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Fuzzy Totally Semi α-irresolute Mappings

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ABSTRACT: The aim of this article is to introduce two new classes of mappings called fuzzy totally semi α-irresolute mapping and fuzzy totally almost irresolute mapping. Moreover, their characterizations , examples and compositions of these mappings, their relationships between other fuzzy mappings are studied.

Key Words: Fuzzy semiclopen, fuzzy β-clopen, fuzzy totally irresolute, fuzzy totally semi α-irresolute mapping, fuzzy totally almost irresolute.

Contents

1 Introduction and Preliminaries **1**

2 Fuzzy totally semi α-irresolute mapping **2**

3 Compositions of fuzzy totally semi α-irresolute mappings **4**

4 Fuzzy totally semi α-irresolute semiopen mapping **5**

5 Some preservation results **5**

1. Introduction and Preliminaries

Zadeh introduced fuzzy sets in his classical paper in [16]. Using fuzzy sets, Chang [5] first introduced the concept of fuzzy topology in 1968. Consequently, certain types of weak mappings have been generalised by many authors. The remarkable notions in fuzzy topology, namely fuzzy semicontinuity and fuzzy precontinuity respectively introduced by Azad [3] and Bin Shanha [4]. The concept of totally continuous function was introduced in [7] and as a consequence of this totally semicontinuous functions was defined and studied in [10]. The notions of fuzzy totally continuous functions and fuzzy totally semicontinuous functions were introduced by Anjan Mukherjee in [2]. The idea of fuzzy semi α-irresolute mapping was investigated by Srinivasan and Balasubramanian in [11].

In this paper, we introduce the new classes of mappings called fuzzy totally semi α-irresolute and fuzzy totally almost irresolute mappings and since some examples and relationships between these new classes with other classes of fuzzy mappings are obtained. In section 3, the compositions of the thses new mappings are given. In section 4, fuzzy totally semi α-irresolute semiopen mapping and fuzzy totally almost irresolte semiopen mappings are studied with some examples. Finally, in section 5, few properties of these mappings are studied.

We begin by recalling some new and known definitions related with this paper.

Definition 1.1. Let η be a fuzzy subset of a fts $(X, \mathcal{F})$, then
(i)η is called fuzzy semiopen [3] if $\eta \leq \mathrm{Cl}(\mathrm{Int}(\eta))$.
(ii)η is called fuzzy α-open [12] if $\eta \leq$IntClInt(η).
(iii)η is called fuzzy preopen [4] if $\eta \leq$ IntCl(η).
(iv)η is called fuzzy β-open [6] if $\eta \leq$ ClIntCl(η).

The complement of a fuzzy semiopen set(fuzzy α-open, fuzzy preopen, fuzzy β-open resp.) is called fuzzy semiclosed(fuzzy α-closed, fuzzy preclosed, fuzzy β-closed resp.).

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Definition 1.2. Let $f : (X, \mathcal{F}) \to (Y, \mathcal{F}')$ be a function from fts $(X, \mathcal{F})$ to another fts $(Y, \mathcal{F}')$, then
(i)f is called fuzzy semicontinuous [3] if $f^{-1}(\vartheta)$ is a fuzzy semiopen set on X for any fuzzy open set ϑ on Y.
(ii)f is called fuzzy irresolute [9] if $f^{-1}(\vartheta)$ is a fuzzy semiopen set on X for any fuzzy semiopen set ϑ on Y.
(iii) f is called fuzzy semi α-irresolute [11] if $f^{-1}(\vartheta)$ is fuzzy semiopen set on X for any fuzzy α-open set ϑ on Y.
(iv) f is called fuzzy almost irresolute [11] if $f^{-1}(\vartheta)$ is fuzzy β-open set on X for any fuzzy semiopen set ϑ on Y.
(v) f is called fuzzy totally semicontinuous [2] if $f^{-1}(\vartheta)$ is a fuzzy semiclopen set on X for any fuzzy open set ϑ on Y.
(vi) f is called fuzzy totally irresolute [14] if $f^{-1}(\vartheta)$ is a fuzzy semiclopen set on X for any fuzzy semiopen set ϑ on Y.

