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A Note on Strong Forms Of Fuzzy Soft Pre-Continuity

Huseyin Cakalli, Ahu Acikgoz and Ferhat Esenbel

abstract: In this paper, we adapt strong $\theta-$pre-continuity into fuzzy soft topology and investigate its properties. Also, the relations with the other types of continuities in fuzzy soft topological spaces are analized. Moreover, we give some new definitions.

Key Words: Fuzzy soft pre-$\theta-$open, fuzzy soft strong $\theta-$pre-continuity, fuzzy soft pre-$\theta-$ closure and pre-$\theta-$interior points, fuzzy soft pre-regular and p-regular spaces, graph of a fuzzy soft function.

Contents

1. Introduction

Sometimes, we can not use traditional classical methods to handle some problems in some parts of real life such as medical sciences, social sciences, economics, engineering etc., because these problems involve various types of uncertainities. To cope with these problems, some new theories were given by scholars. Two of them are the theory of fuzzy sets and the theory of soft sets which were initiated by Zadeh [27] and Molodstov [16] in 1965 and 1999, respectively. These theories have always been used for dealing with these problems and constituted research areas for scientists to make investigations as in [3,4,8,9,11]. But, both of these theories have their inherent difficulties. Because of these difficulties, some new mathematical tools were required. Then, Maji [15] presented the concept of fuzzy soft set in 2001 as a new mathematical tool and investigated its properties such as De Morgan Law, the complement of a fuzzy soft set, fuzzy soft union, fuzzy soft intersection. By using the theory of fuzzy soft sets, the topological structures in geographic information systems (GIS) were analized in [10,11,12]. Also, some results on an application of fuzzy soft sets in decision making problem were presented by Roy and Maji in [23]. Ahmad and Kharal [2] made some additions to these properties and improved them. Tanay and Kandemir [26] investigated topological structures of fuzzy soft sets. Then, Roy and Samanta [24] introduced the definition of fuzzy soft topology over the initial universe in 2011. Scholars adapted many definitions and terms to fuzzy soft topology which had already been used in soft topology and fuzzy topology such as separation axioms, quasi-coincidence, q-neighbourhood, interior point, cluster point etc. Also, a lot of attempts were made to redefine many types of continuity of functions, which had already been used in general topology, in fuzzy soft topology.

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In the present paper, our purpose is to adapt strong $\theta-$pre-continuity, which was introduced in general topology by Noiri [18] in 2001, to fuzzy soft topology. Moreover, we study on relations with other types of continuity and focus on its several properties. Also, some new definitions are given.

2. Preliminaries

In this paper, we give some definitions in fuzzy soft set theory. Throughout this paper, let X be a nonempty set refereed to as the universe, E the set of all parameters for the universe X and $A, B \subseteq E$.

Analogously to different ideas, some definitions are offered by B. Pazar Varol and H. Aygun as follows:

Definition 2.1. *[20] A fuzzy soft set f_A on the universe X is a mapping from the parameter set E to I^X, i.e., $f_A : E \to I^X$, where $f_A(e) = 0_X$ if $e \notin A$, and 0_X is empty fuzzy set on X.*

Definition 2.2. *[20] Let f_A, g_B be fuzzy soft sets over X, where E is a parameter set. Then f_A is called a fuzzy soft subset of g_B if $f_A(e) \leq g_B(e)$, for each $e \in E$, and we write $f_A \sqsubseteq g_B$. Also f_A is called a fuzzy soft superset of g_B if g_B is a fuzzy soft subset of f_A and we write $f_A \sqsupseteq g_B$.*

Definition 2.3. *[20] Let f_A, g_B be fuzzy soft sets over X, where E is a parameter set. Then f_A and g_B are said to be equal, denoted by $f_A = g_B$, $f_A \sqsubseteq g_B$ and $g_B \sqsubseteq f_A$.*

Definition 2.4. *[20] Let f_A, g_B be fuzzy soft sets over X, where E is a parameter set. The union of f_A and g_B, denoted by $f_A \sqcup g_B$, is the fuzzy soft set $h_{A\cup B}$ defined by $h_{A\cup B}(e) = f_A(e) \vee g_B(e)$, $\forall e \in E$. That is, $h_{A\cup B} = f_A \sqcup g_B$.*

Definition 2.5. *[20] Let f_A, g_B be fuzzy soft sets over X, where E is a parameter set. The intersection of f_A and g_B, denoted by $f_A \sqcap g_B$, is the fuzzy soft set $h_{A\cap B}$ defined by $h_{A\cap B}(e) = f_A(e) \wedge g_B(e)$, $\forall e \in E$. That is, $h_{A\cap B} = f_A \sqcap g_B$.*

Definition 2.6. *[20] Let f_A be a fuzzy soft set over X, where E is a parameter set. Then the complement of f_A, denoted by ${f_A}^c$, is the fuzzy soft set defined by ${f_A}^c(e) = 1_X - f_A(e)$, $\forall e \in E$.*

Definition 2.7. *[20]Let f_E be a fuzzy soft set over X, where E is a parameter set. The fuzzy soft set f_E is called the null fuzzy soft set, denoted by $\tilde{0}_E$, if $f_E(e) = 0_X$, $\forall e \in E$.*

Definition 2.8. *[20] Let f_E be a fuzzy soft set over X, where E is a parameter set. The fuzzy soft set f_E is called the universal fuzzy soft set, denoted by $\tilde{1}_E$, if $f_E(e) = 1_X$, $\forall e \in E$.*

The concepts of fuzzy soft topological space and fuzzy soft topological subspace are offered by B. Tanay and M. B. Kandemir as below:

Definition 2.9. *[26] A fuzzy soft topological space is a pair (X, τ), where X is a nonempty set and τ a family of fuzzy soft sets over X satisfying the following properties:*
(1) $\tilde{0}_E$ $\tilde{1}_E \in \tau$,
(2) If f_A, $g_B \in \tau$, the $f_A \sqcap g_B \in \tau$,
(3) If $(f_A)_i \in \tau$, $\forall i \in J$, $\bigsqcup_{i\in J}(f_A)_i \in \tau$.
τ is called a topology of fuzzy soft sets on X. Every member of τ is called fuzzy soft open. g_B is called fuzzy soft closed in (X, τ) if the complement of g_B is a member of τ.

Definition 2.10. *[26] Let (X, τ) be a fuzzy soft topological space and f_A be a fuzzy soft set in this space. Then the collection $\tau_{f_A} = \{f_A \sqcap g_B : g_B \in \tau\ \}$ is called fuzzy soft subspace topology and (f_A, τ_{f_A}, E) is called a fuzzy soft subspace of (X, τ).*

In this paper, fuzzy soft topological spaces (X, τ_1) and (Y, τ_2) whose parameter sets are E and K respectively, are denoted by X_E and Y_K. Let (f, A) be a fuzzy soft set in X_E. For our convenience, we will use the notation f_A instead of (f, A). The fuzzy soft closure [16] of f_A, denoted by $Fcl(f_A)$, is the intersection of all fuzzy closed soft super sets of f_A, i.e.

$$Fcl\,(f_A) = \sqcap\{h_D : h_D \text{ is fuzzy closed soft set and } f_A \sqsubseteq h_D\}.$$

The fuzzy soft interior [20] of g_B, denoted by $Fint\,(g_B)$, is the union of all fuzzy open soft subsets of g_B, i.e.

$$Fint\,(g_B) = \sqcup\{h_D : h_D \text{ is fuzzy open soft set and } h_D \sqsubseteq g_B\}.$$

A fuzzy soft set f_A is said to be fuzzy soft pre-open [1] (resp. fuzzy soft semiopen [13]) if $f_A \sqsubseteq Fint\,(Fcl\,(f_A))$ (resp. $f_A \sqsubseteq Fcl(Fint(f_A))$). The complement of a fuzzy soft pre-open set is called fuzzy soft pre-closed [1]. The fuzzy soft pre-closure [1] of f_A, denoted by $Fpcl\,(f_A)$, is the intersection of all fuzzy pre-closed soft supersets of f_A, i.e.

$$Fpcl(f_A) = \sqcap\{h_D : h_D \text{ is a fuzzy pre-closed soft set and } f_A \sqsubseteq h_D\}.$$

The fuzzy soft pre-interior [1] of g_B, denoted by $Fpint\,(g_B)$, is the union of all fuzzy open soft subsets of g_B, i.e.

$$Fpint\,(g_B) = \sqcup\{h_D : h_D \text{ is a fuzzy pre-open soft set and } h_D \sqsubseteq g_B\}.$$

The fuzzy soft set f_A in X_E is called fuzzy soft point [5] if $A = \{e\} \subseteq E$ and $f_A\,(e)$ is a fuzzy point in X i.e. there exists $x \in X$ such that $f_A\,(e)\,(x) = \alpha$ $(0 < \alpha \leq 1)$ and $f_A(e)(y) = 0$ for all $y \in X - \{x\}$. This fuzzy soft point will be denoted by e_x^α.

