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Anti T_2-Generalized Topological Spaces

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ABSTRACT: In this paper, we investigated non strong hyperconnected generalized topological spaces. Ekici [9] and Devi [17] have provided the results of hyperconnectedness for strong generalized topological spaces. We generalized these results for arbitrary generalized topological spaces. Through the notion of hyperconnectedness of arbitrary generalized topological spaces, we constructed an example which fails Hausdorff characterization of topological spaces "A first countable spaces is Hausdorff if and only if every convergent sequence has unique limit". This example also serves the purpose of constructing Anti Hausdorff Fréchet space in which every convergent sequence has unique limit required by Novak in [15].

Key Words: Generalized topological spaces, connectedness, hyperconnectedness, extremally disconnectedness.

Contents

1. Introduction

In 1968, Levine [11] introduced the notion of hyperconnectedness in topological spaces. A topological space X is said to be hyperconnected if every nonempty open set is dense in X. Majumdar et al. [12] studied the Anti Hausdorff spaces in general topology. In [2,3], Császár introduced the notion of generalized topological spaces which are of great importance not only in the field of pure mathematics but also in the field of applied mathematics like mathematical psychology, combinatorial chemistry. The notion of hyperconnectedness in generalized topological spaces was introduced by Ekici in [9]. This concept is nothing but the irreducibility of generalized topological spaces, introduced and studied by Shen [18]. After that Renukadevi [17] produced some results for hyperconnectedness in generalized topological spaces. Ekici and Renukadevi provide numerous results about hyperconnectedness for strong generalized topological spaces only. On the other hand, Tyagi et al. [19,20,21,23] studied semi open sets, β-open sets, connectedness, extremally disconnectedness, separation axioms in generalized topological spaces in more general way. In this paper, we proved the results of Ekici and Renukadevi for arbitrary generalized topological spaces using the approach of Tyagi et al. [19,20,21,22,23].

In 1971, Franklin et al. [10] tried to construct an example of Anti Hausdorff Fréchet space in which convergent sequence have a unique limit but the Example 3.2 in [10] is not Fréchet. So the problem was not encounter. The Example 4.3 in this paper encounter this problem in generalized topological spaces.

2. Preliminaries

Let X be a set and $\mathcal{P}(X)$ be its power set. A subset μ of $\mathcal{P}(X)$ is called a generalized topology (GT) on X if μ is closed under arbitrary union. The pair (X, μ) is called a generalized topological space (GTS). The elements of μ are called μ-open sets and their complements in X are called μ-closed sets. Let $M_\mu = \cup\{U : U \in \mu\}$. In general, $M_\mu \neq X$. In case $X = M_\mu$, GTS (X, μ) is called strong. Let i_μ and

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c_μ be the usual interior and closure operators. That is, $i_\mu A$ is the union of all μ-open sets contained in A and $c_\mu A$ is the intersection of all μ-closed sets containing A.

Theorem 2.1. *Let (X, μ) be a GTS and $A, B \subseteq X$. Then the following statements hold:*

1. $A \subseteq c_\mu A$ *and* $i_\mu A \subseteq A$.
2. $A \subseteq B$ *implies* $c_\mu A \subseteq c_\mu B$ *and* $i_\mu A \subseteq i_\mu B$.
3. $c_\mu c_\mu A = c_\mu A$ *and* $i_\mu i_\mu A = i_\mu A$.
4. $i_\mu A = X - c_\mu(X - A)$.
5. $x \in c_\mu A$ *if and only if* $x \in U \in \mu$ *implies* $U \cap A \neq \emptyset$.

