Bol. Soc. Paran. Mat.
©SPM –ISSN-2175-1188 on line
SPM: www.spm.uem.br/bspm

(3s.) **v. 2023 (41)** : 1–12.
ISSN-0037-8712 in press
doi:10.5269/bspm.51003

Binary Bipolar Soft Sets

Naime Demirtaş and Orhan Dalkılıç

ABSTRACT: In this paper, we give on interesting connection between two mathematical approaches to vagueness: bipolar soft sets and binary soft sets. The nation of binary bipolar soft set over two universal sets and a parameter set is proposed. The complement, union, intersection, restricted union, restricted intersection, null binary bipolar soft set, absolute binary bipolar soft set, difference of two binary bipolar soft sets, "AND", "OR" operations are defined on the binary bipolar soft sets. The basic properties of binary bipolar soft sets are also investigated and discussed. Finally we give a characteristic function of binary bipolar soft set.

Key Words: Bipolar soft set, binary soft set, binary bipolar soft set.

Contents

1. Introduction

To overcome complex problems containing uncertainty has made many studies. For this; the concept of soft sets was initiated by Molodtsov [1]. This set theory has become an important theory presented against uncertainty in many areas in economics, engineering, social science, medical science, etc. Later Maji et al. [8] presented some new definitions on soft sets such as a subset, the complement of a soft set. Furthermore, many studies on set theoretical aspects of soft sets were made. Some of them can be seen in references [4,5,6] and [7].

Concept of bipolar soft set and its operations such as union, intersection and complement were first defined by Shabir and Naz [2]. Also Shabir and Naz [2] defined bipolar soft sets and presented an application of bipolar soft sets in a decision making problem. Then, Karaaslan and Karataş [3] redefined bipolar soft sets with a new approximation providing opportunity to study on topological structures of bipolar soft sets.

Açıkgöz and Tas [9] introduced the concept of binary soft set theory on two initial universal sets. Then, Benchalli et al. [10] related basic properties which are defined over two initial universal sets with suitable parameters.

In this paper, we propose a novel concept of binary bipolar soft set which is an extension of bipolar soft sets and binary soft sets. We present its basic operations, namely complement, union, intersection, AND, OR and investigate its basic properties. In the last section, we identified a characteristic function for binary bipolar soft sets which utilizing characteristic function given by Açıkgöz and Tas [9].

2. Preliminaries

First we recall some basic notions in soft sets ve bipolar soft sets.

Let U be an initial universe and E be a set of parameters. Let $P(U)$ denotes the power set of U and A, B, C be non-empty subsets of E.

2010 *Mathematics Subject Classification:* 03E20,03E72,28E15.
Submitted November 15, 2019. Published May 10, 2021

Definition 2.1. *[1] A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe X . For $e \in A$, $F(e)$ may be considered as the set of e-approximate elements of the soft set (F, A).*

Definition 2.2. *[8] Let $E = \{e_1, e_2, ..., e_n\}$ be a set of parameters. The NOT set of E denoted by $\neg E$ is defined by $\neg E = \{\neg e_1, \neg e_2, ..., \neg e_n\}$ where, $\neg e_i =$ not e_i for all i.*

Definition 2.3. *[2] A triplet (F, G, A) is called a bipolar soft set over U, where F and G are mappings, given by $F : A \to P(U)$ and $G : \neg A \to P(U)$ such that $F(e) \cap G(\neg e) = \emptyset$ for all $e \in A$.*

Definition 2.4. *[2] For two bipolar soft sets (F, G, A) and (F_1, G_1, B) over a universe U, we say that (F, G, A) is a bipolar soft subset of (F_1, G_1, B) if*
(1) $A \subseteq B$ and
(2) $F(e) \subseteq F_1(e)$ and $G_1(\neg e) \subseteq G(\neg e)$ for all $e \in A$.
This relationship is denoted by $(F, G, A)\tilde{\subseteq}(F1, G1, B)$. Similarly (F, G, A) is said to be a bipolar soft superset of (F_1, G_1, B) if (F_1, G_1, B) is a bipolar soft subset of (F, G, A). We denote it by

$$(F, G, A)\tilde{\supseteq}(F_1, G_1, B).$$

Definition 2.5. *[2] Two bipolar soft sets (F, G, A) and (F_1, G_1, B) over a universe U are said to be equal if (F, G, A) is a bipolar soft subset of (F_1, G_1, B) and (F_1, G_1, B) is a bipolar soft subset of (F, G, A).*

Definition 2.6. *[2] The complement of a bipolar soft set (F, G, A) is denoted by $(F, G, A)^c$ and is defined by $(F, G, A)^c = (F^c, G^c, A)$ where F^c and G^c are mappings given by $F^c(e) = G(\neg e)$ and $G^c(\neg e) = F(e)$ for all $e \in A$.*

Definition 2.7. *[2] A bipolar soft set over U is said to be a relative null bipolar soft set, denoted by $(\Phi, \mathfrak{U}, A)$ if for all $e \in A$, $\Phi(e) = \emptyset$ and $\mathfrak{U}(\neg e) = U$, for all $e \in A$.*

Definition 2.8. *[2] A bipolar soft set over U is said to be a absolute null bipolar soft set, denoted by $(\mathfrak{U}, \Phi, A)$ if for all $e \in A$, $\mathfrak{U}(e) = U$ and $\Phi(\neg e) = \emptyset$, for all $e \in A$.*

Definition 2.9. *[2] If (F, G, A) and (F_1, G_1, B) are two bipolar soft sets over U then " (F, G, A) and (F_1, G_1, B)" denoted by $(F, G, A) \wedge (F_1, G_1, B)$ is defined by*

$$(F, G, A) \wedge (F_1, G_1, B) = (H, I, A \times B)$$

where $H(a, b) = F(a) \cap F_1(b)$ and $I(\neg a, \neg b) = G(\neg a) \cup G_1(\neg b)$, for all $(a, b) \in A \times B$.

Definition 2.10. *[2] If (F, G, A) and (F_1, G_1, B) are two bipolar soft sets over U then " (F, G, A) or (F_1, G_1, B)" denoted by $(F, G, A) \vee (F_1, G_1, B)$ is defined by*

$$(F, G, A) \vee (F_1, G_1, B) = (H, I, A \times B)$$

where $H(a, b) = F(a) \cup F_1(b)$ and $I(\neg a, \neg b) = G(\neg a) \cap G_1(\neg b)$, for all $(a, b) \in A \times B$.

Definition 2.11. *[2] Extended Union of two bipolar soft sets (F, G, A) and (F_1, G_1, B) over the common universe U is the bipolar soft set (H, I, C) over U, where $C = A \cup B$ and for all $e \in C$,*

$$H(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ F_1(e) & \text{if } e \in B - A \\ F(e) \cup F_1(e) & \text{if } e \in A \cap B \end{cases}$$

$$I(\neg e) = \begin{cases} G(\neg e) & \text{if } \neg e \in (\neg A) - (\neg B) \\ G_1(\neg e) & \text{if } \neg e \in (\neg B) - (\neg A) \\ G(\neg e) \cap G_1(\neg e) & \text{if } \neg e \in (\neg A) \cap (\neg B) \end{cases}$$

We denote it by $(F, G, A)\tilde{\cup}(F_1, G_1, B) = (H, I, C)$.