Let f_A be a fuzzy soft set and e_x^α be a fuzzy soft point in X_E. We say $e_x^\alpha \,\widetilde{\in}\, f_A$ read as e_x^α belongs to f_A if $\alpha \leq f_A(e)(x)$. Let f_A and g_B be fuzzy soft sets in X_E. f_A is said to be soft quasi-coincident [5] with g_B, denoted by $f_A q g_B$, if there exist $e \in X$ and $x \in X$ such that $f_A(e)(x) + g_B(e)(x) > 1$. If f_A is not quasi-coincident with g_B, then we write $f_A \overline{q} g_B$. A fuzzy soft point e_x^α of X_E is called a fuzzy soft $\theta-$cluster point [17] of f_A if $Fcl(g_B) q f_A$ for every fuzzy soft open set g_B containing e_x^α.The union of all fuzzy soft $\theta-$cluster points of f_A is called fuzzy soft $\theta-$closure [17] of f_A and denoted by $Fcl_\theta(f_A)$. A fuzzy soft set f_A is said to be fuzzy soft $\theta-$closed [17] if $f_A = Fcl_\theta(f_A)$. The complement of a fuzzy soft $\theta-$closed set is said to be fuzzy soft $\theta-$open [17]. Let $\varphi : X \to Y$ and $\psi : E \to K$ be two functions. Then, the pair (φ, ψ) is called a fuzzy soft mapping [6,14] from X_E to Y_K and denoted by $(\varphi, \psi) : X_E \to Y_K$. The image of each $f_A \in X_E$ under the fuzzy soft function (φ, ψ) will be denoted by $(\varphi, \psi)(f_A) = \varphi(f)_{\psi(A)}$ and the membership function of $\varphi(f)(\beta)$, for each β of $\psi(A)$, is defined as

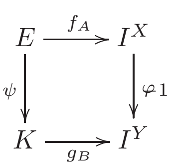
$$\varphi\,(f)\,(\beta) = \begin{cases} \bigvee\limits_{x \in \Psi^{-1}(y)} \left(\bigvee\limits_{x \in \Psi^{-1}(\beta) \cap A} f_\alpha(x) \right), & \Psi^{-1}(y) \neq \emptyset\ ,\ \Psi^{-1}(\beta) \cap A \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

for every $y \in Y$.

The inverse image of each $g_B \in Y_K$ will be denoted by $(\varphi, \psi)^{-1}(g_B) = (\varphi^{-1}(g), \psi^{-1}(B))$ and the membership function of $\varphi^{-1}(g)(\alpha)$, for each $\alpha \in \psi^{-1}(B)$, is defined as

$$\psi^{-1}(g)_\alpha(x) = \begin{cases} g_{\psi(\alpha)}(\varphi(x)), & \psi(\alpha) \in B \\ 0\ , & \text{otherwise} \end{cases}$$

for every $x \in X$.



In the diagram, f_A and g_B are fuzzy soft sets in X_E and Y_K, respectively. $\overrightarrow{\varphi} : I^X \to I^Y$ is the forward powerset operator [18], that is $\overrightarrow{\varphi}(h) : \varphi(h)$ for all $h \in I^X$. If φ and ψ are injective, then the fuzzy soft mapping (φ, ψ) is said to be injective. If φ and ψ are surjective, then the fuzzy soft mapping (φ, ψ) is said to be surjective. If φ and ψ are constant, then the fuzzy soft mapping (φ, ψ) is said to be constant. Also, (φ, ψ) is said to be fuzzy soft continuous [5] if $(\varphi, \psi)^{-1}$ (g_B) is fuzzy soft open in X_E for any fuzzy soft open set $g_B \in Y_K$.

3. Characterizations of Fuzzy Soft Pre-Continous Functions

In this section, we discuss some properties of various kinds of functions. Implications between these functions are indicated with a diagram. Also, it is shown with counterexamples that converse statements are not always true.

Definition 3.1. *A fuzzy soft point e_x^α of X_E is called a fuzzy soft pre-$\theta-$cluster point of f_A if $Fpcl(g_B)qf_A$ for every fuzzy soft pre-open set g_B containing e_x^α. The union of all fuzzy soft pre-$\theta-$cluster points is of f_A is called fuzzy soft pre-$\theta-$closure of f_A and denoted by $Fpcl_\theta(f_A)$. A fuzzy soft set f_A is said to be fuzzy soft pre-$\theta-$closed if $f_A = Fpcl_\theta(f_A)$. The complement of a fuzzy soft pre-$\theta-$closed set is said to be fuzzy soft pre-$\theta-$open.*

In [21], fuzzy soft pre-continuity is defined by A.Ponselvakumari and R.Selvi. We define fuzzy soft pre-continuity in a different way as given below.

Definition 3.2. *A fuzzy soft function $(\varphi, \psi) : X_E \to Y_K$ is said to be fuzzy soft pre-continuous or fuzzy soft almost continuous (resp. fuzzy soft weakly pre-continuous or fuzzy soft almost weakly continuous) if for each e_x^α of X_E and each fuzzy soft open set g_B of Y_K containing $(\varphi,\psi)(e_x^\alpha)$, there exists a fuzzy soft pre-open set f_A containing e_x^α such that $(\varphi,\psi)(f_A) \sqsubseteq g_B$ (resp. $(\varphi,\psi)(f_A) \sqsubseteq Fcl(g_B)$).*

Definition 3.3. *A fuzzy soft function $(\varphi, \psi) : X_E \to Y_K$ is said to be fuzzy soft strongly $\theta-$ continuous if for each e_x^α of X_E and each fuzzy soft open set g_B of Y_K containing $(\varphi,\psi)(e_x^\alpha)$, there exists a fuzzy soft pre-open set f_A containing e_x^α such that $(\varphi,\psi)(Fcl(fA)) \sqsubseteq g_B$.*

Definition 3.4. *A fuzzy soft function $(\varphi, \psi) : X_E \to Y_K$ is said to be fuzzy soft strongly θ- pre-continuous (briefly f.s.st.θ.p.c.) if for each e_x^α of X_E and each fuzzy soft open set g_B of Y_K containing $(\varphi,\psi)(e_x^\alpha)$, there exists a fuzzy soft pre-open set f_A containing e_x^α such that $(\varphi,\psi)(Fpcl(fA)) \sqsubseteq g_B$.*

Remark 3.5. *For a fuzzy soft function function $(\varphi, \psi) : X_E \to Y_K$ the following implications hold:*

$$Fuzzy\ Soft\ Strong\ \theta-\text{ continuity}$$
$$\downarrow$$
$$Fuzzy\ Soft\ Strong\ \theta-\text{ pre-continuity}$$
$$\downarrow$$
$$Fuzzy\ Soft\ Pre-continuity$$

Converse statements are not always true as shown in the examples below.

Example 3.6. *Let $X = \{x, y\}, Y = \{m, n\}, E = \{a, b\}$ and $K = \{d, e\}$. Consider fuzzy soft sets*

$$f_E^1 = \{(a, \{(x, 0.3), (y, 0.3)\}), (b, \{(x, 0.3), (y, 0.3)\})\},$$
$$f_E^2 = \{(a, \{(x, 0.1), (y, 0)\})\},$$
$$g_K = \{(d, \{(m, 0.3), (n, 0.3)\}), (e, \{(m, 0.3), (n, 0.3)\})\}.$$

Then (X, τ_1, E) and (Y, τ_2, K) are fuzzy soft topological spaces, where $\tau_1 = \{\widetilde{0}_E, \widetilde{1}_E, f_E^1, f_E^2\}$ and $\tau_2 = \{\widetilde{0}_K, \widetilde{1}_K, g_K\}$. Let φ and ψ be functions defined as $\varphi = \{(x, m), (y, n)\}$, $\psi = \{(a, d), (b, e)\}$. Then $(\varphi, \psi) : X_E \to Y_K$ is fuzzy soft pre-continuous, but not fuzzy soft strongly $\theta-$pre-continuous.

Example 3.7. *Let $X = \{x, y\}, Y = \{m, n\}, E = \{a, b\}$ and $K = \{d, e\}$. Consider fuzzy soft sets*

$$f_E = \{(a, \{(x, 0.3), (y, 0.3)\}), (b, \{(x, 0.3), (y, 0.3)\})\},$$
$$g_K = \{(d, \{(m, 0.3), (n, 0.3)\}), (e, \{(m, 0.3), (n, 0.3)\})\}.$$

Then (X, τ_1, E) and (Y, τ_2, K) are fuzzy soft topological spaces, where $\tau_1 = \{\widetilde{0}_E, \widetilde{1}_E, f_E\}$ and $\tau_2 = \{\widetilde{0}_K, \widetilde{1}_K, g_K\}$. Let φ and ψ be functions defined as $\varphi = \{(x, m), (y, n)\}, \psi = \{(a, d), (b, e)\}$. Then, $(\varphi, \psi) : X_E \to Y_K$ is fuzzy soft strongly $\theta-$pre-continuous, but not fuzzy soft strongly $\theta-$continuous.

