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Some Specific Differential Identities in 3-prime Near-rings

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ABSTRACT: In this paper we investigate 3-prime near-rings with left generalized semiderivations satisfying certain differential identities. Consequently, some well-known results existing in literature have been generalized. We also show how the constraints placed on the hypothesis of various results are really not redundant.

Key Words: 3-prime near-ring, generalized semiderivations, semiderivations, right multipliers, semigroup ideals, commutativity theorems.

Contents

1. Introduction

Throughout this paper, $\mathcal{N}$ will be a zero-symmetric left near-ring with multiplicative center $Z(\mathcal{N})$, and usually $\mathcal{N}$ will be 3-prime, that is, if for $x, y \in \mathcal{N}$ will have the property that, $x\mathcal{N}y = \{0\}$ implies $x = 0$ or $y = 0$. Note that $\mathcal{N}$ is a zero-symmetric if $0x = 0$ for all $x \in \mathcal{N}$, (recall that left distributive yields $x0 = 0$). Recalling that $\mathcal{N}$ is called 2-torsion free if $2x = 0$ implies $x = 0$ for all $x \in \mathcal{N}$. A nonempty subset U of $\mathcal{N}$ is called semigroup left ideal (resp. semigroup right ideal) if $\mathcal{N}U \subseteq U$ (resp. $U\mathcal{N} \subseteq U$) and if U is both a semigroup left ideal and a semigroup right ideal, it will be called a semigroup ideal. Let α and β be maps from $\mathcal{N}$ to $\mathcal{N}$. Granted $x, y \in \mathcal{N}$, we write $[x, y]_{(\alpha,\beta)} = \beta(x)\alpha(y) - \alpha(y)\beta(x)$ and $(x \circ y)_{(\alpha,\beta)} = \beta(x)\alpha(y) + \alpha(y)\beta(x)$, in particular $[x, y]_{(I_{\mathcal{N}},I_{\mathcal{N}})} = [x, y]$ and $(x \circ y)_{(I_{\mathcal{N}},I_{\mathcal{N}})} = x \circ y$ in the usual sense, where $I_{\mathcal{N}}$ is the identity map of $\mathcal{N}$. An additive mapping $H : \mathcal{N} \to \mathcal{N}$ is said to be a right (resp. left) multiplier if $H(xy) = xH(y)$ (resp. $H(xy) = H(x)y$) holds for all $x, y \in \mathcal{N}$. H is said to be a multiplier if it is both left as well as right multiplier.

In (2013), A. Boua and al. [4] have introduced the notion of semiderivation of a near-ring $\mathcal{N}$ in the following way :

Definition 1.1. *An additive mapping $d : \mathcal{N} \to \mathcal{N}$ is called semiderivation if there exists an additive map $g : \mathcal{N} \to \mathcal{N}$ such that $d(xy) = d(x)g(y) + xd(y) = d(x)y + g(x)d(y)$ and $d(g(x)) = g(d(x))$ for all $x, y \in \mathcal{N}$.*

The notions of left generalized semiderivation and right generalized semiderivation are introduced as follows:

Definition 1.2. *Let $\mathcal{N}$ be a near-ring and d be a semiderivation associated with an additive mapping g of $\mathcal{N}$. An additive mapping $F : \mathcal{N} \longrightarrow \mathcal{N}$ is called a left generalized semiderivation associated with d if it satisfies $F(xy) = d(x)g(y) + xF(y) = d(x)y + g(x)F(y)$ and $F(g(x)) = g(F(x))$ for all $x, y \in \mathcal{N}$.*

Definition 1.3. *Let $\mathcal{N}$ be a near-ring and d be a semiderivation associated of $\mathcal{N}$ with an additive mapping g. An additive mapping $F : \mathcal{N} \longrightarrow \mathcal{N}$ is called a right generalized semiderivation associated with d if it satisfies $F(xy) = F(x)g(y) + xd(y) = F(x)y + g(x)d(y)$ and $F(g(x)) = g(F(x))$ for all $x, y \in \mathcal{N}$.*

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Definition 1.4. *Let $\mathcal{N}$ be a near-ring and d be a semiderivation of $\mathcal{N}$ associated with an additive mapping g. An additive mapping $F : \mathcal{N} \longrightarrow \mathcal{N}$ is called a generalized semiderivation associated with d if it is both a left as well as a right generalized semiderivation associated with d.*

Example 1.5. *Let S be a left zero-symmetric near-ring, and*

$$\mathcal{N} = \left\{ \begin{pmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{pmatrix} \mid a, b, c, 0 \in S \right\}.$$

We define the maps $d, g, F : \mathcal{N} \longrightarrow \mathcal{N}$ as follow:

$$d \begin{pmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & a & b \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \ g \begin{pmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & c \\ 0 & 0 & 0 \end{pmatrix}$$

and

$$F \begin{pmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & b \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

It is straightforward to check that $\mathcal{N}$ is a zero-symmetric left near-ring, d is a semiderivation of $\mathcal{N}$ associated with g, and F is a left generalized semiderivation associated with d, but F is not a right generalized semiderivation associated with d on $\mathcal{N}$.

Example 1.6. *Let S be a left zero-symmetric near-ring, and*

$$\mathcal{N} = \left\{ \begin{pmatrix} 0 & 0 & 0 \\ a & 0 & b \\ c & 0 & 0 \end{pmatrix} \mid a, b, c, 0 \in S \right\}.$$

Let us consider the maps $d, g, F : \mathcal{N} \longrightarrow \mathcal{N}$ given by:

$$d \begin{pmatrix} 0 & 0 & 0 \\ a & 0 & b \\ c & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ a & 0 & b \\ 0 & 0 & 0 \end{pmatrix}, \ g \begin{pmatrix} 0 & 0 & 0 \\ a & 0 & b \\ c & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ c & 0 & 0 \end{pmatrix}$$

and

$$F \begin{pmatrix} 0 & 0 & 0 \\ a & 0 & b \\ c & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & b \\ c & 0 & 0 \end{pmatrix}.$$

It is easy to see that $\mathcal{N}$ is a zero-symmetric left near-ring, d is a semiderivation associated with g of $\mathcal{N}$, and F is a right generalized semiderivation associated with d, but F is not a left generalized semiderivation associated with d on $\mathcal{N}$.