Definition 1.3. Let $f : (X, \mathcal{F}) \to (Y, \mathcal{F}')$ be a function from fts $(X, \mathcal{F})$ to another fts $(Y, \mathcal{F}')$, then
(i)f is called fuzzy semiopen [3] if $f(\vartheta)$ is a fuzzy semiopen set on Y for any fuzzy open set ϑ on X.
(ii)f is called fuzzy irresolute semiopen [14] if $f(\vartheta)$ is a fuzzy semiopen set on Y for any fuzzy semiopen set ϑ on X.
(iii) f is called fuzzy totally irresolute semiopen [14] if $f(\vartheta)$ is a fuzzy semiclopen set on Y for any fuzzy semiopen set ϑ on X.

Definition 1.4. A fuzzy topological space $(X, \mathcal{F})$ is called
(i)fuzzy semi-compact [8] if every fuzzy semiopen cover has a finite subcover.
(ii)fuzzy α-compact [15] if every fuzzy α-open cover has a finite subcover.
(iii)fuzzy β-compact [6] if every fuzzy β-open cover has a finite subcover.
(iv)fuzzy s-closed [13] if every fuzzy semiclopen cover has a finite subcover.

2. Fuzzy totally semi α-irresolute mapping

In this section, the new classes of mappings called fuzzy totally semi α-irresolute and fuzzy totally almost irresolute are introduced. Also, their characterizations, examples and their relationships with other fuzzy mappings are established.

Definition 2.1. A mapping $f : X \to Y$ is called fuzzy totally almost irresolute if $f^{-1}(\vartheta)$ is fuzzy β-clopen set on X for any fuzzy semiopen set ϑ on Y.

Definition 2.2. A mapping $f : X \to Y$ is called fuzzy totally semi α-irresolute if $f^{-1}(\vartheta)$ is fuzzy semiclopen set on X for any fuzzy α-open set ϑ on Y.

Definition 2.3. A mapping $f : X \to Y$ is called fuzzy totally β-continuous if $f^{-1}(\vartheta)$ is fuzzy β-clopen set on X for any fuzzy open set ϑ on Y.

Remark 2.4. The following reverse implications are false as shown in [11]:
(a) Every fuzzy irresolute is a fuzzy semi α-irresolute but not conversely.
(b) Every fuzzy semi α-irresolute is a fuzzy semicontinuous but not conversely.
(c) Every fuzzy semicontinuous is a fuzzy almost irresolute but not conversely.
(d) Every fuzzy irresolute is a fuzzy almost irresolute but not conversely.

The following implications hold and none of these implications can be reversed in general:

It is clear that every fuzzy totally irresolute mapping is a fuzzy totally semi α-irresolute mapping. Every fuzzy totally semi α-irresolute mapping is a fuzzy semi α-irresolute mapping. Every fuzzy totally semicontinuous mapping is a fuzzy totally almost irrsesolute mapping. Every fuzzy totally almost irresolute mapping is a fuzzy almost irresolute mapping.

Example 2.5. Let $K_1(x)$, $K_2(x)$ and $K_3(x)$ be fuzzy sets on $I = [0, 1]$ defined as follows:

$$K_1(x) = \begin{cases} 0, & 0 \le x \le \frac{1}{2} \\ 2x - 1, & \frac{1}{2} \le x \le 1 \end{cases}$$

$$K_2(x) = \begin{cases} 1, & 0 \le x \le \frac{1}{2} \\ -4x+2, & \frac{1}{2} \le x \le \frac{3}{4} \\ 0, & \frac{3}{4} \le x \le 1 \end{cases}$$

$$K_3(x) = \begin{cases} 0, & 0 \le x \le \frac{1}{4} \\ \frac{1}{3}(4x-1), & \frac{1}{4} \le x \le 1 \end{cases}$$

