional curvature $((\alpha^2 - \rho) \neq 0)$, we obtain $L_{B^e} \neq 0$.
Hence we conclude the following corollary:

Corollary 3.3. *An E-Bochner pseudo-symmetric $(LCS)_n$-manifold with $(\alpha^2 - \rho) \neq 0$ never reduces to E-Bochner semi-symmetric manifold ($L_{B^e} \neq 0$).*

4. $(LCS)_n$-manifold satisfying $B^e(U,V)\xi = 0$

Let us consider an $(LCS)_n$-manifold satisfying $B^e(U,V)\xi = 0$, then it follows from an equation (1.2) that

$$2B(U,V)\xi - \eta(U)B(\xi,V)\xi - \eta(V)B(U,\xi)\xi = 0. \tag{4.1}$$

By virtue of (1.1) in (4.1), we get

$$\begin{aligned} 0 = \; & \frac{4(\alpha^2-\rho)+2p-4}{n+3}[3\eta(V)U - 3\eta(U)V] \\ & + \frac{6}{n+3}[\eta(U)QV - \eta(V)QU] \\ & - \frac{p+n-1}{n+3}[2g(U,V)\xi + 2\eta(U)\eta(V)\xi]. \end{aligned} \tag{4.2}$$

Replacing V by ξ in (4.2) gives that

$$\begin{aligned} S(U,W) = \; & \frac{2(n+1)(\alpha^2-\rho) + r - (n+3)}{n+1} g(U,W) \\ & + \frac{(n+1)(3-n)(\alpha^2-\rho) + r - (n+3)}{n+1} \eta(U)\eta(W). \end{aligned} \tag{4.3}$$

Thus we have state the following result:

Theorem 4.1. *An $(LCS)_n$-manifold $(n > 1)$ satisfying $B^e(U,V)\xi = 0$, always turns into η-Einstein manifold.*

Furthermore, on contracting the expression (4.3) yields that

$$r = \frac{3(n^2-1)(\alpha^2-\rho)}{2} - \frac{(n-1)(n+3)}{2}. \tag{4.4}$$

This leads us to the following result:

Theorem 4.2. *In an $(LCS)_n$-manifold $(n > 1)$ satisfying $B^e(U,V)\xi = 0$, the scalar curvature and ξ-sectional curvature are linearly related to each other.*

Next by taking covariant derivative of (4.4) over the arbitrary vector field V, we have

$$(\nabla_V r) = \frac{3(n^2-1)\nabla_V(\alpha^2-\rho)}{2} = \frac{3(n^2-1)(2\alpha\rho-\beta)\eta(V)}{2}. \tag{4.5}$$

Hence we can state the following result:

Theorem 4.3. *In an $(LCS)_n$-manifold $(n > 1)$ satisfying $B^e(U,V)\xi = 0$, the scalar curvature is constant if and only if ξ-sectional curvature is constant.*

On replacing U by ϕU and W by ϕW in (4.3) yields the following relation

$$S(\phi U, \phi W) = \frac{2(n+1)(\alpha^2-\rho)+r-(n+3)}{n+1} g(\phi U, \phi W). \tag{4.6}$$

Differentiating (4.6) covariantly along the arbitrary vector field X, we obtain

$$(\nabla_X S)(\phi U, \phi W) = \frac{2(n+1)(2\alpha\rho-\beta)\eta(X)+dr(X)}{n+1} g(\phi U, \phi W). \tag{4.7}$$

If we consider an $(LCS)_n$-manifold with constant scalar curvature or ξ-sectional curvature, we have

$$(\nabla_X S)(\phi U, \phi W) = 0.$$

Thus we can easily get the following result:

Corollary 4.4. *An $(LCS)_n$-manifold $(n > 1)$, satisfying $B^e(U,V)\xi = 0$ always admits an η-parallel Ricci tensor provided scalar curvature or ξ-sectional curvature are constant.*

5. $(LCS)_n$-manifold satisfying $B^e(\xi, X) \cdot B^e = 0$

Let us consider an $(LCS)_n$-manifold satisfying $(B^e(\xi,X)\cdot B^e)(U,V)W = 0$. Then we can easily see that

$$\begin{aligned} 0 \;=\; & B^e(\xi,X)B^e(U,V)W - B^e(B^e(\xi,X)U,V)W \\ & -B^e(U,B^e(\xi,X)V)W - B^e(U,V)B^e(\xi,X)W. \end{aligned} \tag{5.1}$$