Example 1.7. *Let* $\mathcal{N} = \left\{ \begin{pmatrix} 0 & 0 & a \\ 0 & 0 & b \\ 0 & 0 & 0 \end{pmatrix} \mid a, b, 0 \in S \right\}$, *where S is a left zero-symmetric near-ring. Define the maps $d, g, F : \mathcal{N} \longrightarrow \mathcal{N}$ by:*

$$d \begin{pmatrix} 0 & 0 & a \\ 0 & 0 & b \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & b \\ 0 & 0 & a \\ 0 & 0 & 0 \end{pmatrix}, \ g \begin{pmatrix} 0 & 0 & a \\ 0 & 0 & b \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & b \\ 0 & 0 & a \\ 0 & 0 & 0 \end{pmatrix}$$

and

$$F\begin{pmatrix} 0 & 0 & a \\ 0 & 0 & b \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & b \\ 0 & 0 & 0 \end{pmatrix}.$$

Clearly, $\mathcal{N}$ is zero-symmetric left near-ring, d is a semiderivation of $\mathcal{N}$ associated with g, and F is a generalized semiderivation associated with d on $\mathcal{N}$.

The presence of certain various types of derivations and the link between rings and near-rings commutativity has piqued the interest of researchers. Many authors, including [4], [6], [8] and others, have recently obtained the commutativity of prime rings and near-rings using generalized semiderivations satisfying specified polynomial and differential constants.

In 2015, M. Ashraf and M. A. Siddeeque [2] proved that a 3-prime near-ring must be commutative ring if it admits a left generalized derivation F associated with a nonzero derivation, satisfies one of the following properties: $(i)F([x,y]) = 0$, $(ii)F([x,y]) = \pm[x,y]$, $(iii)F(x \circ y) = 0$, $(iv)F(x \circ y) = \pm(x \circ y)$, $(v)F([x,y]) = \pm(x \circ y)$, $(vi)F(x \circ y) = \pm[x,y]$ for all x, y in a nonzero semigroup ideal U. In this paper, we generalize the above-mentioned results. More precisely, we study the following theorem on commutativity of 3-prime near-rings involving left generalized semiderivations F, right multipliers H, β and an automorphism α, that satisfies the following conditions:

(i) $F([x,y]_{(\alpha,\beta)}) = 0$,	(ii) $F((x \circ y)_{(\alpha,\beta)}) = 0$
(iii) $F([x,y]_{(\alpha,\beta)}) = H([x,y]_{(\alpha,\beta)})$,	(iv) $F((x \circ y)_{(\alpha,\beta)}) = H((x \circ y)_{(\alpha,\beta)})$,
(v) $F([x,y]_{(\alpha,\beta)}) = H((x \circ y)_{(\alpha,\beta)})$,	(vi) $F((x \circ y)_{(\alpha,\beta)}) = H([x,y]_{(\alpha,\beta)})$,

for all $x, y \in U$.

2. Some preliminaries

Lemma 2.1. [3, Lemma 1.2 (i), Lemma 1.2 (iii), Lemma 1.3 (iii)] *Let $\mathcal{N}$ be a 3-prime near-ring.*

(i) If $z \in Z(\mathcal{N}) \setminus \{0\}$, then z is not a zero divisor.

(ii) If $z \in Z(\mathcal{N}) \setminus \{0\}$ and $zx \in Z(\mathcal{N})$, then $x \in Z(\mathcal{N})$.

(iii) If z centralizes a nonzero semigroup right ideal, then $z \in Z(\mathcal{N})$.

Lemma 2.2. [3, Lemma 1.3 (i)] *Let $\mathcal{N}$ be a 3-prime near-ring. If U is a nonzero semigroup right ideal (resp. semigroup left ideal) and x is an element of $\mathcal{N}$ such that $Ux = \{0\}$ (resp. $xU = \{0\}$), then $x = 0$.*

Lemma 2.3. [3, Lemma 1.4 (i)] *Let $\mathcal{N}$ be a 3-prime near-ring, and U a nonzero semigroup ideal of $\mathcal{N}$. If $x, y \in \mathcal{N}$, and $xUy = \{0\}$, then $x = 0$ or $y = 0$.*

Lemma 2.4. [3, Lemma 1.5] *Let $\mathcal{N}$ be a 3-prime near-ring. If $Z(\mathcal{N})$ contains a nonzero semigroup left ideal or a nonzero semigroup right ideal of $\mathcal{N}$, then $\mathcal{N}$ is a commutative ring.*

Lemma 2.5. [1, Lemma 2.4] *Let $\mathcal{N}$ be a 3-prime near-ring and U be a nonzero semigroup ideal of $\mathcal{N}$. If $\mathcal{N}$ admits a nonzero semiderivation d associated with a map g, then $d(U) \neq \{0\}$.*

Lemma 2.6. [6, Theorems 1] *Let $\mathcal{N}$ be a 3-prime near-ring, U a nonzero semigroup ideal of $\mathcal{N}$, and d be a nonzero semiderivation associated with an automorphism g of $\mathcal{N}$. Then the following conditions are equivalent:*

(i) $d(U) \subseteq Z(\mathcal{N})$

(ii) $\mathcal{N}$ is a commutative ring.

Lemma 2.7. *Let $\mathcal{N}$ be a near-ring and d be a nonzero semiderivation associated with an additive map g of $\mathcal{N}$. If $\mathcal{N}$ admits an additive mapping F, then the following statements are equivalent:*

(i) $F(xy) = d(x)g(y) + xF(y) = d(x)y + g(x)F(y)$ *for all* $x, y \in \mathcal{N}$.

(ii) $F(xy) = xF(y) + d(x)g(y) = g(x)F(y) + d(x)y$ *for all* $x, y \in \mathcal{N}$.

