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Some Membership Functions via Neighborhood Systems: Application to a Rough Set Decision Making *

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ABSTRACT: Neighborhood structures can represent information about relationships between universe's objects. In other words, such objects are somewhat similar to that element in an element's neighborhood. Pawlak presented the idea of rough sets as useful tools for learning computer science and information systems. Neighborhood structures used this principle to be generalized and studied. This paper used a neighborhood method to solve several rough set theory problems. By using a neighborhood of objects in information system and illustrative examples to apply it, we introduce some new definitions of attributes, membership function and accuracy measurement. A decision making of our method gives an accurate decision and helps with decision correlation to calculate the accuracy of each attribute that builds an approach to decision making.

Key Words: Information system, neighborhood system, rough set, graph theory, membership functions and accuracy.

Contents

1. Introduction and preliminaries

Through studying the history of mathematics, we found that all mathematical principles and their branches were beginning to solve problems that faced people such as geometry and earth problems (division of land around the Nile River in Egypt) and Fabiaosh rabbit development inequalities. In discovering and solving problems, classical mathematics achieved many developments. Now our world, developed information and communication revolution, and all the problems of existence are becoming complicated and uncertain, so we need to translate our problems into topological theories, fractional and uncertainty theories such as fuzzy sets, rough sets and soft sets. The German mathematician Felix Hausdorff propagated the importance of the notion of neighborhood and its utility in the description of a topological space [1]. Nowadays, we have the ability to collect information about any problems that need to be addressed in our lives [2]. Mathematics play important role to construct suitable models to obtain the best decision [3,4].

Rough set theory is the new method of debriefing and making decisions or knowledge extraction. The rough set theory is based on the assumption that some data knowledge is associated with each object of the discourse universe. Objects with the same information are identical in view of the information available about them. Any set of all similar objects is referred to as an element set and forms a simple atom of knowledge. Pawlak space approximation [5,6,7,30,31] relied on equivalence relation and attribute reduction [8] performing the main role of information extraction [9,10,11]. Slowinski [12] and El-Bably et al. [13] introduced some topological structures which depend on generalized rough approximations based similarity. A rough collection method [26,27] when choosing attributes for partitioning. Partitioning of data mining is carried out on attributes to increase the concentration of data. A rough set-based technique

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that calculates the crispness of partitioning is provided to pick successful partitioning attributes. This technique is based on the theory of rough sets in mathematics. The method is a simple and efficient approach to calculating the partition's crispiness in data mining [28,29]. The starting point of rough set theory is the relationship of indiscernibility created by information on objects of interest. The relationship of indiscernibility illustrates the fact that we are unable to differentiate other objects using available information due to a lack of information. This means that we cannot deal with each particular object in general, but we must consider clusters of indiscernible objects as a fundamental basis for our theory.

Neighborhood [17,18,19] plays an important role in every aspect of mathematics. The term "neighborhood" has different levels of significance in mathematics, typically point x neighborhood is all points inside a ball with center x, and the set includes an open neighborhood set of x as well. In the graph [20,21], the neighborhood of a vertex x is the set of all vertices shared by x. The neighborhood of every object in the universe is the set that provides information about this object, so that it allows us to know everything about this object and we can establish the degree of correlation with respect to the space of approximation. Using an object's neighborhood by situation and decision attributes from the information system [22,23,24] shows the full view of objects. In addition, neighborhood of any point in the universe is the important concepts in a topological space and related areas of mathematics, is closely defining the concepts of open set that it is necessary to obtain in topological spaces in the interior and closure region. In the approximation space open neighborhood of a point identified by the equivalence class [25] consisting of all objects shredding in properties information data to this point.

In this paper, using several definitions as completely similarity relations [12,13], our new definitions of the neighborhood of condition attributes in terms of decision attributes and membership function in data processing [14,15,16]. All of these ideas will turn problems into qualitative and quantitative mathematical issues. We describe the approximation space attribute accuracy by using the neighborhood view instead of the classical view and modify the neighborhood shape definition from the Cartesian form. The article was about giving a space approximation study. The rough set idea is based on the approximation space definition. In the first section, we present some definitions and preliminaries used through this thesis and give an overview of the rough set concepts in Pawlak. The second section's primary task is to introduce the new term. The third section addresses an important illustrated example.