Remark 3.8. *Fuzzy soft strong $\theta-$pre-continuity and fuzzy soft continuity are independent of each other. In Example 16, the given fuzzy soft function is fuzzy soft continuous, but not fuzzy soft strongly $\theta-$pre-continuous. In the following example, it is seen that the given fuzzy soft function is fuzzy soft strongly $\theta-$pre-continuous, but not fuzzy soft continuous.*

Example 3.9. *Let $X = \{x, y\}$, $Y = \{m, n\}$, $E = \{a, b\}$ and $K = \{d, e\}$. Consider fuzzy soft sets*

$$f_E = \{(a, \{(x, 0.3), (y, 0.3)\}), (b, \{(x, 0.3), (y, 0.3)\})\},$$

$$g^1_K = \{(d, \{(m, 0.3), (n, 0.3)\}), (e, \{(m, 0.3), (n, 0.3)\})\},$$

$$g^2_K = \{(d, \{(m, 0.2), (n, 0.2)\}), (e, \{(m, 0.2), (n, 0.2)\})\}.$$

Then (X, τ_1, E) and (Y, τ_2, K) are fuzzy soft topological spaces, where τ_1=$\{\widetilde{0}_E, \widetilde{1}_E, f_E\}$ and τ_2=$\{\widetilde{0}_K, \widetilde{1}_K, g^1_K, g^2_K\}$. Let φ and ψ be functions defined as $\varphi = \{(x, m), (y, n)\}$, $\psi = \{(a, d), (b, e)\}$. Then $(\varphi, \psi) : X_E \to Y_K$ is fuzzy soft strongly θ-pre-continuous, but not fuzzy soft continuous.

Theorem 3.10. *Let X_E and Y_K be fuzzy soft topological spaces. Then the following properties are equivalent for a function $(\varphi, \psi) : X_E \to Y_K$;*

(1) (φ, ψ) is f.s.st.θ.p.c.;
(2) $(\varphi, \psi)^{-1}(g_B)$ is fuzzy soft pre-$\theta-$open in X_E for every fuzzy soft open set g_B of Y_K;
(3) $(\varphi, \psi)^{-1}(g_B)$ is fuzzy soft pre-$\theta-$closed in X_E for every fuzzy soft closed set g_B of Y_K;
(4) $(\varphi, \psi)(Fpcl_\theta(f_A)) \sqsubseteq Fcl((\varphi, \psi)(f_A))$ for every fuzzy soft subset f_A of X_E;
(5) $Fpcl_\theta$ $((\varphi, \psi)^{-1}(g_B))\sqsubseteq (\varphi, \psi)^{-1}(Fcl(g_B))$ for every fuzzy soft subset g_B of Y_K .

Proof. (1) $\Longrightarrow$ (2). Let g_B be a fuzzy soft open set in Y_K. Suppose that $e^\alpha_x \widetilde{\in} (\varphi, \psi)^{-1}(g_B)$, where $e \in K$ and x_α is a fuzzy point of X. There exists a fuzzy soft pre-open set f_A such that $e^\alpha_x \widetilde{\in}\ f_A$ and $(\varphi, \psi)(Fpcl(f_A)) \sqsubseteq g_B$. Therefore we have $e^\alpha_x \widetilde{\in}\ f_A \sqsubseteq Fpcl(f_A) \sqsubseteq (\varphi, \psi)^{-1}(g_B)$. This shows that $(\varphi, \psi)^{-1}(g_B)$ is fuzzy soft pre-open in X_E.

(2) $\Longrightarrow$ (3). This is obvious.

(3) $\Longrightarrow$ (4). Let f_A be any fuzzy soft subset in X_E. Since $Fcl((\varphi, \psi)(f_A))$ is fuzzy soft closed in Y_K, $(\varphi, \psi)^{-1}$ $(Fcl((\varphi, \psi)(f_A)))$ is fuzzy soft pre-θ-closed by (3). Then we have

$$Fpcl_\theta(f_A) \sqsubseteq Fpcl_\theta((\varphi, \psi)^{-1}\ ((\varphi, \psi)(f_A)))$$

$$\sqsubseteq Fpcl_\theta((\varphi, \psi)^{-1}\ (Fcl((\varphi, \psi)\ (f_A)))) = (\varphi, \psi)^{-1}(Fcl((\varphi, \psi)\ (f_A))).$$

Therefore, we obtain $(\varphi, \psi)(Fpcl_\theta(f_A)) \sqsubseteq Fcl((\varphi, \psi)(f_A))$.

(4) $\Longrightarrow$ (5). Let g_B be a fuzzy soft subset in Y_K. By (4), we obtain

$$(\varphi, \psi)(Fpcl_\theta((\varphi, \psi)^{-1}(g_B))) \sqsubseteq Fcl((\varphi, \psi)((\varphi, \psi)^{-1}(g_B))) \sqsubseteq Fcl(g_B)$$

and hence

$$Fpcl_\theta((\varphi, \psi)^{-1}(g_B)) \sqsubseteq (\varphi, \psi)^{-1}(Fcl(g_B)).$$

(5) $\Longrightarrow$ (1). Let e^α_x be a fuzzy soft point in X_E, where $e \in E$ and x_α is a fuzzy point of X and g_B be any fuzzy soft open neighbourhood of $(\varphi, \psi)(e^\alpha_x)$. Since g^C_B is fuzzy soft closed in Y_K, we have $Fpcl_\theta((\varphi, \psi)^{-1}(g^C_B)) \sqsubseteq (\varphi, \psi)^{-1}(Fcl(g^C_B)) = (\varphi, \psi)^{-1}(g^C_B)$. Therefore, $(\varphi, \psi)^{-1}(g^C_B)$ is fuzzy soft pre-θ-closed in X_E and $(\varphi, \psi)^{-1}(g_B)$ is a fuzzy soft pre-θ-open set containing e^α_x. There exists a fuzzy soft pre-open set f_A containing e^α_x such that $Fpcl(f_A) \sqsubseteq (\varphi, \psi)^{-1}(g_B)$; hence $(\varphi, \psi)(Fpcl(f_A)) \sqsubseteq g_B$. This shows that f is f.s.st. θ.p.c. □

Definition 3.11. *[25] A fuzzy soft topological space X_E is said to be fuzzy soft regular if for each pair of a fuzzy soft point e^α_x and a fuzzy soft closed set g_C in X_E such that $e^\alpha_x \widetilde{\notin}\ g^C_C$, there exist fuzzy soft open sets f_A and f_B such that $e^\alpha_x \widetilde{\in}\ f_A$, $g_C \sqsubseteq f_B$ and $f_A \overline{q}\ f_B$.*

Theorem 3.12. *Let Y_K be a fuzzy soft regular topological space. Then the following properties are equivalent for a function $(\varphi, \psi) : X_E \to Y_K$;*
(1) (φ, ψ) is fuzzy soft weakly pre-continuous;
(2) (φ, ψ) is fuzzy soft pre-continuous;
(3) (φ, ψ) is f.s.st.θ.p.c.

Proof. (1)$\Longrightarrow$(2) Let e_x^α be a fuzzy soft point in X_E, where $e \in E$ and x_α is a fuzzy point of X and let g_B be a fuzzy soft open set in Y_K containing $(\varphi, \psi)(e_x^\alpha)$. Since Y_K is a fuzzy soft regular topological space, there exists a fuzzy soft open set g_C such that
$(\varphi, \psi)(e_x^\alpha) \widetilde{\in} g_C \sqsubseteq Fcl(g_C) \sqsubseteq g_B$. Since (φ, ψ) is fuzzy soft weakly pre-continuous, there exists a fuzzy soft pre-open set f_A containing e_x^α such that $(\varphi, \psi)(f_A) \sqsubseteq Fcl(g_C)$. Therefore, we have $(\varphi, \psi)(f_A) \sqsubseteq g_B$.

(2)$\Longrightarrow$(3) Let e_x^α be a fuzzy soft point in X_E, where $e \in E$. Let x_α be a fuzzy point of X and g_B be a fuzzy soft open set in Y_K containing $(\varphi, \psi)(e_x^\alpha)$. Since Y_K is a fuzzy soft regular topological space, there exists a fuzzy soft open set g_C such that $(\varphi, \psi)(e_x^\alpha) \widetilde{\in} g_C \sqsubseteq Fcl(g_C) \sqsubseteq g_B$. Since (φ, ψ) is fuzzy soft pre-continuous, there exists a fuzzy soft pre-open set f_A containing e_x^α such that $(\varphi, \psi)(f_A) \sqsubseteq g_C$. We shall show that $(\varphi, \psi)(Fpcl(f_A)) \sqsubseteq Fcl(g_C)$. Let k_y^β be a fuzzy soft point where $k \in K$ and y_β is a fuzzy point of Y such that $k_y^\beta \widetilde{\in} (\varphi, \psi)(Fpcl(f_A))$. Then $(\varphi, \psi)^{-1}(k_y^\beta) \widetilde{\in} Fpcl(f_A)$. So, any fuzzy soft pre-open neighbourhood of $(\varphi, \psi)^{-1}(k_y^\beta)$ is soft quasi-coincident with f_A. This means that k_y^β is soft quasi-coincident with $(\varphi, \psi)(f_A)$. So, k_y^β is soft quasi-coincident with g_C and $k_y^\beta \widetilde{\in} Fcl(g_C)$. Consequently, $(\varphi, \psi)(Fpcl(f_A)) \sqsubseteq Fcl(g_C) \sqsubseteq g_B$.