Let $\mathfrak{I}_1 = \{0, K_1, K_2, K_1 \vee K_2, 1\}$ be a fuzzy topology on I. Let $f : (I, \mathfrak{I}_1) \to (I, \mathfrak{I}_1)$ be a function defined by $f(x) = \frac{x}{2}$ for each $x \in I$. We can see that for fuzzy α-open sets K_1,K_2 and $K_1 \vee K_2$ on $(I, \mathfrak{I}_1)$, $f^{-1}(K_1) = 0$,$f^{-1}(K_2) = K_1^{'} = f^{-1}(K_1 \vee K_2)$. Since $K_1^{'}$ is a fuzzy semiopen set on $(I, \mathfrak{I}_1)$. Therefore f is fuzzy totally semi α-irresolute mapping. But for a fuzzy open set K_3 on $(I, \mathfrak{I}_2)$, $f^{-1}(K_3) = M(x) = K_3 f(\frac{x}{2}) = \begin{cases} 0, & 0 \le x \le \frac{1}{2} \\ \frac{1}{3}(2x-1), & \frac{1}{2} \le x \le 1 \end{cases}$ for each $x \in I$ which is not fuzzy semiclopen set on $(I, \mathfrak{I}_1)$. Hence f is not fuzzy totally irresolute mapping.

Example 2.6. Let K_1 and K_2 and $K_1 \vee K_2$ be fuzzy sets on I as described in 2.1. Consider the fuzzy topologies $\mathfrak{I}_1 = \{0, K_1, K_2, K_1 \vee K_2, 1\}$ and $\mathfrak{I}_2 = \{0, K_3^{'}, 1\}$ on I and a mapping $f : (I, \mathfrak{I}_2) \to (I, \mathfrak{I}_1)$ defined by $f(x) = \frac{x}{2}$ for each $x \in I$. It is observed that, for the fuzzy α-open sets K_1, K_2 and $K_1 \vee K_2$ on $(I, \mathfrak{I}_1)$, $f^{-1}(K_1) = 0$,$f^{-1}(K_2) = K_1^{'} = f^{-1}(K_1 \vee K_2)$ are fuzzy semiopen sets on $(I, \mathfrak{I}_2)$. Therefore, f is fuzzy semi α-irresolute. But for a fuzzy α-open set K_2 on $(I, \mathfrak{I}_2)$, $f^{-1}(K_2) = K_1^{'}$ which is fuzzy semiopen set but not fuzzy semiclosed set on $(I, \mathfrak{I}_2)$. Hence f is not fuzzy totally semi α-irresolute mapping.

Example 2.7. As described in example 2.1, it is observed that fuzzy semi-open sets K_1,K_2 and $K_1 \vee K_2$ on $(I, \mathfrak{I}_1)$, $f^{-1}(K_1) = 0$,$f^{-1}(K_2) = K_1^{'} = f^{-1}(K_1 \vee K_2)$ is fuzzy β-clopen set on $(I, \mathfrak{I}_1)$. Therefore f is fuzzy totally almost irresolute. But for a fuzzy open set K_3, $f^{-1}(K_3) = M(x)$ which is not fuzzy semiclopen set on $(I, \mathfrak{I}_1)$. Hence f is not fuzzy totally semicontinuous mapping.

Example 2.8. Let K_1 and K_2 be fuzzy sets in I as described in example 2.1. Consider the fuzzy topology $\mathfrak{I}_3 = \{0, K_1, K_2, 1\}$ on I and a mapping $f : (I, \mathfrak{I}_3) \to (I, \mathfrak{I}_3)$ defined by $f(x) = x$ for each $x \in I$. It is easily verified that for a fuzzy β-open sets K_2 on $(I, \mathfrak{I}_3)$, $f^{-1}(K_1) = K_1$ and $f^{-1}(K_2) = K_2$ which are fuzzy β-open sets on $(I, \mathfrak{I}_1)$ and hence f is fuzzy irresolute. Since for a fuzzy β-open set K_1 on $(I, \mathfrak{I}_3)$, $f^{-1}(K_1) = K_1$ is fuzzy β-open set but not fuzzy semiclosed set on $(I, \mathfrak{I}_3)$ and f is not fuzzy totally irresolute mapping.

