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Somewhat Precontinuous Mappings via Grill

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ABSTRACT: This article introduces the concepts of somewhat $\mathcal{G}$-precontinuous mapping and somewhat $\mathcal{G}$-preopen mappings. Using these notions, some examples and few interesting properties of those mappings are discussed by means of grill topological spaces.

Key Words: $\mathcal{G}$-continuous mapping, somewhat $\mathcal{G}$-continuous mapping, $\mathcal{G}$-precontinuous mapping, somewhat $\mathcal{G}$-precontinuous mapping, somewhat $\mathcal{G}$-preopen mapping, $\mathcal{G}$-predense set.

Contents

1. Introduction and Preliminaries

Dhananjay Mandal and Mukherjee studied the notion of $\mathcal{G}$-precontinuous mappings in [2]. The study of somewhat continuous functions was first initiated by Karl Gentry et al in [4]. Although somewhat continuous functions are not at all continuous mappings it has been studied and developed considerably by some authors using topological properties. In 1947, Choquet [1] established the notion of a grill which has been milestone of developing topology via grills. Almost all the foremost concepts of general topology have been tried to a certain extent in grill notions by various intellectuals. It is widely known that in many aspects, grills are more effective than a certain similar concepts like nets and filters. Hatir and Jafari introduced the idea of $\mathcal{G}$-continuous functions in [3] and they showed that the concept of open and $\mathcal{G}$-open are independent of each other.

Our aim of this paper is to introduce and study new concepts namely somewhat $\mathcal{G}$-precontinuous mapping and somewhat $\mathcal{G}$-semiopen mapping. Also, their characterizations, interrelations and examples are studied.

Throughout this paper, X stands for a topological space with no separation axioms assumed unless explicitly given. For a subset H of X, the closure of H and the interior of H denoted by $\mathrm{Cl}(H)$ and $\mathrm{Int}(H)$ repectively. The power set of X denoted by $P(X)$. The definitions and results which are used in this paper concerning topological and grill topological spaces have already taken some standard shape. We recall those definitions and basic properties as follows:

Definition 1.1. A mapping $f : (X, \mathfrak{F}) \to (Y, \mathfrak{F}^{'})$ is called somewhat continuous [4] if there exists an open set $\mathcal{A} \neq \phi$ on $(X, \mathfrak{F})$ such that $\mathcal{A} \subseteq f^{-1}(\mathcal{B}) \neq \phi$ for any open set $\mathcal{B} \neq \phi$ on $(Y, \mathfrak{F}^{'})$.

Definition 1.2. A non-empty collection $\mathcal{G}$ of subsets of a topological spaces X is said to be a grill [1] on X if (i) $\phi \notin \mathcal{G}$ (ii) $\mathcal{C} \in \mathcal{G}$ and $\mathcal{C} \subseteq \mathcal{D} \subseteq X \Rightarrow \mathcal{D} \in \mathcal{G}$ and (iii) $\mathcal{C}, \mathcal{D} \subseteq X$ and $\mathcal{C} \cup \mathcal{D} \in \mathcal{G} \Rightarrow \mathcal{C} \in \mathcal{G}$ or $\mathcal{D} \in \mathcal{G}$.

A topological space $(X, \mathfrak{F})$ with a grill $\mathcal{G}$ on X denoted by $(X, \mathfrak{F}, \mathcal{G})$ is called a grill topological space.

Definition 1.3. Let $(X, \mathfrak{F})$ be a topological space and $\mathcal{G}$ be a grill on X. An operator $\Phi : \mathcal{P}(X) \to \mathcal{P}(X)$, denoted by $\Phi_{\mathcal{G}}(\mathcal{C}, \mathfrak{F})$ (for $\mathcal{C} \in \mathcal{P}(X)$) or $\Phi_{\mathcal{G}}(\mathcal{C})$ or simply $\Phi(\mathcal{C})$, called the operator associated with the grill $\mathcal{G}$ and the topology $\mathfrak{F}$ defined by [5]

$$\Phi_{\mathcal{G}}(\mathcal{C}) = \{x \in X : U \cap \mathcal{C} \in \mathcal{G}, \forall U \in \mathfrak{F}(x)\}$$

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Then the operator $\Psi(\mathcal{C}) = \mathcal{C} \cup \Phi(\mathcal{C})$(for $\mathcal{C} \subseteq X$), was also known as Kuratowski's operator [5], defining a unique topology $\mathfrak{F}_{\mathcal{G}}$ such that $\mathfrak{F} \subseteq \mathfrak{F}_{\mathcal{G}}$.