Proposition 2.2. *If μ and ν are two generalized topologies on a set X, then $\mu \subseteq \nu$ implies $c_\nu A \subseteq c_\mu A$ for all $A \in P(X)$.*

Definition 2.3. *A subset A of a GTS (X, μ) is called*

1. *μ-regular open (or r_μ-open) [6] if $i_\mu c_\mu A = A$.*
2. *μ-semi open (or s_μ-open) [23] if $A \subseteq c_\mu i_\mu A \cap M_\mu$.*
3. *μ-preopen (or p_μ-open) [7] if $A \subseteq i_\mu c_\mu A$.*
4. *μ-α-open (or α_μ-open) [7] if $A \subseteq i_\mu c_\mu i_\mu A$.*
5. *μ-β-open (or β_μ-open) [23] if $A \subseteq c_\mu i_\mu c_\mu A \cap M_\mu$.*

For each $t \in \{r, s, p, \alpha, \beta\}$, the collection of all t_μ-open sets is denoted by t_μ. The complement of t_μ-open set is called t_μ-closed. The t_μ-interior of a set A, denoted by $i_{t_\mu} A$, is the union of all t_μ-open sets contained in A and the t_μ-closure of A, denoted by$c_{t_\mu} A$, is the intersection of all t_μ-closed sets containing A.

From Theorem 2.1, it follows that a subset A of GTS (X, μ) is r_μ-closed if and only if $c_\mu i_\mu A = A$; A is s_μ-open if and only if $c_\mu A = c_\mu i_\mu A$ and $A \subseteq M_\mu$. A set A is s_μ-closed if and only if $i_\mu c_\mu A \subseteq A$ and $X - M_\mu \subseteq A$; A is p_μ-closed if and only if $c_\mu i_\mu A \subseteq A$; A is α_μ-closed if and only if $c_\mu i_\mu c_\mu A \subseteq A$; A is β_μ-closed if and only if $i_\mu c_\mu i_\mu A \subseteq A$ and $X - M_\mu \subseteq A$.

Theorem 2.4. *[20] For a GTS (X, μ), α_μ, s_μ, p_μ and β_μ are GTS and*

1. $\mu \subseteq \alpha_\mu \subseteq s_\mu \subseteq \beta_\mu$;
2. $\alpha_\mu \subseteq p_\mu \subseteq \beta_\mu$.

Definition 2.5. *[3] A function $f : (X, \mu) \to (Y, \nu)$ is said to be (μ, ν)-continuous if the inverse image under f of each ν-open set is μ-open.*

It may be remarked that if f is (μ, ν)-continuous, then $f(X - M_\mu) \subseteq Y - M_\nu$. If in addition to (μ, ν)-continuity of f, $f(M_\mu) \subseteq M_\nu$, then it follows that $f^{-1}(Y - M_\nu) = X - M_\mu$ and $f^{-1}(M_\nu) = M_\mu$.

Definition 2.6. *[4] Let (X, μ) be GTS. Two subsets U and V of X are said to be μ-separated relative to X if $c_\mu U \cap V = \emptyset$ and $U \cap c_\mu V = \emptyset$.*

Definition 2.7. *[19] A set $A \subseteq X$ is said to be μ-connected if $A \cap M_\mu = U \cup V$, and U and V are μ-separated relative to X, implies $U = \emptyset$ or $V = \emptyset$. The GTS (X, μ) is said to be μ-connected if it is μ-connected subset of itself.*

Definition 2.8. *A sequence $< a_n >$ in a GTS (X, μ) is said to be converge to $a \in X$ if for every μ-open set U containing a, there exists $m \in \mathbb{N}$ such that $a_n \in U$ for all $n \geq m$.*

Definition 2.9. *A point a in a GTS (X, μ) is said to be limit point of sequence $< a_n >$ if there exists a subsequence of $< a_n >$ converging to a.*

It is observed that the sequential limit always belongs to M_μ.