(3)$\Longrightarrow$(1) This is obvious. □

Definition 3.13. *A fuzzy soft topological space X_E is said to be fuzzy soft pre-regular (resp. fuzzy soft p-regular) if for each fuzzy soft pre-closed (resp. fuzzy soft closed) set f_A where $A \subseteq E$ and each fuzzy soft point $e_x^\alpha \widetilde{\in} f_A^c$, there exists fuzzy soft pre-open sets g_B and g_C such that $f_A \sqsubseteq g_B$, $e_x^\alpha \widetilde{\in} g_C$ and $g_B \overline{q} g_C$.*

Theorem 3.14. *A fuzzy soft continuous function $(\varphi, \psi) : X_E \longrightarrow Y_K$ is f.s.st.θ.p.c. if and only if X_E is fuzzy soft p-regular.*

Proof. ($\Longrightarrow$) Let $(\varphi, \psi) : X_E \to X_E$ be the fuzzy soft identity function. So, (φ, ψ) is fuzzy soft continuous. This means that (φ, ψ) is f.s.st.θ.p.c. from our hypothesis. For any fuzzy soft closed set f_A and fuzzy soft point $e_x^\alpha \widetilde{\in} f_A^c$, we have $(\varphi, \psi)(e_x^\alpha) = e_x^\alpha \widetilde{\in} f_A^c$. Then there exists a fuzzy pre-open set g_B containing e_x^α such that $(\varphi, \psi)(Fpcl(g_B)) \sqsubseteq f_A^c$. So, $Fpcl(g_B) \sqsubseteq f_A^c$ and $Fpcl(g_B) \overline{q}\ f_A$. It is easily seen that $f_A \sqsubseteq (Fpcl(g_B))^c$, $(FpCl(g_B))^c$ is a fuzzy soft pre-open set in X_E and $(Fpcl(g_B))^c \overline{q} g_B$. Therefore, X_E is fuzzy soft p-regular.

($\Longleftarrow$) Suppose that $(\varphi, \psi) : X_E \to Y_K$ is fuzzy soft continuous and X_E is fuzzy soft p-regular. For any fuzzy soft point e_x^α in X_E and fuzzy soft open neighbourhood g_C of $(\varphi, \psi)(e_x^\alpha)$, $(\varphi, \psi)^{-1}(g_C)$ is fuzzy soft open in X_E and $e_x^\alpha \widetilde{\in} (\varphi, \psi)^{-1}(g_C)$. So, $e_x^\alpha \widetilde{\in} [((\varphi, \psi)^{-1}(g_C))^c]^c$ and $((\varphi, \psi)^{-1}(g_C))^c$ is fuzzy soft closed in X_E. Since X_E is fuzzy soft p-regular, there exist fuzzy soft pre-open sets f_A and f_B such that $((\varphi, \psi)^{-1}(g_C))^c \sqsubseteq f_A$, $e_x^\alpha \widetilde{\in} f_B$ and $f_A \overline{q} f_B$. This means that $e_x^\alpha \widetilde{\in} f_B \sqsubseteq f_A^c \sqsubseteq (\varphi, \psi)^{-1}(g_C)$. Since f_A^c is fuzzy soft pre-closed, $e_x^\alpha \widetilde{\in} f_B \sqsubseteq Fpcl(f_B) \sqsubseteq (\varphi, \psi)^{-1}(g_C)$. Therefore, (φ, ψ) is f.s.st.θ.p.c. □

Theorem 3.15. *Let X_E be a fuzzy soft pre-regular space. Then (φ, ψ): $X_E \longrightarrow Y_K$ is f.s.st.θ.p.c. if and only if (φ, ψ) is fuzzy soft pre-continuous.*

Proof. ($\Longrightarrow$) Suppose that X_E is a fuzzy soft pre-regular space and (φ, ψ): $X_E \longrightarrow Y_K$ is f.s.st. θ.p.c.Let e_x^α be a fuzzy soft point in X_E and g_B be a fuzzy soft open set in Y_K containing $(\varphi, \psi)(e_x^\alpha)$. Since (φ, ψ) is f.s.st.θ.p.c., there exists a fuzzy soft pre-open set f_A *in* X_E containing e_x^α such that $(\varphi, \psi)(Fpcl(f_A)) \sqsubseteq g_B$. Then $Fpcl(f_A) \sqsubseteq (\varphi, \psi)^{-1}(g_B)$. Clearly,
$e_x^\alpha \widetilde{\in} f_A \sqsubseteq (\varphi, \psi)^{-1}(g_B)$ and (φ, ψ) is f.s.st.θ.p.c.

($\Leftarrow$) Suppose that X_E is a fuzzy soft pre-regular space and (φ, ψ): $X_E \longrightarrow Y_K$ is fuzzy soft pre-continuous. Let e_x^α be a fuzzy soft point in X_E and g_B be a fuzzy soft open set in Y_K containing $(\varphi, \psi)(e_x^\alpha)$. Since (φ, ψ) is fuzzy soft pre-continuous, $(\varphi, \psi)^{-1}(g_B)$ is fuzzy soft pre-open in X_E and contains

e_x^α. So, $((\varphi,\psi)^{-1}(g_B))^c$ is fuzzy soft pre-closed and $e_x^\alpha \widetilde{\in}[((\varphi,\psi)^{-1}(g_B))^c]^c$. Since X_E is a fuzzy soft pre-regular space, there exist fuzzy soft pre-open sets f_A and f_C such that $e_x^\alpha \widetilde{\in} f_A$, $((\varphi,\psi)^{-1}(g_B))^c \sqsubseteq f_C$ and $f_A \overline{q} f_C$. For f_C^c is fuzzy soft pre-closed and $f_A \sqsubseteq f_C^c$, $Fpcl(f_A) \sqsubseteq f_C^c$. This means that $Fpcl(f_A) \sqsubseteq (\varphi,\psi)^{-1}(g_B)$ and $(\varphi,\psi)(Fpcl(f_A)) \sqsubseteq g_B$. Therefore, f is f.s.st.θ.p.c. □

4. Some Properties

In this section, properties of f.s.st.θ.p.c. functions are investigated in fuzzy soft preregular spaces and fuzzy soft p-regular spaces. Also, some other types of functions are introduced and their properties are investigated.

Definition 4.1. *[28] Let X_E and Y_K be two fuzzy soft topological spaces. Let $f_E \in X_E$ and $g_K \in Y_K$. The Cartesian product of f_E and g_K, denoted by $f_E \widetilde{\otimes} g_K$, is a fuzzy soft set over XxY with regards to parameter set ExK defined as below:*

$$\begin{array}{rccl} f_E\widetilde{\otimes}g_K: & E\times K & \to & I^X\times I^Y \\ & (e,k) & \to & f(e)\times g(k) \end{array}$$

such that $f(e)\times g(k)$ is the fuzzy product of fuzzy sets $f(e)$ and $g(k)$ where

$$\begin{array}{rccl} f(e)\times g(k): & X\times Y & \to & [0,1] \\ & (x,y) & \to & \min\{f(e)(x), g(k)(y)\} \end{array}$$

Definition 4.2. *[28] Let X_E and Y_K be two fuzzy soft topological spaces. Then $X_E \widetilde{\otimes} Y_K$={ $f_E\widetilde{\otimes}g_K : f_E \in X_E, g_K \in Y_K$} is called the fuzzy soft catesian product of fuzzy soft topological spaces X_E and Y_K.*

Definition 4.3. *For a fuzzy soft function $(\varphi,\psi): X_E \longrightarrow Y_K$, the subset $\{e_x^\alpha\widetilde{\otimes}(\varphi,\psi)(\ e_x^\alpha)$: $e_x^\alpha \in X_E\}$ of $X_E\widetilde{\otimes}Y_K$ is called the graph of (φ,ψ) and is denoted by $G_{(\varphi,\psi)}$.*

Theorem 4.4. *Let (φ,ψ):$X_E \longrightarrow Y_K$ be a function and $G_{(\varphi,\psi)}$: $X_E \longrightarrow X_E\widetilde{\otimes}Y_K$ be the graph function of (φ,ψ). Then the following properties hold:*
(1) If $G_{(\varphi,\psi)}$ is f.s.st.θ.p.c., then (φ,ψ) is f.s.st.θ.p.c. and X_E is fuzzy soft p-regular.
(2) If (φ,ψ) is f.s.st.θ.p.c. and X_E is fuzzy soft pre-regular, then $G_{(\varphi,\psi)}$ is f.s.st.θ.p.c.