Definition 2.12. *[2] Extended Intersection of two bipolar soft sets (F,G,A) and (F_1,G_1,B) over the common universe U is the bipolar soft set (H,I,C) over U, where $C=A\cup B$ and for all $e\in C$,*

$$H(e)=\begin{cases} F(e) & \text{if } e\in A-B \\ F_1(e) & \text{if } e\in B-A \\ F(e)\cap F_1(e) & \text{if } e\in A\cap B \end{cases}$$

$$I(\neg e)=\begin{cases} G(e) & \text{if } e\in (\neg A)-(\neg B) \\ G_1(e) & \text{if } e\in (\neg B)-(\neg A) \\ G(e)\cup G_1(e) & \text{if } e\in (\neg A)\cap(\neg B) \end{cases}$$

We denote it by $(F,G,A)\tilde{\cap}(F_1,G_1,B)=(H,I,C)$.

Definition 2.13. *[2] Restricted Union of two bipolar soft sets (F,G,A) and (F_1,G_1,B) over the common universe U is the bipolar soft set (H,I,C), where $C=A\cap B$ is non-empty and for all $e\in C$*

$$H(e)=F(e)\cup F_1(e) \quad \text{and} \quad I(\neg e)=G(\neg e)\cap G_1(\neg e)$$

We denote it by $(F,G,A)\cup_{\mathfrak{R}}(F_1,G_1,B)=(H,I,C)$.

Definition 2.14. *[2] Restricted Intersection of two bipolar soft sets (F,G,A) and (F_1,G_1,B) over the common universe U is the bipolar soft set (H,I,C), where $C=A\cap B$ is non-empty and for all $e\in C$*

$$H(e)=F(e)\cap F_1(e) \quad \text{and} \quad I(\neg e)=G(\neg e)\cup G_1(\neg e)$$

We denote it by $(F,G,A)\cap_{\mathfrak{R}}(F_1,G_1,B)=(H,I,C)$.

Now let's recall some basic definitions for binary soft sets. Let U_1, U_2 be two initial universe sets and E be a set of parameters. Let $P(U_1)$, $P(U_2)$ denote the power set of U_1, U_2, respectively. Also, let $A,B,C\subseteq E$.

Definition 2.15. *[9] A pair (F,A) is said to be a binary soft set over U_1, U_2, where F is defined as below:*
$F:A\to P(U_1)\times P(U_2)$, $F(e)=(X,Y)$ for each $e\in A$ such that $X\subseteq U_1$, $Y\subseteq U_2$.

Definition 2.16. *[9] A binary soft set (F,A) over U_1, U_2 is called a binary null soft set, denoted by $\tilde{\tilde{\phi}}$ if $F(e)=(\phi,\phi)$ for each $e\in A$.*

Definition 2.17. *[9] A binary soft set (G,A) over U_1, U_2 is called a binary absolute soft set, denoted by $\tilde{\tilde{A}}$ if $F(e)=(U_1,U_2)$ for each $e\in A$.*

Definition 2.18. *[9] The complement of a binary soft set (F,A) is denoted by $(F,A)^c$ and is defined $(F,A)^c=(F^c,\neg A)$, where $F^c:\neg A\to P(U_1)\times P(U_2)$ is a mapping given by $F^c(e)=(U_1-X,U_2-Y)$ such that $F(e)=(X,Y)$. Clearly, $((F,A)^c)^c=(F,A)$.*

Definition 2.19. *[9] The union of two binary soft sets of (F,A) and (G,B) over the common niverse U_1, U_2 is the binary soft set (H,C) , where $C=A\cup B$ and for all $e\in C$,*

$$H(e)=\begin{cases} (X_1,Y_1) & \text{if } e\in A-B \\ (X_2,Y_2) & \text{if } e\in B-A \\ (X_1\cup X_2, Y_1\cup Y_2) & \text{if } e\in A\cap B \end{cases}$$

such that $F(e)=(X_1,Y_1)$ for each $e\in A$ and $G(e)=(X_2,Y_2)$ for each $e\in B$. We denote it as $(F,A)\tilde{\tilde{\cup}}(G,A)=(H,C)$.

Definition 2.20. *[9] The intersection of two binary soft sets (F,A) and (G,B) over a common U_1, U_2 is the binary soft set (H,C), where $C=A\cap B$, and $(H,E)=(X_1\cap X_2,Y_1\cap Y_2)$ for each $e\in C$ such that $F(e)=(X_1,Y_1)$ for each $e\in A$ and $G(e)=(X_2,Y_2)$ for each $e\in B$. We denote it as $(F,A)\tilde{\tilde{\cap}}(G,B)=(H,C)$.*

Definition 2.21. *[9] Let (F, A) and (G, B) be two binary soft sets over a common U_1, U_2. (F, A) is called a binary soft subset of (G, B) if*
(i) $A \subseteq B$,
(ii) $X_1 \subseteq X_2$ and $Y_1 \subseteq Y_2$ such that $F(e) = (X_1, Y_1)$, $G(e) = (X_2, Y_2)$ for each $e \in A$. We denote it as $(F, A)\widetilde{\widetilde{\subseteq}}(G, B)$.

Definition 2.22. *[9] The difference of two binary soft sets (F, A) and (G, A) over the common U_1, U_2 is the binary soft set (H, A), where $H(e) = (X_1 - X_2, Y_1 - Y_2)$ for each $e \in A$ such that $(F, A) = (X_1, Y_1)$ and $(G, A) = (X_2, Y_2)$.*

Definition 2.23. *[9] If (F, A) and (G, B) are two binary soft sets then "(F, A) AND (G, B)" denoted by $(F, A)\widetilde{\widetilde{\wedge}}(G, B)$ is defined by $(F, A)\widetilde{\widetilde{\wedge}}(G, B) = (H, A \times B)$, where $H(e, f) = (X_1 \cap X_2, Y_1 \cap Y_2)$ for each $(e, f) \in A \times B$ such that $F(e) = (X_1, Y_1)$ and $G(e) = (X_2, Y_2)$.*

Definition 2.24. *[9] If (F, A) and (G, B) are two binary soft sets then "(F, A) OR (G, B)" denoted by $(F, A)\widetilde{\widetilde{\vee}}(G, B)$ is defined by $(F, A)\widetilde{\widetilde{\vee}}(G, B) = (O, A \times B)$, where $O(e, f) = (X_1 \cup X_2, Y_1 \cup Y_2)$ for each $(e, f) \in A \times B$ such that $F(e) = (X_1, Y_1)$ and $G(e) = (X_2, Y_2)$.*

3. Binary Bipolar Soft Sets

Likewise, let U_1, U_2 be two initial universe sets and E be a set of parameters. Let $P(U_1)$, $P(U_2)$ denote the power set of U_1, U_2, respectively. Also, let $A, B, C \subseteq E$.

