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Applications of L_{gfs}-closed Sets In Mixed Fuzzy Soft Ideal Topological Spaces

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ABSTRACT: The primary goal of this treatise is to introduce a new kind of generalized closed sets termed as L_{gfs}-closed sets in the light of fuzzy soft local function in a mixed fuzzy soft ideal topological space. We also define the notion of fuzzy soft $*$-separated set. In addition, we procure the idea of fuzzy soft regularity, normality and fuzzy soft compactness. We also study their behavior in terms of L_{gfs}-closed sets.

Key Words: L_{gfs}-closed set, $*$ dense-in-itself, fuzzy soft $*$-separated set, L_{gfs}-regularity, L_{gfs}-normality, L_{gfs}-compactness.

Contents

1. Introduction and preliminaries

The ever great scholar L. A Zadeh [29] introduced the concept of fuzzy set in 1965. By the use of this notion of fuzzy sets, Chang [7] in 1968 and Lowen [13] in 1976 have introduced the theory of fuzzy topology with different objectives. Molodtsov [18] in 1999, initiated the idea of soft sets for handling uncertainties. He additionally carried out the soft set theory in operation research, game theory, theory of probability, and so on. Since then many researchers have worked on soft sets.

Levine [12] presented the concept of a generalised closed set, established its basic characteristics, and demonstrated some of its applications in general topological spaces. Generalized fuzzy closed set concept and its applications in fuzzy topological spaces (fts, for short) were given by Balasubramanian and Sundaram [2] in 1997. Different kind of generalized closed sets are being studied by different authors in various environments, for instance [19,4,3,20].

The notion of soft topological spaces introduced and studied in [24]. Maji et al. [16] initiated the idea of fuzzy soft sets. Indeed, a fuzzy soft set, where soft set is defined over the fuzzy set, is a combination of fuzzy and soft sets. The topological structure of fuzzy soft sets was first developed in 2011 by Tanay et al. [26]. In general topological spaces, the introduction of notion of ideal was given by Kuratowski [11] and then further studied by [28] and so on in fuzzy topological spaces, it had been introduced by Sarkar [23]. In fuzzy soft topological spaces (FSTSs), the fuzzy soft ideal was first introduced by Kandil et al. [9] in 2016.

The idea of mixed fuzzy topology given by Das and Baishya [8] in 1995 and by Tripathy and Ray [27] in 2012 in different ways and using these concepts, Borah and Hazarika [5] introduced mixed fuzzy soft topological spaces. Further, the study involving the notion of mixed fuzzy soft ideal topology was given by Borah and Hazarika [6].

The main objective of this present article is to introduce a new type of generalized closed set via fuzzy soft local function in a mixed fuzzy soft ideal topological space (MFSITS). The collection of such closed sets is found to contain the collection of all generalized fuzzy soft closed sets. As an application of the said closed sets we consider some extended version of fuzzy soft regularity, normality and compactness.

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Before moving on to the main part, let's provide some basic and preliminary concepts about the prevailing definitions and results that play an important role in this study.

In this article, We denote initial universal set by X and the set of all possible parameters for X by E.

Definition 1.1 [22] Let $A \subseteq E$. A pair (F_A, E), denoted by F_A is said to be a fuzzy soft set over X if $F_A : E \to I^X$ is a mapping defined by $F_A(e) = \mu^e_{F_A}$, where $\mu^e_{F_A} = \bar{0}$ if $e \in E - A$ and $\mu^e_{F_A} \neq \bar{0}$ if $e \in A$, where $\bar{0}(x) = 0, \forall x \in X$. By $FS(X)_E$ we mean the family of all fuzzy soft set over X.

Definition 1.2 [22] The complement of a fuzzy soft set (F_A, E) on (X, E) is a fuzzy soft set (F^c_A, E), denoted by F^c_A, where $F^c_A : E \to I^X$ is defined by $\mu^e_{F^c_A} = \bar{1}$ if $e \in E - A$ and $\mu^e_{F^c_A} = 1 - \mu^e_{F_A}$ if $e \in A$, where $\bar{1}(x) = 1, \forall x \in X$.

Definition 1.3 [26] A fuzzy soft set F_E over X is defined to be a null fuzzy soft set, denoted by $\tilde{0_E}$ if $F_E(e) = \bar{0}, \forall e \in E$.

Definition 1.4 [26] A fuzzy soft set F_E over X is defined to be an absolute fuzzy soft set, denoted by $\tilde{1_E}$ if $F_E(e) = \bar{1}, \forall e \in E$.

Definition 1.5 [22] Let $F_A, G_B \in FS(X)_E$. Then F_A is fuzzy soft subset of G_B, denoted by $F_A \tilde{\subseteq} G_B$ if $\mu^e_{F_A}(x) \leq \mu^e_{F_A}(x)$, $\forall e \in E$, $\forall x \in X$.

Theorem 1.6 [22] Let $F_A, G_B \in FS(X)_E$. The union of F_A, G_B over X is a fuzzy soft set H_C, denoted by $F_A \tilde{\cup} G_B$ and is defined as $H_C(e) = \mu^e_{H_C} = \mu^e_{F_A} \cup \mu^e_{G_B}$ for all $e \in E$, where $C = A \cup B$.

Definition 1.7 [22] Let $F_A, G_B \in FS(X)_E$. The intersection of F_A, G_B over X is a fuzzy soft set H_C, denoted by $F_A \tilde{\cap} G_B$ and is defined as $H_C(e) = \mu^e_{H_C} = \mu^e_{F_A} \cap \mu^e_{G_B}$ for all $e \in E$, where $C = A \cap B$.

Definition 1.8 [21] Let (X, τ) be an FSTS and $F_A \in FS(X)_E$. The fuzzy soft closure of F_A is denoted by $cl(F_A)$, is the intersection of all fuzzy soft closed sets which contains F_A.

Theorem 1.9 [21] Let (X, τ) be an FSTS and $F_A \in FS(X)_E$. The fuzzy soft interior of F_A is denoted by $int(F_A)$, is the union of all fuzzy soft open sets which contained in F_A.