Theorem 1.4. *[2]* Let $(X, \mathfrak{F})$ be a topological space and $\mathcal{G}$ be a grill on X. Then for any $\mathcal{C}, \mathcal{D} \subseteq X$ the following hold:
(a)$\mathcal{C}, \mathcal{D} \Rightarrow \Phi(\mathcal{C}) \subseteq \Phi(\mathcal{D})$.
(b)$\Phi(\mathcal{C} \cup \mathcal{D}) = \Phi(\mathcal{C}) \cup \Phi(\mathcal{D})$.
(c)$\Phi(\Phi(\mathcal{C})) \subseteq \Phi(\mathcal{C}) = Cl(\Phi(\mathcal{C})) \subseteq Cl(\mathcal{C})$.

Definition 1.5. Let $(X, \mathfrak{F}, \mathcal{G})$ be a grill topological space. A subset $\mathcal{C}$ in X is said to be
(i)$\mathcal{G}$-open [6] (or Φ-open [3]) if $\mathcal{C} \subseteq \text{Int}\Phi(\mathcal{C})$.
(ii)$\mathcal{G}$-preopen [3] if $\mathcal{C} \subseteq Int\Psi(\mathcal{C})$.

Definition 1.6. A mapping $f : (X, \mathfrak{F}, \mathcal{G}) \to (Y, \mathfrak{F}')$ is called
(i)$\mathcal{G}$-continuous [3] if $f^{-1}(\mathcal{B})$ is a $\mathcal{G}$-open set on $(X, \mathfrak{F}, \mathcal{G})$ for any open set $\mathcal{B}$ on $(Y, \mathfrak{F}')$.
(ii)$\mathcal{G}$-precontinuous [3] if $f^{-1}(\mathcal{B})$ is a $\mathcal{G}$-preopen set on $(X, \mathfrak{F}, \mathcal{G})$ for any open set $\mathcal{B}$ on $(Y, \mathfrak{F}')$.

Definition 1.7. A mapping $f : (X, \mathfrak{F}, \mathcal{G}) \to (Y, \mathfrak{F}')$ is called somewhat $\mathcal{G}$-continuous [7] if there exists a $\mathcal{G}$-open set $\mathcal{A} \neq \phi$ on $(X, \mathfrak{F}, \mathcal{G})$ such that $\mathcal{A} \subseteq f^{-1}(\mathcal{B}) \neq \phi$ for any open set $\mathcal{B} \neq \phi$ on $(Y, \mathfrak{F}')$.

Definition 1.8. Let $(X, \mathfrak{F}, \mathcal{G})$ be a grill topological space. A subset $H \subset X$ is called $\mathcal{G}$-dense [3] in X if $\Psi(H) = X$.

2. Somewhat $\mathcal{G}$-precontinuous mappings

In this section, the concept of somewhat $\mathcal{G}$-precontinuous mapping is introduced. The notion of somewhat $\mathcal{G}$-precontinuous mapping are independent of somewhat precontinuous mapping. Also, we characterize a somewhat $\mathcal{G}$-precontinuous mapping.

Definition 2.1. A mapping $f : (X, \mathfrak{F}) \to (Y, \mathfrak{F}')$ is called somewhat precontinuous if there exists preopen set $\mathcal{A} \neq \phi$ on $(X, \mathfrak{F})$ such that $\mathcal{A} \subseteq f^{-1}(\mathcal{B}) \neq \phi$ for any open set $\mathcal{B} \neq \phi$ on $(Y, \mathfrak{F}')$.