Proof. $(i) \Rightarrow (ii)$ Assume that $F(xy) = d(x)g(y) + xF(y)$ for all $x, y \in \mathcal{N}$. Thus $F(x(y + y)) = d(x)g(y + y) + xF(y + y)$ for all $x, y \in \mathcal{N}$. So $F(x(y + y)) = d(x)g(y) + d(x)g(y) + xF(y) + xF(y)$ for all $x, y \in \mathcal{N}$. On the other hand, we have $F(x(y+y)) = F(xy)+F(xy) = d(x)g(y)+xF(y)+d(x)g(y)+xF(y)$ for all $x, y \in \mathcal{N}$.
Comparing the two equations, we find that $d(x)g(y) + xF(y) = xF(y) + d(x)g(y)$ for all $x, y \in \mathcal{N}$. Similarly, we can prove that $d(x)y + g(x)F(y) = g(x)F(y) + d(x)y$ for all $x, y \in \mathcal{N}$. Hence $F(xy) = xF(y) + d(x)g(y) = g(x)F(y) + d(x)y$ for all $x, y \in \mathcal{N}$.
$(ii) \Rightarrow (i)$ We obtain the proof by employing the identical techniques as those given in $(i) \Rightarrow (ii)$. □

We can show the following result in a similar way:

Lemma 2.8. *Let* $\mathcal{N}$ *be a near-ring and* d *be a nonzero semiderivation associated with an additive map* g *of* $\mathcal{N}$. *If* $\mathcal{N}$ *admits an additive mapping* F, *then the following statements are equivalent:*

(i) $F(xy) = F(x)g(y) + xd(y) = F(x)y + g(x)d(y)$ *for all* $x, y \in \mathcal{N}$.

(ii) $F(xy) = xd(y) + F(x)g(y) = g(x)d(y) + F(x)y$ *for all* $x, y \in \mathcal{N}$.

Lemma 2.9. *Let* $\mathcal{N}$ *be a near-ring and* d *be a nonzero semiderivation associated with an additive map* g *of* $\mathcal{N}$. *If* F *is a left generalized semiderivation associated with a semiderivation* d, *then* $\mathcal{N}$ *satisfies the following partial distributive laws:*

(i) $(d(x)g(y) + xF(y))z = d(x)g(y)z + xF(y)z$ *for all* $x, y, z \in \mathcal{N}$.

(ii) $(d(x)y + g(x)F(y))z = d(x)yz + g(x)F(y)z$ *for all* $x, y, z \in \mathcal{N}$.

Proof. From the computation of $F(x(yz))$ and $F((xy)z)$, we obtain the required results. □

Similary we can prove the next result:

Lemma 2.10. *Let* $\mathcal{N}$ *be a near-ring and* d *be a nonzero semiderivation associated with an additive map* g *of* $\mathcal{N}$. *If* F *is a right generalized semiderivation associated with a semiderivation* d, *then* $\mathcal{N}$ *satisfies the following partial distributive laws:*

(i) $(F(x)g(y) + xd(y))z = F(x)g(y)z + xd(y)z$ *for all* $x, y, z \in \mathcal{N}$.

(ii) $(F(x)y + g(x)d(y))z = F(x)yz + g(x)d(y)z$ *for all* $x, y, z \in \mathcal{N}$.

Lemma 2.11. *Let* $\mathcal{N}$ *be a near-ring. If* d *is a semiderivation associated with epimorphism* g *of* $\mathcal{N}$, *then* $d(Z(\mathcal{N})) \subseteq Z(\mathcal{N})$.

Proof. Let $z \in Z(\mathcal{N})$, we have $d(zx) = d(xz)$, for all $x \in \mathcal{N}$. Using Lemma 2.7 and the definition of d, we get $d(zx) = d(z)g(x) + zd(x) = d(xz) = g(x)d(z) + d(x)z$ for all $x \in \mathcal{N}$. Thus $d(z)g(x) = g(x)d(z)$ for all $x \in \mathcal{N}$. Since g is an epimorphism of $\mathcal{N}$, it follows that $xd(z) = d(z)x$ for all $x \in \mathcal{N}$. So, $d(z) \in Z(\mathcal{N})$ for all $z \in Z(\mathcal{N})$. Hence $d(Z(\mathcal{N})) \subseteq Z(\mathcal{N})$. □

3. Some results for right multipliers and semigroup ideals

In this section, it is assumed that α is an automorphism of the near-ring $\mathcal{N}$.

Lemma 3.1. *Let* $\mathcal{N}$ *be a* 3*-prime near-ring and* U *be a nonzero right semigroup ideal of* $\mathcal{N}$. *If* H *is a nonzero right multiplier of* $\mathcal{N}$, *then* $H(U) \neq \{0\}$. *Moreover, if* $H(U) \subseteq Z(\mathcal{N})$, *then* $\mathcal{N}$ *is a commutative ring.*

Proof. Assume that $H(x) = 0$ for all $x \in U$. Taking xt instead of x, where $t \in \mathcal{N}$, in the last expression, we get $UH(t) = \{0\}$ for all $t \in \mathcal{N}$. By Lemma 2.2 we get $H = 0$; a contradiction.
Now, suppose that $H(x) \in Z(\mathcal{N})$ for all $x \in U$. Substituting ux for x in the last expression, we get $uH(x) \in Z(\mathcal{N})$ for all $x, u \in U$. By Lemma 2.1 (ii), we obtain $U \subseteq Z(\mathcal{N})$ or $H(U) = \{0\}$. Since $H(U) \neq \{0\}$, we have $U \subseteq Z(\mathcal{N})$, so by using Lemma 2.4, we conclude that $\mathcal{N}$ is a commutative ring. □

Theorem 3.2. *Let $\mathcal{N}$ be a 3-prime near-ring and U be a nonzero semigroup ideal of $\mathcal{N}$. If $\mathcal{N}$ admits a nonzero right multiplier β, then the following assertions are equivalent:*

(i) $[x, y]_{(\alpha,\beta)} \in Z(\mathcal{N})$ *for all* $x, y \in U$.