Definition 1.10 [14] Let $F_A \in FS(X)_E$. Then F_A is called a fuzzy soft point if there exist $x \in X$ and $e \in E$ such that $F_A(a) = x_\alpha$ and $F_A(a) = \bar{0}$ for $a \in A - \{a\}$. This is denoted by x^e_α or $e(F_A)$. We say that $x^e_\alpha \in F_A$, if for $e \in A$ and $x \in X$, $\alpha \leq \mu^e_{F_A}(x)$.

Definition 1.11 [14] A fuzzy soft point x^e_α is called q-coincident with $F_A \in FS(X)_E$, denoted by $x^e_\alpha q F_A$, if $\alpha + \mu^e_{F_A}(x) > 1$ for some $x \in X$.

Definition 1.12 [14] Let $F_A, G_B \in FS(X)_E$. Then F_A is q-coincident with G_B, denoted by $F_A q G_B$, if there exists $x \in X$ such that $\mu^e_{F_A}(x) + \mu^e_{G_B}(x) > 1$. When F_A and G_B are not q-coincident, we denote it by $F_A \bar{q} G_B$.

Proposition 1.13 [25] Let $F_A, G_A \in FS(X)_E$. Then $F_A \tilde{\subseteq} G_A$ if and only if $F_A \bar{q} G^c_A$.

Definition 1.14 [9] A collection of non-empty fuzzy soft sets $\tilde{I}$ over X is called a fuzzy soft ideal on X if the following conditions are met:
(i) If $F_A, G_B \in \tilde{I}$, then $F_A \tilde{\cup} G_B \in \tilde{I}$;
(ii) If $F_A \in \tilde{I}$ and $G_B \tilde{\subseteq} F_A$, then $G_B \in \tilde{I}$.

Definition 1.15 [5] Let (X, τ_1) and (X, τ_2) be two FSTSs. Consider the collection: $\tau_1(\tau_2) = \{F_A \in FS(X)_E$: for any $G_B \in FS(X)_E$ with $F_A q G_B$, then there exists $H_C \in \tau_2$ such that $G_B q H_C$ and τ_1 closure, $cl(H_C)\tilde{\subseteq} F_A\}$. Then this family will form a topology and the topology generated by this is called a mixed fuzzy soft topology.

Definition 1.16 [10] Let $(X, \tau, \tilde{I})$ be fuzzy soft ideal topological space and $F_A \in FS(X)_E$. Then $F_A^*(\tilde{I}, \tau)$ or $F_A^* = \tilde{\cup}\{x_\alpha^e \in FS(X)_E : F_A \tilde{\cap} G_B \notin \tilde{I}, \forall G_B \in \tau(x_\alpha^e)\}$.

Definition 1.17 [9] Let $(X, \tau, \tilde{I})$ be a fuzzy soft ideal topological space and $F_A \in FS(X)_E$. Then the fuzzy soft local function $F_A^*(\tilde{I}, \tau)$ (or F_A^*) of F_A is the union of all fuzzy soft points x_α^e such that if $G_B \in N(x_\alpha^e)$ and $H_C \in \tilde{I}$, then there exist at least one $x \in X$ and $e \in E$ for which $\mu_{F_A}^e(x) + \mu_{G_B}^e(x) - 1 > \mu_{H_C}^e(x)$.

Definition 1.18 [6] Let $(X, \tau_1(\tau_2))$ be a mixed fuzzy soft topological space and let $\tilde{I}$ be a fuzzy soft ideal over X. Then $(X, \tau_1(\tau_2), \tilde{I})$ is called a mixed fuzzy soft ideal topological space (MFSITS).

Theorem 1.19 [6] Let $(X, \tau_1(\tau_2), \tilde{I})$ be an MFSITS. Let $H_A, K_A \in FS(X)_E$, then
(i) $H_A \tilde{\subseteq} K_A \Rightarrow H_A^* \tilde{\subseteq} K_A^*$,
(ii) $H_A^* = cl(H_A^*) \tilde{\subseteq} cl(H_A)$,
(iii) $(H_A^*)^* \tilde{\subseteq} H_A^*$,
(iv) $(H_A \tilde{\cap} K_A)^* \tilde{\subseteq} H_A^* \tilde{\cap} K_A^*$,
(v) $(H_A \tilde{\cup} K_A)^* = H_A^* \tilde{\cup} K_A^*$,
(vi) H_A^* is fuzzy soft closed in X.

2. Main results

This particular section deals with the idea of generalized closed set in the light of local function in an MFSITS and we emphasize that this approach is something different from the conventional directions. Nonetheless, we introduce gfs-closed set in the sense of [2] to find the relationship among the newly defined notion with the same in an MFSITS.

Definition 2.1 A fuzzy soft set H_A in an MFSITS X is called generalized fuzzy soft closed set (gfs-closed, in short) if $cl(H_A) \leq U_A$, whenever $H_A \leq U_A$ and U_A is fuzzy soft open set in X. H_A is called generalized fuzzy soft open set (gfs-open, in short) if its complement, H_A^c is gfs-closed set.

The following example shows the existence of the above defined notion.

Example 2.2 Let $X = \{x, y\}$ and $E = A = \{e_1, e_2\}$.
Let $\tau_1 = \{\tilde{0_E}, \tilde{1_E}, H_A, K_A\}$ and $\tau_2 = \{\tilde{0_E}, \tilde{1_E}, F_A, G_A\}$, where $H_A = \{H(e_1) = \{(x, 0.1), (y, 0.2)\}$, $H(e_2) = \{(x, 0.3), (y, 0.2)\}\}$, $K_A = \{K(e_1) = \{(x, 0.8), (y, 0.7)\}, K(e_2) = \{(x, 0.6), (y, 0.7)\}\}$, $F_A = \{F(e_1) = \{(x, 0.9), (y, 0.8)\}$, $F(e_2) = \{(x, 0.7), (y, 0.8)\}\}$ and $G_A = \{G(e_1) = \{(x, 0.2), (y, 0.3)\}$, $G(e_2) = \{(x, 0.4), (y, 0.3)\}\}$. Let us consider any fuzzy soft ideal $\tilde{I}$.
Then $\tau_1(\tau_2) = \{\tilde{0_E}, \tilde{1_E}, F_A, G_A\}$ and consequently $(X, \tau_1(\tau_2), \tilde{I})$ is an MFSITS.
Let $Q_A = \{Q(e_1) = \{(x, 0.7), (y, 0.6)\}, Q(e_2) = \{(x, 0.5), (y, 0.2)\}\}$. Here, we claim that Q_A is gfs-closed. As $Q_A \tilde{\subseteq} F_A$ and $cl(Q_A) \tilde{\subseteq} F_A$.