Definition 2.2. A mapping $f : (X, \mathfrak{F}, \mathcal{G}) \to (Y, \mathfrak{F}')$ is called somewhat $\mathcal{G}$-precontinuous if there exists a $\mathcal{G}$-preopen set $\mathcal{A} \neq \phi$ on $(X, \mathfrak{F}, \mathcal{G})$ such that $\mathcal{A} \subseteq f^{-1}(\mathcal{B}) \neq \phi$ for any open set $\mathcal{B} \neq \phi$ on $(Y, \mathfrak{F}')$.

Remark 2.3. (a) (a)The concept of open and $\mathcal{G}$-open are independent of each other. Hence the notions of continuous and $\mathcal{G}$-continuous are indepedent [2].
(b)The notion of $\mathcal{G}$-continuous and $\mathcal{G}$-precontinuous are independent from each other [3].

Remark 2.4. The following reverse implications are false:
(a)Every continuous mapping is a somewhat continuous mapping [4].
(b)Every $\mathcal{G}$-continuous is somewhat $\mathcal{G}$-continuous [7].

It is clear that every precontinuous mapping is a somewhat precontinuous mapping but not conversely. Every $\mathcal{G}$-precontinuous mapping is a somewhat $\mathcal{G}$-precontinuous mapping but the converses are not true in general as the following examples show.

Example 2.5. Let $X = \{x, y, z\}$, $\mathfrak{F} = \{\phi, \{x\}, \{z\}, \{x, z\}, X\}$ and $Y = \{a, b\}$, $\mathfrak{F}' = \{\phi, \{b\}, Y\}$. We define a function $f : (X, \mathfrak{F}) \to (Y, \mathfrak{F}')$ as follows: $f(x) = a$ and $f(y) = f(z) = b$. Then for open set $\{y\}$ on $(Y, \mathfrak{F}')$, we have $\{z\} \subseteq f^{-1}\{b\} = \{y, z\}$; hence $\{z\}$ is a preopen set on $(X, \mathfrak{F})$. Therfore f is somewhat precontinuous function. But for open set $\{b\}$ on $(Y, \mathfrak{F}')$, $f^{-1}\{b\} = \{y, z\}$ which is not a precontinuous on $(X, \mathfrak{F})$.

Example 2.6. Let $X = \{x, y, z, w\}$, $\mathfrak{F} = \{\phi, \{x\}, \{y, w\}, \{x, y, w\}, X\}$ and $\mathcal{G} = \{\{x\}, \{x, y\}, \{x, z\}, \{x, w\}, \{x, y, z\}, \{x$ $Y = \{a, b\}$ and $\mathfrak{F}' = \{\phi, \{a\}, Y\}$. We define a function $f : (X, \mathfrak{F}, \mathcal{G}) \to (Y, \mathfrak{F}')$ as follows: $f(x) = f(z) = a$ and $f(y) = f(w) = b$. Then for open set $\{a\}$ on $(Y, \mathfrak{F}')$, we have $\{x\} \subseteq f^{-1}\{a\} = \{x, z\}$; hence $\{x\}$ is a $\mathcal{G}$-preopen set on $(X, \mathfrak{F}, \mathcal{G})$. Therfore f is somewhat $\mathcal{G}$-precontinuous function. But for open set $\{a\}$ on $(Y, \mathfrak{F}')$, $f^{-1}\{a\} = \{x, z\}$ which is not a $\mathcal{G}$-precontinuous on $(X, \mathfrak{F}, \mathcal{G})$.

Theorem 2.7. If $f : (X, \mathfrak{F}, \mathcal{G}) \to (Y, \mathfrak{F}^{'})$ is somewhat $\mathcal{G}$-precontinuous and $g : (Y, \mathfrak{F}^{'}) \to (Z, \mathcal{J})$ is continuous, then $g \circ f$: $(X, \mathfrak{F}, \mathcal{G}) \to (Z, \mathcal{J})$ is somewhat $\mathcal{G}$-precontinuous.