(ii) $\mathcal{N}$ *is a commutative ring.*

Proof. The implication $(ii) \Rightarrow (i)$ is obvious.
$(i) \Rightarrow (ii)$ Assume that $[x, y]_{(\alpha,\beta)} \in Z(\mathcal{N})$ for all $x, y \in U$. Substituting $\alpha(y)x$ for x in the last expression we get $\alpha(y)[x, y]_{(\alpha,\beta)} \in Z(\mathcal{N})$ for all $x, y \in U$. Using Lemma 2.1 (ii), we obtain $[x, y]_{(\alpha,\beta)} = [\beta(x), \alpha(y)] = 0$ or $\alpha(y) \in Z(\mathcal{N})$ for all $x, y \in U$. Thus, $[x, y]_{(\alpha,\beta)} = 0$ for all $x, y \in U$. Which can be rewritten as $[\beta(x), \alpha(y)] = 0$ for all $x, y \in U$. By Lemma 2.1 (iii), we get $\beta(U) \subseteq Z(\mathcal{N})$. Applying Lemma 3.1, we conclude that $\mathcal{N}$ is a commutative ring. □

Theorem 3.3. *Let $\mathcal{N}$ be a 3-prime near-ring and U be a nonzero semigroup ideal of $\mathcal{N}$. If $\mathcal{N}$ admits a nonzero right multiplier β, then the following assertions are equivalent:*

(i) $(x \circ y)_{(\alpha,\beta)} = 0$ *for all* $x, y \in U$.

(ii) $\mathcal{N}$ *is a commutative ring with* $2\mathcal{N} = \{0\}$.

Proof. Clearly $(ii) \Rightarrow (i)$.
$(i) \Rightarrow (ii)$ Assume that

$$(x \circ y)_{(\alpha,\beta)} = 0 \text{ for all } x, y \in U. \tag{3.1}$$

That is $\beta(x)\alpha(y) = -\alpha(y)\beta(x)$ for all $x, y \in U$. Substituting $\alpha^{-1}(t)y$ for y in the last relation, we obtain $\beta(x)t\alpha(y) = -t\alpha(y)\beta(x) = t\alpha(y)\beta(-x) = t(\beta(-x))\alpha(y)$ for all $t, x, y \in U$, which implies $[\beta(-x), t]\alpha(U) = \{0\}$ for all $t, x \in U$. So by Lemma 2.3 and Lemma 2.1 (iii), it follows that $\beta(-U) \subseteq Z(\mathcal{N})$. By using the fact that $-U$ is a nonzero semigroup right ideal and Lemma 3.1, we have $\mathcal{N}$ is a commutative ring.
So, (3.1) becomes $\beta(x)\alpha(y + y) = 0$ for all $x, y \in U$. Replacing y by $y\alpha^{-1}(t)$ in the last equation, where $t \in \mathcal{N}$, we get $\beta(x)\alpha(y)(t + t) = 0$ for all $x, y \in U, t \in \mathcal{N}$. Which gives $\beta(x)\alpha(U)(t + t) = \{0\}$ for all $x \in U, t \in \mathcal{N}$. Since $\beta(U) \neq \{0\}$, by using Lemma 2.3, we conclude that $2\mathcal{N} = \{0\}$. □

Corollary 3.4. *Let $\mathcal{N}$ be a 3-prime near-ring and U be a nonzero semigroup ideal of $\mathcal{N}$, then the following assertions are equivalent:*

(i) $x \circ y = 0$ *for all* $x, y \in U$.

(ii) $\mathcal{N}$ *is a commutative ring with* $2\mathcal{N} = \{0\}$.

Theorem 3.5. *Let $\mathcal{N}$ be a 2-torsion free 3-prime near-ring and U be a nonzero semigroup ideal of $\mathcal{N}$. If $\mathcal{N}$ admits a nonzero right multiplier β, then the following assertions are equivalent:*

(i) $(x \circ y)_{(\alpha,\beta)} \in Z(\mathcal{N})$ *for all* $x, y \in U$.

(ii) $\mathcal{N}$ *is a commutative ring.*

Proof. It is easy to check that $(ii) \Rightarrow (i)$.
$(i) \Rightarrow (ii)$ Assume that $(x \circ y)_{(\alpha,\beta)} \in Z(\mathcal{N})$ for all $x, y \in U$. Substituting $\alpha(y)x$ for x in the last expression we get $\alpha(y)(x \circ y)_{(\alpha,\beta)} \in Z(\mathcal{N})$ for all $x, y \in U$. So, by Lemma 2.1 (ii), we obtain

$$(x \circ y)_{(\alpha,\beta)} = 0 \text{ or } \alpha(y) \in Z(\mathcal{N}) \text{ for all } x, y \in U. \tag{3.2}$$

Suppose that $Z(\mathcal{N})\cap U=\{0\}$, then (3.2) becomes $(x\circ y)_{(\alpha,\beta)}=0$ for all $x,y\in U$. Thus by Theorem 3.3, we get $\mathcal{N}$ is a commutative ring with $2\mathcal{N}=\{0\}$; a contradiction.
Hence $Z(\mathcal{N})\cap U\neq\{0\}$. Let $z\in Z(\mathcal{N})\cap U\smallsetminus\{0\}$. From $(t\circ z)_{(\alpha,\beta)}\in Z(\mathcal{N})$ for all $t\in U$, it follows that $\beta(t+t)\alpha(z)\in Z(\mathcal{N})$ for all $t\in U$. By Lemma 2.1 (ii), we obtain

$$\beta(t+t)\in Z(\mathcal{N}) \text{ for all } t\in U. \tag{3.3}$$

Replacing t by ut in (3.3), we arrive at

$$u\beta(t+t)\in Z(\mathcal{N}) \text{ for all } t,u\in U.$$

Using Lemma 2.1 (iii), we have $2\beta(t)=0$ for all $t\in U$ or $U\subseteq Z(\mathcal{N})$. If $2\beta(t)=0$ for all $t\in U$, then by using the 2-torsion freeness of $\mathcal{N}$, we obtain $\beta(U)=\{0\}$; a contradiction. Hence $U\subseteq Z(\mathcal{N})$, so, we conclude that $\mathcal{N}$ is a commutative ring according to Lemma 2.4. □

4. Some results for left generalized semiderivations

In this section, it is assumed that α is an automorphism and that d is a semiderivation associated with an automorphism g of the near-ring $\mathcal{N}$.