Lemma 2.9. *[3]* Let $f : X \to Y$, be a mapping and $\{\lambda_\alpha\}$ be a family of fuzzy sets in Y. Then (a)$f^{-1}(\bigvee \beta_\alpha) = \bigvee f^{-1}(\beta_\alpha)$ and (b)$f^{-1}(\bigwedge \beta_\alpha) = \bigwedge f^{-1}(\beta_\alpha)$.

Lemma 2.10. *[3]* For mappings $f_i : X_i \to Y_i$ and fuzzy sets β_i in Y, $i = 1, 2$ we have $(f_1 \times f_2)^{-1}(\beta_1 \times \beta_2) = f^{-1}(\beta_1) \times f^{-1}(\lambda_2)$.

Lemma 2.11. *[3]* Let $g : X \to X \times Y$ be the graph of a mapping $f : X \to Y$. If K is a fuzzy set in X and ν is a fuzzy set in Y, then $g^{-1}(K \times \nu) = K \wedge f^{-1}(\nu)$.

Lemma 2.12. *[4]* Let X and Y be fuzzy topological spaces such that X is product related to Y. Then the product $K \times L$ of a fuzzy α-open(preopen) set K in X and fuzzy α-open(preopen) set L in Y is a fuzzy α-open(preopen set) in the fuzzy product space $X \times Y$.

Theorem 2.13. If $g_1 : X_1 \to Y_1$ and $g_2 : X_2 \to Y_2$ are fuzzy totally semi α-irresolute mappings, then $g_1 \times g_2 : X_1 \times X_2 \to Y_1 \times Y_2$ is also fuzzy totally semi α-irresolute mapping.

Proof. Let $\lambda = \bigvee_{i,j}(\delta_i \times \sigma_j)$ be a fuzzy α-open set on $Y_1 \times Y_2$ where δ_i and σ_j are fuzzy α-open sets on Y_1 and Y_2 respectively. Since Y_1 is product related to Y_2 , by lemma 3.4 that $\lambda = \bigvee_{i,j}(\delta_i \times \sigma_j)$ be a fuzzy α-open set on $Y_1 \times Y_2$. Using lemmas 3.1 and 3.2, we obtain $(g_1 \times g_2)^{-1}(\lambda) = (g_1 \times g_2)^{-1}(\bigvee_{i,j}(\delta_i \times \sigma_j)) = \bigvee_{i,j}(g_1^{-1}(\delta_i) \times g_2^{-1}(\sigma_j))$. Since g_1 and g_2 are fuzzy totally semi α-irresolute,we can conclude that $(g_1 \times g_2)^{-1}(\lambda)$ is a fuzzy semiclopen in $X_1 \times X_2$ and hence $(g_1 \times g_2)$ is fuzzy totally semi α-irresolute. □

Corollary 2.14. Let $\mathcal{B}_1$, $\mathcal{B}_2$, and $\mathcal{B}$ be fuzzy topological spaces and $p_i : \mathcal{B}_1 \times \mathcal{B}_2 \to \mathcal{B}_i (i = 1, 2)$ be the projection of $\mathcal{B}_1 \times \mathcal{B}_2$ onto $\mathcal{B}_i$. Then if $f : \mathcal{B} \to \mathcal{B}_1 \times \mathcal{B}_2$ is a fuzzy totally semi α-irrsesolute mapping, then $p_i \circ f$ of f is also a fuzzy totally semi α-irrsesolute mapping.

Proof. Proof is obvious. □

Theorem 2.15. For any two fuzzy totally semi α-irrsesolute mappings f, g from a topological space (X, G_1) into a fuzzy topological space (Y, G_2), the mapping $(f, g) : (X, G_1) \to (Y \times Y, G_1 \times G_2)$ is also fuzzy totally semi α-irrsesolute mapping, where $(f, g)(x) = (f(x), g(x)), \forall x \in X$.