3. μ-Hyperconnectednes

Definition 3.1. *[9] A GTS (X, μ) is called μ-hyperconnected if every nonempty μ-open set is μ-dense.*

Remark 3.2. *If a GTS (X, μ) is μ-hyperconnected, then it is μ-connected.*

Example 3.3. *Let $X = \{a, b, c\}$ and $\mu = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$. Then GTS (X, μ) is μ-connected but it is not μ-hyperconnected.*

Proposition 3.4. *In a μ-hyperconnected GTS (X, μ), a nonempty set is μ-semi open if and only if it contains a nonempty μ-open set.*

Theorem 3.5. *[9] Let (X, μ) be a GTS. Then the following statements are equivalent:*

1. *(X, μ) is μ-hyperconnected.*
2. *G is μ-dense or μ-nowhere dense for every subset G of X.*
3. *$G \cap H \neq \emptyset$ for every nonempty μ-open subsets G and H of X.*

Theorem 3.6. *Let (X, μ) be a GTS. Then the following statements are equivalent:*

1. *(X, μ) is μ-hyperconnected.*
2. *G is μ-dense for every nonempty μ-preopen subset of X.*
3. *$c_{s_\mu} G = X$ for every nonempty μ-preopen subset of X.*
4. *$c_{p_\mu} G = X$ for every nonempty μ-semi open subset of X.*

Proof. $(i) \Rightarrow (ii)$. The proof follows from [Theorem 10 (i), [9]].

$(ii) \Rightarrow (iii)$. Suppose that there is a nonempty μ-preopen set G such that $c_{s_\mu} G \neq X$. Then there exists a nonempty μ-semi open set A such that $G \cap A = \emptyset$. Then $G \cap i_\mu A = \emptyset$ and so $c_\mu G \cap i_\mu A = \emptyset$. Therefore, $X \cap i_\mu A = i_\mu A = \emptyset$ by (ii). So $c_\mu \emptyset = c_\mu i_\mu A$. Hence, $(X - M_\mu) \cap M_\mu = (c_\mu i_\mu A) \cap M_\mu$. Thus, $\emptyset = (c_\mu i_\mu A) \cap M_\mu$, a contradiction.

$(iii) \Rightarrow (iv)$. Suppose that there is a nonempty μ-semi open set G such that $c_{p_\mu} G \neq X$. Then there exists a nonempty μ-preopen set A such that $G \cap A = \emptyset$. Then $G \cap i_\mu A = \emptyset$ and so $\emptyset = c_\mu G \cap i_\mu A \supseteq c_{s_\mu} G \cap i_\mu A$.

$(iv) \Rightarrow (i)$. The proof follows from [Theorem 10 (iv), [9]]. □

Proposition 3.7. *[23] For a subset $A \subseteq X$, $c_{s_\mu} A = A \cup i_\mu c_\mu A \cup (X - M_\mu)$.*

Proposition 3.8. *If A is a β_μ-open subset of a GTS (X, μ) then $c_\mu A = c_\mu i_\mu c_\mu A$.*

Proof. Since A is β_μ-open, $A \subseteq c_\mu i_\mu c_\mu A \cap M_\mu \subseteq c_\mu i_\mu c_\mu A$. Therefore, $c_\mu A \subseteq c_\mu i_\mu c_\mu A$. Thus, $c_\mu i_\mu c_\mu A \subseteq c_\mu A \subseteq c_\mu i_\mu c_\mu A$. □

Theorem 3.9. *Let (X, μ) be a GTS. Then the following statements are equivalent:*

1. *(X, μ) is μ-hyperconnected.*
2. *A is μ-dense for every nonempty β_μ-open set $A \subseteq X$.*
3. *$c_{s_\mu} A = X$ for every nonempty β_μ-open $A \subseteq X$.*

Proof. $(i) \Rightarrow (ii)$. Let A be a nonempty β_μ-open set then $A \subseteq c_\mu i_\mu c_\mu A \cap M_\mu$. From the definition of β_μ-open set, it is clear that $i_\mu c_\mu A \neq \emptyset$. Otherwise, $A \subseteq c_\mu(\emptyset) \cap M_\mu = (X - M_\mu) \cap M_\mu = \emptyset$, which is a contradiction. Therefore by the hypothesis and Proposition 3.8, $X = c_\mu i_\mu c_\mu A = c_\mu A$.