Proof. (1) Suppose that $G_{(\varphi,\psi)}$ is f.s.st.θ.p.c. First, we show that (φ,ψ) is f.s.st.θ.p.c. Let $e_x^\alpha \widetilde{\in} X_E$ and g_K be a fuzzy soft open neighbourhood of $(\varphi,\psi)(\ e_x^\alpha)$. $X_E\widetilde{\otimes}g_K$ is a fuzzy soft open neighbourhood of $G_{(\varphi,\psi)}\ (e_x^\alpha)$. Since $G_{(\varphi,\psi)}$ is f.s.st.θ.p.c., there exists a fuzzy soft pre-open set $f_E \in X_E$ containing e_x^α such that $G_{(\varphi,\psi)}\ (Fpcl(f_E)) \sqsubseteq X_E x g_K$. So, $(\varphi,\psi)(Fpcl(f_E)) \sqsubseteq g_K$. Therefore, (φ,ψ) is f.s.st.θ.p.c. Next, we show that X_E is fuzzy soft p-regular. Let f_E be a fuzzy soft closed set in X_E and $e_x^\alpha \in f_E^c$, where f_E^c is fuzzy soft open in X_E. Since $G_{(\varphi,\psi)}\ (e_x^\alpha\ )\in f_E^c\widetilde{\otimes}Y_K$ and $f_E^c\widetilde{\otimes}Y_K$ is fuzzy soft open in $X_E\widetilde{\otimes}Y_K$, there exists a fuzzy soft pre-open set h_E in X_E containing e_x^α such that $G_{(\varphi,\psi)}\ (Fpcl(h_E)) \sqsubseteq f_E^c\widetilde{\otimes}Y_K$. This means that $e_x^\alpha\ \widetilde{\in} h_E \sqsubseteq Fpcl(h_E) \sqsubseteq f_E^c$. Then, $e_x^\alpha\widetilde{\in}\ h_E$, $f_E \sqsubseteq (Fpcl(h_E))^c$ and $h_E\overline{q}(Fpcl(h_E))^c$, where h_E and $(pCl(h_E))^c$ are fuzzy soft pre-open. This shows that X_E is fuzzy soft p-regular.

(2) Let $e_x^\alpha \widetilde{\in} X_E$ and $f_E\widetilde{\otimes}g_K$ be a fuzzy soft open set of $X_E\widetilde{\otimes}Y_K$ containing $G_{(\varphi,\psi)}\ (e_x^\alpha)$. Since (φ,ψ) $(e_x^\alpha)\ \widetilde{\in}\ g_K$ and (φ,ψ) is f.s.st.θ.p.c., there exists a fuzzy soft open set h_E such that $e_x^\alpha \widetilde{\in} h_E$, $(\varphi,\psi)(Fpcl(h_E)) \sqsubseteq g_K$. Since $f_E \sqcap h_E$ is a fuzzy soft pre-open set in X_E containing e_x^α, $(f_E\sqcap h_E)^c$ is a fuzzy soft pre-closed set and $e_x^\alpha\ \widetilde{\in}\ [(f_E\sqcap h_E)^c]^c$. Since X_E is fuzzy soft pre-regular, there exist fuzzy soft pre-open sets k_E and p_E such that $(f_E\sqcap h_E)^c \sqsubseteq k_E$, $e_x^\alpha\ \widetilde{\in} p_E$ and $k_E\overline{q}p_E$. Then $p_E \sqsubseteq k_E^c \sqsubseteq f_E\sqcap\ h_E$, where k_E^c is fuzzy soft pre-closed. So, $Fpcl(p_E) \sqsubseteq f_E$ and $Fpcl(p_E) \sqsubseteq h_E \sqsubseteq Fpcl(h_E)$. Therefore, we obtain

$$G_{(\varphi,\psi)}(Fpcl(p_E)) \sqsubseteq f_E\widetilde{\otimes}(\varphi,\psi)(Fpcl(h_E)) \sqsubseteq f_E\widetilde{\otimes}g_K.$$

This shows that $G_{(\varphi,\psi)}$ f.s.st.θ.p.c. □

Corollary 4.5. *Let X_E be a fuzzy soft pre-regular space. Then a function $(\varphi,\psi) : X_E \longrightarrow Y_K$ is f.s.st.θ.p.c. if and only if the graph function $G_{(\varphi,\psi)} : X_E \longrightarrow X_E x Y_K$ is f.s.st.θ.p.c.*

Theorem 4.6. *If $(\varphi,\psi) : X_E \longrightarrow Y_K$ is f.s.st.θ.p.c. and f_A is a fuzzy soft semi-open subset of X_E, then the restriction $(\varphi,\psi)_{f_A} : f_A \longrightarrow Y_K$ is f.s.st. θ.p.c.*

Proof. For any $e_x^\alpha \widetilde{\in} f_A$ and any fuzzy soft open neighbourhood g_B of $(\varphi,\psi)(e_x^\alpha)$, there exists a fuzzy pre-open set f_C containing e_x^α such that $(\varphi,\psi)(Fpcl(f_C)) \sqsubseteq g_A$ since (φ,ψ) is f.s.st.θ.p.c. It is easily seen that

$$\begin{gathered} f_A \sqcap f_C \sqsubseteq f_A \sqcap Fint(Fcl(f_C)) = Fint_{f_A}(f_A \sqcap Fint_{f_A}(Fcl(f_C))) \\ \sqsubseteq Fint_{f_A}(Fcl(Fint(f_A)) \sqcap Fint(Fcl(f_C))) \sqsubseteq Fint_{f_A}(Fcl(Fint(f_A) \sqcap Fint(Fcl(f_C)))) \\ \sqsubseteq Fint_{f_A}(Fcl(Fint(f_A) \sqcap Fcl(f_C))) \subseteq Fint_{f_A}(Fcl(f_A \sqcap f_C)). \end{gathered}$$

So,

$$f_A \sqcap f_C \sqsubseteq Fint_{f_A}(Fcl(f_A \sqcap f_C)) \sqcap f_A \sqsubseteq Fint_{f_A}(Fcl(f_A \sqcap f_C) \sqcap f_A) \sqsubseteq Fint_{f_A}(Fcl_{f_A}(f_A \sqcap f_C)).$$

This means that $f_A \sqcap f_C$ is fuzzy soft pre-open in fuzzy soft subspace (f_A, τ_{f_A}, E). It is also easily seen that $Fpcl_{f_A}(f_A \sqcap f_C) \sqsubseteq Fpcl(f_A \sqcap f_C)$. Therefore, we obtain

$$\begin{gathered} (\varphi,\psi)_{f_A}(Fpcl_{f_A}(f_A \sqcap f_C)) = (\varphi,\psi)(Fpcl_{f_A}(f_A \sqcap f_C)) \\ \sqsubseteq (\varphi,\psi)(Fpcl(f_A \sqcap f_C)) \sqsubseteq (\varphi,\psi)((Fpcl(f_C)) \sqsubseteq g_A. \end{gathered}$$

This implies that $(\varphi,\psi)_{f_A}$ is f.s.st.θ.p.c. □

Theorem 4.7. *A function $(\varphi,\psi) : X_E \longrightarrow Y_K$ is f.s.st.θ.p.c., if for each $e_x^\alpha \widetilde{\in} X_E$ there exists fuzzy soft pre-open set f_A containing e_x^α such that $(\varphi,\psi)_{f_A}$ $A : f_A \to Y_K$ is f.s.st.θ.p.c.*

Proof. Let $e_x^\alpha \widetilde{\in} X_E$ and g_C be a fuzzy soft open neighbourhood of (φ,ψ) (e_x^α). From our assumption, there exists a fuzzy soft pre-open set f_A containing e_x^α such that $(\varphi,\psi)_{f_A}$: $f_A \longrightarrow Y_K$ is f.s.st.θ.p.c. Then there exists a fuzzy soft pre open set f_B in fuzzy soft subspace (f_A, τ_{f_A}, E) containing e_x^α such that $(\varphi,\psi)_{f_A}(Fpcl_{f_A}(f_B)) \sqsubseteq g_C$. It is known that $f_B \sqsubseteq Fint_{f_A}(Fcl_{f_A}(f_B))$ and $Fint_{f_A}(Fcl_{f_A}(f_B))$ is fuzzy soft open in the fuzzy soft subspace (f_A, τ_{f_A}, E). So, there exists a fuzzy soft open set f_C in X_E such that $f_A \sqcap f_C = Fint_{f_A}(Fcl_{f_A}(f_B))$.
It is easily seen that

$$\begin{gathered} f_B \sqsubseteq f_A \sqcap f_C \sqsubseteq Fint(Fcl(f_A)) \sqcap f_C \sqsubseteq Fint(Fcl(f_A \sqcap f_C)) \\ = Fint(Fcl(Fint_{f_A}(Fcl_{f_A}(f_B)))) \sqsubseteq Fint(Fcl(Fcl_{f_A}(f_B))) \\ \sqsubseteq Fint(Fcl(Fcl(f_B))) = Fint(Fcl(f_B)). \end{gathered}$$

This means that f_B is a fuzzy soft pre-open set in X_E containing e_x^α. Hence,

$$\begin{gathered} (\varphi,\psi)(Fpcl(f_B)) = (\varphi,\psi)_{f_A}(Fpcl(f_B)) \\ \sqsubseteq (\varphi,\psi)_{f_A}(Fpcl_{f_A}(f_B)) \sqsubseteq g_C. \end{gathered}$$

This shows that (φ,ψ) is f.s.st.θ.p.c. □

Definition 4.8. *A fuzzy soft function $(\varphi,\psi) : X_E \longrightarrow Y_K$ is said to be*

(1) fuzzy soft pre-irresolute, if for each $e_x^\alpha \in X_E$ and each fuzzy soft pre-open set g_B set in Y_K containing $(\varphi,\psi)(e_x^\alpha)$, there exists a fuzzy soft pre-open set f_A containing e_x^α such that $(\varphi,\psi)(f_A) \sqsubseteq g_B$.

(2) fuzzy soft M-pre-open, if $(\varphi,\psi)(f_A)$ is fuzzy soft pre-open in Y_K for every fuzzy soft pre-open set f_A in X_E.