Definition 1.1. *[30] Let U be a finite set and R be an equivalence relation on U. The pair $K = (U, R)$ is said to be an approximation space. Some subsets of U are definable in the given approximation space and others are undefinable. Thus, a set $X \subseteq U$ is definable or exact, if it is a union of some equivalence classes. Otherwise, X is undefinable. Rough set or undefinable set can be defined approximately in any approximation space which depend on two exact sets referred to as lower approximation and upper approximation.*

Definition 1.2. *[31] Let $K = (U, R)$ be an approximation space and $X \subseteq U$. Then, the accuracy is measured $\mu(X)$ of X is given by $\mu(X) = \frac{|L(X)|}{|u(X)|}$, $|u(X)| \neq 0$.*

Definition 1.3. *[4] Let $\mathrm{K} = (\mathrm{U}, \mathrm{AT} = \{\mathrm{C},\ \mathrm{D}\}, \mathrm{v_i} \in \mathrm{V})$ be an information attribute-value system, where U is a nonempty set of finite objects . Then, $\mathrm{AT} = \{\mathrm{C},\ \mathrm{D}\}$ are a nonempty finite set of conditions and decision and V is the set of values that attribute will be taken.*

Definition 1.4. *[8] Let $\mathrm{K} = (\mathrm{U},\ \mathrm{AT} = \{\mathrm{C},\ \mathrm{D}\}, \mathrm{v_i} \in \mathrm{V})$ be an information system, then a neighborhood of every object given by $\mathrm{N_i^A}(\mathrm{x}) = \{\mathrm{y} :\ \mathrm{a}(\mathrm{x}) = \mathrm{a}(\mathrm{y}) = \mathrm{v_i},\ \forall\ \mathrm{v_i} \in \mathrm{V}\}$, where $\mathrm{i} \in \mathrm{AT}$.*

2. A classification of data in information system

In Table 1 information system, each object has a neighborhood with respect to each attribute containing all objects with the same properties and a membership function relies on two sets of the object's neighborhood and the set intersects with another neighborhood. That implies, each object identified by two sets in approximation space by using condition attributes neighborhood with regard to decision attributes. Both sets are lower object neighborhood object $|N(x) \cap A|$ and neighborhood of object $|N(x)|$.

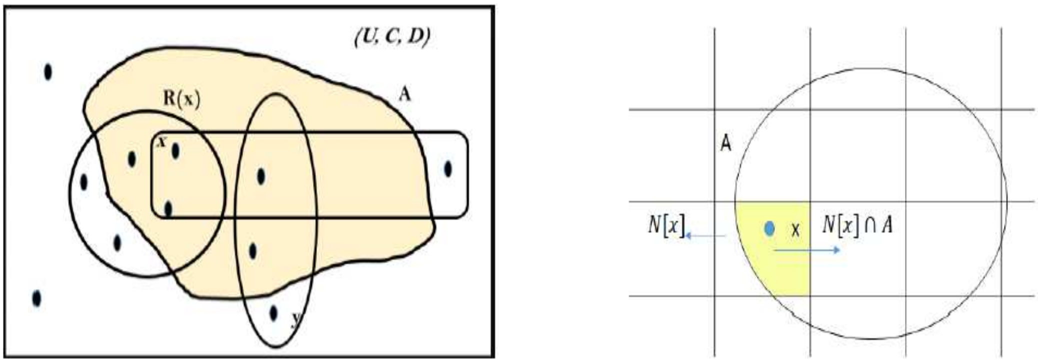

Figure 1: A classification of information system

Definition 2.1. *Let* $K = (U, AT = \{C, D\}, v_i \in V)$ *be an information system and for every* $x \in U$, $A \in \frac{U}{D}$. *Then,*

(i) The quality of the inner region of neighborhood of x given by $P_L(x) = \frac{|N(x) \cap A|}{|U|}$.

(ii) The quality of the outer region of neighborhood of x given by $P_u(x) = \frac{|N(x)|}{|U|}$.

(iii) The membership function of neighborhood of x given by $\mu(x) = \frac{P_L(x)}{P_u(x)} = \frac{|N(x) \cap A|}{|N(x)|}$.