$(ii) \Rightarrow (iii)$. Let A be a nonempty β_μ-open set. By the Proposition 3.7, $c_{s_\mu} A = A \cup i_\mu c_\mu A \cup (X - M_\mu) = A \cup i_\mu X \cup (X - M_\mu) = A \cup M_\mu \cup (X - M_\mu) = X$.

$(iii) \Rightarrow (i)$. Let O be a nonempty μ-open subset of X, then $O \in \beta_\mu$. By the hypothesis, $c_{s_\mu} O = X$. By the Proposition 2.2, it follows that $c_\mu O = X$. □

Corollary 3.10. *Let (X, μ) be a GTS. Then the following statements are equivalent:*

1. *(X, μ) is μ-hyperconnected.*
2. *$A \cap B \neq \emptyset$ for $\emptyset \neq A \in s_\mu$ and $\emptyset \neq B \in \beta_\mu$.*
3. *$A \cap B \neq \emptyset$ for $\emptyset \neq A \in \mu$ and $\emptyset \neq B \in \beta_\mu$.*

Theorem 3.11. *Let (X, μ) be a GTS. Then the following statements are equivalent:*

1. *(X, μ) is μ-hyperconnected.*
2. *A is μ-dense for $\emptyset \neq A \in s_\mu$.*
3. *$c_{s_\mu} A = X$ for $\emptyset \neq A \in s_\mu$.*

Proof. The proof is on the similar lines of Theorem 3.9. □

Definition 3.12. *[22] A GTS (X, μ) is called extremally μ-disconnected if $c_\mu U \cap M_\mu \in \mu$ for every $U \in \mu$.*

Theorem 3.13. *A GTS (X, μ) is extremally μ-disconnected if and only if for every r_μ-closed set A, $A \cap M_\mu$ is μ-open.*

Proof. Let (X, μ) be extremally μ-disconnected GTS. If A is r_μ-closed then $A = c_\mu i_\mu A$. Also $i_\mu A$ is μ-open then by the hypothesis, $c_\mu(i_\mu A) \cap M_\mu \in \mu$. Hence, $A \cap M_\mu \in \mu$. Conversely, let $U \in \mu$. Since, $c_\mu U = c_\mu(i_\mu U)$ is r_μ-closed, then by the hypothesis, $c_\mu U \cap M_\mu$ is μ-open. □

Theorem 3.14. *Let (X, μ) be GTS. Then X is extremally μ-disconnected if and only if $s_\mu \subseteq p_\mu$.*

Proof. Suppose X is extremally μ-disconnected GTS. If $A \in s_\mu$, then $A \subseteq c_\mu i_\mu A \cap M_\mu$. Since X is extremally μ-disconnected then $c_\mu i_\mu A \cap M_\mu \in \mu$. So $A \subseteq c_\mu i_\mu A \cap M_\mu = i_\mu(c_\mu i_\mu A \cap M_\mu) \subseteq i_\mu c_\mu i_\mu A \subseteq i_\mu c_\mu A$. Hence, $s_\mu \subseteq p_\mu$. Conversely, let F be a r_μ-closed subset of X then $F = c_\mu i_\mu F$. Now $F \cap M_\mu = c_\mu i_\mu F \cap M_\mu = c_\mu i_\mu(F \cap M_\mu) \cap M_\mu$. So $F \cap M_\mu \in s_\mu \subseteq p_\mu$. Thus, $(F \cap M_\mu) \subseteq i_\mu c_\mu(F \cap M_\mu) \subseteq i_\mu c_\mu(c_\mu i_\mu F \cap M_\mu) \subseteq i_\mu c_\mu c_\mu i_\mu F = i_\mu c_\mu i_\mu F = i_\mu F = i_\mu(F \cap M_\mu)$. So $F \cap M_\mu \in \mu$. Hence, by Theorem 3.13, X is extremally μ-disconnected. □

Theorem 3.15. *Every μ-hyperconnected GTS is is extremally μ-disconnected.*

Proof. Let (X, μ) be a μ-hyperconnected GTS. Then for every $\emptyset \neq U \in \mu$, $c_\mu U = X$. Therefore $c_\mu U \cap M_\mu = X \cap M_\mu = M_\mu$. Hence, $c_\mu U \cap M_\mu \in \mu$. □