Proof. To prove the mapping is fuzzy totally semi α-irrsesolute mapping, it is enough to show that the inverse image of each fuzzy α-open subset in $Y \times Y$ is fuzzy semiclopen in X. Now if $\sigma \times \nu$ is any fuzzy α-open subset in $Y \times Y$, then

$$\delta_{(f,g)^{-1}(\sigma\times\nu)}(x) = \delta_{(\sigma\times\nu)}(f(x), g(x))$$

$$=\text{inf}[\delta_\sigma f(x), \delta_\nu g(x)]$$

$$=\text{inf}[\delta_{f^{-1}(\sigma)}(x), \delta_{g^{-1}(\nu)}(x)]$$

$$=\delta_{f^{-1}(\sigma)(x)\cap g^{-1}(\nu)}(x)$$

Then

$$(f, g)^{-1}(\sigma \times \nu) = f^{-1}(\sigma) \cap g^{-1}(\nu)$$

By the definition of the fuzzy totally semi α-irrsesolute mapping of f and g, it is clear that $f^{-1}(\sigma)$ and $g^{-1}(\nu)$ are fuzzy semiclopen in X, also $f^{-1}(\sigma) \cap g^{-1}(\nu)$ is again fuzzy semiclopen. Then (f, g) is fuzzy totally semi α-irrsesolute mapping. □

Theorem 2.16. Let $g : X \to Y$ be a mapping and assume that X is product related to Y. If the graph $h : X \to X \times Y$ of g is a fuzzy totally semi α-irresolute, then g is also fuzzy totally semi α-irresolute.

Proof. Let ν be a fuzzy α-open set on Y. Then $g^{-1}(\vartheta) = 1 \wedge g^{-1}(\vartheta) = g^{-1}(1 \times \vartheta)$. Now $(1 \times \vartheta)$ is a fuzzy α-open set in $X \times Y$. Since g is fuzzy totally semi α-irresolute mapping, $h^{-1}(1 \times \vartheta)$ is a fuzzy semiclopen set in X. Hence $g^{-1}(\vartheta)$ is a fuzzy semiclopen set in X. Therefore, g is fuzzy totally semi α-irresolute mapping. □

Theorem 2.17. If a function $f : X \to Y$ is fuzzy totally semi α-irresolute, then $P_i \circ f : X \to Y$ is fuzzy totally semi α-irresolute, where P_i is the projection of $\prod Y_i$ onto Y_i.

Proof. Let λ_i be any fuzzy α-open set in Y_i. Since P_i is a fuzzy continuous and fuzzy open set, it is a fuzzy α-open set. Now $P_i : \prod Y_i \to Y_i$, $P_i^{-1}(\lambda_i)$ is fuzzy α-open in $\prod Y_i$. Therefore, P_i is a fuzzy totally semi α-irresolute function. Now $(P_i \circ f)^{-1}(\lambda) = f^{-1}(P_i(\lambda_i))$, since f is fuzzy totally semi α-irresolute. Hence $f^{-1}(P_i^{-1}(\lambda_i))$ is a fuzzy α-open set, since $P_i^{-1}(\lambda_i)$ is a fuzzy semiclopen set. Hence $P_i \circ f$ is fuzzy totally semi α-irresolute. □

3. Compositions of fuzzy totally semi α-irresolute mappings

In this section the composition of fuzzy totally semi α-irresolute mappings with other fuzzy mappings are studied.

Theorem 3.1. (i) If $f : X \to Y$ is fuzzy totally semi α-irresolute and $g : Y \to Z$ is fuzzy α-continuous, then $g \circ f : X \to Z$ is fuzzy totally semicontinuous.
(ii) If $f : X \to Y$ is fuzzy totally semi α-irresolute and $g : Y \to Z$ is fuzzy α-irresolute, then $g \circ f : X \to Z$ is fuzzy totally semi α-irresolute.

Proof. Obvious and omitted. □

Theorem 3.2. (i) If $f : X \to Y$ is fuzzy totally almost irresolute and $g : Y \to Z$ is fuzzy irresolute, then $g \circ f : X \to Z$ is fuzzy totally almost irresolute.
(ii) If $f : X \to Y$ is fuzzy totally almost irresolute and $g : Y \to Z$ is fuzzy semicontinuous, then $g \circ f : X \to Z$ is fuzzy totally β-continuous.