Proof. Let K be a non-empty open set in Z. Since g is continuous, $g^{-1}(K)$ is open in Y. Now $(g \circ f)^{-1}(K) = f^{-1}(g^{-1}(K)) \neq \phi$. Since $g^{-1}(K)$ is open in Y and f is somewhat $\mathcal{G}$-precontinuous, then there exists a $\mathcal{G}$-preopen set $H \neq \phi$ in X such that $H \subseteq f^{-1}(g^{-1}(K)) = (g \circ f)^{-1}(K)$. Hence $g \circ f$ is somewhat $\mathcal{G}$-precontinuous. □

Definition 2.8. Let $(X, \mathfrak{F}, \mathcal{G})$ be a grill topological space. A subset $H \subset X$ is called $\mathcal{G}$-predense in X if pre-$\Psi(H) = X$.

Theorem 2.9. If $f : (X, \mathfrak{F}, \mathcal{G}) \to (Y, \mathfrak{F}^{'})$ is somewhat $\mathcal{G}$-precontinuous and A is a $\mathcal{G}$-pre dense subset of X and $\mathcal{G}_H$ is the induced grill topology for H, then $f|_H : (X, \mathfrak{F}, \mathcal{G}_H) \to (Y, \mathfrak{F}^{'})$ is somewhat $\mathcal{G}$-precontinuous.

The following example is enough to justify the restriction is somewhat $\mathcal{G}$-precontinuous.

Example 2.10. Let $X = \{x, y, z, w\}$, $\mathfrak{F} = \{\phi, \{x\}, \{y, w\}, \{x, y, w\}, X\}$ and $\mathcal{G} = \{\{x\}, \{x, y\}, \{x, z\}, \{x, w\}, \{x, y, z\}, \{x, y, w\}, \{x, z, w\}, X\}$; Let $Y = \{a, b\}$ and $\mathfrak{F}^{'} = \{\phi, \{b\}, Y\}$; Let $H = \{x, y, w\}$ be a subset of $(X, \mathfrak{F}, \mathcal{G})$ and the induced grill topology for $\mathcal{G}_H$ is $\mathcal{G}_H = \{\{x\}, \{x, y\}, \{x, w\}, H\}$. Then $f : (X, \mathfrak{F}, \mathcal{G}) \to (Y, \mathfrak{F}^{'})$ is defined as follows: $f(x) = f(z) = a$ and $f(y) = f(w) = b$. Hence $f : (X, \mathfrak{F}, \mathcal{G}) \to (Y, \mathfrak{F}^{'})$ is somewhat $\mathcal{G}$-precontinuous function. But there is no non-empty $\mathcal{G}$-preopen set smaller than $f^{-1}\{b\} = \{x, z\}$. Hence $f|_H : (X, \mathfrak{F}, \mathcal{G}_H) \to (Y, \mathfrak{F}^{'})$ is not somewhat $\mathcal{G}$-precontinuous function.

Theorem 2.11. If $f : (X, \mathfrak{F}, \mathcal{G}) \to (Y, \mathfrak{F}^{'})$ be a mapping, then the following are equivalent:
(1) f is somewhat $\mathcal{G}$-precontinuous.
(2) If $\mathcal{B}$ is a closed set of $(Y, \mathfrak{F}^{'})$ such that $f^{-1}(\mathcal{B}) \neq X$, then there exists a $\mathcal{G}$-preclosed set $\mathcal{A} \neq X$ of $(X, \mathfrak{F}, \mathcal{G})$ such that $f^{-1}(\mathcal{B}) \subseteq \mathcal{A}$.
(3) If $\mathcal{A}$ is a $\mathcal{G}$-predense set on $(X, \mathfrak{F}, \mathcal{G})$, then $f(\mathcal{A})$ is a dense set on $(Y, \mathfrak{F}^{'})$.