Remark 3.16. *Clearly, from Example 3.3, converse of the Theorem 3.15 is not true. Since in this example, GTS (X, μ) is extremally μ-disconnected but not μ-hyperconnected.*

Theorem 3.17. *In a GTS (X, μ), if $\emptyset \neq A \in s_\mu$, then $i_\mu A \neq \emptyset$.*

Proof. Since A is μ-semi open set then $A \subseteq c_\mu i_\mu A \cap M_\mu$. If $i_\mu A = \emptyset$, $A \subseteq c_\mu(\emptyset) \cap M_\mu = (X - M_\mu) \cap M_\mu = \emptyset$, a contradiction. □

Theorem 3.18. *Let (X, μ) be a μ-hyperconnected GTS. If $\emptyset \neq A \subseteq X$ be such that $i_\mu A \neq \emptyset$, then $A \cap M_\mu \in s_\mu$.*

Proof. Let $i_\mu A = U$. Then by hypothesis, $c_\mu U = X$. Therefore, $U \subseteq A \subseteq X$. Thus, $U \subseteq A \cap M_\mu \subseteq c_\mu U \cap M_\mu$ and hence, $A \cap M_\mu \in s_\mu$. □

Theorem 3.19. *Let (X, μ) be GTS. Then X is μ-hyperconnected if and only if for every $A \neq \emptyset \in s_\mu$ and $B \neq \emptyset \in p_\mu$, $A \cap B \neq \emptyset$.*

Proof. If possible $A \cap B = \emptyset$, then by Theorem 3.6, $c_{s_\mu} B = X$. Since, $X = c_{s_\mu} B \subseteq c_{s_\mu}(X - A) = X - A$ then $A = \emptyset$, a contradiction. Conversely, let $A, B \in \mu$ such that $A \neq \emptyset \neq B$,. Then by Theorem 2.4, taking A as s_μ-open and B as p_μ-open, (X, μ) is μ-hyperconneceted. □

Definition 3.20. *[16] Let X be any nonempty set. Then a sub collection $\mathcal{F}(X)$ of power set $P(X)$ of X is called g-filter if*

1. *$\emptyset \notin \mathcal{F}(X)$.*
2. *if $A \in \mathcal{F}(X)$ and $A \subseteq B$ then $B \in \mathcal{F}(X)$.*

A topological space (X, τ) is hyperconnected iff $SO(X, \tau) \setminus \{\emptyset\}$ is a filter on X [13] where $SO(X, \tau)$ is the collection of all semi open subsets of X. The converse part of this result is not always true in generalized topological spaces which is clear from Example 3.22

Theorem 3.21. *If a strong GTS (X, μ) is μ-hyperconnected then $s_\mu \setminus \emptyset$ is a μ-filter on X.*

Proof. Let (X, μ) be a hyperconnected strong GTS. Let $\emptyset \neq A \in s_\mu$ and $A \subseteq B$. Then there exists $\emptyset \neq O \in \mu$ such that $O \subseteq A$. Therefore, $O \subseteq B$. Hence, by Proposition 3.4 $B \in s_\mu \setminus \emptyset$. □

Example 3.22. *Let $X = \{a, b, c\}$ and $\mu = \{\{\emptyset\}, \{a\}, \{b\}, \{a, b\}, X\}$ be a GT on X. Then, $s_\mu = \{\{\emptyset\}, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}, X\}$. Clearly, $s_\mu \setminus \emptyset$ is a μ-filter on X but GTS (X, μ) is not μ-hyperconnected.*

Definition 3.23. *[23] A mapping $f : (X, \mu) \to (Y, \nu)$ is said to be (μ, ν)-semi-continuous at a point $x \in X$ if for each ν-open set V containing $f(x)$, there exists a μ-semi open set U containing x such that $f(U) \subseteq V$. If f is (μ, ν)-semi-continuous at each point of X then f is called (μ, ν)-semi-continuous on X.*