Definition 3.1. *A triplet (F, G, A) is called a binary bipolar soft set over U_1, U_2, where F and G are mappings, given by $F : A \to P(U_1) \times P(U_2)$ and $G : \neg A \to P(U_1) \times P(U_2)$ such that $\hat{X}, \check{X} \subseteq U_1$, $\hat{Y}, \check{Y} \subseteq U_2$ and $F(e)\widetilde{\cap}G(\neg e) = (\hat{X}, \hat{Y})\widetilde{\cap}(\check{X}, \check{Y}) = \emptyset$ for all $e \in A$.*

Example 3.2. *Let $U_1 = \{j_1, j_2, j_3, j_4\}$, $U_2 = \{t_1, t_2, t_3\}$ be the universes containing four jackets and three t-shirts, respectively. Also, let $E = \{e_1, e_2, e_3, e_4\} = \{cheap, traditional, large, colored\}$ and $\neg E = \{\neg e_1, \neg e_2, \neg e_3, \neg e_4\} = \{expensive, classic, small, colorless\}$ be the sets of parameters.*

The binary bipolar soft set (F, G, A) describes the "requirements of both the jackets and the t-shirts" which Mr. X is going to buy , where $A = \{e_1, e_3, e_4\} \subseteq E$. (F, G, A) is a binary bipolar soft set over U_1, U_2 defined as follows:

$$F(e_1) = (\{j_1, j_3\}, \{t_2\}),\ F(e_3) = (\{j_2\}, \{t_1, t_3\}),\ F(e_4) = (\{j_1, j_4\}, \{t_2\}),$$

$$G(\neg e_1) = (\{j_2, j_4\}, \{t_1, t_3\}),\ G(\neg e_3) = (\{j_1\}, \{t_2\}),\ G(\neg e_4) = (\{j_2, j_3\}, \{t_3\}).$$

So, we can say the binary soft set

$$\begin{aligned}(F, G, A) = \{&cheap\ jackets,\ t-shirts : resp.\ \{j_1, j_3\}, \{t_2\},\\ &large\ jackets,\ t-shirts : resp.\{j_2\}, \{t_1, t_3\},\\ &colored,\ jackets,\ t-shirts : resp.\{j_1, j_4\}, \{t_2\},\\ &expensive,\ jackets,\ t-shirts : resp.\{j_2, j_4\}, \{t_1, t_3\},\\ &small,\ jackets,\ t-shirts : resp.\{j_1\}, \{t_2\},\\ &colorless,\ jackets,\ t-shirts : resp.\{j_2, j_3\}, \{t_3\}\}\end{aligned}$$

We denote the binary soft set (F, G, A) as below:

$$(F, G, A) = (e_1, (\{j_1, j_3\}, \{t_2\})), (e_3, (\{j_2\}, \{t_1, t_3\})), (e_4, (\{j_1, j_4\}, \{t_2\})),$$

$$(\neg e_1, (\{j_2, j_4\}, \{t_1, t_3\})), (\neg e_3, (\{j_1\}, \{t_2\})), (\neg e_4, (\{j_2, j_3\}, \{t_3\}))$$

In this example, we can see the views of Mr. X who wants or does not wants to buy both jackets and t-shirts under contrasting sets of parameters.

Definition 3.3. *For two bipolar soft sets (F, G, A) and (F_1, G_1, B) over a universe U_1, U_2, we say that (F, G, A) is a binary bipolar soft subset of (F_1, G_1, B), if*
(1) $A \subseteq B$,
(2) $X_1 \subseteq X_2$ and $Y_1 \subseteq Y_2$ such that $F(e)(= (\hat{X}_1, \hat{Y}_1)) \tilde{\tilde{\subseteq}} F_1(e)(= (\hat{X}_2, \hat{Y}_2))$ and

$$G_1(\neg e)(= (\check{X}_2, \check{Y}_2)) \tilde{\tilde{\subseteq}} G(\neg e)(= (\check{X}_1, \check{Y}_1))$$

for all $e \in A$.
This relationship is denoted by $(F, G, A) \tilde{\tilde{\sqsubseteq}} (F1, G1, B)$. Similarly (F, G, A) is said to be a binary bipolar soft superset of (F_1, G_1, B), if (F_1, G_1, B) is a binary bipolar soft subset of (F, G, A). We denote it by $(F, G, A) \tilde{\tilde{\sqsupseteq}} (F_1, G_1, B)$.

Example 3.4. *Let $U_1 = \{m_1, m_2, m_3, m_4\}$, $U_2 = \{n_1, n_2, n_3\}$, $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ and $\neg E = \{\neg e_1, \neg e_2, \neg e_3, \neg e_4, \neg e_5, \neg e_6\}$. Let $A = \{e_3, e_5\}$ and $B = \{e_1, e_3, e_5, e_6\}$. (F, G, A), (F_1, G_1, B) are two binary bipolar soft sets over U_1, U_2 defined as follows:*

$$(F, G, A) = \{(e_3, (\{m_2, m_4\}, \{n_2\})), (\neg e_3, (\{m_1, m_3\}, \{n_1\})), (e_5, (\{m_3\}, \{n_1\})), (\neg e_5, (\{m_2\}, \{n_3\}))$$

$$(F_1, G_1, B) = \{(e_1, (\{m_3, m_4\}, \{n_1\})), (\neg e_1, (\{m_1, m_2\}, \{n_2, n_3\})), (e_3, (\{m_1, m_2, m_4\}, \{n_2\})),$$
$$(\neg e_3, (\{m_3\}, \{n_1\})), (e_5, (\{m_1, m_3\}, \{n_1, n_2\})), (\neg e_5, (\{\emptyset\}, \{n_3\})), (e_6, (\{m_1\}, \{n_1, n_2\})),$$
$$(\neg e_6, (\{m_2\}, \{n_3\}))\}$$

Therefore, $(F, G, A) \tilde{\tilde{\sqsubseteq}} (F1, G1, B)$.

Definition 3.5. *Two binary bipolar soft sets (F, G, A) and (F_1, G_1, B) over a universe U are said to be equal if (F, G, A) is a binary bipolar soft subset of (F_1, G_1, B) and (F_1, G_1, B) is a binary bipolar soft subset of (F, G, A).*

Definition 3.6. *The complement of a binary bipolar soft set (F, G, A) is denoted by $(F, G, A)^c$ and is defined by $(F, G, A)^c = (F^c, G^c, A)$ where $F^c : A \to P(U_1) \times P(U_2)$ and $G^c : \neg A \to P(U_1) \times P(U_2)$ are mappings given by $F^c(e) = (U_1 - \hat{X}, U_2 - \hat{Y}) = (\check{X}, \check{Y}) = G(\neg e)$ and $G^c(\neg e) = (U_1 - \check{X}, U_2 - \check{Y}) = (\hat{X}, \hat{Y}) = F(e)$ for all $e \in A$.*

Example 3.7. *Consider Example 3.2. Then*

$$(F, G, A)^c = \{not\ cheap\ jackets,\ t-shirts : resp.\ \{j_2, j_4\}, \{t_1, t_3\},$$
$$not\ large\ jackets,\ t-shirts : resp.\{j_1, j_3, j_4\}, \{t_2\},$$
$$not\ colored,\ jackets,\ t-shirts : resp.\{j_2, j_3\}, \{t_1, t_3\},$$
$$not\ expensive,\ jackets,\ t-shirts : resp.\{j_1, j_3\}, \{t_2\},$$
$$not\ small,\ jackets,\ t-shirts : resp.\{j_2, j_3, j_4\}, \{t_1, t_3\},$$
$$not\ colorless,\ jackets,\ t-shirts : resp.\{j_1, j_4\}, \{t_1, t_2\}\}$$