Lemma 4.1. *Let $\mathcal{N}$ be a 3-prime near-ring, U a nonzero semigroup right ideal of $\mathcal{N}$, and β be a right multiplier of $\mathcal{N}$. If $\mathcal{N}$ admits a left generalized semiderivation F associated with a nonzero semiderivation d of $\mathcal{N}$, such that $F(\beta(U))=\{0\}$, then $d=0$ or $\beta=0$.*

Proof. Assume that $F(\beta(x))=0$ for all $x\in U$. Taking xy in place of x in the last expression and using the definition of F, we get $d(x)\beta(y)=0$ for all $x,y\in U$. Replacing y by ty in the above equation, we find $d(x)t\beta(y)=0$ for all $x,y,t\in U$, which gives $d(x)U\beta(y)=\{0\}$ for all $x,y\in U$. By Lemma 2.3, it follows that $\beta(U)=\{0\}$ or $d(U)=\{0\}$. Hence, according to Lemma 2.5 and Lemma 3.1, we have $\beta=0$ or $d=0$. □

Theorem 4.2. *Let $\mathcal{N}$ be a 3-prime near-ring, U a nonzero semigroup ideal of $\mathcal{N}$, and β be a nonzero right multiplier of $\mathcal{N}$. If $\mathcal{N}$ admits a left generalized semiderivation F associated with a nonzero semiderivation d of $\mathcal{N}$, then the following assertions are equivalent:*

- *(i)* $F([x,y]_{(\alpha,\beta)})=0$ *for all* $x,y\in U$.
- *(ii)* $\mathcal{N}$ *is a commutative ring.*

Proof. It is easy to see that $(ii)\Rightarrow(i)$.
$(i)\Rightarrow(ii)$ Assume that

$$F([x,y]_{(\alpha,\beta)})=0 \text{ for all } x,y\in U. \tag{4.1}$$

Replacing y by $\alpha^{-1}(\beta(x))y$ in (4.1), we get $F(\beta(x)[x,y]_{(\alpha,\beta)})=0$ for all $x,y\in U$. Previous equation implies that

$$d(\beta(x))g([x,y]_{(\alpha,\beta)})=0 \text{ for all } x,y\in U. \tag{4.2}$$

That is,

$$d(\beta(x))g(\beta(x))g(\alpha(y))=d(\beta(x))g(\alpha(y))g(\beta(x)) \text{ for all } x,y\in U. \tag{4.3}$$

Putting ty in place of y in (4.3), and using it, we get

$$\begin{aligned} d(\beta(x))g(\beta(x))g(\alpha(t))g(\alpha(y)) &= d(\beta(x))g(\alpha(t))g(\alpha(y))g(\beta(x)) \\ &= d(\beta(x))g(\alpha(t))g(\beta(x))g(\alpha(y)) \text{ for all } x,y,t\in U. \end{aligned}$$

Which means that $d(\beta(x))g\circ\alpha(U)g([\alpha(y),\beta(x)])=\{0\}$ for all $x,y\in U$. As a result of Lemma 2.1 (ii) and Lemma 2.1 (iii), we obtain

$$d(\beta(x))=0 \text{ or } \beta(x)\in Z(\mathcal{N}) \text{ for all } x\in U. \tag{4.4}$$

Consider the case where $d(Z(\mathcal{N})) = \{0\}$. Thus, $d(\beta(U)) = \{0\}$ is implied by (4.4). By Lemma 4.1, we get $d = 0$ or $\beta = 0$, which is a contradiction.
Therefore $d(Z(\mathcal{N})) \neq \{0\}$. Let $z \in Z(\mathcal{N}) \setminus \{0\}$ such that $d(\alpha(z)) \neq 0$. Taking zy instead of y in (4.1), we arrive at $F(\alpha(z)[x,y]_{(\alpha,\beta)}) = 0$ for all $x, y \in U$, implying $d(\alpha(z))g([x,y]_{(\alpha,\beta)}) = 0$ for all $x, y \in U$. By Lemma 2.11, we have $d(\alpha(z)) \in Z(\mathcal{N}) \setminus \{0\}$, which implies that $[x,y]_{(\alpha,\beta)} = 0$ for all $x, y \in U$. According to Theorem 3.2, we conclude that $\mathcal{N}$ is a commutative ring. □

Theorem 4.3. *Let $\mathcal{N}$ be a 3-prime near-ring, U a nonzero semigroup ideal of $\mathcal{N}$, and H, β are nonzero right multipliers of $\mathcal{N}$. If $\mathcal{N}$ admits a left generalized semiderivation F associated with a nonzero semiderivation d of $\mathcal{N}$, then the following assertions are equivalent:*

(i) $F([x,y]_{(\alpha,\beta)}) = H([x,y]_{(\alpha,\beta)})$ for all $x, y \in U$.

(ii) $\mathcal{N}$ is a commutative ring.