From Definition 2.1, every rough set has a boundary region depending on the rate between the lower and upper approximation that we can define as a value from zero to one, and it depended entirely on the accuracy measure that, in fact, depended on the membership function of each object neighborhood. We cannot have a classified boundary line by giving knowledge and information can be seen by providing information about knowledge that leads us to use neighborhoods.

Proposition 2.2. *Let* $K = (U, AT = \{C, D\}, v_i \in V)$ *be an information system and for every* $x \in U$, $A \in \frac{U}{D}$. *Then, the membership function of neighborhood of x for all condition attributes with respect to decision attributes given by* $\mu^K_{A \in \frac{U}{D}}(x) = \frac{\sum_i |N_i(x) \cap A|}{\sum_i |N_i(x)|}$, $i = 1, 2, \ldots$.

Proof. The result will be proved by mathematical induction. At $n = 2$, the membership function of x with respect to two condition attributes, $P_L(x) = \frac{|N_1(x) \cap A|}{|U|} + \frac{|N_2(x) \cap A|}{|U|} = \frac{|N_1(x) \cap A| + |N_2(x) \cap A|}{|U|}$ and $P_u(x) = \frac{|N_1(x)| + |N_2(x)|}{|U|}$. Then the membership function of x given by $\mu(x) = \frac{|N_1(x) \cap A| + |N_2(x) \cap A|}{|N_1(x)| + |N_2(x)|}$. Assume that the membership function is true at k conditional attributes i.e
$P_L(x) = \frac{|N_1(x) \cap A| + |N_2(x) \cap A| + \ldots\ldots + |N_k(x) \cap A|}{|U|}$ $P_u(x) = \frac{|N_1(x)| + |N_2(x)| + \ldots\ldots + |N_k(x)|}{|U|}$ and $\mu^K_{A \in \frac{U}{D}}(x) = \frac{|N_1(x) \cap A| + |N_2(x) \cap A| + \ldots\ldots + |N_k(x) \cap A|}{|N_1(x)| + |N_2(x)| + \ldots\ldots + |N_k(x)|}$. Now, we prove the equation of membership at $n = k + 1$. Since

$$P_L(x) = \frac{|N_1(x) \cap A| + \ldots\ldots + |N_k(x) \cap A|}{|U|} + \frac{|N_{k+1}(x) \cap A|}{|U|} = \frac{\sum_i |N_{k+1}(x) \cap A|}{|U|},$$

$$P_u(x) = \frac{|N_1(x)| + |N_2(x)| + \ldots\ldots\ldots\ldots + |N_k(x)|}{|U|} + \frac{|N_{k+1}(x)|}{|U|} = \frac{\sum_i |N_{k+1}(x)|}{|U|},$$

Therefore, $\mu^K_{A \in \frac{U}{D}}(x) = \frac{|N_1(x) \cap A| + |N_2(x) \cap A| + \ldots\ldots + |N_{k+1}(x) \cap A|}{|N_1(x)| + |N_2(x)| + \ldots\ldots + |N_{k+1}(x)|} = \frac{\sum_i |N_{k+1}(x) \cap A|}{\sum_i |N_{k+1}(x)|}$. □

It is easy to prove Proposition 2.3, so we omit the proof.

Proposition 2.3. *Let* $\mathrm{K} = (\mathrm{U},\ \mathrm{AT} = \{\mathrm{C},\ \mathrm{D}\},\ \mathrm{v_i} \in \mathrm{V})$ *be an information system and for every* $\mathrm{x} \in \mathrm{U}$, $\mathrm{A} \in \frac{\mathrm{U}}{\mathrm{D}}$. *Then, the membership function* $\mu^{\mathrm{K}}_{\mathrm{A} \in \frac{\mathrm{U}}{\mathrm{D}}}(\mathrm{x})$ *of neighborhood of x for all condition attributes with respect to decision attributes, then*

(i) If $\mathrm{N_i}(\mathrm{x}) \bigcap \mathrm{A} = \mathrm{N_i}(\mathrm{x})$, *then* $\mu^{\mathrm{K}}_{\mathrm{A} \in \frac{\mathrm{U}}{\mathrm{D}}}(\mathrm{x}) = 1$. *This means that the neighborhood of x in all condition attributes are completely lie in A.*

(ii) If $\mathrm{N_i}(\mathrm{x}) \cap \mathrm{A} = \emptyset$, *then* $\mu^{\mathrm{K}}_{\mathrm{A} \in \frac{\mathrm{U}}{\mathrm{D}}}(\mathrm{x}) = 0$, *i.e there are no intersect between neighborhood of x and A.*

(iii) Otherwise, $0 < \mu^{\mathrm{K}}_{\mathrm{A} \in \frac{\mathrm{U}}{\mathrm{D}}}(\mathrm{x}) < 1$, *where* $\mathrm{N_i}(\mathrm{x}) \cap \mathrm{A} \neq \emptyset$.