Lemma 4.9. *If (φ,ψ):$X_E \longrightarrow Y_K$ is fuzzy soft pre-irresolute and g_B is a fuzzy soft pre-$\theta-$open set in Y_K, then $(\varphi,\psi)^{-1}(g_B)$ is fuzzy soft pre-$\theta-$open in X_E.*

Proof. Let g_B be a fuzzy soft pre-θ-open set in Y_K and $e_x^\alpha \widetilde{\in} (\varphi,\psi)^{-1}(g_B)$. There exists a fuzzy soft pre-open set g_C in Y_K such that (φ,ψ) (e_x^α) $\widetilde{\in} g_C \sqsubseteq Fpcl(g_C) \sqsubseteq g_B$. Since (φ,ψ) is fuzzy soft pre-irresolute, $(\varphi,\psi)^{-1}(g_C)$ is fuzzy soft pre-open in X_E and

$$Fpcl((\varphi,\psi)^{-1}(g_C)) \sqsubseteq (\varphi,\psi)^{-1}(Fpcl(g_C)).$$

Then we have $e_x^\alpha \widetilde{\in}$ $(\varphi,\psi)^{-1}(g_C) \sqsubseteq Fpcl((\varphi,\psi)^{-1}(g_C)) \sqsubseteq (\varphi,\psi)^{-1}(g_B)$ and $(\varphi,\psi)^{-1}(g_B)$ is fuzzy soft pre-θ-open in X_E. □

Theorem 4.10. *Let (φ_1,ψ_1):$X_{E_1} \longrightarrow Y_{E_2}$ and (φ_2,ψ_2): $Y_{E_2} \longrightarrow Z_{E_3}$ be fuzzy soft functions. Then the following properties hold:*

(1) If (φ_1,ψ_1) is f.s.st.θ.p.c. and (φ_2,ψ_2) is fuzzy soft continuous, then the composition

$$(\varphi_2 o \varphi_1, \psi_2 o \psi_1) : X_{E_1} \longrightarrow Z_{E_3}$$

is f.s.st.θ.p.c.

(2) If (φ_1,ψ_1) is fuzzy soft pre-irresolute and (φ_2,ψ_2) is f.s.st.θ.p.c., then $(\varphi_2 o\varphi_1, \psi_2 o\psi_1)$ is f.s.st.θ.p.c.

(3) If $(\varphi_1,\psi_1) : X_{E_1} \longrightarrow Y_{E_2}$ is a fuzzy soft M-pre-open bijection and $(\varphi_2 o\varphi_1, \psi_2 o\psi_1)$: $X_{E_1} \longrightarrow Z_{E_3}$ is f.s.st.θ.p.c., then (φ_2,ψ_2) is f.s.st.θ.p.c.

Proof. (1) This is obvious from Theorem 20.

(2) This follows from Theorem 20 and Lemma 34.

(3) Let g$_A$ be a fuzzy soft open set in Z_{E_3} . Since $(\varphi_1 o\varphi_1, \psi_2 o\psi_1)$ is f.s.st.θ.p.c., $(\varphi_1 o\varphi_1, \psi_2 o\psi_1)^{-1}(g_A)$ is fuzzy soft pre-$\theta-$open in X_{E_1} . Since (φ_1,ψ_1) is fuzzy soft M-pre-open and bijective, $(\varphi_1,\psi_1)^{-1}$ is fuzzy soft pre-irresolute. By Lemma 34, we have $(\varphi_2,\psi_2)^{-1}(g_A) = (\varphi_1 o\psi_1)((\varphi_2 o\varphi_1,\ \psi_2 o\psi_1)^{-1}(g_A))$ is fuzzy soft pre-$\theta-$open in Y_{E_2} . By Theorem 20, (φ_2,ψ_2) is f.s.st.θ.p.c.

Let $\{X_{E_k} : k \in A\}$ be a family of fuzzy soft topological spaces, f_{A_k} a nonempty fuzzy soft subset of X_{E_k} for each $k \in A$ and denote $X_E = \prod\{\ X_{E_k}\ : k \in A\}$ the fuzzy soft product space, where A is nonempty. □

Lemma 4.11. *Let n be a positive integer and $f_A = \prod\limits_{j=1}^{n} A_{k_j} \times \prod\limits_{k \neq k_j} X_{E_k}$.*

(1)*f_A is fuzzy soft open in X_E if and only if A_{k_j} is fuzzy soft open in X_{k_j} for each $1, 2, 3, \ldots..n$*

(2) *$Fpcl(\prod_{\ k\in A}\ f_{A_k}) \sqsubseteq \prod\limits_{k\in A} Fpcl(f_{A_k})$.*

Theorem 4.12. *If a function (φ_k,ψ_k): $X_{E_K} \longrightarrow Y_{E_K}$ is f.s.st.θ.p.c. for each $k \in A$, then the product function $(\varphi,\psi) : \prod X_{E_k} \longrightarrow \prod Y_{E_k}$, defined by*

$$(\varphi,\psi)((e_{k_1}, e_{k_2}, e_{k_3}, \ldots\ \ldots)\min\{\alpha_{k_1}, \alpha_{k_2}, \alpha_{k_1} \ldots\ldots\}(x_{k_1}, x_{k_2}, x_{k_1} \ldots\ldots) =$$

$$(\psi_1(e_{k_1}), \psi_2(e_{k_2}), \psi_3(e_{k_3}), \ldots\ \ldots)\min\{\alpha_{k_1}, \alpha_{k_2}, \alpha_{k_1} \ldots\ldots\}(\varphi_1(x_{k_1}), \varphi_2(x_{k_2}), \varphi_3(x_{k_1}) \ldots\ldots)$$

for each $e_x^\alpha = (e_{k_1}, e_{k_2}, e_{k_3}, \ldots\ \ldots)\min\{\alpha_{k_1}, \alpha_{k_2}, \alpha_{k_1} \ldots\ldots\}(x_{k_1}, x_{k_2}, x_{k_1} \ldots\ldots)$, is f.s.st.$\theta$.p.c.

Proof. Let $e_x^\alpha = (e_{k_1}, e_{k_2}, e_{k_3}, \ldots\ \ldots)\min\{\alpha_{k_1}, \alpha_{k_2}, \alpha_{k_1} \ldots\ldots\}(x_{k_1}, x_{k_2}, x_{k_1} \ldots\ldots) \widetilde{\in} \prod X_{E_K}$ and g_C be any fuzzy soft open set of $\prod$ Y$_{E_k}$ containing $(\varphi,\psi)(e_x^\alpha)$. Then there exists a fuzzy soft open set $g_{B_{k_j}}$ of $Y_{E_{k_j}}$ such that $(\varphi,\psi)(e_x^\alpha) =$ $(\psi_1(e_{k_1}),\ \psi_2(e_{k_2}\ ),\ \psi_3(e_{k_3}), \ldots\ )\min\{\alpha_{k_1}, \alpha_{k_2}, \alpha_{k_1} \ldots\}(\varphi_1(x_{k_1}), \varphi_2(x_{k_2}), \varphi_3(x_{k_1}) \ldots)$ $\widetilde{\in} \prod_{j=1}^{n} g_{B_{k_j}}$ x $\prod_{k\neq k_j} X_{E_K} \sqsubseteq g_C$ Since $(\varphi_k,\ \psi_k)$ is f.s.st.θ.p.c. for each k, there exists a fuzzy soft pre-open set $f_{A_{k_j}}$ in $X_{E_{k_j}}$ containing $e_{k_{j_{x_{k_j}}}}^{\alpha_{k_j}}$ such that $(\varphi_{k_j},\psi_{k_j})(Fpcl(f_{A_{k_j}})) \sqsubseteq g_{B_{k_j}}$ for $j = 1, 2, 3, \ldots\ldots..n$. Now, put $f_A = \prod_{j=1}^{n} f_{A_{k_j}} \times \prod_{k\neq k_j} X_{E_k}$. Then it follows from Lemma 36 that f_A is a fuzzy soft pre-open set containing e_x^α in $\prod X_{E_k}$. Moreover, we have $(\varphi,\psi)(Fpcl(f_A))$ $\sqsubseteq (\varphi,\psi)(\prod_{j=1}^{n} Fpcl(f_{A_{k_j}})x\prod_{k\neq k_j} X_{E_k}\ ) \sqsubseteq \prod_{j=1}^{n}(\varphi_{k_j}\ ,\ \psi_{k_j})(Fpcl(f_{A_{k_j}}))$x $\prod_{k\neq k_j} X_{E_k} Y_{E_k}$ $\sqsubseteq \prod_{j=1}^{n} g_{B_{k_j}}x\prod_{k\neq k_j} Y_{E_k} \sqsubseteq g_C$. This shows that (φ,ψ) is f.s.st.θ.p.c. □

5. Fuzzy Soft St. θ.p.c. Functions and Seperation Axioms

In this section, we examine relationships of f.s.st.θ.p.c. functions with fuzzy soft separation axioms.