Proof. The implication $(ii) \Rightarrow (i)$ is obvious.
$(i) \Rightarrow (ii)$ Assume that

$$F([x,y]_{(\alpha,\beta)}) = H([x,y]_{(\alpha,\beta)}) \text{ for all } x, y \in U. \tag{4.5}$$

Taking $\alpha^{-1}(\beta(x))y$ instead of y in (4.5), we get

$$F(\beta(x)[x,y]_{(\alpha,\beta)}) = H(\beta(x)[x,y]_{(\alpha,\beta)}) \text{ for all } x, y \in U.$$

That gives

$$d(\beta(x))g([x,y]_{(\alpha,\beta)}) = 0 \text{ for all } x, y \in U, \tag{4.6}$$

which is identical with the equation (4.2) of Theorem 4.2. We may now conclude that $\mathcal{N}$ is a commutative ring by arguing in the same way as in Theorem 4.2. □

Corollary 4.4. [2, Theorem 1] *Let $\mathcal{N}$ be a 3-prime near-ring and U be a nonzero semigroup ideal of $\mathcal{N}$. If $\mathcal{N}$ admits a left generalized derivation (F, d) satisfying either of the following identities $(i) F([x,y]) = 0$, for all $x, y \in U$ or $(ii) F([x,y]) = \pm[x,y]$ for all $x, y \in U$, then $\mathcal{N}$ is a commutative ring.*

Theorem 4.5. *Let $\mathcal{N}$ be a 3-prime near-ring, U a nonzero semigroup ideal of $\mathcal{N}$, and β be a nonzero right multiplier of $\mathcal{N}$. If $\mathcal{N}$ admits a left generalized semiderivation F associated with a nonzero semiderivation d of $\mathcal{N}$, then the following assertions are equivalent:*

(i) $F((x \circ y)_{(\alpha,\beta)}) = 0$ for all $x, y \in U$.

(ii) $\mathcal{N}$ is a commutative ring with $2\mathcal{N} = \{0\}$.

Proof. Clearly $(ii) \Rightarrow (i)$.
$(i) \Rightarrow (ii)$ Assume that

$$F((x \circ y)_{(\alpha,\beta)}) = 0 \text{ for all } x, y \in U. \tag{4.7}$$

Putting $\alpha^{-1}(\beta(x))y$ instead of y in (4.7), we arrive at $F(\beta(x)(x \circ y)_{(\alpha,\beta)}) = 0$ for all $x, y \in U$, which gives,

$$d(\beta(x))g((x \circ y)_{(\alpha,\beta)}) = 0 \text{ for all } x, y \in U. \tag{4.8}$$

Equivalently,

$$d(\beta(x))g(\beta(x))g(\alpha(y)) = -d(\beta(x))g(\alpha(y))g(\beta(x)) \text{ for all } x, y \in U. \tag{4.9}$$

Taking yt in place of y in (4.9), and using it, we get

$$\begin{aligned} -d(\beta(x))g(\alpha(y))g(\alpha(t))g(\beta(x)) &= d(\beta(x))g(\beta(x))g(\alpha(y))g(\alpha(t)) \\ &= d(\beta(x))g(\alpha(y))g(\beta(-x))g(\alpha(t)) \text{ for all } t, x, y \in U. \end{aligned}$$

Which means that $d(\beta(x))g\circ\alpha(U)g([\beta(-x),\alpha(t)])=\{0\}$ for all $x,t\in U$. Consequently, by Lemma 2.3 and Lemma 2.1 (iii), we find

$$d(\beta(x))=0 \text{ or } \beta(-x)\in Z(\mathcal{N}) \text{ for all } x\in U. \tag{4.10}$$

Suppose that $d(Z(\mathcal{N}))=\{0\}$. Then (4.10) gives $d(\beta(-U))=\{0\}$. In light of Lemma 4.1 we obtain $d=0$ or $\beta=0$; a contradiction. Therefore $d(Z(\mathcal{N}))\neq\{0\}$.
Let $z\in Z(\mathcal{N})\setminus\{0\}$ such that $d(\alpha(z))\neq 0$. Replacing y by zy in (4.7), we get $F(\alpha(z)(x\circ y)_{(\alpha,\beta)})=0$ for all $x,y\in U$, which implies that $d(\alpha(z))g((x\circ y)_{(\alpha,\beta)})=0$ for all $x,y\in U$. Using Lemma 2.11, we have $d(\alpha(z))\in Z(\mathcal{N})\setminus\{0\}$, which gives $(x\circ y)_{(\alpha,\beta)}=0$ for all $x,y\in U$. Hence, $\mathcal{N}$ is a commutative ring with $2\mathcal{N}=\{0\}$, by Theorem 3.3. □

Theorem 4.6. *Let $\mathcal{N}$ be a 3-prime near-ring, U a nonzero semigroup ideal of $\mathcal{N}$, and H,β are nonzero right multipliers of $\mathcal{N}$. If $\mathcal{N}$ admits a left generalized semiderivation F associated with a semiderivation d of $\mathcal{N}$, then the following assertions are equivalent:*

- *(i) $F((x\circ y)_{(\alpha,\beta)})=H((x\circ y)_{(\alpha,\beta)})$ for all $x,y\in U$.*
- *(ii) $\mathcal{N}$ is a commutative ring with $2\mathcal{N}=\{0\}$.*

Proof. It is easy to check that $(ii)\Rightarrow(i)$.
$(i)\Rightarrow(ii)$ Suppose that

$$F((x\circ y)_{(\alpha,\beta)})=H((x\circ y)_{(\alpha,\beta)}) \text{ for all } x,y\in U. \tag{4.11}$$

Replacing y by $\alpha^{-1}(\beta(x))y$ in (4.11), we arrive at

$$F(\beta(x)(x\circ y)_{(\alpha,\beta)})=H(\beta(x)(x\circ y)_{(\alpha,\beta)}) \text{ for all } x,y\in U.$$

Which yields

$$d(\beta(x))g((x\circ y)_{(\alpha,\beta)})=0 \text{ for all } x,y\in U.$$

Since this equation is identical with (4.8) of Theorem 4.5, by arguing in the same way as in Theorem 4.5, we may conclude that $\mathcal{N}$ is a commutative ring with $2\mathcal{N}=\{0\}$. □

Corollary 4.7. [2, Theorem 2] *Let $\mathcal{N}$ be a 3-prime near-ring and U be a nonzero semigroup ideal of $\mathcal{N}$. If $\mathcal{N}$ admits a left generalized derivation (F,d) satisfying either of the following identities $(i)F(x\circ y)=0$, for all $x,y\in U$ or $(ii)F(x\circ y)=\pm(x\circ y)$ for all $x,y\in U$, then $\mathcal{N}$ is a commutative ring.*