We denote the binary bipolar soft set $(F, G, A)^c$ as below:

$$(F, G, A)^c = (\neg e_1, (\{j_2, j_4\}, \{t_1, t_3\})), (\neg e_3, (\{j_1, j_3, j_4\}, \{t_2\})), (\neg e_4, (\{j_2, j_3\}, \{t_1, t_3\})),$$
$$(e_1, (\{j_1, j_3\}, \{t_2\})), (e_3, (\{j_2, j_3, j_4\}, \{t_1, t_3\})), (e_4, (\{j_1, j_4\}, \{t_1, t_2\}))$$

Definition 3.8. *A binary bipolar soft set over U_1, U_2 is said to be a null binary bipolar soft set, denoted by $\widetilde{\widetilde{(\Phi, \mathfrak{U}, A)}}$ if for all $e \in A$, $\Phi(e) = (\emptyset, \emptyset)$ and $\mathfrak{U}(\neg e) = (U_1, U_2)$, for all $e \in A$.*

Example 3.9. *Let* $U_1 = \{m_1, m_2\}$, $U_2 = \{n_1, n_2, n_3\}$, $A = \{e_1, e_2\}$ *and* $\neg A = \{\neg e_1, \neg e_2\}$. *Let* (F, G, A) *be a binary bipolar soft set as follows:*

$$(F, G, A) = \{(e_1, (\emptyset, \emptyset)), (\neg e_1, (U_1, U_2)), (e_2, (\emptyset, \emptyset)), (\neg e_2, (U_1, U_2))\}.$$

Therefore, (F, G, A) *is a null binary bipolar soft set.*

Definition 3.10. *A binary bipolar soft set over* U_1, U_2 *is said to be a absolute binary bipolar soft set, denoted by* $\widetilde{\widetilde{(\mathfrak{U}, \Phi, A)}}$ *if for all* $e \in A$, $\mathfrak{U}(e) = (U_1, U_2)$ *and* $\Phi(\neg e) = (\emptyset, \emptyset)$, *for all* $e \in A$.

Example 3.11. *Let* U_1, U_2, A *and* $\neg A$ *be sets as in Example 3.9. Let* (F, G, A) *be a binary bipolar soft set as follows:*

$$(F, G, A) = \{(e_1, (U_1, U_2)), (\neg e_1, (\emptyset, \emptyset)), (e_2, (U_1, U_2)), (\neg e_2, (\emptyset, \emptyset))\}.$$

Therefore, (F, G, A) *is a absolute binary bipolar soft set. Clearly,* $\widetilde{\widetilde{(\Phi, \mathfrak{U}, A)}}^c = \widetilde{\widetilde{(\mathfrak{U}, \Phi, A)}}$ *and* $\widetilde{\widetilde{(\mathfrak{U}, \Phi, A)}}^c = \widetilde{\widetilde{(\Phi, \mathfrak{U}, A)}}$.

Definition 3.12. *The difference of two binary bipolar soft sets* (F, G, A) *and* (F_1, G_1, A) *over the common universe* U_1, U_2 *is the binary bipolar soft set* (H, I, A), *where* $H(e) = (\hat{X}_1 - \hat{X}_2, \hat{Y}_1 - \hat{Y}_2)$ *and* $I(\neg e) = (\check{X}_1 - \check{X}_2, \check{Y}_1 - \check{Y}_2)$ *for each* $e \in A$ *such that* $F(e) = (\hat{X}_1, \hat{Y}_1)$, $G(\neg e) = (\check{X}_1, \check{Y}_1)$, $F_1(e) = (\hat{X}_2, \hat{Y}_2)$ *and* $G_1(\neg e) = (\check{X}_2, \check{Y}_2)$.

Definition 3.13. *If* (F, G, A) *and* (F_1, G_1, B) *are two binary bipolar soft sets over* U_1, U_2 *then "*(F, G, A) *and* (F_1, G_1, B)*" denoted by* $(F, G, A)\widetilde{\widetilde{\wedge}}(F_1, G_1, B)$ *is defined by*

$$(F, G, A)\widetilde{\widetilde{\wedge}}(F_1, G_1, B) = (H, I, A \times B)$$

where $H(a, b) = F(a)\widetilde{\cap}F_1(b) = (\hat{X}_1, \hat{Y}_1)\widetilde{\cap}(\hat{X}_2, \hat{Y}_2)$ *and* $I(\neg a, \neg b) = G(\neg a)\widetilde{\cup}G_1(\neg b) = (\check{X}_1, \check{Y}_1)\widetilde{\cup}(\check{X}_2, \check{Y}_2)$, *for all* $(a, b) \in A \times B$.

Definition 3.14. *If* (F, G, A) *and* (F_1, G_1, B) *are two binary bipolar soft sets over* U_1, U_2 *then "*(F, G, A) *or* (F_1, G_1, B)*" denoted by* $(F, G, A)\widetilde{\widetilde{\vee}}(F_1, G_1, B)$ *is defined by*

$$(F, G, A)\widetilde{\widetilde{\vee}}(F_1, G_1, B) = (H, I, A \times B)$$

where $H(a, b) = F(a)\widetilde{\cup}F_1(b) = (\hat{X}_1, \hat{Y}_1)\widetilde{\cup}(\hat{X}_2, \hat{Y}_2)$ *and* $I(\neg a, \neg b) = G(\neg a)\widetilde{\cap}G_1(\neg b) = (\check{X}_1, \check{Y}_1)\widetilde{\cap}(\check{X}_2, \check{Y}_2)$, *for all* $(a, b) \in A \times B$.

Proposition 3.15. *If* (F, G, A) *and* (F_1, G_1, B) *are two binary bipolar soft sets over* U_1, U_2 *then*
(1) $((F, G, A)\widetilde{\widetilde{\wedge}}(F_1, G_1, B))^c = (F, G, A)^c\widetilde{\widetilde{\vee}}(F_1, G_1, B)^c$
(2) $((F, G, A)\widetilde{\widetilde{\vee}}(F_1, G_1, B))^c = (F, G, A)^c\widetilde{\widetilde{\wedge}}(F_1, G_1, B)^c$

Proof. It is obvious from Definitions 3.6, 3.13 and 3.14. □

Definition 3.16. *Extended Union of two binary bipolar soft sets* (F, G, A) *and* (F_1, G_1, B) *over the common universes* U_1, U_2 *is the binary bipolar soft set* (H, I, C), *where* $C = A \cup B$ *and for all* $e \in C$,

$$H(e) = \begin{cases} (\hat{X}_1, \hat{Y}_1) & \text{if } e \in A - B \\ (\hat{X}_2, \hat{Y}_2) & \text{if } e \in B - A \\ (\hat{X}_1, \hat{Y}_1)\widetilde{\cup}(\hat{X}_2, \hat{Y}_2) & \text{if } e \in A \cap B \end{cases}$$

$$I(\neg e) = \begin{cases} (\check{X}_1, \check{Y}_1) & \text{if } \neg e \in (\neg A) - (\neg B) \\ (\check{X}_2, \check{Y}_2) & \text{if } \neg e \in (\neg B) - (\neg A) \\ (\check{X}_1, \check{Y}_1)\widetilde{\cap}(\check{X}_2, \check{Y}_2) & \text{if } \neg e \in (\neg A) \cap (\neg B) \end{cases}$$

such that $F(e) = (\hat{X}_1, \hat{Y}_1)$, $G(\neg e) = (\check{X}_1, \check{Y}_1)$ *for each* $e \in A$ *and* $F_1(e) = (\hat{X}_2, \hat{Y}_2)$, $G_1(\neg e) = (\check{X}_2, \check{Y}_2)$ *for each* $e \in B$. *In addition* $(\hat{X}_1, \hat{Y}_1)\widetilde{\cup}(\hat{X}_2, \hat{Y}_2) = (\hat{X}_1 \cup \hat{X}_2, \hat{Y}_1 \cup \hat{Y}_2)$ *and* $(\check{X}_1, \check{Y}_1)\widetilde{\cap}(\check{X}_2, \check{Y}_2) = (\check{X}_1 \cup \check{X}_2, \check{Y}_1 \cup \check{Y}_2)$. *We denote it by* $(F, G, A)\widetilde{\widetilde{\sqcup}}(F_1, G_1, B) = (H, I, C)$.