Proof. Obvious. □

4. Fuzzy totally semi α-irresolute semiopen mapping

In this section, we introduce two new classes of mappings namely, fuzzy totally semi α-irresolute semiopen mapping and fuzzy totally semi semiopen mapping. Also, we characterize a fuzzy totally semi α-irresolute semiopen mapping with other fuzzy mappings.

Definition 4.1. Let $f : (X, \mathcal{F}) \to (Y, \mathcal{F}^{'})$ be a function from fts $(X, \mathcal{F})$ to another fts $(Y, \mathcal{F}^{'})$, then
(i)f is called fuzzy almost irresolute semiopen if $f(\vartheta)$ is a fuzzy β-open set on Y for any fuzzy semiopen set ϑ on X.
(ii)f is called fuzzy semi α-irresolute semiopen if $f(\vartheta)$ is a fuzzy semiopen set on Y for any fuzzy α-open set ϑ on X.
(iii) f is called fuzzy totally almost irresolute semiopen if $f(\vartheta)$ is a fuzzy β-clopen set on Y for any fuzzy semiopen set ϑ on X.
(iv)f is called fuzzy totally semi α-irresolute semiopen if $f(\vartheta)$ is a fuzzy semiclopen set on Y for any fuzzy α-open set ϑ on X.
(v)f is called fuzzy totally semiopen if $f(\vartheta)$ is a fuzzy semiclopen set on Y for any fuzzy semiopen set ϑ on X.

It is clear that every fuzzy totally irresolute semiopen mapping is a fuzzy totally semi α-irresolute semiopen mapping. Every fuzzy totally semi α-irresolute semiopen mapping is a fuzzy semi α-irresolute semiopen mapping. Every fuzzy totally semiopen mapping is a fuzzy totally almost irrsesolute semiopen mapping. Every fuzzy totally almost irresolute semiopen mapping is a fuzzy almost irresolute semiopen mapping.

Example 4.2. Consider the fuzzy sets as described in Example 2.1 and take $\mathfrak{J}_1 = \{0,_1, K_2, K_1 \vee K_2, 1\}$ be a fuzzy topology on I. Let $f : (I, \mathfrak{J}_1) \to (I, \mathfrak{J}_1)$ be a function defined by $f(x) = \frac{x}{2}$ for each $x \in I$. From this we can obtain that for fuzzy α-open sets K_1,K_2 and $K_1 \vee K_2$ on $(I, \mathfrak{J}_1)$, $f(K_1) = 0$,$f(K_2) = K_1^{'} = f(K_1 \vee K_2)$ which are fuzzy semiclopen set on $(I, \mathfrak{J}_1)$ and hence f is fuzzy totally semi α-irresolute semiopen . But for a fuzzy open set K_3 on $(I, \mathfrak{J}_1)$, $f(K_3) = M(x)$ which is not a fuzzy semiclopen set on $(I, \mathfrak{J}_1)$, f is not fuzzy totally semi α-irresolute semiopen.

Example 4.3. As described in Example 2.2, the two fuzzy topologies $\mathfrak{J}_1 = \{0, K_1, K_2, K_1 \vee K_2, 1\}$ and $\mathfrak{J}_2 = \{0, K_3^{'}, 1\}$ on I and a mapping $f : (I, \mathfrak{J}_1) \to (I, \mathfrak{J}_2)$ defined by $f(x) = \frac{x}{2}$ for each $x \in I$. Seeing that for the α-open sets for the fuzzy α-open sets K_1, K_2 and $K_1 \vee K_2$ on $(I, \mathfrak{J}_1)$, $f(K_1) = 0$,$f(K_2) = K_1^{'} = f(K_1 \vee K_2)$ which are fuzzy semiopen sets on $(I, \mathfrak{J}_3)$, f is fuzzy semi α-irresolute semiopen. But $f(K_2) = K_2$, which is fuzzy semiopen but not semoclosed on $(I, \mathfrak{J}_3)$. Therefore f is not fuzzy totally semi α-irresolute semiopen mapping.