Proposition 2.4. *Let* $\mathrm{K} = (\mathrm{U},\ \mathrm{AT} = \{\mathrm{C},\ \mathrm{D}\},\ \mathrm{v_i} \in \mathrm{V})$ *be an information system and for every* $\mathrm{x} \in \mathrm{U}$, $\{\mathrm{x}\} \in \frac{\mathrm{U}}{\mathrm{C}}$ and $\mathrm{x} \in \mathrm{A}$. *Then, the membership function of x is the unity* $\mu(\mathrm{x}) = 1$.

Proof. $\mu(\mathrm{x}) = \frac{\mathrm{P_L}(\mathrm{x})}{\mathrm{P_u}(\mathrm{x})} = \frac{|\{\mathrm{x}\} \cap \mathrm{A}|}{|\{\mathrm{x}\}|} = \frac{|\{\mathrm{x}\}|}{|\{\mathrm{x}\}|} = 1$ □

Remark 2.5. *(1) The membership function of neighborhood of x is unity in two cases:*
(i) The neighborhood of x is a singleton.
(ii) The neighborhood of x coincides in all condition attributes and decision attributes at the same time.

(2) We can contract the condition attributes when has the same membership function of objects.

Proposition 2.6. *Let* $\mathrm{K} = (\mathrm{U},\ \mathrm{AT} = \{\mathrm{C},\ \mathrm{D}\},\ \mathrm{v_i} \in \mathrm{V})$ *be an information system and for every* $\mathrm{x} \in \mathrm{U}$, $\mathrm{D} \in \frac{\mathrm{U}}{\mathrm{D}}$. *Then, the membership function of neighborhood of x for all condition attributes with respect to decision attributes given by*

$$\mu^{\mathrm{K}}_{\mathrm{D}}(\mathrm{x}) = \frac{\sum_{\mathrm{i,j}} |\mathrm{N_i}(\mathrm{x}) \cap \mathrm{D_j}|}{\sum_{\mathrm{i}} |\mathrm{N_i}(\mathrm{x})|},\ \forall\ \mathrm{x} \in \mathrm{D_j},\ \mathrm{i},\ \mathrm{j} = 1,\ 2,\ \ldots$$

Proof. Similar to Proposition 2.2. □

3. Different accuracies in terms of topology

The following procedures will give the first common description of a topological Hausdorff space as a collection or set of elements for each of which certain subsets of the set of so-called neighborhoods correspond:

(i) To each point x, there corresponds at least one neighbourhood $R(x)$, and $R(x)$ contains x.

(ii) If $\mathrm{R_1}(\mathrm{x})$ and $\mathrm{R_2}(\mathrm{x})$ are neighbourhoods of x, there exists another neighbourhood of x, $R(x)$, which is a subset of $\mathrm{R_1}(\mathrm{x})$ and of $\mathrm{R_2}(\mathrm{x})$.

(iii) If y is in $R(x)$, there is a neighbourhood $R(y)$ of y such that $R(y)$ is a subset of $R(x)$.

(iv) For two distinct points x and y, there exist two neighborhoods, $R(x)$ and $R(y)$ with no points in common.

Remark 3.1. *(i) Every attribute of the approximation space in an information system is a base for a quasi discrete topology which every set is open and closed the same time.*

(ii) The number of topologies which form of any information system is 2^n, *where n is the number of attributes in the information system.*

(iii) The accuracy measure between information table attributes is $\alpha_D(C) = \frac{\sum |l_P(X_i)|}{\sum |u_P(X_i)|}$, $X_i \epsilon \frac{U}{C}$, *expresses the ratio of correct possibility of topologies with respect decision attributes.*

(iv) The quality of the approximation space which generated from an information system between topologies is $\gamma_D(C) = \frac{\sum |l_P(X_i)|}{|U|}$, $X_i \epsilon \frac{U}{C}$, *express the ratio of correct classification.*

(v) Using the accuracy measure and quality to choose the best attributes in classification and retracted the weak one and choose decision attribute which have the best ratio.