Remark 3.24. *[23] Note that if $f : (X, \mu) \to (Y, \nu)$ is a mapping and $f(x) \in Y - M_\nu$ then f is trivially (μ, ν)-semi-continuous at x. If $x \in X - M_\mu$ and $f(x) \in M_\nu$, then f is not (μ, ν)-semi-continuous at x since there is no μ-semi open set U containing x. Thus, for f to be (μ, ν)-semi-continuous it is necessary that $f(X - M_\mu) \subseteq Y - M_\nu$.*

Theorem 3.25. *[23] For a mapping $f : (X, \mu) \to (Y, \nu)$, the following statements are equivalent.*

1. *f is (μ, ν)-semi-continuous.*
2. *$f^{-1}(V)$ is μ-semi open for each ν-open set V.*
3. *$f^{-1}(F)$ is μ-semi closed for each ν-closed set F.*
4. *$f(c_{s_\mu} A) \subseteq c_\nu(f(A))$ for any subset A of X.*
5. *$c_{s_\mu}(f^{-1}(B)) \subseteq f^{-1}(c_\nu B)$ for any subset B of Y.*
6. *$f^{-1}(i_\nu(B)) \subseteq i_{s_\mu}(f^{-1}(B))$ for any subset B of Y.*

Definition 3.26. *A function $f : (X, \mu) \to (Y, \nu)$ is called almost feebly (μ, ν)-continuous if for each r_ν-open set U, $f^{-1}(U) \neq \emptyset$ implies $i_\nu(f^{-1}(U)) \neq \emptyset$.*

Theorem 3.27. *Every (μ,ν)-semi-continuous $f:(X,\mu)\to(Y,\nu)$ is almost feebly (μ,ν)-continuous.*

Proof. Proof is similar to Theorem 18 in [9]. □

Theorem 3.28. *Let (X,μ) be μ-hyperconnected GTS. If $f:(X,\mu)\to(Y,\nu)$ is almost feebly (μ,ν)-continuous and $f(M_\mu)\subseteq M_\nu$ then (Y,ν) is ν-hyperconnected.*

Proof. If possible suppose (Y,ν) is not ν-hyperconnected then there exist ν-open sets U and V such that $U\cap V=\emptyset$. Let $A=i_\nu c_\nu U$ and $B=i_\nu c_\nu V$. Then A and B are nonempty r_ν-open sets such that $A\cap B=\emptyset$. So $i_\mu(f^{-1}(A))\cap i_\mu(f^{-1}(B))\subseteq f^{-1}(A)\cap f^{-1}(B)=\emptyset$. Since f is a almost feebly (μ,ν)-continuous then $i_{s_\mu}(f^{-1}(A))\neq\emptyset$ and $i_{s_\mu}(f^{-1}(B))\neq\emptyset$. Hence, (X,μ) is not μ-hyperconnected by Corollary 3.10, which is a contradiction. □

Corollary 3.29. *Let (X,μ) be μ-hyperconnected GTS. If $f:(X,\mu)\to(Y,\nu)$ is a (μ,ν)-semi-continuous, then (Y,ν) is ν-hyperconnected.*

4. Example of hyperconnected GTS in which convergent sequence has unique limit

It is well known that in Hausdorff topological spaces every convergent sequence has unique limit and converse follows if the space is first countable. Note that in first countable topological spaces, if every convergent sequence has unique limit then necessarily Hausdorff but in case of GTS, we have an example of hyperconnected GTS which is first countable and every convergent sequence have unique limit in GTS. It is observed that the class of hyperconnected GTS is disjoint from the class of μ-T_2 GTS. So the characterization of μ-T_2 space through sequences fails. It is a natural question to ask in which space uniqueness of limit of convergent sequence implies or characterized μ-T_2.