Definition 5.1. *Let X_E be a fuzzy soft topological space. Let $e^{\alpha_1}_{1_{x_1}}$ and $e^{\alpha_2}_{2_{x_2}}$ be fuzzy soft points in X_E. It is said that $e^{\alpha_1}_{1_{x_1}}$ and $e^{\alpha_2}_{2_{x_2}}$ are distinct fuzzy soft points if and only if $e_1 \neq e_2$ or $x_1 \neq x_2$.*

Definition 5.2. *A fuzzy soft topological space X_E is fuzzy soft pre-T_2 (resp. fuzzy soft pre-Urysohn) if for each pair of distinct fuzzy soft points $e^{\alpha_1}_{1_{x_1}}$ and $e^{\alpha_2}_{2_{x_2}}$ in X_E, there exist fuzzy soft pre-open sets f_A and f_B such that $e^{\alpha_1}_{1_{x_1}} \widetilde{\in} f_A \sqsubseteq (e^{\alpha_2}_{2_{x_2}})^c$, $e^{\alpha_2}_{2_{x_2}} \widetilde{\in} f_B \sqsubseteq (e^{\alpha_1}_{1_{x_1}})^c$ and $f_A \overline{q} f_B$ (resp. $Fpcl(f_A)\overline{q}Fpcl(f_B)$).*

Definition 5.3. *[25] A fuzzy soft topological space X_E is fuzzy soft T_0 if for each pair of distinct fuzzy soft points $e^{\alpha_1}_{1_{x_1}}$ and $e^{\alpha_2}_{2_{x_2}}$ in X_E, there exist fuzzy soft open sets f_A and f_B such that $e^{\alpha_1}_{1_{x_1}} \widetilde{\in} f_A \sqsubseteq (e^{\alpha_2}_{2_{x_2}})^c$, $e^{\alpha_2}_{2_{x_2}} \widetilde{\in} f_B \sqsubseteq (e^{\alpha_1}_{1_{x_1}})^c$.*

Definition 5.4. *[25] A fuzzy soft topological space X_E is fuzzy soft Hausdorff if for each pair of distinct fuzzy soft points $e^{\alpha_1}_{1_{x_1}}$ and $e^{\alpha_2}_{2_{x_2}}$ in X_E, there exist fuzzy soft open sets f_A and f_B such that $e^{\alpha_1}_{1_{x_1}} \widetilde{\in} f_A \sqsubseteq (e^{\alpha_2}_{2_{x_2}})^c$, $e^{\alpha_2}_{2_{x_2}} \widetilde{\in} f_B \sqsubseteq (e^{\alpha_1}_{1_{x_1}})^c$ and $f_A \overline{q} f_B$.*

Theorem 5.5. *If a fuzzy soft function (φ, ψ):$X_E \longrightarrow Y_K$ is f.s.st.θ.p.c. injection and Y_K is fuzzy soft T_0 (resp. fuzzy soft Hausdorff), then X_E is fuzzy soft pre-T_2(resp. fuzzy soft pre-Urysohn).*

Proof. (1) Suppose that Y_K is fuzzy soft T_0. Let $e^{\alpha_1}_{1_{x_1}}$ and $e^{\alpha_2}_{2_{x_2}}$ be any fuzzy soft distinct points of X_E. Since (φ, ψ) is injective, $(\varphi, \psi)(e^{\alpha_1}_{1_{x_1}}) \neq (\varphi, \psi)(e^{\alpha_2}_{2_{x_2}})$ and there exists either a fuzzy soft open neighbourhood g_B of $(\varphi, \psi)(e^{\alpha_1}_{1_{x_1}})$ such that $(\varphi, \psi)(e^{\alpha_2}_{2_{x_2}}) \widetilde{\in} g^c_B$, or a fuzzy soft open neighbourhood g_C of $(\varphi, \psi)(e^{\alpha_2}_{2_{x_2}})$ such that (φ, ψ) $(e^{\alpha_1}_{1_{x_1}}) \widetilde{\in}$ g^c_C . If the first case holds, then there exists a fuzzy soft pre-open set f_A containing $e^{\alpha_1}_{1_{x_1}}$ such that $(\varphi, \psi)(Fpcl(f_A)) \sqsubseteq g_B$. Therefore, we obtain $(\varphi, \psi)(e^{\alpha_2}_{2_{x_2}}) \widetilde{\in} [(\varphi, \psi)(Fpcl(f_A))]^c$. Hence, $(Fpcl(f_A))^c$ is a fuzzy soft pre-open set containing $e^{\alpha_2}_{2_{x_2}}$. If the second case holds, then we obtain a similar result. Therefore, X_E is fuzzy soft pre-T_2.

(2) Suppose that Y_K is fuzzy soft Hausdorff. Let $e^{\alpha_1}_{1_{x_1}}$ and $e^{\alpha_2}_{2_{x_2}}$ be any fuzzy soft distinct points of X_E. Then, $(\varphi, \psi)(e^{\alpha_1}_{1_{x_1}}) \neq (\varphi, \psi)(e^{\alpha_2}_{2_{x_2}})$. Since Y_K is fuzzy soft Hausdorff, there exist fuzzy soft open sets g_C and g_D such that $g_C \overline{q} g_D$,$(\varphi, \psi)(e^{\alpha_1}_{1_{x_1}}) \widetilde{\in}$ g_C and $(\varphi, \psi)(e^{\alpha_2}_{2_{x_2}})$ $\widetilde{\in} g_D$. Since (φ, ψ) is f.s.st.θ.p.c., there exist fuzzy soft pre-open sets f_A and f_B containing $e^{\alpha_1}_{1_{x_1}}$ and $e^{\alpha_2}_{2_{x_2}}$, respectively, such that $(\varphi, \psi)(Fpcl(f_A)) \sqsubseteq g_C$ and $(\varphi, \psi)(Fpcl(f_B)) \sqsubseteq g_D$. It follows that $Fpcl\,(f_A)\,\overline{q} Fpcl(f_B)$. This means that X_E is fuzzy soft pre-Urysohn. □

Theorem 5.6. *If a fuzzy soft function $(\varphi, \psi) : X_E \longrightarrow Y_K$ is f.s.st.θ.p.c and Y_K is fuzzy soft Hausdorff, then the subset $E = \{f_E \widetilde{\otimes} g_E : (\varphi, \psi)(f_E) = (\varphi, \psi)(g_E)\}$ is fuzzy soft pre-$\theta-$closed in $X_E \widetilde{\otimes} X_E$.*

Proof. Suppose that $e^{\alpha_1}_{1_{x_1}} \widetilde{\otimes} e^{\alpha_2}_{2_{x_2}} \in E^c$. It follows that $(\varphi, \psi)(e^{\alpha_1}_{1_{x_1}}) \neq (\varphi, \psi)(e^{\alpha_2}_{2_{x_2}})$. Since Y_K is fuzzy soft Hausdorff, there exist fuzzy soft open sets g_C and g_D in Y_K such that
$g_C \overline{q} g_D$, $(\varphi, \psi)(e^{\alpha_1}_{1_{x_1}}) \widetilde{\in} g_C$ and $(\varphi, \psi)(e^{\alpha_2}_{2_{x_2}}) \widetilde{\in} g_D$. Since (φ, ψ) is f.s.st.θ.p.c. there exist fuzzy soft pre-open sets f_A and f_B in X_E containing $e^{\alpha_1}_{1_{x_1}}$ and $e^{\alpha_2}_{2_{x_2}}$, respectively, such that
$(\varphi, \psi)(Fpcl(f_A)) \sqsubseteq g_C$ and $(\varphi, \psi)(Fpcl(f_B)) \sqsubseteq g_D$. Then it follows that $f_A \widetilde{\otimes} f_B$ is a fuzzy soft pre-open set in $X_E \widetilde{\otimes} X_E$ containing $e^{\alpha_1}_{1_{x_1}} \widetilde{\otimes} e^{\alpha_2}_{2_{x_2}}$.

Since $Fpcl(f_A \widetilde{\otimes} f_B) \sqsubseteq Fpcl(f_A) \widetilde{\otimes} Fpcl(f_B)$ and $[Fpcl(f_A) \widetilde{\otimes} Fpcl(f_B)]\overline{q}E$, $Fpcl(f_A \widetilde{\otimes} f_B)\overline{q}E$. Therefore, E is fuzzy soft pre-$\theta-$closed in $X_E \overline{\widetilde{\otimes}} X_E$. □

Definition 5.7. *The graph $G_{(\varphi,\psi)}$ of a fuzzy soft function $(\varphi, \psi) : X_E \longrightarrow Y_K$ is said to be fuzzy soft strongly pre-closed if for each point $e^{\alpha_1}_{1_{x_1}} \widetilde{\otimes} e^{\alpha_2}_{2_{x_2}}$ $\widetilde{\in}$ $(G_{(\varphi,\psi)})^c$, there exist a fuzzy soft pre-open set f_A in X_E containing $e^{\alpha_1}_{1_{x_1}}$ and a fuzzy soft open set g_B in Y_K containing $e^{\alpha_2}_{2_{x_2}}$ such that $(Fpcl(f_A) \widetilde{\otimes} g_B)\overline{q} G_{(\varphi,\psi)}$.*

Lemma 5.8. *The graph $G_{(\varphi,\psi)}$ of a fuzzy soft function $(\varphi,\psi): X_E \longrightarrow Y_K$ is fuzzy soft strongly pre-closed in $X_E \widetilde{\otimes} Y_K$ if only if for each point $e^{\alpha_1}_{1_{x_1}} \widetilde{\otimes} e^{\alpha_2}_{2_{x_2}} \widetilde{\in} (G_{(\varphi,\psi)})^c$, there exist a fuzzy soft pre-open set f_A in X_E containing $e^{\alpha_1}_{1_{x_1}}$ and a fuzzy soft open set g_B in Y_K containing $e^{\alpha_2}_{2_{x_2}}$ such that $(\varphi,\psi)(Fpcl(f_A))\overline{q}g_B$.*