Proposition 3.2. *Let* K = (U, AT ={C, D}, v_i ∈V) *be an information system define quasi discrete topologies and for every $x,\ y \in U,\ x \neq y$ and $N(x),\ N(y)$ are neighbourhood of x and y. If $N(x)$ is bottom roughly definable, then $N(y)$ is top roughly definable.*

Proof. Let $x \in N(x)$ *and* $L(N(x)) \neq \emptyset$, then there is $x \in G \subset N(x)$ which implies $G \bigcap N(y) = \emptyset$, then $G \bigcap u(N(y)) = \emptyset$. Thus, $u(N(y)) \neq U$. □

Proposition 3.3. *Let* K = (U, AT ={C, D }, v_i∈V) *be an information system define quasi discrete topologies and for every $x,\ y \in U,\ x \neq y$ and $N(x),\ N(y)$ are neighborhood of x and y. If the membership function of $N(x)$ is unity, then the membership function of $N(y)$ is zero.*

Proof. The membership function of neighborhood of x is $\mu(x) = \frac{|N(x)\cap A|}{|N(x)|} = 1$, where $N(x) = A$. Since $\neq y$ then $N(x) \neq N(y)$ also, $N(x) \bigcap N(y) = \emptyset$. Thus, $A \bigcap N(y) = \emptyset$. Therefore, the membership function of y $\mu(y) = \frac{|N(y)\cap A|}{|N(y)|} = \frac{|\emptyset|}{|N(y)|} = 0$ □

Example 3.4. ***(Application information system)***

*The process starts with the attribute description for each class. The pre-set was modified during the test phase of the prototype power plant. Table 1 provides eight examples with the attributes: Frequency (**Freq.**), Amplitude (**Amp.**), Harmonic Level of Distortion (**THD**) and Distortion (**Dis.**). The outputs are normal, warning and danger. The universe set is $U = \{x_1\ ,\ x_2\ ,\ x_3\ ,\ x_4\ ,\ x_5\ ,\ x_6\ ,\ x_7\ ,\ x_8\ \}$ and the attribute set is { Freq., Amp., Dis., THD}.*

	Freq.	**Amp.**	**TDH**	**Dis.**	**Output**
$\boldsymbol{X}_1$	Low	Normal	Normal	Normal	Normal
$\boldsymbol{X}_2$	Low	Medium	Medium	Normal	Normal
$\boldsymbol{X}_3$	Low	Medium	Normal	High	Normal
$\boldsymbol{X}_4$	Medium	Medium	Normal	Medium	Warm
$\boldsymbol{X}_5$	Medium	Medium	Normal	High	Warm
$\boldsymbol{X}_6$	Medium	High	Normal	High	Danger
$\boldsymbol{X}_7$	High	High	Medium	Medium	Danger
$\boldsymbol{X}_8$	Medium	High	Normal	Medium	Danger

Table 1: Test phase in the power plant.

Pawlak approximation spaces depend on the complete similarity between objects in relation to each attribute and each object in the same neighborhood if they have the same attribute values in the information system, so this operation forms space partitions. This partition is a special type of topology called quasi discrete topology, every neighbourhood is at the same time open and closed.

In this example, we try to obtain topological spaces of decision attributes regarding condition attributes on a several basis of objects. Use object's neighbourhood belongs to U with the following attributes and decision attributes being recognized as follows:

$$N_i^A(x) = \{y :\quad a(x) = a(y) = v_i,\ \forall\ v_i \in \quad V\}$$

Then, from the last table, we have all the attributes of the situation on all the neighbourhoods:

$$N_{Freq.}^A(x) = \{\{x_1,\ x_2,\ x_3\}\ ,\ \{x_4,\ x_5,\ x_6,\ x_8\}\ ,\ \{x_7\}\}$$

$$N_{Amp.}^A(x) = \{\ \{x_1\}, \{\ x_2,\ x_3,\ x_4,\ x_5\}\ , \{x_6, x_7,\ x_8\}\}$$

$$\mathrm{N}_{\mathrm{TDH}}^{\mathrm{A}}(x) = \{\ \{x_1,\ x_3,\ x_4,\ x_5,\ x_6,\ x_8\}\ ,\ \{x_2,\ x_7\}\}$$

$$\mathrm{N}^{\mathrm{A}}_{\mathrm{Dis}}(x) = \{\{x_1, x_2\}, \{x_3, x_5, x_6\}, \{x_4, x_7, x_8\}\}$$