Definition 3.17. *Extended Intersection of two binary bipolar soft sets (F,G,A) and (F_1,G_1,B) over the common universes U_1, U_2 is the binary bipolar soft set (H,I,C), where $C = A\cup B$ and for all $e\in C$,*

$$H(e) = \begin{cases} (\hat{X}_1,\hat{Y}_1) & \text{if } e\in A-B \\ (\hat{X}_2,\hat{Y}_2) & \text{if } e\in B-A \\ (\hat{X}_1,\hat{Y}_1)\widetilde{\sqcap}(\hat{X}_2,\hat{Y}_2) & \text{if } e\in A\cap B \end{cases}$$

$$I(\neg e) = \begin{cases} (\check{X}_1,\check{Y}_1) & \text{if } \neg e\in (\neg A)-(\neg B) \\ (\check{X}_2,\check{Y}_2) & \text{if } \neg e\in (\neg B)-(\neg A) \\ (\check{X}_1,\check{Y}_1)\widetilde{\sqcup}(\check{X}_2,\check{Y}_2) & \text{if } \neg e\in (\neg A)\cap(\neg B) \end{cases}$$

such that $F(e) = (\hat{X}_1,\hat{Y}_1)$, $G(\neg e) = (\check{X}_1,\check{Y}_1)$ for each $e\in A$ and $F_1(e) = (\hat{X}_2,\hat{Y}_2)$, $G_1(\neg e) = (\check{X}_2,\check{Y}_2)$ for each $e\in B$. In addition $(\hat{X}_1,\hat{Y}_1)\widetilde{\sqcap}(\hat{X}_2,\hat{Y}_2) = (\hat{X}_1\cap\hat{X}_2,\hat{Y}_1\cap\hat{Y}_2)$ and $(\check{X}_1,\check{Y}_1)\widetilde{\sqcup}(\check{X}_2,\check{Y}_2) = (\check{X}_1\cap\check{X}_2,\check{Y}_1\cap\check{Y}_2)$. We denote it by $(F,G,A)\tilde{\tilde{\sqcap}}(F_1,G_1,B) = (H,I,C)$.

Definition 3.18. *Restricted Union of two binary bipolar soft sets (F,G,A) and (F_1,G_1,B) over the common universes U_1, U_2 is the binary bipolar soft set (H,I,C), where $C = A\cap B$ is non-empty and for all $e\in C$*

$$H(e) = F(e)\widetilde{\sqcup}F_1(e) \quad \text{and} \quad I(\neg e) = G(\neg e)\widetilde{\sqcap}G_1(\neg e)$$

such that $F(e) = (\hat{X}_1,\hat{Y}_1)$, $G(\neg e) = (\check{X}_1,\check{Y}_1)$ for each $e\in A$ and $F_1(e) = (\hat{X}_2,\hat{Y}_2)$, $G_1(\neg e) = (\check{X}_2,\check{Y}_2)$ for each $e\in B$. We denote it by $(F,G,A)\tilde{\tilde{\sqcup}}_{\mathfrak{R}}(F_1,G_1,B) = (H,I,C)$.

Definition 3.19. *Restricted Intersection of two binary bipolar soft sets (F,G,A) and (F_1,G_1,B) over the common universes U_1, U_2 is the binary bipolar soft set (H,I,C), where $C = A\cap B$ is non-empty and for all $e\in C$*

$$H(e) = F(e)\widetilde{\sqcap}F_1(e) \quad \text{and} \quad I(\neg e) = G(\neg e)\widetilde{\sqcup}G_1(\neg e)$$

such that $F(e) = (\hat{X}_1,\hat{Y}_1)$, $G(\neg e) = (\check{X}_1,\check{Y}_1)$ for each $e\in A$ and $F_1(e) = (\hat{X}_2,\hat{Y}_2)$, $G_1(\neg e) = (\check{X}_2,\check{Y}_2)$ for each $e\in B$. We denote it by $(F,G,A)\tilde{\tilde{\sqcap}}_{\mathfrak{R}}(F_1,G_1,B) = (H,I,C)$.

Proposition 3.20. *Let (F,G,A) and (F_1,G_1,A) be two binary bipolar soft sets over a common universes U_1, U_2. Then the following are true*
(1) $((F,G,A)\tilde{\tilde{\sqcup}}(F_1,G_1,B))^c = (F,G,A)^c\tilde{\tilde{\sqcap}}(F_1,G_1,B)^c$,
(2) $((F,G,A)\tilde{\tilde{\sqcap}}(F_1,G_1,B))^c = (F,G,A)^c\tilde{\tilde{\sqcup}}(F_1,G_1,B)^c$,
(3) $((F,G,A)\tilde{\tilde{\sqcup}}_{\mathfrak{R}}(F_1,G_1,B))^c = (F,G,A)^c\tilde{\tilde{\sqcap}}_{\mathfrak{R}}(F_1,G_1,B)^c$,
(4) $((F,G,A)\tilde{\tilde{\sqcap}}_{\mathfrak{R}}(F_1,G_1,B))^c = (F,G,A)^c\tilde{\tilde{\sqcup}}_{\mathfrak{R}}(F_1,G_1,B)^c$.

Proof. (1) Let $(F,G,A)\tilde{\tilde{\sqcup}}(F_1,G_1,B) = (H,I,A\cup B)$ where for each $e\in A\cup B$

$$H(e) = \begin{cases} (\hat{X}_1,\hat{Y}_1) & \text{if } e\in A-B \\ (\hat{X}_2,\hat{Y}_2) & \text{if } e\in B-A \\ (\hat{X}_1\cup\hat{X}_2,\hat{Y}_1\cup\hat{Y}_2) & \text{if } e\in A\cap B \end{cases}$$

$$I(\neg e) = \begin{cases} (\check{X}_1,\check{Y}_1) & \text{if } \neg e\in (\neg A)-(\neg B) \\ (\check{X}_2,\check{Y}_2) & \text{if } \neg e\in (\neg B)-(\neg A) \\ (\check{X}_1\cap\check{X}_2,\check{Y}_1\cap\check{Y}_2) & \text{if } \neg e\in (\neg A)\cap(\neg B) \end{cases}$$