Using (2.13) in (5.1), we have the following equation

$$\begin{aligned} & \frac{4(\alpha^2-\rho)+2p-4}{n+3}[2g(X,B^e(U,V)W)\xi - 3\eta(B^e(U,V)W)X \\ & +3\eta(U)B^e(X,V)W - \eta(X)\eta(B^e(U,V)W)\xi \\ & -2g(X,U)B^e(\xi,V)W + \eta(X)\eta(U)B^e(\xi,V)W \\ & -2g(X,V)B^e(U,\xi)W + 3\eta(V)B^e(U,X)W \\ & +\eta(X)\eta(V)B^e(U,\xi)W - 2g(X,W)B^e(U,V)\xi \\ & +3\eta(W)B^e(U,V)X + \eta(X)\eta(W)B^e(U,V)\xi] \\ & +\frac{6}{n+3}[\eta(B^e(U,V)W)QX - S(X,B^e(U,V)W)\xi \\ & -\eta(U)B^e(QX,V)W + S(X,U)B^e(\xi,V)W \\ & -\eta(V)B^e(U,QX)W + S(X,V)B^e(U,\xi)W \\ & -\eta(W)B^e(U,V)QX + S(X,W)B^e(U,V)\xi] = 0. \end{aligned} \tag{5.2}$$

Replacing $U = W = \xi$ in (5.2) and then by taking an account of (2.12)-(2.15), we obtain

$$\begin{aligned} & -(4(\alpha^2-\rho)+2p-4)[(4(\alpha^2-\rho)+2p-4)\eta(V)X \\ & +4\eta(X)QV] + 6[S(X,QV)\xi - S(QX,V)\xi] = 0. \end{aligned} \tag{5.3}$$

Again replacing $X = \xi$ in (5.3) and then by using (2.9) gives,
either $4(\alpha^2-\rho)+2p-4 = 0$ or

$$4QV = (4(\alpha^2-\rho)+2p-4)\eta(V)\xi. \tag{5.4}$$

Now consider $4(\alpha^2-\rho)+2p-4 = 0$, we have

$$r = (n+3) - 2(n+1)(\alpha^2-\rho). \tag{5.5}$$

Hence we can state the following:

Theorem 5.1. *In an $(LCS)_n$-manifold satisfying $B^e(\xi, X) \cdot B^e = 0$, the scalar curvature and ξ-sectional curvature are linearly related to each other.*

On the other hand by considering (5.4), we have

$$S(V, Y) = \frac{r + 2(n+1)(\alpha^2 - \rho) - (n+3)}{2(n+1)} \eta(V)\eta(Y). \tag{5.6}$$

Thus we have state the following result:

Theorem 5.2. *An $(LCS)_n$-manifold satisfying $B^e(\xi, X) \cdot B^e = 0$, always turns into special type of η-Einstein manifold.*

Further, replacing V and Y by ϕV and ϕY in (5.6), we get

$$S(\phi V, \phi Y) = 0. \tag{5.7}$$

On differentiating (5.7) covariantly along the vector field X, gives

$$(\nabla_X S)(\phi V, \phi Y) = 0. \tag{5.8}$$

Hence from the above expression, Theorem 5.1. and Theorem 5.2., we can able to conclude the following:

Corollary 5.3. *In an $(LCS)_n$-manifold satisfying $B^e(\xi, X) \cdot B^e = 0$, either the scalar curvature and ξ-sectional curvature are linearly related to each other or the manifold turns into special type of η-Einstein and hence the manifold always admits an η-parallel Ricci tensor.*

6. $(LCS)_n$-manifold satisfying $B^e(\xi, X) \cdot S = 0$

Theorem 6.1. *Let M^n be an $(LCS)_n$-manifold satisfying $B^e(\xi, Y) \cdot S = 0$. Then the Ricci tensor S and the Ricci operator Q are given by the equations (6.2) and (6.3) respectively.*

Proof. In an $(LCS)_n$-manifold satisfying $B^e(\xi, Y) \cdot S = 0$, we can easily see that

$$S(B^e(\xi, Y)U, V) + S(U, B^e(\xi, Y)V) = 0. \tag{6.1}$$

On plugging $V = \xi$ in (6.1) and then by considering(2.9) and (2.13) follows that

$$\begin{aligned} S(QY, U) &= M' S(Y, U) + N'[-2g(Y, U) + \eta(Y)\eta(U)], \\ \text{where,} \quad M' &= \frac{6(n+1)(3n+1)(\alpha^2 - \rho) + 6(r - n - 3)}{6(n+1)}, \\ N' &= \frac{(n-1)(\alpha^2 - \rho)[4(n+1)((\alpha^2 - \rho) - 1) + 2(r + n - 1)]}{6(n+1)}. \end{aligned} \tag{6.2}$$

Contracting above expression over Y and U, we have

$$\|Q\|^2 = \frac{\begin{array}{c} r[3(r - n - 3) + (\alpha^2 - \rho)(7n^2 + 13n + 4)] \\ -(n-1)(2n+1)(\alpha^2 - \rho)[(n-1) + 2(n+1)((\alpha^2 - \rho) - 1)] \end{array}}{3(n+1)}. \tag{6.3}$$

□

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