Then, from Table 1, we have all neighborhoods with respect to decision attribute:

$$\mathrm{N}^{\mathrm{A}}_{\mathrm{Out}}(\mathrm{x}) = \{\{\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3\}, \{\mathrm{x}_4, \mathrm{x}_5\}, \{\mathrm{x}_6, \mathrm{x}_7, \mathrm{x}_8\}\}$$

(i) The membership function of several neighbourhoods of x with respect to $A = \{x_1, x_2, x_3\}$.

$$\mu_{\mathrm{freq.}}(\mathrm{x}_1) = \frac{|\{\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3\} \cap \{\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3\}|}{\{\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3\}} = 1 = \mu_{\mathrm{freq.}}(\mathrm{x}_2) = \mu_{\mathrm{freq.}}(\mathrm{x}_3)$$

$$\mu_{\mathrm{freq.}}(\mathrm{x}_4) = \frac{|\{\mathrm{x}_4, \mathrm{x}_5, \mathrm{x}_6, \mathrm{x}_8\} \cap \{\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3\}|}{|\{\mathrm{x4}, \mathrm{x5}, \mathrm{x6}, \mathrm{x8}\}|} = 0 = \mu_{\mathrm{freq.}}(\mathrm{x}_5) = \mu_{\mathrm{freq.}}(\mathrm{x}_6) = \mu_{\mathrm{freq.}}(\mathrm{x}_8)$$

(ii) The membership function of singleton neighbourhood of x

$$\mu_{\mathrm{amp.}}(\mathrm{x}_1) = \frac{|\{\mathrm{x}_1\} \cap \{\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3\}|}{|\{\mathrm{x}_1\}|} = 1 \ , \ \mu_{\mathrm{amp.}}(\mathrm{x}_2) = \frac{|\{\mathrm{x}_2, \mathrm{x}_3, \mathrm{x}_4, \mathrm{x}_5\} \cap \{\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3\}|}{|\{\mathrm{x}_2, \mathrm{x}_3, \mathrm{x}_4, \mathrm{x}_5\}|} = \frac{1}{2}$$

(iii) The accuracy measure and quality of information system attributes.

$$\alpha_{\mathrm{output}}(\mathrm{freq.}) = \frac{2+0+0}{5+6+3} = \frac{1}{7}, \ \ \gamma_{\mathrm{out}}(\mathrm{freq.}) = \frac{2+0+0}{8} = 1/4$$

$$\alpha_{\mathrm{freq.}}(\mathrm{output}) = \frac{1+0+0}{5+7+3} = \frac{1}{15}, \ \ \gamma_{\mathrm{freq}}(\mathrm{output}) = \frac{1+0+0}{8} = 1/8$$

(iv) The membership function of information system neighbourhood with respect to all decisions based on Proposition 2.2.

$$\mu^{\mathrm{A}}_{\mathrm{E}\in\frac{\mathrm{U}}{\mathrm{D}}}(\mathrm{x}) = \frac{\sum_{\mathrm{i}} |\mathrm{N}_{\mathrm{i}}(\mathrm{x}) \cap \mathrm{E}|}{\sum_{\mathrm{i}} |\mathrm{N}_{\mathrm{i}}(\mathrm{x})|}, \ \mathrm{i} = 1, \ 2, \ \ldots..$$