such that $F(e) = (\hat{X}_1,\hat{Y}_1)$, $G(\neg e) = (\check{X}_1,\check{Y}_1)$ for each $e\in A$ and $F_1(e) = (\hat{X}_2,\hat{Y}_2)$, $G_1(\neg e) = (\check{X}_2,\check{Y}_2)$ for each $e\in B$. Hence, $((F,G,A)\tilde{\tilde{\sqcup}}(F_1,G_1,B))^c = (H,I,A\cup B)^c = (H^c,I^c,A\cup B)$.
Now, $H^c(e) = (U_1-\hat{X},U_2-\hat{Y})$ and $I^c(e) = (U_1-\check{X},U_2-\check{Y})$ for each $e\in A\cup B$ such that $H(e) = (\hat{X},\hat{Y})$ and $I(\neg e) = (\check{X},\check{Y})$. Therefore

$$H^c(e) = \begin{cases} (U_1-\hat{X}_1,U_2-\hat{Y}_1) & \text{if } e\in A-B \\ (U_1-\hat{X}_2,U_2-\hat{Y}_2) & \text{if } e\in B-A \\ (U_1-(\hat{X}_1\cup\hat{X}_2),U_2-(\hat{Y}_1\cup\hat{Y}_2)) & \text{if } e\in A\cap B \end{cases}$$

$$I^c(\neg e) = \begin{cases} (U_1 - \check{X}_1, U_2 - \check{Y}_1) & \text{if } \neg e \in (\neg A) - (\neg B) \\ (U_1 - \check{X}_2, U_2 - \check{Y}_2) & \text{if } \neg e \in (\neg B) - (\neg A) \\ (U_1 - (\check{X}_1 \cap \check{X}_2), U_2 - (\check{Y}_1 \cap \check{Y}_2)) & \text{if } \neg e \in (\neg A) \cap (\neg B) \end{cases}$$

Now, $(F, G, A)^c \tilde{\tilde{\sqcap}} (F_1, G_1, B)^c = (F^c, G^c, A) \tilde{\tilde{\sqcap}} (F_1^c, G_1^c, B) = (K, L, A \cup B)$, where

$$K(e) = \begin{cases} (U_1 - \hat{X}_1, U_2 - \hat{Y}_1) & \text{if } e \in A - B \\ (U_1 - \hat{X}_2, U_2 - \hat{Y}_2) & \text{if } e \in B - A \\ (U_1 - (\hat{X}_1 \cup \hat{X}_2), U_2 - (\hat{Y}_1 \cup \hat{Y}_2)) & \text{if } e \in A \cap B \end{cases}$$

$$L(\neg e) = \begin{cases} (U_1 - \check{X}_1, U_2 - \check{Y}_1) & \text{if } \neg e \in (\neg A) - (\neg B) \\ (U_1 - \check{X}_2, U_2 - \check{Y}_2) & \text{if } \neg e \in (\neg B) - (\neg A) \\ (U_1 - (\check{X}_1 \cap \check{X}_2), U_2 - (\check{Y}_1 \cap \check{Y}_2)) & \text{if } \neg e \in (\neg A) \cap (\neg B) \end{cases}$$

Finally, "H^c and K" and "I^c and L" are same. So, proof is completed.

(2), (3) and (4) options are proved by a similar way. □

Proposition 3.21. *If $\widetilde{\widetilde{(\Phi, \mathfrak{U}, A)}}$ is a null binary bipolar soft set, $\widetilde{\widetilde{(\mathfrak{U}, \Phi, A)}}$ an absolute binary bipolar soft set, and (F, G, A), (F_1, G_1, A) are binary bipolar soft sets over U_1, U_2, then*
(1) $(F, G, A) \tilde{\tilde{\sqcup}} (F_1, G_1, A) = (F, G, A) \tilde{\tilde{\sqcup}}_{\mathfrak{R}} (F_1, G_1, A)$,
(2) $(F, G, A) \tilde{\tilde{\sqcap}} (F_1, G_1, A) = (F, G, A) \tilde{\tilde{\sqcap}}_{\mathfrak{R}} (F_1, G_1, A)$,
(3) $(F, G, A) \tilde{\tilde{\sqcup}} (F, G, A) = (F, G, A)$; $(F, G, A) \tilde{\tilde{\sqcap}} (F, G, A) = (F, G, A)$,
(4) $(F, G, A) \tilde{\tilde{\sqcup}} \widetilde{\widetilde{(\Phi, \mathfrak{U}, A)}} = (F, G, A)$; $(F, G, A) \tilde{\tilde{\sqcap}} \widetilde{\widetilde{(\Phi, \mathfrak{U}, A)}} = \widetilde{\widetilde{(\Phi, \mathfrak{U}, A)}}$,
(5) $(F, G, A) \tilde{\tilde{\sqcup}} \widetilde{\widetilde{(\mathfrak{U}, \Phi, A)}} = \widetilde{\widetilde{(\mathfrak{U}, \Phi, A)}}$; $(F, G, A) \tilde{\tilde{\sqcap}} \widetilde{\widetilde{(\mathfrak{U}, \Phi, A)}} = (F, G, A)$.

Proof. Straightforward. □

Proposition 3.22. *Let (F_1, G_1, A), (F_2, G_2, B) and (F_3, G_3, C), be three binary bipolar soft sets. Then we have the following results:*
(1) $(F_1, G_1, A) \tilde{\tilde{\sqcup}} ((F_2, G_2, B) \tilde{\tilde{\sqcap}} (F_3, G_3, C)) = ((F_1, G_1, A) \tilde{\tilde{\sqcup}} (F_2, G_2, B)) \tilde{\tilde{\sqcap}} ((F_1, G_1, A) \tilde{\tilde{\sqcup}} (F_3, G_3, C))$. (2) $(F_1, G_1, A) \tilde{\tilde{\sqcap}} ((F_2, G_2, B) \tilde{\tilde{\sqcup}} (F_3, G_3, C)) = ((F_1, G_1, A) \tilde{\tilde{\sqcap}} (F_2, G_2, B)) \tilde{\tilde{\sqcup}} ((F_1, G_1, A) \tilde{\tilde{\sqcap}} (F_3, G_3, C))$.

Proof. It is obvious from Definition 3.16 and 3.17. □

Example 3.23. *Consider the following sets:*
$U_1 = \{s_1, s_2, s_3, s_4, s_5\}$ *is the set of t-shirts.*
$U_2 = \{t_1, t_2, t_3, t_4\}$ *is the set of ties.*
$E = \{e_1, e_2, e_3, e_4\}$ *and* $\neg E = \{\neg e_1, \neg e_2, \neg e_3, \neg e_4\}$ *is the sets of parameters, where* $e_1 : colorful, \neg e_1 : plain, e_2 : sport, \neg e_2 : classic, e_3 : cheap, \neg e_3 : expensive, e_4 : modern, \neg e_4 : traditional.$

Suppose that $A = \{e_1, e_2, e_3\}$, *and* $B = \{e_2, e_3, e_4\}$. *The binary bipolar soft sets* (F, G, A) *and* (F_1, G_1, B) *describe the "the special features of both the t-shirts and the ties" which Mr. X and Mr. Y are going to buy respectively. Suppose that*

$$F(e_1) = (\{s_1, s_3\}, \{t_2, t_4\}),\ F(e_2) = (\{s_2, s_3\}, \{t_1\}),\ F(e_3) = (\{s_5\}, \{t_3, t_4\}),$$

$$G(\neg e_1) = (\{s_2, s_4\}, \{t_1, t_3\}),\ G(\neg e_2) = (\{s_1, s_5\}, \{t_2\}),\ G(\neg e_3) = (\{s_4\}, \{t_1\}).$$

and

$$F_1(e_2) = (\{s_3, s_5\}, \{t_1, t_3\}),\ F_1(e_3) = (\{s_2, s_5\}, \{t_2, t_4\}),\ F_1(e_4) = (\{s_3\}, \{t_4\}),$$

$$G_1(\neg e_2) = (\{s_1, s_3\}, \{t_2, t_4\}),\ G_1(\neg e_3) = (\{s_4\}, \{t_1, t_3\}),\ G_1(\neg e_4) = (\{s_1\}, \{t_2, t_3\}).$$

Now, we approximate the resulting binary bipolar soft sets obtained by applying the above mentioned operations on (F, G, A) *and* (F_1, G_1, B).