We now propose the definition of L_{gfs}-closed set in $(X, \tau_1(\tau_2), \tilde{I})$ as follows.

Definition 2.3. Let X be an MFSITS. A fuzzy soft set H_A over X is called a generalized fuzzy soft closed set with respect to a fuzzy soft local function (in short, L_{gfs}-closed) if $H_A^* \tilde{\subseteq} U_A$, whenever $H_A \tilde{\subseteq} U_A$ and U_A is fuzzy soft open set. H_A is called a generalized fuzzy soft open set with respect to a fuzzy soft local function (in short, L_{gfs}-open) if its complement, H_A^c is L_{gfs}-closed set.

Existence of such closed sets are very interesting in this domain and we illustrate the following as ready reference.

Example 2.4. Let $X = \{x, y\}$ and $E = A = \{e_1, e_2\}$.

Let $F_A = \{F(e_1) = \{(x, 1), (y, 1)\},\ F(e_2) = \{(x, 0.6), (y, 0.6)\}\},\ G_A = \{G(e_1) = \{(x, 0), (y, 0)\}, G(e_2) = \{(x, 0.4), (y, 0.4)\}\}$.

Let $\tau_1 = \{\tilde{0_E}, \tilde{1_E}, G_A\}$, $\tau_2 = \{\tilde{0_E}, \tilde{1_E}, F_A\}$. Then $\tau_1(\tau_2) = \{\tilde{0_E}, \tilde{1_E}, F_A\}$.
Let $H_A = \{H(e_1) = \{(x, 0), (y, 0)\}, H(e_2) = \{(x, 0.5), (y, 0.5)\}\}$ and $\tilde{I} = \{\tilde{0_E}, K_A\}$ where $K_A = \{K(e_1) = \{(a, 0), (b, 0)\}, K(e_2) = \{(x, 0.7), (y, 0.7)\}\}$.

Now, to compute H_A^*, we consider the fuzzy soft point $e_1(X_A) = \{(e_1) = \{(x, 1), (y, 1)\}, (e_2) = \{(x, 0), (y, 0)\}\}$. The open sets in $\tau_1(\tau_2)$ containing $e_1(X_A)$ are F_A and $\tilde{1_E}$. Here, $F_A \tilde{\cap} H_A = H_A \in \tilde{I}$ and $\tilde{1_E} \tilde{\cap} H_A = H_A \in \tilde{I}$.
Which implies that $e_1(X_A) \notin H_A^*$.
Again, we consider the fuzzy point $e_2(X_A) = \{(e_1) = \{(x, 0), (y, 0)\}, (e_2) = \{(x, 1), (y, 1)\}\}$ and the open sets in $\tau_1(\tau_2)$ containing $e_2(H_A)$ is $\tilde{1_E}$ only. Clearly $\tilde{1_E} \tilde{\cap} H_A = H_A \in \tilde{I}$, and consequently we have $e_2(X_A) \notin H_A^*$. Then $H_A^* \neq \tilde{1_E}$ as well as $H_A^* \neq \tilde{0_E}$, so we have $H_A^* = \{H(e_1) = \{(x, 0), (y, 0)\}, H(e_2) = \{(x, 0.4), (y, 0.4)\}\}$, as H_A^* is fuzzy soft closed in X. Consequently, we have $H_A \tilde{\subseteq} F_A$ implies $H_A^* \tilde{\subseteq} F_A$. Therefore, H_A is an L_{gfs}-closed set in X.

Theorem 2.5. Every fuzzy soft closed as well as gfs-closed set in an MFSITS X is an L_{gfs}-closed.
Proof: Let H_A be generalized fuzzy soft closed set (gfs-closed) such that $H_A \tilde{\subseteq} U_A$, where U_A is fuzzy soft open set. Then $cl(H_A) \tilde{\subseteq} U_A$ and consequently $H_A^* \tilde{\subseteq} U_A$(by Theorem 1.19(ii)) and therefore H_A is L_{gfs}-closed.

Again, since every fuzzy soft closed set is gfs-closed and hence the theorem is completed.

However, the converses are not necessarily true. The following example demonstrates our claim.

Example 2.6. From the above Example 2.4, we have H_A is L_{gfs}-closed set but not a gfs-closed set and also not a fuzzy soft closed set, since $cl(H_A) = \tilde{1_E}$ and $H_A \tilde{\subseteq} F_A$.

But in particular, if $\tilde{I} = \{\tilde{0_E}\}$, then $H_A^* = cl(H_A), \forall H_A \in FS(X)_E$ [9], and hence every L_{gfs}-closed set is gfs-closed set.

Theorem 2.7. If H_A and K_A are two L_{gfs}-closed sets in an MFSITS X, then $H_A \tilde{\cup} K_A$ is also an L_{gfs}-closed set therein.
Proof: Suppose $H_A \tilde{\cup} K_A \tilde{\subseteq} U_A$, for some fuzzy soft open set U_A. Then $H_A \tilde{\subseteq} U_A$ and $K_A \tilde{\subseteq} U_A$ and hence $H_A^* \tilde{\subseteq} U_A$ and $K_A^* \tilde{\subseteq} U_A$, as H_A and K_A are two L_{gfs}-closed sets. Now, $(H_A \tilde{\cup} K_A)^* = H_A^* \tilde{\cup} K_A^* \tilde{\subseteq} U_A$(by Theorem 1.19(v)). This shows that $H_A \tilde{\cup} K_A$ is also L_{gfs}-closed set.