Let $\mathrm{E}_1 = \{\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3\}$

$$\mu^{\mathrm{A}}_{\mathrm{E1}\in\frac{\mathrm{U}}{\mathrm{D}}}(\mathrm{x}_{\mathrm{i}}) = \frac{|\mathrm{N}_{\mathrm{freq.}}(\mathrm{x}_{\mathrm{i}}) \cap \mathrm{E1}| + |\mathrm{N}_{\mathrm{Amp}}(\mathrm{x}_{\mathrm{i}}) \cap E_1| + |\mathrm{N}_{\mathrm{TDH}}(\mathrm{x}_{\mathrm{i}}) \cap \mathrm{E1}| + |\mathrm{N}_{\mathrm{Dis.}}(\mathrm{x}_{\mathrm{i}}) \cap \mathrm{E}_1|}{|\mathrm{N}_{\mathrm{freq.}}(\mathrm{x}_{\mathrm{i}})| + |\mathrm{N}_{\mathrm{Amp.}}(\mathrm{x}_{\mathrm{i}})| + |\mathrm{N}_{\mathrm{TDH}}(\mathrm{x}_{\mathrm{i}})| + |\mathrm{N}_{\mathrm{Dis.}}(\mathrm{x}_{\mathrm{i}})|}$$

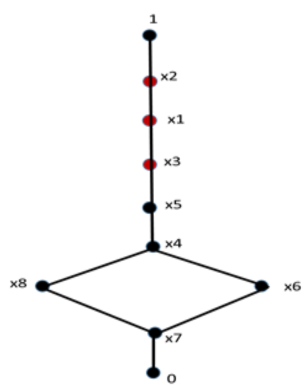

Figure 2: Lattice for E_1 in Table 2.

Therefore, $\mathrm{M} = \frac{\sum_{\mathrm{i}=1}^{\mathrm{i}=8} \mu^{\mathrm{A}}_{\mathrm{E1}}(x_i)}{|\mathrm{U}|} = 0.36.$

$\mu^A_{E1}(x_1)$	0.66	$\mu^A_{E1}(x_2)$	0.72
$\mu^A_{E1}(x_3)$	0.50	$\mu^A_{E1}(x_4)$	0.24
$\mu^A_{E1}(x_5)$	0.29	$\mu^A_{E1}(x_6)$	0.19
$\mu^A_{E1}(x_7)$	0.11	$\mu^A_{E1}(x_8)$	0.19

Table 2: The accuracy for $E_1 = \{x_1, x_2, x_3\}$ in Figure 2.

Let $E_2 = \{x_4, x_5\}$

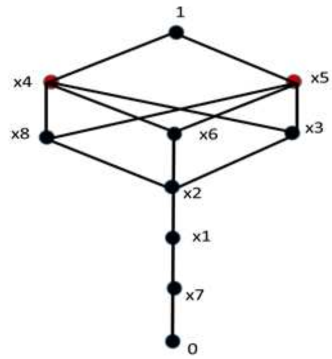

Figure 3: Lattice for E_2 in Table 3.

$\mu^A_{E2}(x_1)$	0.17	$\mu^A_{E2}(x_2)$	0.18
$\mu^A_{E2}(x_3)$	0.31	$\mu^A_{E2}(x_4)$	0.41
$\mu^A_{E2}(x_5)$	0.41	$\mu^A_{E2}(x_6)$	0.31
$\mu^A_{E2}(x_7)$	0.11	$\mu^A_{E2}(x_8)$	0.31

Table 3: The accuracy for $E_2 = \{x_4, x_5\}$ in Figure 3.

Therefore, $M = \frac{\sum_{i=1}^{i=8} \mu^A_{E2}(x_i)}{|U|} = 0.28$.

Let $E_3 = \{x_6, x_7, x_8\}$.

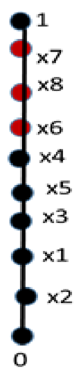

Figure 4: Lattice for E_3 in Table 4.

$\mu^{A}_{E3}(x_1)$	0.17	$\mu^{A}_{E3}(x_2)$	0.29
$\mu^{A}_{E3}(x_3)$	0.10	$\mu^{A}_{E3}(x_4)$	0.50
$\mu^{A}_{E3}(x_5)$	0.19	$\mu^{A}_{E3}(x_6)$	0.78
$\mu^{A}_{E3}(x_7)$	0.35	$\mu^{A}_{E3}(x_8)$	0.56

Table 4: The accuracy for $E_3 = \{x_6, x_7, x_8\}$ in Figure 4.

Therefore, $M = \frac{\sum_{i=1}^{i=8} \mu^{A}_{E3}(x_i)}{|U|} = 0.37$.