Theorem 4.8. *Let $\mathcal{N}$ be a 3-prime near-ring, U a nonzero semigroup ideal of $\mathcal{N}$, and H,β are nonzero right multipliers of $\mathcal{N}$. If $\mathcal{N}$ admits a left generalized semiderivation F associated with semiderivation d of $\mathcal{N}$, then the following assertions are equivalent:*

- *(i) $F([x,y]_{(\alpha,\beta)})=H((x\circ y)_{(\alpha,\beta)})$ for all $x,y\in U$.*
- *(ii) $\mathcal{N}$ is a commutative ring with $2\mathcal{N}=\{0\}$.*

Proof. It is easy to see that $(ii)\Rightarrow(i)$.
$(i)\Rightarrow(ii)$ Assume that

$$F([x,y]_{(\alpha,\beta)})=H((x\circ y)_{(\alpha,\beta)}) \text{ for all } x,y\in U. \tag{4.12}$$

Substituting $\alpha^{-1}(\beta(x))y$ for y in (4.12), we obtain $F(\beta(x)[x,y]_{(\alpha,\beta)})=H(\beta(x)(x\circ y)_{(\alpha,\beta)})$ for all $x,y\in U$, from which it follows easily that

$$d(\beta(x))g([x,y]_{(\alpha,\beta)})=0 \text{ for all } x,y\in U.$$

This is the same as the equation (4.2) of Theorem 4.2. By arguing similarly to Theorem 4.2, we obtain that $\mathcal{N}$ is a commutative ring.
Consequently, (4.12) becomes $H(\alpha(y)+\alpha(y))\beta(x)=0$ for all $x,y\in U$. Putting tuy in place of y and vx in place of x in last equation, we get $\alpha(t+t)\alpha(u)H(\alpha(y))v\beta(x)=0$ for all $u,v,x,y\in U,t\in\mathcal{N}$. Which means that $\alpha(t+t)\alpha(U)H(\alpha(y))U\beta(x)=\{0\}$ for all $x,y\in U,t\in\mathcal{N}$. According to Lemma 2.3 and Lemma 3.1, we conclude that $2\mathcal{N}=\{0\}$. □

Corollary 4.9. [2, Theorem 3] *Let $\mathcal{N}$ be a 3-prime near-ring and U be a nonzero semigroup ideal of $\mathcal{N}$. If $\mathcal{N}$ admits a left generalized derivation (F, d) satisfying $F([x, y]) = \pm(x \circ y)$ for all $x, y \in U$, then $\mathcal{N}$ is a commutative ring.*

Theorem 4.10. *Let $\mathcal{N}$ be a 3-prime near-ring, U a nonzero semigroup ideal, and H, β are nonzero right multipliers of $\mathcal{N}$. If $\mathcal{N}$ admits a left generalized semiderivation F associated with a nonzero semiderivation d of $\mathcal{N}$, then the following assertions are equivalent:*

(*i*) *$F((x \circ y)_{(\alpha,\beta)}) = H([x, y]_{(\alpha,\beta)})$ for all $x, y \in U$.*

(*ii*) *$\mathcal{N}$ is a commutative ring with $2\mathcal{N} = \{0\}$.*

Proof. The implication $(ii) \Rightarrow (i)$ is obvious.
$(i) \Rightarrow (ii)$ Assume that

$$F((x \circ y)_{(\alpha,\beta)}) = H([x, y]_{(\alpha,\beta)}) \text{ for all } x, y \in U. \tag{4.13}$$

Putting $\alpha^{-1}(\beta(x))y$ in place of y in (4.13), we find $F(\beta(x)(x \circ y)_{(\alpha,\beta)}) = H(\beta(x)[x, y]_{(\alpha,\beta)})$ for all $x, y \in U$. This implies

$$d(\beta(x))g((x \circ y)_{(\alpha,\beta)}) = 0 \text{ for all } x, y \in U.$$

This is the same as equation (4.8) of Theorem 4.5. By arguing in the same way as in Theorem 4.5, we can prove that $\mathcal{N}$ is a commutative ring with $2\mathcal{N} = \{0\}$. □

Corollary 4.11. [2, Theorem 4] *Let $\mathcal{N}$ be a 3-prime near-ring, U a nonzero semigroup ideal of $\mathcal{N}$. If $\mathcal{N}$ admits a left generalized derivation (F, d) satisfying $F(x \circ y) = \pm[x, y]$ for all $x, y \in U$, then $\mathcal{N}$ is a commutative ring.*

The following example shows that the condition of 3-primeness of $\mathcal{N}$ imposed on the assumptions of the above theorems is not redundant.

Example 4.12. *Let S be a left zero-symmetric near-ring and*

$$\mathcal{N} = \left\{ \begin{pmatrix} 0 & 0 & a \\ 0 & 0 & b \\ 0 & 0 & 0 \end{pmatrix} \mid a, b, 0 \in S \right\}.$$

If we set

$$U = \left\{ \begin{pmatrix} 0 & 0 & u \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \mid u, 0 \in S \right\},$$

then it is easy to check that $\mathcal{N}$ is a left zero-symmetric near-ring and U is a nonzero semigroup ideal of $\mathcal{N}$. Define the maps $\alpha = g, d, F, \beta, H : \mathcal{N} \longrightarrow \mathcal{N}$ by:

$$g\begin{pmatrix} 0 & 0 & a \\ 0 & 0 & b \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & b \\ 0 & 0 & a \\ 0 & 0 & 0 \end{pmatrix}, \; d\begin{pmatrix} 0 & 0 & a \\ 0 & 0 & b \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & b \\ 0 & 0 & a \\ 0 & 0 & 0 \end{pmatrix},$$

$$F\begin{pmatrix} 0 & 0 & a \\ 0 & 0 & b \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & a \\ 0 & 0 & b \\ 0 & 0 & 0 \end{pmatrix}, \beta\begin{pmatrix} 0 & 0 & a \\ 0 & 0 & b \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & a \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

and

$$H\begin{pmatrix} 0 & 0 & a \\ 0 & 0 & b \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & a \\ 0 & 0 & 0 \end{pmatrix}.$$