It is pertinent to note that the arbitrary union of L_{gfs}-closed sets may not be an L_{gfs}-closed set. This follows from the fact that in an MFSITS X if we consider $\tilde{I} = \{\tilde{0_E}\}$, then $H_A^* = cl(H_A), \forall H_A \in FS(X)_E$ [9] and the fuzzy soft closure of infinite unions may not be equal to the union of closures. However, even the intersection of two L_{gfs}-closed sets may not be an L_{gfs}-closed set. To show this we consider the following example.

Example 2.8. Let $X = \{x, y\}$ and $E = A = \{e_1, e_2\}$.

Let $F_A = \{F(e_1) = \{(x, 0.6), (y, 0.6)\}, F(e_2) = \{(x, 0.3), (y, 0.3)\}\}$, $G_A = \{G(e_1) = \{(x, 0.4), (y, 0.4)\}, G(e_2) = \{(x, 0.7), (y, 0.7)\}\}$.

Let $\tau_1 = \{\tilde{0_E}, \tilde{1_E}, G_A\}$, $\tau_2 = \{\tilde{0_E}, \tilde{1_E}, F_A\}$. Then $\tau_1(\tau_2) = \{\tilde{0_E}, \tilde{1_E}, F_A\}$.

We consider the ideal $\tilde{I} = \{\tilde{0_E}, K_A\}$ where $K_A = \{K(e_1) = \{(x, 0), (y, 0)\}, K(e_2) = \{(x, 0.3), (y, 0.3)\}\}$.

Let $H_A = \{H(e_1) = \{(x, 0.3), (y, 0.3)\}, H(e_2) = \{(x, 0.9), (y, 0.9)\}\}$, $K_A = \{K(e_1) = \{(x, 0.7), (y, 0.7)\}, K(e_2) = \{(x, 0.2), (y, 0.2)\}\}$. Then we have $H_A^* = \tilde{1_E}$ and $K_A^* = \tilde{1_E}$ and consequently H_A and K_A are L_{gfs}-closed sets.

Again, we have $H_A\tilde{\cap}K_A = \{(e_1) = \{(x,0.3),(y,0.3)\},\ (e_2) = \{(x,0.2),(y,0.2)\}\}$ and one can show that $(H_A\tilde{\cap}K_A)^* = 1_{\tilde{E}}$. Since here, $H_A\tilde{\cap}K_A\tilde{\subseteq}F_A$ and $(H_A\tilde{\cap}K_A)^*\tilde{\nsubseteq}F_A$, which is showing that $H_A\tilde{\cap}K_A$ is not an L_{gfs}-closed.

Our concern is to find out that required condition with which an L_{gfs}-closed set becomes a gfs-closed set. For the purpose, we define the following notion that is of interest in itself.

Definition 2.9. A fuzzy soft set H_A in an MFSITS X is said to be $*$ dense-in-itself if $H_A\tilde{\subseteq}H_A^*$.

Theorem 2.10. If H_A be an L_{gfs}-closed set and $*$ dense-in-itself in X, then it is gfs-closed set.

Proof: Let $U_A \in \tau_1(\tau_2)$ such that $H_A\tilde{\subseteq}U_A$. We have $H_A^*\tilde{\subseteq}U_A$ and $H_A\tilde{\subseteq}H_A^*$. Since H_A^* is fuzzy soft closed and $cl(H_A)$ is the smallest closed set containing H_A, we get $H_A\tilde{\subseteq}cl(H_A)\tilde{\subseteq}H_A^*\tilde{\subseteq}U_A$ and hence $cl(H_A)\tilde{\subseteq}U_A$. Therefore, H_A is gfs-closed set.

Definition 2.11. Let $(X,\tau_1(\tau_2),\tilde{I})$ be an MFSITS. The fuzzy soft closure and interior operator of a fuzzy soft set H_A in X is defined as $cl^*(H_A) = H_A\tilde{\cup}H_A^*$, $Int^*(H_A) = H_A\tilde{\cap}(((H_A)^c)^*)^c$ respectively.

Theorem 2.12. Let H_A be any L_{gfs}-closed set and K_A be any fuzzy soft set in an MFSITS $(X,\tau_1(\tau_2),\tilde{I})$, then
(i) if $H_A\tilde{\subseteq}K_A\tilde{\subseteq}H_A^*$, then K_A is L_{gfs}-closed set.
(ii) if $H_A\tilde{\subseteq}K_A\tilde{\subseteq}cl^*(H_A)$, then K_A is L_{gfs}-closed set.
Proof: (i) Suppose $K_A\tilde{\subseteq}U_A$, for some fuzzy soft open set U_A. Then $H_A\tilde{\subseteq}U_A$ and H_A being L_{gfs}-closed set, $H_A^*\tilde{\subseteq}U_A$. Now $K_A\tilde{\subseteq}H_A^*$ implies $K_A^*\tilde{\subseteq}(H_A^*)^*\tilde{\subseteq}H_A^*$. Thus $K_A^*\tilde{\subseteq}U_A$. Therefore, K_A is L_{gfs}-closed set.
(ii) Suppose $K_A\tilde{\subseteq}V_A$, for some fuzzy soft open set V_A. Then $H_A\tilde{\subseteq}V_A$ and H_A being L_{gfs}-closed set, $H_A^*\tilde{\subseteq}V_A$. Now $K_A\tilde{\subseteq}cl^*(H_A) = H_A\tilde{\cup}H_A^*$, we have $K_A^*\tilde{\subseteq}(H_A\tilde{\cup}H_A^*)^*\tilde{\subseteq}H_A^*\tilde{\subseteq}V_A$. Thus $K_A^*\tilde{\subseteq}V_A$ and consequently K_A is L_{gfs}-closed set.

Corollary 2.13 If H_A and K_A are two fuzzy soft sets such that $H_A\tilde{\subseteq}K_A\tilde{\subseteq}H_A^*$ and if H_A is L_{gfs}-closed set. Then both H_A and K_A are gfs-closed set.
Proof. It follows from Theorem 2.12(i) and Theorem 2.10.