Definition 4.1. *[1] A GTS (X,μ) is called a μ-first countable GTS if there is a countable μ-local base at every $p\in M_\mu$.*

Definition 4.2. *[14] A GTS (X,μ) is called μ-T_2 if $x,y\in M_\mu$, $x\neq y$ implies the existence of disjoint μ-open sets U_1 and U_2 containing x and y, respectively.*

Example 4.3. *Let $X=\mathbb{R}$ and $\mu=\{\emptyset\}\cup\{(a-1/n,a+n);a\in\mathbb{R},n\in\mathbb{N}\}$. Then X is μ-first countable and every convergent sequece has unique limit point but this space fails to be μ-T_2. So this gives third type of space in which every convergent sequence has unique limit. First type of spaces is known as Hausdorff space.*

References

1. J. B. T. Ayawan and S. R. Canoy jr., Axioms of Countability in Generalized Topologial Spaces, *International Mathematical Forum*, **8**, no. 31, (2013), 1523–1530.
2. Á. Császár, Generalized open sets, *Acta Math. Hungar.,* **75** (1997), 65–87.
3. Á. Császár, Generalized topology, generalized continuity, *Acta Math. Hungar.*, **96** (2002), 351–357.
4. Á. Császár, γ-connected sets, *Acta Math. Hungar.*, **101**(2) (2003), 273–279.
5. Á. Császár, Separation axioms for generalized topologies, *Acta Math. Hungar.*, **104** (2004), 63–69.
6. Á. Császár, Separation properties θ-modifications of topologies, Acta Math. Hungar. **102** (2004), no. 1-2, 151–157.
7. Á. Császár, Generalized open sets generalized topologies, *Acta Math. Hungar.*, **106**(1-2) (2005), 53–66.
8. Á. Császár, Product of generalized topologies, *Acta Math. Hungar.*, **123**(1-2) (2009), 127–132.
9. E. Ekici, Generalized hyperconnectedness, *Acta Math. Hungar.*, **133** (2011), 140–147.
10. S. P. Franklin and M. Rajagopalan, Some examples in topology, *Transactions of the American Mathematical Society*, **155**, (2), (1971), 305–314.
11. N. Levine, Dense topologies, *Amer. Math. Monthly*, **75** (1968), 847–852.
12. S. Majumdar and M. Mitra, Anti- Hausdorff spaces, *Journal of Physical Sciences*, **16** (2012), 117–123.

13. P. M. Mathew, On hyperconnected spaces, *Indian J. Pure Appl. Math.*, **19**(12) (1988), 1180–1184.

14. W. K. Min, Remarks on separation axioms on generalized topological space, Chungcheong mathematical society, **23** (2) (2010), 293–298.

15. J. Novak, Surles expaces et surles produits cartesiens, *Publ. Fac. Sci. Univ. Masaryk*, **273** (1968), 847–852.

16. S. Palaniammal and M. Murugalingam, Generalized Filters, *International Mathematical Forum*, **9**(36) (2014), 1751–1756.

17. V. Renukadevi, Remark on Generalized hyperconnectedness, *Acta Math. Hungar.*, **136** (3) (2012), 157–164.

18. R. X. Shen, A note on generalized connectedness, *Acta Math. Hungar.*, **122** (2009), 231–235.

19. B. K. Tyagi, H. V. S. Chauhan, R. Choudhary, On γ- connected sets, *International Journal of Computer Applications,* **113**(16) (2015), 1–3.

20. B. K. Tyagi and H. V. S. Chauhan, A remark on extremally μ-disconnected generalized topological spaces, Math. Appl. (Brno), **5** (2016), no. 1, 83–90.

21. B. K. Tyagi, H. V. S. Chauhan, On generalized closed sets in generalized topological spaces, *Cubo*, **18** (01) (2016), 27–45.

22. B. K. Tyagi, and Harsh V. S. Chauhan, On some separation axioms in generalized topological spaces, *Questions Answers in Gen. topology* , 36 (2018), 9-29.

23. B. K. Tyagi, and Harsh V. S. Chauhan, On Semi-open sets and Feebly open sets in generalized topological spaces, *Proyecciones Journal of Mathematics*, (2019), (In press).

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