Clearly d is a semiderivation associated with g, F is a left generalized semiderivation associated with d, H and β are nonzero right multipliers satisfying the conditions:

(i) $F([x,y]_{(\alpha,\beta)})=0$,	(ii) $F((x\circ y)_{(\alpha,\beta)})=0$
(iii) $F([x,y]_{(\alpha,\beta)})=H([x,y]_{(\alpha,\beta)})$,	(iv) $F((x\circ y)_{(\alpha,\beta)})=H((x\circ y)_{(\alpha,\beta)})$,
(v) $F([x,y]_{(\alpha,\beta)})=H((x\circ y)_{(\alpha,\beta)})$,	(vi) $F((x\circ y)_{(\alpha,\beta)})=H([x,y]_{(\alpha,\beta)})$,

for all $x,y\in U$, but $\mathcal{N}$ is not a commutative ring.

Theorem 4.13. *Let $\mathcal{N}$ be a 3-prime near-ring, U a nonzero semigroup ideal of $\mathcal{N}$, and β be a nonzero right multiplier of $\mathcal{N}$. If $\mathcal{N}$ admits a left generalized semiderivation F associated with a nonzero semiderivation d of $\mathcal{N}$, such that $d(Z(\mathcal{N}))\neq\{0\}$, then the following assertions are equivalent:*

(i) *$F([x,y]_{(\alpha,\beta)})\in Z(\mathcal{N})$ for all $x,y\in U$.*

(ii) *$\mathcal{N}$ is a commutative ring.*

Proof. Clearly $(ii)\Rightarrow(i)$.
$(i)\Rightarrow(ii)$ Assume that

$$F([x,y]_{(\alpha,\beta)})\in Z(\mathcal{N}) \text{ for all } x,y\in U. \tag{4.14}$$

If $Z(\mathcal{N})=\{0\}$, it follows that $F([x,y]_{(\alpha,\beta)})=0$ for all $x,y\in U$. In view of Theorem 4.6, we obtain $\mathcal{N}$ is a commutative ring. So $\mathcal{N}=Z(\mathcal{N})=\{0\}$; a contradiction.
Thus $Z(\mathcal{N})\neq\{0\}$. Let $z\in Z(\mathcal{N})\setminus\{0\}$ such that $d(\alpha(z))\neq 0$. Replacing y by zy in (4.14), we get

$$d(\alpha(z))g([x,y]_{(\alpha,\beta)})+zF([x,y]_{(\alpha,\beta)})\in Z(\mathcal{N}) \text{ for all } x,y\in U, \tag{4.15}$$

which together with (4.14) gives

$$d(\alpha(z))g([x,y]_{(\alpha,\beta)})\in Z(\mathcal{N}) \text{ for all } x,y\in U. \tag{4.16}$$

Due to $d(\alpha(z))\in Z(\mathcal{N})$, by using Lemma 2.1 (ii), we find

$$[x,y]_{(\alpha,\beta)}\in Z(\mathcal{N}) \text{ for all } x,y\in U. \tag{4.17}$$

Hence, by Theorem 3.2, $\mathcal{N}$ is a commutative ring. □

Corollary 4.14. [2, Theorem 6] *Let $\mathcal{N}$ be a 3-prime near-ring and U be a nonzero semigroup ideal of $\mathcal{N}$. Let (F,d) be a left generalized derivation of $\mathcal{N}$ such that $d(Z(\mathcal{N}))\neq\{0\}$ and $F([x,y])\in Z(\mathcal{N})$ for all $x,y\in U$, then $\mathcal{N}$ is a commutative ring.*

The restriction of $d(Z(\mathcal{N}))\neq\{0\}$ imposed on the hypothesis of the Theorem 4.13 is not redundant in the situation of arbitrary near-rings, as shown in the following example:

Example 4.15. *Let*

$$\mathcal{R}=\left\{\begin{pmatrix} a & b\\ 0 & c\end{pmatrix} \mid a,b,c,0\in\mathbb{Z}\right\}$$

It is easy to see that $\mathcal{R}$ is prime ring with the center $Z=\left\{\begin{pmatrix} x & 0\\ 0 & x\end{pmatrix} \mid 0,x\in\mathbb{Z}\right\}$. Also it can be verified that $U=\left\{\begin{pmatrix} p & n\\ 0 & t\end{pmatrix} \mid p,n,t,0\in 2\mathbb{Z}\right\}$ is a nonzero semigroup ideal of $\mathcal{R}$, where $2\mathbb{Z}$ denotes the set of even integers. Define $\alpha=g,\beta,d,F:\mathcal{R}\to\mathcal{R}$ as following,

$$g\begin{pmatrix} a & b\\ 0 & c\end{pmatrix}=\begin{pmatrix} a & a+b-c\\ 0 & c\end{pmatrix},\ \beta\begin{pmatrix} a & b\\ 0 & c\end{pmatrix}=\begin{pmatrix} a & 0\\ 0 & 0\end{pmatrix},$$

$$d\begin{pmatrix} a & b\\ 0 & c\end{pmatrix}=\begin{pmatrix} 0 & c-a\\ 0 & 0\end{pmatrix} \text{ and } F\begin{pmatrix} a & b\\ 0 & c\end{pmatrix}=\begin{pmatrix} 0 & c+a\\ 0 & 0\end{pmatrix}.$$

It can be easily proved that d is a semiderivation associated with g, F is a left generalized semiderivation associated with d of $\mathcal{R}$, and β is a nonzero right miltiplier satisfying the conditions, $d(Z(\mathcal{N}))=\{0\}$ and $F([x,y]_{(\alpha,\beta)})\in Z(\mathcal{N})$ for all $x,y\in U$. However $\mathcal{R}$ is not a commutative ring.

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