Theorem 2.14. A fuzzy soft set H_A is L_{gfs}-open if and only if $K_A\tilde{\subseteq}Int^*(H_A)$ whenever K_A is fuzzy soft closed and $K_A\tilde{\subseteq}H_A$.
Proof: Let H_A be L_{gfs}-open set and K_A is fuzzy soft closed set such that $K_A\tilde{\subseteq}H_A$. Then H_A^c is L_{gfs}-closed set and $H_A^c\tilde{\subseteq}K_A^c$. As H_A^c is L_{gfs}-closed and K_A^c is fuzzy soft open, we have $(H_A^c)^*\tilde{\subseteq}K_A^c$ and then $cl^*(H_A^c)\tilde{\subseteq}K_A^c$. Eventually, $K_A\tilde{\subseteq}(cl^*(H_A^c))^c = Int^*(H_A)$(it can be proved easily by Definition 2.11 and Theorem 1.19(v)).

Conversely, let $U_A \in \tau_1(\tau_2)$ such that $H_A^c\tilde{\subseteq}U_A$. Then U_A^c is fuzzy soft closed and $U_A^c\tilde{\subseteq}H_A$ and by the given condition, we have $U_A^c\tilde{\subseteq}Int^*(H_A)$. This implies $cl^*(H_A^c) = (Int^*(H_A))^c\tilde{\subseteq}U_A$ and from this we have, $(H_A^c)^*\tilde{\subseteq}U_A$. Hence, H_A^c is L_{gfs}-closed set and accordingly H_A is L_{gfs}-open set.

Theorem 2.15. If H_A and K_A are two L_{gfs}-open sets, then $H_A\tilde{\cap}K_A$ is L_{gfs}-open set.
Proof: Let H_A and K_A be two L_{gfs}-open sets. Suppose that $(H_A\tilde{\cap}K_A)^c\tilde{\subseteq}U_A$, for some fuzzy soft open set U_A. Then $H_A^c\tilde{\cup}K_A^c\tilde{\subseteq}U_A$. This implies $H_A^c\tilde{\subseteq}U_A$ and $K_A^c\tilde{\subseteq}U_A$. Now, both H_A^c and K_A^c being L_{gfs}-closed sets, giving that $(H_A^c)^*\tilde{\subseteq}U_A$, $(K_A^c)^*\tilde{\subseteq}U_A$ and so, $(H_A\tilde{\cap}K_A)^c)^* = (H_A^c)^*\tilde{\cup}(K_A^c)^*\tilde{\subseteq}U_A$(by Theorem 1.19(v)). This implies $(H_A\tilde{\cap}K_A)^c$ is L_{gfs}-closed set and thus $H_A\tilde{\cap}K_A$ is L_{gfs}-open set.

Remark 2.16. In general union of two L_{gfs}-open sets is not an L_{gfs}-open set. It can be easily established in view of the Example 2.8 itself.

Theorem 2.17. If $Int^*(H_A)\tilde{\subseteq}K_A\tilde{\subseteq}H_A$ and if H_A is an L_{gfs}-open set. Then K_A is also an L_{gfs}-open set.

Proof: Let H_A be L_{gfs}-open set and $Int^*(H_A)\tilde{\subseteq}K_A\tilde{\subseteq}H_A$. Then $(H_A)^c$ is L_{gfs}-closed set and $H_A^c\tilde{\subseteq}K_A^c$ $\tilde{\subseteq}(Int^*(H_A))^c = cl^*(H_A^c)$. By Theorem 2.12(ii), K_A^c is L_{gfs}-closed set and so K_A is L_{gfs}-open set.

We conclude this section by showing that condition under which union of two L_{gfs}-open sets is again an L_{gfs}-open set.

We have seen that union of two L_{gfs}-open sets may not be an L_{gfs}-open set. However, if we consider the pair H_A and K_A as fuzzy soft Q-separated set in the sense of Abd El-Latif [1], the following holds true.

Proposition 2.18. If H_A and K_A are two Q-separated L_{gfs}-open sets, then their union is an L_{gfs}-open set.

Proof. It can be easily established and hence omitted.

Now, to define separated set in this context of fuzzy soft local function we need the following preliminary.

Definition 2.19. [17] In an ideal topological space (X, τ, I) two non-empty subsets H, K of X are said to be $*_*$ separated if $H^* \cap K = H \cap K^* = H \cap K = \phi$.

Definition 2.20. Two non-zero fuzzy soft subsets H_A, K_A of an MFSITS $(X, \tau_1(\tau_2), \tilde{I})$ are called fuzzy soft $*$-separated if $H_A^*\tilde{\cap}K_A = H_A\tilde{\cap}K_A^* = H_A\tilde{\cap}K_A = \tilde{0_E}$.

Using the above definition 2.20 of fuzzy soft $*$-separated set, the union of two L_{gfs}-open sets may not be an L_{gfs}-open set. Nevertheless, we searched that required condition under which the union of two L_{gfs}-open sets will be again an L_{gfs}-open set.

Theorem 2.21. If F_A and G_A are two fuzzy soft $*$-separated L_{gfs}-open sets in an MFSITS X, then $F_A\tilde{\cup}G_A$ is L_{gfs}-open set if both F_A and G_A are $*$ dense-in-itself in X.

Proof: Let both F_A and G_A be fuzzy soft $*$-separated L_{gfs}-open sets as well as $*$ dense-in-itself in X. Let K_A be a fuzzy soft closed set such that $K_A\tilde{\subseteq}(F_A\tilde{\cup}G_A)$. Now, $K_A\tilde{\subseteq}(F_A\tilde{\cup}G_A)$ implies $K_A\tilde{\cap}F_A^*\tilde{\subseteq}(F_A\tilde{\cup}G_A)$ $\tilde{\cap}F_A^* = (F_A\tilde{\cap}F_A^*)\tilde{\cup}(G_A\tilde{\cap}G_A^*) = F_A\tilde{\cup}\tilde{0_E} = F_A$. Since $K_A\tilde{\cap}F_A^*$ is fuzzy soft closed and $K_A\tilde{\cap}F_A^*\tilde{\subseteq}F_A$ and by the Theorem 2.14, we have $K_A\tilde{\cap}F_A^*\tilde{\subseteq}Int^*(F_A)$. Similarly, one can show that $K_A\tilde{\cap}G_A^*\tilde{\subseteq}Int^*(G_A)$.