Let $(F, G, A)\tilde{\tilde{\sqcup}}(F_1, G_1, B) = (H_1, I_1, A \cup B)$. *Then*

$$H_1(e_1) = (\{s_1, s_3\}, \{t_2, t_4\}),\ H_1(e_2) = (\{s_2, s_3, s_5\}, \{t_1, t_3\}),$$

$$H_1(e_3) = (\{s_2, s_5\}, \{t_2, t_3, t_4\}),\ H_1(e_4) = (\{s_3\}, \{t_4\}),$$

and

$$I_1(\neg e_1) = (\{s_2, s_4\}, \{t_1, t_3\}), I_1(\neg e_2) = (\{s_1\}, \{t_2\}),$$
$$I_1(\neg e_3) = (\{s_4\}, \{t_1\}),\ I_1(\neg e_4) = (\{s_1\}, \{t_2, t_3\}).$$

Let $(F, G, A)\tilde{\tilde{\sqcap}}(F_1, G_1, B) = (H_2, I_2, A \cup B)$. *Then*

$$H_2(e_1) = (\{s_1, s_3\}, \{t_2, t_4\}),\ H_2(e_2) = (\{s_2\}, \{t_1\}),$$

$$H_2(e_3) = (\{s_5\}, \{t_4\}),\ H_2(e_4) = (\{s_3\}, \{t_4\}),$$

and

$$I_2(\neg e_1) = (\{s_2, s_4\}, \{t_1, t_3\}), I_2(\neg e_2) = (\{s_1, s_3, s_5\}, \{t_2, t_4\}),$$
$$I_2(\neg e_3) = (\{s_4\}, \{t_1, t_3\}),\ I_2(\neg e_4) = (\{s_1\}, \{t_2, t_3\}).$$

Let $(F, G, A)\tilde{\tilde{\sqcup}}_{\mathfrak{R}}(F_1, G_1, B) = (H_3, I_3, A \cap B)$. *Then*

$$H_3(e_2) = (\{s_2, s_3, s_5\}, \{t_1, t_3\}),\ H_3(e_3) = (\{s_2, s_5\}, \{t_2, t_3, t_4\}),$$

$$I_3(\neg e_2) = (\{s_1\}, \{t_2\}),\ I_3(\neg e_3) = (\{s_4\}, \{t_1\}).$$

Let $(F, G, A)\tilde{\tilde{\sqcap}}_{\mathfrak{R}}(F_1, G_1, B) = (H_4, I_4, A \cap B)$. *Then*

$$H_4(e_2) = (\{s_2\}, \{t_1\}),\ H_4(e_3) = (\{s_5\}, \{t_4\}),$$

$$I_4(\neg e_2) = (\{s_1, s_3, s_5\}, \{t_2, t_4\}),\ I_4(\neg e_3) = (\{s_4\}, \{t_1, t_3\}).$$

Let $(F, G, A)\tilde{\tilde{\vee}}(F_1, G_1, B) = (H_5, I_5, A \times B)$. *Then*

$$H_5(e_1, e_2) = (\{s_1, s_2, s_3, s_5\}, U_2),\ H_5(e_1, e_3) = (\{s_1, s_2, s_3, s_5\}, \{t_1, t_2, t_4\}),$$

$$H_5(e_1, e_4) = (\{s_1, s_3\}, \{t_2, t_4\}), H_5(e_2, e_2) = (\{s_2, s_3, s_5\}, \{t_1, t_3\}),$$

and

$$I_5(e_1, e_2) = (\emptyset, \emptyset),\ I_5(e_1, e_3) = (\{s_4\}, \{t_3\}),$$
$$I_5(e_1, e_4) = (\emptyset, \{t_3\}), I_5(e_2, e_2) = (\{s_1\}, \{t_2\}),$$

and so on.

Let $(F, G, A)\tilde{\tilde{\wedge}}(F_1, G_1, B) = (H_6, I_6, A \times B)$. *Then*

$$H_6(e_1, e_2) = (\emptyset, \emptyset),\ H_6(e_1, e_3) = (\emptyset, \{t_2, t_4\}),$$

$$H_6(e_1, e_4) = (\{s_3\}, \{t_4\}), H_6(e_2, e_2) = (\{s_2\}, \{t_1\}),$$

and

$$I_6(e_1, e_2) = (\{s_1, s_2, s_3, s_4\}, U_2),\ I_6(e_1, e_3) = (\{s_2, s_4\}, \{t_1, t_3\}),$$
$$I_6(e_1, e_4) = (\{s_1, s_2, s_4\}, \{t_1, t_2, t_3\}), I_6(e_2, e_2) = (\{s_1, s_3, s_5\}, \{t_2, t_4\}),$$

and so on.

4. A Characteristic Function of the Binary Bipolar Soft Set

A characteristic function of the binary bipolar soft set may be represented in a similar way as characteristic function of binary soft sets is given by Açıkgöz and Taş in [9]. The characteristic function is as follows:

Consider the following sets:

$$U_1 = \{h_j : 1 \leq j \leq n\},$$
$$U_2 = \{t_k : 1 \leq k \leq m\},$$
$$E = \{e_i : 1 \leq i \leq p\},$$
$$\neg E = \{\neg e_i : 1 \leq i \leq p\}.$$

Let $(F, G, E) = \{(e_i, (\hat{X}_i, \hat{Y}_i)), (\neg e_i, (\check{X}_i, \check{Y}_i)) : 1 \leq i \leq p; \hat{X}_i, \check{X}_i \subseteq U_1; \hat{Y}_i, \check{Y}_i \subseteq U_2\}$ is the binary bipolar soft set.

$$\hat{f}_E(e_i)_j = \begin{cases} 1 & h_j \in \hat{X}_i \\ 0 & h_j \notin \hat{X}_i \end{cases} \qquad \hat{f}_E(e_i)_k = \begin{cases} 1 & t_k \in \hat{Y}_i \\ 0 & t_k \notin \hat{Y}_i \end{cases}$$

$$\check{g}_E(\neg e_i)_j = \begin{cases} -1 & h_j \in \check{X}_i \\ 0 & h_j \notin \check{X}_i \end{cases} \qquad \check{g}_E(\neg e_i)_k = \begin{cases} -1 & t_k \in \check{Y}_i \\ 0 & t_k \notin \check{Y}_i \end{cases}$$