Proof. (1)⇒(2) :Let $\mathcal{B}$ be a closed set on Y such that $f^{-1}(\mathcal{B}) \neq X$. Then $\mathcal{B}^c$ is an open set in Y and $f^{-1}(\mathcal{B}^c) = (f^{-1}(\mathcal{B}))^c \neq \phi$. Since f is somewhat $\mathcal{G}$-precontinuous, there exists a $\mathcal{G}$-preopen set $\mathcal{A} \neq \phi$ on X such that $\mathcal{A} \subseteq f^{-1}(\mathcal{B}^c)$. Let $\mathcal{A} = \mathcal{B}^c$. Then $\mathcal{A} \subseteq X$ is $\mathcal{G}$-preclosed such that $f^{-1}(\mathcal{B}) = X - f^{-1}(\mathcal{B}^c) \subseteq X - \mathcal{A}^c = \mathcal{A}$.

(2)⇒(3): Let $\mathcal{A}$ be a $\mathcal{G}$-predense set on X and suppose $f(\mathcal{A})$ is not dense on Y . Then there exists a closed set $\mathcal{B}$ on Y such that $f(\mathcal{A}) \subset \mathcal{B} \subset X$. Since $\mathcal{B} \subset X$ and $f^{-1}(\mathcal{B}) \neq X$, there exists a $\mathcal{G}$-preclosed set $\mathcal{W} \neq X$ such that $\mathcal{A} \subseteq f^{-1}(f(\mathcal{A})) \subset f^{-1}(\mathcal{B}) \subset \mathcal{W}$. This contradicts to the assumption that $\mathcal{A}$ is a $\mathcal{G}$-predense set on X. Hence $f(\mathcal{A})$ is a dense set on Y .

(3)⇒(1): Let $\mathcal{B} \neq \phi$ be an open set on Y and $f^{-1}(\mathcal{B}) \neq \phi$. Suppose there exists no $\mathcal{G}$-preopen $\mathcal{A} \neq \phi$ on X such that $\mathcal{A} \subseteq f^{-1}(\mathcal{B})$. Then $(f^{-1}(\mathcal{B}))^c$ is a set on X such that there is no $\mathcal{G}$-preclosed set $\mathcal{W}$ on X with $(f^{-1}(\mathcal{B}))^c \subset \mathcal{W} \subset X$. In fact, if there exists a $\mathcal{G}$-preopen set $\mathcal{W}^c$ such that $\mathcal{W}^c \subseteq f^{-1}(\mathcal{B})$, then it is a contradiction. So $(f^{-1}(\mathcal{B}))^c$ is a $\mathcal{G}$-predense set on X. Then $f((f^{-1}(\mathcal{B}))^c)$ is a dense set on Y . But $f((f^{-1}(\mathcal{B}))^c) = f((f^{-1}(\mathcal{B}))^c)) \neq \mathcal{B}^c \subset X$. This contradicts to the fact that $f((f^{-1}(\mathcal{B}))^c)$ is fuzzy dense on Y . Hence there exists a $\mathcal{G}$-preopen set $\mathcal{A} \neq \phi$ on X such that $\mathcal{A} \subseteq f^{-1}(\mathcal{B})$. Consequently, f is somewhat $\mathcal{G}$-precontinuous. □

Theorem 2.12. Let $(X_1, \mathfrak{F}_1, \mathcal{G})$,$(X_2, \mathfrak{F}_2, \mathcal{G})$,$(Y_1, \mathfrak{F}_1^{'}, \mathcal{G})$ and $(Y_2, \mathfrak{F}_2^{'}, \mathcal{G})$ be grill topological spaces. Let $(X_1, \mathfrak{F}_1, \mathcal{G})$ be product related to $(X_2, \mathfrak{F}_2, \mathcal{G})$ and let $(Y_1, \mathfrak{F}_1^{'}, \mathcal{G})$ be product related to $(Y_2, \mathfrak{F}_2^{'}, \mathcal{G})$. If $f_1 : (X_1, \mathfrak{F}_1, \mathcal{G}) \to (Y_1, \mathfrak{F}_1^{'}, \mathcal{G})$ and $f_2 : (X_2, \mathfrak{F}_2, \mathcal{G}) \to (Y_2, \mathfrak{F}_2^{'}, \mathcal{G})$ are somewhat $\mathcal{G}$-precontinuous, then the product $f_1 \times f_2 : (X_1, \mathfrak{F}_1, \mathcal{G}) \times (X_2, \mathfrak{F}_2, \mathcal{G}) \to (Y_1, \mathfrak{F}_1^{'}, \mathcal{G}) \times (Y_2, \mathfrak{F}_2^{'}, \mathcal{G})$ is also somewhat $\mathcal{G}$-precontinuous mappings.