Now, $K_A = K_A\tilde{\cap}(F_A\tilde{\cup}G_A)\tilde{\subseteq}(K_A\tilde{\cap}F_A^*)\tilde{\cup}(K_A\tilde{\cap}G_A^*)$ (by the given condition of dense-in-itself). This implies that $K_A\tilde{\subseteq}Int^*(F_A)\ \tilde{\cup}Int^*(F_A)\tilde{\subseteq}Int^*(F_A\tilde{\cup}G_A)$ (it follows from Definition 2.11 and Theorem 1.19(iv),(v)) and then again by Theorem 2.14, $F_A\tilde{\cup}G_A$ is an L_{gfs}-open set.

Corollary 2.22. If H_A and K_A are two L_{gfs}-closed sets and if H_A^c and K_A^c are $*$-separated fuzzy soft sets, then $H_A\tilde{\cap}K_A$ is L_{gfs}-closed set.

3. Some applications of L_{gfs}-closed sets

In this section, we introduce generalized fuzzy soft regular, fuzzy soft normal spaces and generalized fuzzy soft compactness in the light of L_{gfs}-closed sets.

Definition 3.1 [15] An FSTS (X, τ) is said to be fuzzy soft regular if for every fuzzy soft point $e(F_A)$ and fuzzy soft closed set G_A not containing $e(F_A)$, there exist disjoint fuzzy soft open sets U_A and V_A such that $e(F_A)\tilde{\subseteq}U_A$, $G_A\tilde{\subseteq}V_A$.

Definition 3.2. Let X be an MFSITS. Then X is said to be L_{gfs}-regular if for any fuzzy soft point $e(F_A)$ and for each L_{gfs}-closed set G_A in X with $e(F_A)\bar{q}G_A$, there exist fuzzy soft open sets U_A and V_A such that $e(F_A)\tilde{\subseteq}U_A$, $G_A\tilde{\subseteq}V_A$ and $U_A\bar{q}V_A$.

We now give some characterizations of L_{gfs}-regularity:

Theorem 3.3. The followings are equivalent in an MFSITS X:
(a) X is L_{gfs}-regular.
(b) For each L_{gfs}-closed set F_A in X and for each fuzzy soft point $e(Q_A)$ with $e(Q_A)\bar{q}F_A$, there exist fuzzy soft open sets U_A and V_A such that $e(G_A)\tilde{\subseteq}U_A$, $F_A\tilde{\subseteq}V_A$ and $cl(U_A)\bar{q}cl(V_A)$.
(c) For each fuzzy soft point $e(Q_A)$ in X and for each L_{gfs}-open set U_A containing $e(Q_A)$, there exists a fuzzy soft open set V_A such that $e(Q_A)\tilde{\subseteq}V_A\ \tilde{\subseteq}cl(V_A)\tilde{\subseteq}U_A$.
Proof. $(a) \Rightarrow (b)$: Let F_A be any L_{gfs}-closed set and $e(Q_A)$, any fuzzy soft point in X such that $e(Q_A)\bar{q}F_A$. Then by (a) there exist $W_A, V_A \in \tau_1(\tau_2)$ such that $e(Q_A)\tilde{\subseteq}W_A$, $F_A\tilde{\subseteq}V_A$ and $W_A\bar{q}V_A$. Then, we have $W_A\bar{q}cl(V_A)$. Since $cl(V_A)$ is a fuzzy soft closed set, it is L_{gfs}-closed set. Again, since X is L_{gfs}-regular, so there exist $G_A, H_A \in \tau_1(\tau_2)$ such that $e(Q_A)\tilde{\subseteq}G_A$, $cl(V_A)\tilde{\subseteq}H_A$ and $G_A\bar{q}H_A$ and thus $cl(G_A)\bar{q}H_A$.

Let $U_A = W_A\tilde{\cap}G_A$, then both $U_A, V_A \in \tau_1(\tau_2)$ such that $e(Q_A)\tilde{\subseteq}U_A$, $F_A\tilde{\subseteq}V_A$ and $cl(U_A)\bar{q}cl(V_A)$. In fact, $cl(U_A) = cl(W_A\tilde{\cap}G_A)\tilde{\subseteq}cl(G_A)$ and $cl(G_A)\ \bar{q}H_A$, we have $cl(U_A)\bar{q}H_A$ and as $cl(V_A)\tilde{\subseteq}H_A$, consequently, we obtain $cl(U_A)\bar{q}\ cl(V_A)$.

$(b) \Rightarrow (c)$: Let $e(Q_A)$ be any fuzzy soft point and U_A be any L_{gfs}-open set in X with $e(Q_A)\tilde{\subseteq}U_A$. Then we have $e(Q_A)\bar{q}U^c_A$. By the given condition (b), there exist $V_A, W_A \in \tau_1(\tau_2)$ such that $e(Q_A)\tilde{\subseteq}V_A$, $U^c_A\tilde{\subseteq}W_A$ and $cl(V_A)\bar{q}cl(W_A)$. This gives $e(Q_A)\tilde{\subseteq}V_A$, $W^c_A\tilde{\subseteq}U_A$ and $cl(V_A)\bar{q}W_A$ and again this implies $e(Q_A)\tilde{\subseteq}V_A$, $W^c_A \leq U_A$ and $cl(V_A)\tilde{\subseteq}W^c_A$. Thus, $e(Q_A)\tilde{\subseteq}V_A\tilde{\subseteq}cl(V_A)\tilde{\subseteq}W^c_A\tilde{\subseteq}U_A$. Therefore, we get, $e(Q_A)\tilde{\subseteq}V_A\tilde{\subseteq}cl(V_A)\tilde{\subseteq}U_A$.