Theorem 5.9. *If $(\varphi,\psi): X_E \longrightarrow Y_K$ is fuzzy soft st.θ.p.c. and Y_K is fuzzy soft Hausdorff , then $G_{(\varphi,\psi)}$ is soft strongly pre-closed in $X_E \widetilde{\otimes} Y_K$.*

Proof. Let $e^{\alpha_1}_{1_{x_1}} \widetilde{\otimes} e^{\alpha_2}_{2_{x_2}} \widetilde{\in} (G_{(\varphi,\psi)})^c$. It follows that $(\varphi,\psi)(e^{\alpha_1}_{1_{x_1}}) \neq e^{\alpha_2}_{2_{x_2}}$. Since Y_K is fuzzy soft Hausdorff, there exists fuzzy soft open sets g_B and g_C such that $(\varphi,\psi)(e^{\alpha_1}_{1_{x_1}}) \widetilde{\in} g_B$, $e^{\alpha_2}_{2_{x_2}} \widetilde{\in} g_C$ and $g_B \overline{q} g_C$. Since (φ,ψ) is f.s.st.θ.p.c., there exists a fuzzy soft pre-open set f_A containing $e^{\alpha_1}_{1_{x_1}}$ such that $(\varphi,\psi)(Fpcl(f_A)) \sqsubseteq g_B$. Therefore, $(\varphi,\psi)(Fpcl(f_A))\overline{q}g_C$ and $G_{(\varphi,\psi)}$ is fuzzy soft strongly pre-closed in $X_E \widetilde{\otimes} Y_K$. □

6. Preservation Properties

In this section, we discuss preservation of some properties of fuzzy soft topological spaces under various kinds of functions.

Definition 6.1. *A fuzzy soft space X_E is said to be*

(1) Fuzzy soft p-closed (resp. fuzzy soft p-Lindelöf) if every cover of $\widetilde{1}_E$ by fuzzy soft pre-open sets has a finite (resp. countable) subcover whose fuzzy soft pre-closures cover $\widetilde{1}_E$,

(2) Fuzzy soft countably p-closed if every countable cover of $\widetilde{1}_E$ by fuzzy soft pre-open sets has a finite subcover whose fuzzy soft pre-closures cover $\widetilde{1}_E$.

A fuzzy soft subset K of a fuzzy soft space X_E is said to be p-closed relative to X_E, if for every cover $\{f^k_A$: $k \in A\}$ of K by fuzzy soft pre-open sets of X_E, there exists a finite subset A_* of A such that $K \sqsubseteq \sqcup\{$ $Fpcl(f^k_A$:): $k \in A_*\}$.

Definition 6.2. *[19] Let X_E be a fuzzy soft topological space. A fuzzy soft set f_A in X_E is called fuzzy soft compact, if each fuzzy soft open cover of f_A has a finite subcover. Also fuzzy soft topological space X is called compact if each fuzzy soft open cover of $\widetilde{1}_E$ has a finite subcover.*

Definition 6.3. *Let X_E be a fuzzy soft topological space. A fuzzy soft set f_A in X_E is called fuzzy soft countably compact, if each countable fuzzy soft open cover of f_A has a finite subcover. Also a fuzzy soft topological space X_E is called fuzzy soft countably compact if each countable fuzzy soft open cover of $\widetilde{1}_E$ has a finite subcover.*

Theorem 6.4. *If $(\varphi,\psi): X_E \longrightarrow Y_K$ is a f.s.st.θ.p.c. function and K is fuzzy soft p-closed relative to X_E, then $(\varphi,\psi)(K)$ is a fuzzy soft compact set of Y_K.*

Proof. Suppose that $(\varphi,\psi): X_E \longrightarrow Y_K$ is f.s.st.θ.p.c. and K is fuzzy soft p-closed relative to X_E. Let $\{g^k_B$: $k \in A\}$ be a fuzzy soft open cover of $(\varphi,\psi)(K)$. For each point $e^\alpha_x \widetilde{\in} K$, there exists $k(e^\alpha_x) \in A$ such that $(\varphi,\psi)(e^\alpha_x) \widetilde{\in} g^{k(e^\alpha_x)}_B$. Since (φ,ψ) is f.s.st.θ.p.c., there exists a fuzzy soft pre-open set $f^{e^\alpha_x}_A$ containing e^α_x such that $(\varphi,\psi)(Fpcl(f^{e^\alpha_x}_A)) \sqsubseteq g^{k(e^\alpha_x)}_B$. The family $\{f^{e^\alpha_x}_A$: $e^\alpha_x \widetilde{\in} K\}$ is a cover of K by fuzzy soft pre-open sets of X_E and hence there exists a finite subsets K_* of K such that $K \sqsubseteq \sqcup_{e^\alpha_x \widetilde{\in} K_*} Fpcl(f^{e^\alpha_x}_A)$. Therefore, we obtain $(\varphi,\psi)(K) \sqsubseteq \sqcup_{e^\alpha_x \widetilde{\in} K_*} g^{k(e^\alpha_x)}_B$. This shows that $(\varphi,\psi)(K)$ is fuzzy soft compact. □

Corollary 6.5. *Let (φ,ψ):$X_E \longrightarrow Y_K$ be a f.s.st.θ.p.c. surjection. Then the following properties hold:*

(1) If X_E is fuzzy soft p-closed, then Y_K is fuzzy soft compact.

(2) If X_E is fuzzy soft p-Lindelöf, then Y_K is fuzzy soft Lindelöf.

(3) If X_E is fuzzy soft countably p-closed, then Y_K is fuzzy soft countably compact.

Theorem 6.6. *If a fuzzy soft function $(\varphi,\psi): X_E \longrightarrow Y_K$ has a fuzzy soft strongly pre-closed graph, then (φ,ψ) (K) is fuzzy soft closed in Y_K for each fuzzy soft subset K which is fuzzy soft p-closed relative to X_E.*

Proof. Let K be fuzzy soft p-closed relative to X_E and $k_y^\beta \widetilde{\in}((\varphi,\psi)(K))^c$. Then for each $e_x^\alpha \widetilde{\in} K$ we have $e_x^\alpha \widetilde{\otimes} k_y^\beta \widetilde{\in}(G_{(\varphi,\psi)})^c$. By Lemma 45, there exist a fuzzy soft pre-open set $f_A^{e_x^\alpha}$ containing e_x^α and a fuzzy soft open set $g_B^{e_x^\alpha} of Y_K$ containing k_y^β such that $(\varphi,\psi)(Fpcl(f_A^{e_x^\alpha}))\overline{q} g_B^{e_x^\alpha}$. The family $\{f_A^{e_x^\alpha} : e_x^\alpha \widetilde{\in} K\}$ is a cover of K by pre-open sets of X_E. Since K is fuzzy soft p-closed relative to X_E, there exists a finite subset K_* of K such that $K \sqsubseteq \sqcup\{Fpcl(f_A^{e_x^\alpha}: e_x^\alpha \widetilde{\in} K_*\}$. Put $g_B = \sqcap\{g_B^{e_x^\alpha} : e_x^\alpha \widetilde{\in} K\}$. Then g_B is a fuzzy soft open neighbourhood of k_y^β. For each
$e_x^\alpha \widetilde{\in} K_*$, $(\varphi,\psi)(Fpcl(f_A^{e_x^\alpha}))\overline{q} g_B^{e_x^\alpha}$. This means that $[\sqcup\{(\varphi,\psi)(Fpcl(f_A^{e_x^\alpha})) : e_x^\alpha \widetilde{\in} K_*\}]\overline{q} g_B$. As
$(\varphi,\psi)(K) \sqsubseteq \sqcup\{(\varphi,\psi)(Fpcl(f_A^{e_x^\alpha})) : e_x^\alpha \widetilde{\in} K_*\}$, $(\varphi,\psi)(K)\overline{q} g_B$. So, $[(\varphi,\psi)(K)]^c$ is fuzzy soft open. Therefore, $(\varphi,\psi)(K)$ is fuzzy soft closed in Y_K. □

7. Conclusion

In this study, we have given the definition of strongly $\theta-$pre-continuous function in fuzzy soft topology. We have focused on the properties of fuzzy soft strongly $\theta-$pre-continuity in several types of fuzzy soft topological spaces and investigated the relationships with some other continuities which have been supported by a diagram and counter examples. This study is also an attempt to make a new approach to give a different definition for fuzzy soft graph function. Some valuable results that can be used in different disciplines are obtained and analyzed.

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Huseyin Cakalli,
Department of Mathematics,
Maltepe University, 34857, Istanbul
Turkey.
E-mail address: huseyincakalli@maltepe.edu.tr

and

Ahu Acikgoz,
Department of Mathematics,
Balikesir University, 10145 Balikesir,
Turkey.
E-mail address: ahuacikgoz@gmail.com

and

Ferhat Esenbel,
Department of Mathematics,
Balikesir University, 10145 Balikesir,
Turkey.
E-mail address: fesenbel@gmail.com