Proof. Let $G = \bigcup_{i,j}(M_i \times N_j)$ be an open set on $Y_1 \times Y_2$ where $M_i \neq \phi_{Y_1}$ and $N_j \neq \phi_{Y_2}$ are open sets on Y_1 and Y_2 respectively. Then $(f_1 \times f_2)^{-1}(G) = \bigcup_{i,j}(f_1^{-1}(M_i) \times f_2^{-1}(N_j))$. Since f_1 is somewhat $\mathcal{G}$-precontinuous, there exists a $\mathcal{G}$-preopen set $U_i \neq \phi_{X_1}$ such that $U_i \subseteq f_1^{-1}(M_i) \neq \phi_{X_1}$. And, since f_2 is somewhat $\mathcal{G}$-precontinuous, there exists a $\mathcal{G}$-preopen set $V_j \neq \phi_{X_2}$ such that $V_j \subseteq f_2^{-1}(N_j) \neq \phi_{X_2}$. Now $U_i \times V_j \subseteq f_1^{-1}(M_i) \times f_2^{-1}(N_j) = (f_1 \times f_2)^{-1}(M_i \times N_j)$ and $U_i \times V_j \neq \phi_{X_1 \times X_2}$. Hence $\bigcup_{i,j}(M_i \times N_j) \neq \phi_{X_1} \times \phi_{X_2}$ is a $\mathcal{G}$-preopen set on $X_1 \times X_2$ such that $\bigcup_{i,j}(U_i \times V_j) \subseteq \bigcup_{i,j}(f_1^{-1}(M_i) \times f_2^{-1}(N_j)) = (f_1 \times f_2)^{-1}(\bigcup_{i,j}(M_i \times N_j)) = (f_1 \times f_2)^{-1}(G) \neq \phi_{X_1 \times X_2}$. Therefore $f_1 \times f_2$ is somewhat $\mathcal{G}$-precontinuous. □

Theorem 2.13. A mapping $f : (X, \mathfrak{F}, \mathcal{G}) \to (Y, \mathfrak{F}')$ is somewhat $\mathcal{G}$-precontinuous iff the graph function $g : X \to X \times Y$ defined by $g(x) = (x, f(x))$ for each $x \in X$ is a somewhat $\mathcal{G}$-precontinuous.

Proof. Let N be an open set on Y. Then $f^{-1}(N) = X \cap f^{-1}(N) = g^{-1}(X \times N)$. Since g is somewhat $\mathcal{G}$-precontinuous and $X \times N$ is an open set on $X \times Y$, there exists a $\mathcal{G}$-preopen set $M \neq \phi$ on X such that $M \subseteq g^{-1}(X \times N) = f^{-1}(N) \neq \phi$. Therefore, f is somewhat $\mathcal{G}$-precontinuous. □

3. Somewhat $\mathcal{G}$-preopen mappings

In this section, we introduce a somewhat $\mathcal{G}$-preopen mapping which are independent of somewhat open mapping. We discuss some behaviour of somewhat $\mathcal{G}$-preopen mapping.