$(c) \Rightarrow (a)$: Let F_A be any L_{gfs}-closed set and $e(Q_A)$ be any fuzzy soft point in X such that $e(Q_A)\bar{q}F_A$. Then $e(Q_A)\tilde{\subseteq}F^c_A$. Since F^c_A is an L_{gfs}-open set in X, by the condition (c), there exists $V_A \in \tau_1(\tau_2)$ such that $e(Q_A)\tilde{\subseteq}V_A\tilde{\subseteq}cl(V_A)\tilde{\subseteq}F^c_A$ and from this we get $F_A\tilde{\subseteq}(cl(V_A))^c$. Hence, $e(Q_A)\tilde{\subseteq}V_A$, $F_A\tilde{\subseteq}(cl(V_A))^c$ and $V_A\bar{q}(cl(V_A))^c$, eventually X is L_{gfs}-regular.

Definition 3.4 [15] An FSTS (X, τ) is said to be fuzzy soft normal if for every pair of disjoint fuzzy soft closed set F_A and G_A, there exist disjoint fuzzy soft open sets U_A and V_A such that $F_A\tilde{\subseteq}U_A$, $G_A\tilde{\subseteq}V_A$.

Definition 3.5. Let X be an MFSITS. Then X is said to be L_{gfs}-normal if for any two L_{gfs}-closed sets F_A, G_A in X with $F_A\bar{q}G_A$, then there exist fuzzy soft open sets U_A and V_A such that $F_A\tilde{\subseteq}U_A$, $G_A\tilde{\subseteq}V_A$ and $U_A\bar{q}V_A$.

We now give some characterizations of L_{gfs}-normality:

Theorem 3.6. The followings are equivalent in an MFSITS X:
(a) X is L_{gfs}-normal.
(b) For any two L_{gfs}-closed sets F_A, G_A in X with $F_A\bar{q}G_A$, there exist fuzzy soft open sets U_A and V_A such that $F_A\tilde{\subseteq}U_A$, $G_A\tilde{\subseteq}V_A$ and $cl(U_A)\bar{q}cl(V_A)$.
(c) For each L_{gfs}-closed set F_A and for each L_{gfs}-open set H_A containing F_A, there exists a fuzzy soft open set V_A such that $F_A\tilde{\subseteq}V_A\tilde{\subseteq}cl(V_A)\tilde{\subseteq}H_A$.

Proof. $(a) \Rightarrow (b)$: Let F_A and G_A be two L_{gfs}-closed sets in X with $F_A\bar{q}G_A$. Then by (a) there exist $W_A, V_A \in \tau_1(\tau_2)$ such that $F_A\tilde{\subseteq}W_A$, $G_A\tilde{\subseteq}V_A$ and $W_A\bar{q}V_A$. Then $F_A\bar{q}cl(V_A)$ and so $W_A\bar{q}cl(V_A)$. Now $cl(V_A)$ being fuzzy soft closed, it is L_{gfs}-closed set. Thus F_A and $cl(V_A)$ are two L_{gfs}-closed sets in X such that $F_A\bar{q}cl(V_A)$. Again, since X is L_{gfs}-normal, there exist $G_A, H_A \in \tau_1(\tau_2)$ such that $F_A\tilde{\subseteq}G_A$, $cl(V_A)\tilde{\subseteq}H_A$ and $G_A\bar{q}H_A$. Thus $cl(G_A)\bar{q}H_A$.

Let $U_A = W_A\tilde{\cap}G_A$, then $U_A, V_A \in \tau_1(\tau_2)$, such that $F_A\tilde{\subseteq}U_A$, $G_A\tilde{\subseteq}V_A$ and $cl(U_A)\bar{q}cl(V_A)$.

$(b) \Rightarrow (c)$: Let F_A and H_A be any L_{gfs}-closed and L_{gfs}-open set, respectively in X such that $F_A\tilde{\subseteq}H_A$. Then we have $F_A\bar{q}H^c_A$ and H^c_A is L_{gfs}-closed set in X and hence by (b), there exist $U_A, V_A \in \tau_1(\tau_2)$ such that $F_A\tilde{\subseteq}U_A$, $H^c_A\tilde{\subseteq}V_A$ and $cl(U_A)\bar{q}cl(V_A)$. Now from $H^c_A\tilde{\subseteq}V_A\tilde{\subseteq}cl(V_A)$, we have $(cl(V_A))^c)\tilde{\subseteq}H_A$ and $cl(U_A)\bar{q}cl(V_A)$ implies $cl(U_A)\tilde{\subseteq}(cl(V_A))^c\tilde{\subseteq}H_A$. Thus, we have $U_A \in \tau_1(\tau_2)$ such that $F_A \leq U_A\tilde{\subseteq}cl(U_A)\tilde{\subseteq}H_A$, and U_A is the required fuzzy soft open set.

$(c) \Rightarrow (a)$: Let F_A, G_A be two L_{gfs}-closed sets with $F_A\bar{q}G_A$. Then $F_A\tilde{\subseteq}G^c_A$ and G^c_A is L_{gfs}-open set.

By (c), there exists $U_A \in \tau_1(\tau_2)$ such that $F_A \tilde{\subseteq} U_A \tilde{\subseteq} cl(U_A) \tilde{\subseteq} G_A^c$ that is, $F_A \tilde{\subseteq} U_A$, $G_A \tilde{\subseteq} (cl(U_A))^c$ and $U_A \bar{q} (cl(U_A))^c)$ and consequently X is L_{gfs}-normal.

We now introduce L_{gfs}-compactness in an MFSITS $(X, \tau_1(\tau_2), \tilde{I})$ as follows:

Definition 3.7. A collection $\{U_{A_i} : i \in \Lambda\}$ of L_{gfs}-open sets in an MFSITS X is called L_{gfs}-open cover of a fuzzy soft set F_A if $F_A \tilde{\subseteq} \tilde{\cup}_{i \in \Lambda} U_{A_i}$.

Definition 3.8. An MFSITS X is called L_{gfs}-compact if every L_{gfs}-open cover of X has a finite subcover.

Definition 3.9. A fuzzy soft set F_A in an MFSITS X is called L_{gfs}-compact relative to X if for every collection $\{U_{A_i} : i \in \Lambda\}$ of L_{gfs}-open sets of X such that $F_A \tilde{\subseteq} \tilde{\cup}_{i \in \Lambda} U_{A_i}$, there exists a finite subset Λ_o of Λ such that $F_A \tilde{\subseteq} \tilde{\cup}_{i \in \Lambda_o} U_{A_i}$.