(v) The membership function of information system with respect to neighborhood object in condition and decision attributes together (See, Proposition 2.6)

$$\mu_D^A(x) = \frac{\sum_{i,j} |N_i(x) \cap D_j|}{\sum_i |N_i(x)|}, \ \forall\ x \ \in D_j,\ i,\ j = 1, 2, \ldots$$

$$\mu_D^A(x) = \frac{|N_1(x) \cap E_j| + |N_2(x) \cap \ E_j| + \ldots\ldots\ldots\cdots + |N_i(x) \cap E_j|}{|N_1(x)| + |N_2(x)| + \ldots\cdots + |N_i(x)|}, \forall x \in D_j,\ i,\ j = 1, 2, \ldots$$

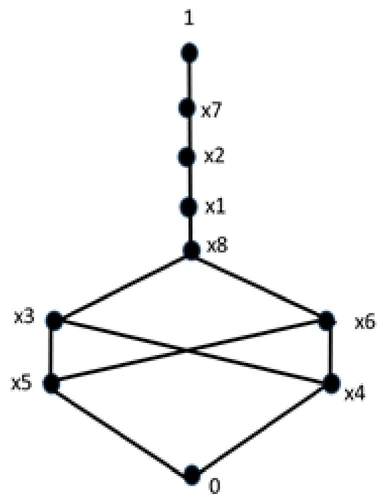

Figure 5: Lattice for U in Table 5.

$\mu^{A}_{E1}(x_3)$	0.50	$\mu^{A}_{E1}(x_4)$	0.41
$\mu^{A}_{E1}(x_5)$	0.41	$\mu^{A}_{E1}(x_6)$	0.50
$\mu^{A}_{E1}(x_7)$	0.78	$\mu^{A}_{E1}(x_8)$	0.56

Table 5: The accuracy for Table 1 in Figure 5.

Therefore, $M = \frac{\sum_{i=1}^{i=8} \mu^{A}_{E1}(xi)}{|U|} = 0.57$.

4. A comparison and explanation results

A comparison between our membership function method and Pawlak method is shown Table 6.

	Pawlak accuracy	Condition accuracy			Decision accuracy
		E1	E2	E3	
x_1	0.49	0.66	0.17	0.17	0.66
x_2	0.43	0.72	0.18	0.29	0.73
x_3	0.43	0.50	0.31	0.10	0.50
x_4	0.38	0.24	0.41	0.50	0.41
x_5	0.43	0.29	0.41	0.19	0.41
x_6	0.33	0.19	0.31	0.78	0.50
x_7	0.80	0.11	0.11	0.35	0.78
x_8	0.38	0.19	0.31	0.56	0.56
M	0.46	0.36	0.28	0.37	0.57

Table 6: A comparison results.

From Table 6, we observe that every object of the universe shows its factor and advantages will be fully determined for each element within space through its object's neighborhood. We have all the information about events that occur in the same neighborhood through similarity relation. Consequently, these objects ' conditions and decision attributes are fully defined. We use two membership functions, say, inner region's quality and outer region's quality for this purpose. Membership functions are used to measure the percentage of the element's participation in each neighborhood and the decision-making characteristics of each equivalence class, which is the ratio of what the neighborhood element shares with an internal set and the strength of the neighborhood itself. It implies that two sets, one internal and the other the same neighborhood, describe the neighborhood between them. The percentage and precision of the item were determined from its neighbourhood. The membership function can also be defined by the union of all neighborhoods of the element through the properties of attributes in the information system and then determined for each element the accuracy of all neighborhoods. Finally, the total accuracy is calculated. From the results, for each element in the information system, the best membership feature in our view is that determined by condition attributes in decision attributes with the neighbourhood of the element.

5. Conclusion

Pawlak approximation spaces depended on a complete information system to obtain an equivalence relation that defined a particular topological space called quasi discrete topology. We have certain topological approaches from the information system in this work. Some topologist used a reduction of topological structures to solve information reduction. We have proved the knowledge reduction for each object through the accuracy and measure the Mathematical Mean for them. We are contrasting these results. The remainder of the approximation reduction will be ignored. In our analysis, a mixture of these data would divide the data in an information system into conditions, decision data and any topic. We also introduce new methods for calculating each object's accurate measurement depending on both conditions and decision data to provide approximately an accurate measure to solve some issues. We have used it to provide totally undefinable lower rough sets or a maximum boundary region. In future, we will apply our results on topological structures induced by neighbourhoods, graphs and the results in [34,35,36,37].

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