Table 1. The Elements of Universal Sets for the Given Parameters

$E - \neg E / U_1 - U_2$	h_1	h_2	$\ldots$	h_n	t_1	t_2	$\ldots$	t_m
e_1	$\hat{f}_E(e_1)_1$	$\hat{f}_E(e_1)_2$	$\ldots$	$\hat{f}_E(e_1)_n$	$\hat{f}_E(e_1)_1$	$\hat{f}_E(e_1)_2$	$\ldots$	$\hat{f}_E(e_1)_m$
e_2	$\hat{f}_E(e_2)_1$	$\hat{f}_E(e_2)_2$	$\ldots$	$\hat{f}_E(e_2)_n$	$\hat{f}_E(e_2)_1$	$\hat{f}_E(e_2)_2$	$\ldots$	$\hat{f}_E(e_2)_m$
e_i	$\vdots$	$\vdots$	$\ldots$	$\vdots$	$\vdots$	$\vdots$	$\ldots$	$\vdots$
e_p	$\hat{f}_E(e_p)_1$	$\hat{f}_E(e_p)_2$	$\ldots$	$\hat{f}_E(e_p)_n$	$\hat{f}_E(e_p)_1$	$\hat{f}_E(e_p)_2$	$\ldots$	$\hat{f}_E(e_p)_m$
$\neg e_1$	$\check{g}_E(e_1)_1$	$\check{g}_E(e_1)_2$	$\ldots$	$\check{g}_E(e_1)_n$	$\check{g}_E(e_1)_1$	$\check{g}_E(e_1)_2$	$\ldots$	$\check{g}_E(e_1)_m$
$\neg e_2$	$\check{g}_E(e_2)_1$	$\check{g}_E(e_2)_2$	$\ldots$	$\check{g}_E(e_2)_n$	$\check{g}_E(e_2)_1$	$\check{g}_E(e_2)_2$	$\ldots$	$\check{g}_E(e_2)_m$
$\neg e_i$	$\vdots$	$\vdots$	$\ldots$	$\vdots$	$\vdots$	$\vdots$	$\ldots$	$\vdots$
$\neg e_p$	$\check{g}_E(e_p)_1$	$\check{g}_E(e_p)_2$	$\ldots$	$\check{g}_E(e_p)_n$	$\check{g}_E(e_p)_1$	$\check{g}_E(e_p)_2$	$\ldots$	$\check{g}_E(e_p)_m$

Now we investigate a following example:

Example 4.1. *Consider the following sets:*
$U_1 = \{t_1, t_2, t_3, t_4\}$ *is the set of trousers,*
$U_2 = \{c_1, c_2, c_3, c_4\}$ *is the set of shoes,*
$E = \{e_1 = expensive, e_2 = sport, e_3 = beautiful\}$ $\neg E = \{\neg e_1 = cheap, \neg e_2 = classic, \neg e_3 = ugly\}$ *is the sets of parameters.*
Let (F, G, E) *is a binary bipolar soft sets as follows:*

$$(F, G, E) = (e_1, (\hat{X}_1 = \{t_1, t_3\}, \hat{Y}_1 = \{c_1, c_2\})), (\neg e_1, (\check{X}_1 = \{t_2\}, \check{Y}_1 = \{c_3, c_4\})),$$
$$(e_2, (\hat{X}_2 = \{t_2, t_4\}, \hat{Y}_2 = \{c_3\})), (\neg e_2, (\check{X}_2 = \{t_1\}, \check{Y}_2 = \{c_1\})),$$
$$(e_3, (\hat{X}_3 = \{t_3\}, \hat{Y}_3 = \{c_2, c_4\})), (\neg e_3, (\check{X}_3 = \{t_2, t_4\}, \check{Y}_3 = \{c_3\})).$$

Then, for $1 \leq i \leq 3$ *and* $1 \leq j, k \leq 4$,

$$\hat{f}_E(e_i)_j = \begin{cases} 1, & t_j \in \hat{X}_i \\ 0, & t_j \notin \hat{X}_i \end{cases} \qquad \check{f}_E(e_i)_k = \begin{cases} 1, & c_k \in \hat{Y}_i \\ 0, & c_k \notin \hat{Y}_i \end{cases}$$

$$\hat{g}_E(\neg e_i)_j = \begin{cases} -1, & t_j \in \check{X}_i \\ 0, & t_j \notin \check{X}_i \end{cases} \qquad \check{g}_E(\neg e_i)_k = \begin{cases} -1, & c_k \in \check{Y}_i \\ 0, & c_k \notin \check{Y}_i \end{cases}$$

Table 2. Trousers and Shoes Providing Parameter Properties

$E - \neg E/U_1 - U_2$	t_1	t_2	t_3	t_4	c_1	c_2	c_3	c_4
e_1	1	0	1	0	1	1	0	0
e_2	0	1	0	1	0	0	1	0
e_3	0	0	1	0	0	1	0	1
$\neg e_1$	0	-1	0	0	0	0	-1	-1
$\neg e_2$	-1	0	0	0	-1	0	0	0
$\neg e_3$	0	-1	0	-1	0	0	-1	0

In the above table, we see some trousers and shoes providing parameter properties. For example, the second trouser is sport but the first and third trouser are not sport. At the same time the second trouser is cheap and ugly but not classic. Similarly, the third shoe is sport but the other shoes are not sport. Again likewise the fist shoe is classic but the other shoes are not classic.

5. Results and Discussion

In this paper, we introduce binary bipolar soft sets by using binary soft sets and bipolar soft sets. We also study basic operations of complement, union, intersection, restricted union, restricted intersection, difference, “AND”, ”OR” on binary bipolar soft sets. Thereafter, the basic properties of these operations are proven along with several examples to illustrate those properties. Finally, the characteristic function defined on binary bipolar soft sets hope that it might be useful for solving decision-making problem.

References

1. D., Molodtsov , *Soft set theory first results*, Comput. Math. Appl., 37, 19-31, (1999).
2. M., Shabir, M., Naz, *On bipolar soft set*, Retrieved from https://arxiv.org/abs/1303.1344, (2013).
3. F., Karaaslan, S., Karatas, *A new approach to bipolar soft sets and its applications*, Discrete Math. Algorithm. Appl., 07, 1550054, (2015).
4. K.Y., Qin, Z.Y., Hong, *On soft equality*, Journal of Computational and Applied Mathematics, 234, 1347-1355, (2010).
5. Z., Xiao, K., Gong, S., Xia, Y., Zou, *Exclusive disjunctive soft sets*, Computers and Mathematics with Applications, 59, 2128-2137, (2010).
6. K., Gong, Z., Xiao, X., Zhang, *The bijective soft set with its operations*, Computers and Mathematics with Applications, 60, 2270-2278, (2010).
7. Y.K., Kima, W.K., Minb, *Full soft sets and full soft decision systems*, Journal of Intelligent and Fuzzy Systems, DOI:10.3233/IFS-130783.
8. P. K., Maji, R., Biswas and A. R., Roy, *Soft set theory*, Computers and Mathematics with Applications, 45, 555–562, (2003).
9. A., Açıkgöz, N., Tas, *Binary Soft Set Theory*, Eur.Jl. of Pure. and Appl. Math., 9, 452-463, (2016).
10. S.S., Benchalli, P.G., Patil, Abeda, S., Dodamani, *On Binary Soft Topological Spaces*, Journal of Inter disciplinary Mathematics, Communicated.

Naime Demirtaş,
Department of Mathematics,
Mersin University,
33343, Mersin, TURKEY.
E-mail address: naimedemirtas@mersin.edu.tr

and

Orhan Dalkılıç,
Department of Mathematics,
Mersin University,
33343, Mersin, TURKEY.
E-mail address: orhandlk952495@hotmail.com