Theorem 3.10. Let F_A be an L_{gfs}-closed subset of an L_{gfs}-compact space X. Then F_A is L_{gfs}-compact.
Proof: Let F_A be an L_{gfs}-closed subset of X such that $F_A \tilde{\subseteq} \bigvee_{i \in \Lambda} U_{A_i}$, where U_{A_i}'s are L_{gfs}-open sets for each $i \in \Lambda$. Then $\{U_{A_i} : i \in \Lambda\} \tilde{\cup} \{F_A^c\}$ is an L_{gfs}-open cover of X. Now, X being L_{gfs}-compact space, there exists a finite subset Λ_0 of Λ such that $X = \{U_{A_i} : i \in \Lambda_0\} \tilde{\cup} \{F_A^c\}$. As F_A and F_A^c are disjoint, hence we have $F_A \tilde{\subseteq} \bigvee_{i \in \Lambda_0} U_{A_i}$. This shows that F_A is L_{gfs}-compact relative to X and thus F_A is L_{gfs}-compact.

Theorem 3.11. If F_A and G_A be two fuzzy soft subsets of an MFSITS X such that F_A is L_{gfs}-compact and G_A is L_{gfs}-closed in X. Then $F_A \tilde{\cap} G_A$ is L_{gfs}-compact.
Proof: Let $\{U_{A_i} : i \in \Lambda\}$ be a L_{gfs}-open cover of $F_A \tilde{\cap} G_A$ in X and as G_A^c is L_{gfs}-open set, $\{U_{A_i} : i \in \Lambda\} \tilde{\cup} \{G_A^c\}$ is a L_{gfs}-open cover of F_A. Now F_A being L_{gfs}-compact, there exists a finite subset Λ_0 of Λ such that $F_A \tilde{\subseteq} \{U_{A_i} : i \in \Lambda_0\} \tilde{\cup} \{G_A^c\}$. This implies that $F_A \tilde{\cap} G_A \tilde{\subseteq} \{U_{A_i} : i \in \Lambda_0\}$. Thus $F_A \tilde{\cap} G_A$ is L_{gfs}-compact.

Theorem 3.12. Every L_{gfs}-closed subset F_A of a fuzzy soft compact space X is fuzzy soft compact if F_A is $*$ dense-in-itself in X.
Proof: Let $F_A \tilde{\subseteq} (\bigvee_{i \in \Lambda} U_{A_i})$, where U_{A_i} is fuzzy open, for each $i \in \Lambda$. Then $F_A^* \tilde{\subseteq} (\bigvee_{i \in \Lambda} U_{A_i})$, as F_A is L_{gfs}-closed set in X. Since F_A^* is fuzzy soft closed in X, it is fuzzy soft compact. Then $F_A^* \tilde{\subseteq} (\bigvee_{i=1}^{n} U_{A_i})$ and consequently $F_A \tilde{\subseteq} F_A^* \tilde{\subseteq} (\bigvee_{i=1}^{n} U_{A_i})$. This establishes that F_A is fuzzy soft compact.

Theorem 3.13. Every L_{gfs}-compact subset F_A of an L_{gfs}-regular space X is always an L_{gfs}-closed set.

Proof: Let F_A be a L_{gfs}-compact set with $F_A \tilde{\subseteq} U_A$, where $U_A \in \tau_1(\tau_2)$. Let $e(G_A)$ be any fuzzy soft point in X such that $e(G_A) \tilde{\subseteq} U_A$. Since U_A is fuzzy soft open set, it is L_{gfs}-open set. By Theorem 3.3(c), L_{gfs}-regularity of X implies that there exists fuzzy soft open set V_{A_i} such that $e(G_A) \tilde{\subseteq} V_{A_i} \tilde{\subseteq} cl(V_{A_i}) \tilde{\subseteq} U_A$ and hence we have $F_A \tilde{\subseteq} \tilde{\cup}_{i \in \Lambda} V_{A_i} \tilde{\subseteq} cl(\tilde{\cup}_{i \in \Lambda} V_{A_i}) \tilde{\subseteq} U_A$, for $e(G_A) \in F_A$, and where each V_{A_i} is L_{gfs}-open sets (as V_{A_i} being fuzzy soft open set). Now, since F_A is an L_{gfs}-compact set, exists a finite subset Λ_0 of Λ such that $F_A \tilde{\subseteq} \tilde{\cup}_{i \in \Lambda_o} V_{A_i} \tilde{\subseteq} cl(\tilde{\cup}_{i \in \Lambda_o} V_{A_i}) \tilde{\subseteq} U_A$. Let $V_A = \tilde{\cup}_{i \in \Lambda_o} V_{A_i}$, then V_A is fuzzy soft open set and $F_A \tilde{\subseteq} V_A \tilde{\subseteq} cl(V_A) \tilde{\subseteq} U_A$. Since $cl(F_A)$ is the smallest fuzzy soft closed set and $F_A^* \tilde{\subseteq} cl(F_A)$, we get $F_A^* \tilde{\subseteq} cl(F_A) \tilde{\subseteq} cl(V_A) \tilde{\subseteq} U_A$ and so $F_A^* \tilde{\subseteq} U_A$, this establishes that F_A is L_{gfs}-closed set.

Remark 3.14. Every fuzzy soft compact subset of an L_{gfs}-regular space X is always an L_{gfs}-closed set.
Proof. Using the above theorem 3.13, it can be proved the theorem easily and thus omitted.

4. Conclusion

In this article, a new type of generalized closed sets is introduced, whose characteristics are different up to some extent from the conventional ones. We have applied L_{gfs}-closed set to study few of the separation axioms and fuzzy soft compactness also. We have traditionally found that a generalized closed subset of a compact topological space is compact but in our context L_{gfs}-closed subset of a fuzzy soft compact space may not be a fuzzy soft compact in general. We emphasize that this traditional result will come true under certain conditions contained in this report.

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