Definition 3.1. A mapping $f : (X, \mathfrak{F}) \to (Y, \mathfrak{F}', \mathcal{G})$ is called somewhat $\mathcal{G}$-preopen if there exists a $\mathcal{G}$-preopen set $\mathcal{B} \neq \phi$ on $(Y, \mathfrak{F}', \mathcal{G})$ such that $\mathcal{B} \subseteq f(\mathcal{A}) \neq \phi$ for any open set $\mathcal{A} \neq \phi$ on $(X, \mathfrak{F})$.

Remark 3.2. Every open mapping is a somewhat open mapping but not conversely [4].

It is clear that every preopen mapping is a somewhat preopen mapping. Every $\mathcal{G}$-preopen mapping is a somewhat $\mathcal{G}$-preopen mapping but the reverse implication is false as the following examples show.

Example 3.3. Let $X = \{x, y, z\}$, $\mathfrak{F} = \{\phi, \{y, z\}, X\}$ and $\mathfrak{F}' = \{\phi, \{x\}, \{z\}, \{x, z\}, X\}$; Consider $f : (X, \mathfrak{F}) \to (X, \mathfrak{F}')$ be an identity function. Then for open set $\{y, z\}$ on $(X, \mathfrak{F})$, we have $\{z\} \subseteq f\{y, z\} = \{y, z\}$; hence $\{z\}$ is a preopen set on $(X, \mathfrak{F}')$. Therfore f is somewhat preopen function. But for open set $\{y, z\}$ on $(X, \mathfrak{F})$, $f\{y, z\} = \{y, z\}$ which is not preopen on $(X, \mathfrak{F}',)$ and hence f is not preopen mapping.

Example 3.4. Let $X = \{x, y, z, w\}$, $\mathfrak{F}' = \{\phi, \{x\}, \{y, w\}, \{x, y, w\}, X\}$, $\mathfrak{F} = \{\phi, \{x, z\}, X\}$ and $\mathcal{G} = \{\{x\}, \{x, y\}, \{x, z\}, \{x, w\}, \{x, y, z\}, \{x, y, w\}, \{x, z, w\}, X\}$; Consider $f : (X, \mathfrak{F}) \to (X, \mathfrak{F}', \mathcal{G})$ be an identity function. Then for open set $\{x, z\}$ on $(X, \mathfrak{F})$, we have $\{x\} \subseteq f\{x, z\} = \{x, z\}$; hence $\{x\}$ is a $\mathcal{G}$-preopen set on $(X, \mathfrak{F}', \mathcal{G})$. Therfore f is somewhat $\mathcal{G}$-preopen function. But for open set $\{x, z\}$ on $(X, \mathfrak{F})$, $f\{x, z\} = \{x, z\}$ which is not $\mathcal{G}$-preopen on $(X, \mathfrak{F}', \mathcal{G})$ and hence f is not $\mathcal{G}$-preopen mapping.

Theorem 3.5. If $f : (X, \mathfrak{F}) \to (Y, \mathfrak{F}')$ is somewhat open and $g : (Y, \mathfrak{F}') \to (Z, \mathfrak{J}, \mathcal{G})$ is somewhat $\mathcal{G}$-preopen, then $g \circ f$: $(X, \mathfrak{F}) \to (Z, \mathfrak{J}, \mathcal{G})$ is somewhat $\mathcal{G}$-preopen.

Proof. Obvious. □

Theorem 3.6. If $f : (X, \mathfrak{F}) \to (Y, \mathfrak{F}', \mathcal{G})$ be a bijection. Then the following are equivalent.
(1) f is somewhat $\mathcal{G}$-preopen.
(2) If $\mathcal{A}$ is a closed set on X such that $f(\mathcal{A}) \neq Y$, then there exists a $\mathcal{G}$-preclosed set $\mathcal{B} \neq Y$ on Y such that $f(\mathcal{A}) \subset \mathcal{B}$.