Example 4.4. It can be easily verified that, by an example 4.1, we have $f(K_1) = 0$,$f(K_2) = K_1^{'} = f(K_1 \vee K_2)$ which are fuzzy β-clopen set on $(I, \mathfrak{J}_1)$. Therfore, f is fuzzy totally almost irresolute semiopen. But for a fuzzy open set K_3 on $(I, \mathfrak{J}_1)$, $f(K_3) = M(x)$ is fuzzy not β-clopen set on $(I, \mathfrak{J}_1)$. Hence f is not fuzzy totally semiopen mapping.

Example 4.5. From example 2.4, consider the fuzzy mapping $f : (I, \mathfrak{J}_3) \to (I, \mathfrak{J}_3)$ defined by $f(x) = x$ for each $x \in I$. It is evidant that for a fuzzy semiopen sets K_1 and K_2 on $(I, \mathfrak{J}_3)$, $f(K_1) = K_1$ and $f(K_2) = K_2$ which are fuzzy β-open sets on $(I, \mathfrak{J}_3)$ and hence f is fuzzy almost irresolute semiopen. But consider a fuzzy β-open sets K_1 and K_2 on $(I, \mathfrak{J}_3)$, $f(K_1) = K_1$ and $f(K_2) = K_2$ are fuzzy β-open but not β-closed set on $(I, \mathfrak{J}_3)$. Hence f is not fuzzy totally almost irresolute semiopen mapping.

5. Some preservation results

In this section by means of fuzzy totally semi α-irresolute and fuzzy totally almost irresolute mapping preservation of some fuzzy topological structures are discussed. In [1], Abd El-Hakeim defined fuzzy weak nearly compact iff every fuzzy clopen cover of X has a finite subcover. From this inducement the concept of fuzzy β-closed space is defined here. Now we study the following results by using these new spaces.

Definition 5.1. A fuzzy topological space $(X, \mathfrak{F})$ is called fuzzy β-closed if every fuzzy β-clopen cover has a finite subcover.

Theorem 5.2. Every surjective fuzzy totally semi α-irresolute image of a fuzzy s-closed space is fuzzy α-compact.

Proof. Let $f : X \to Y$ be a fuzzy totally semi α-irresolute mapping of a fuzzy s-closed space (X, T_1) onto a fuzzy space (Y, T_2). Let $\{W_a : a \in A\}$ be any fuzzy α-open cover of Y. Since f is fuzzy totally semi α-irresolute, $\left\{f^{-1}(W_a) : a \in A\right\}$ is a fuzzy semiclopen cover of X. Since X is a fuzzy s-closed space, then there exists a finite subfamily $\left\{f^{-1}(W_{a_i}) : i = 1, ..., n\right\}$ of $\left\{f^{-1}(W)\right\}$ which covers X. It implies that $\{W_{a_i} : i = 1, ..., n\}$ is a finite subcover of $\{W_a : a \in A\}$ which covers Y. Hence $f(X) = Y$ is fuzzy α-compact. □

Theorem 5.3. Every surjective fuzzy totally almost irresolute image of a fuzzy β-closed space is fuzzy semi-compact.

Proof. Let $f : X \to Y$ be a fuzzy totally almost irresolute of a fuzzy β-closed space (X, T_1) onto a fuzzy space (Y, T_2). Let $\{V_b : b \in A\}$ be any fuzzy semiopen cover of Y. Since f is fuzzy totally almost irresolute, $\left\{f^{-1}(V) : b \in A\right\}$ is a fuzzy β-clopen cover of X. Since X is a fuzzy β-closed space, then there exists a finite subfamily $\left\{f^{-1}(V_{b_i}) : i = 1, ..., n\right\}$ of $\left\{f^{-1}(V) : b \in A\right\}$ which covers X. It implies that $\{V_{b_i} : i = 1, ..., n\}$ is a finite subcover of $\{V_b : b \in A\}$ which covers Y. Hence $f(X) = Y$ is fuzzy semi-compact. □

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