Proof. (1)⇒(2): Let $\mathcal{A}$ be a closed set on X such that $f(\mathcal{A}) \neq Y$. Since f is bijective and $\mathcal{A}^c$ is an open set on X, $f(\mathcal{A}^c) = (f(\mathcal{A}))^c \neq \phi$. And, since f is somewhat $\mathcal{G}$-preopen, there exists a $\mathcal{G}$-preopen set $\mathcal{W} \neq \phi$ on Y such that $\mathcal{W} \subset f(\mathcal{A}^c) = (f(\mathcal{A}))^c$. Consequently, $f(\mathcal{A}) \subset \mathcal{W}^c = \mathcal{B} \neq Y$ and $\mathcal{B}$ is a $\mathcal{G}$-preclosed set on Y .

(2)⇒(1): Let $\mathcal{A}$ be an open set on X such that $f(\mathcal{A}) \neq \phi$. Then $\mathcal{A}^c$ is a closed set on X and $f(\mathcal{A}^c) \neq Y$. Hence there exists a $\mathcal{G}$-preclosed set $\mathcal{B} \neq Y$ on Y such that $f(\mathcal{A}^c) \subset \mathcal{B}$. Since f is bijective, $f(\mathcal{A}^c) = (f(\mathcal{A}))^c \subset \mathcal{B}$. Hence $\mathcal{B}^c \subset f(\mathcal{A})$ and $\mathcal{B}^c \neq \phi$ is a $\mathcal{G}$-preopen set on Y . Therefore, f is somewhat $\mathcal{G}$-preopen. □

Theorem 3.7. If $f:(X,\mathfrak{F})\to(Y,\mathfrak{F}',\mathcal{G})$ be a surjection. Then the following are equivalent.
(1) f is somewhat $\mathcal{G}$-preopen.
(2) If $\mathcal{B}$ is a $\mathcal{G}$-predense set on Y, then $f^{-1}(\mathcal{B})$ is a dense set on X.

Proof. (1)⇒(2): Let $\mathcal{B}$ be a $\mathcal{G}$-predense set on Y . Suppose $f^{-1}(\mathcal{B})$ is not dense on X. Then there exists a closed set $\mathcal{A}$ on X such that $f^{-1}(\mathcal{B})\subset\mathcal{A}\subset X$. Since f is somewhat $\mathcal{G}$-preopen and $\mathcal{A}^c$ is an open set on X, there exists a $\mathcal{G}$-preopen set $\mathcal{W}\neq\phi$ on Y such that $\mathcal{W}\subseteq f(\text{Int}\mathcal{A}^c)\subseteq f(\mathcal{A}^c)$. Since f is surjective, $\mathcal{W}\subseteq f(\mathcal{A}^c)\subset f(f^{-1}(\mathcal{B}^c))=\mathcal{B}^c$. Thus there exists a $\mathcal{G}$-preclosed set $\mathcal{W}^c$ on Y such that $\mathcal{B}\subset\mathcal{W}^c\subset Y$. This is a contradiction. Hence $f^{-1}(\mathcal{B})$ is dense on X.

(2)⇒(1): Let $\mathcal{A}$ be an open set on X and $f(\mathcal{A})\neq\phi$. Suppose there exists no $\mathcal{G}$-preopen set $\mathcal{B}\neq\phi$ on Y such that $\mathcal{B}\subseteq f(\mathcal{A})$. Then $(f(\mathcal{A}))^c$ is a set on Y such that there exists no $\mathcal{G}$-preclosed set $\mathcal{W}$ on Y with $(f(\mathcal{A}))^c\subset\mathcal{W}\subset Y$. This means that $(f(\mathcal{A}))^c$ is $\mathcal{G}$-predense on Y . Thus $f^{-1}((f(\mathcal{A}))^c)$ is dense on X. But $f^{-1}((f(\mathcal{A}))^c)=(f^{-1}(f(\mathcal{A})))^c\subseteq\mathcal{A}^c\subset X$. This contradicts to the fact that $f^{-1}((f(\mathcal{B}))^c$ is dense on X. Hence there exists a $\mathcal{G}$-preopen set $\mathcal{B}\neq\phi$ on Y such that $\mathcal{B}\subseteq f(\mathcal{A})$. Therefore, f is somewhat $\mathcal{G}$-preopen. □

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