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Fuzzy Controller and Stabilizer on Random Operators

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ABSTRACT: In a random operator inequality, by the fuzzy controllers, we make stable an approximately additive odd random operator and find an estimation for such random operators and we solve Hyers-Ulam-Rassias stability problem for a random operator inequality.

Key Words: Random operator inequality, fuzzy controller, stability, approximation, estimation.

Contents

1. Introduction

Let (Γ, Σ, ξ) be a probability measure space. Assume that $(T, \mathfrak{B}_T)$ and $(S, \mathfrak{B}_S)$ are Borel measureable spaces, in which T and S are f-k-NLS and $F : \Gamma \times T^2 \to S$ is a random operator. First, by a fuzzy control function we make stable the random operator F satisfies in the following random operator inequality

$$\begin{aligned} & \nu(F(\gamma, t_1) + F(\gamma, s_1) + F(\gamma, p_1) + F(\gamma, r_1), \ldots, F(\gamma, t_k) + F(\gamma, s_k) \\ + \;& F(\gamma, p_k) + F(\gamma, r_k), \tau) \\ \geq \;& \nu(F(\gamma, t_1 + s_1) + F(\gamma, p_1 + r_1), \ldots, F(\gamma, t_k + s_k) + F(\gamma, p_k + r_k), \tau). \end{aligned} \tag{1.1}$$

Next, we get an approximation of F by an additive random operator.

2. Preliminaries

Inspired by the basic notions of Katsaras' paper [1], using the concept of Minkowski functionals of L-fuzzy sets introduced by Höhle [2], and fuzzy metric space by Kaleva and Seikkala [3], in 1988, Morsi [4] introduced a notion of fuzzy (pseudo) normed spaces. On the other hand, by using the notion of random normed spaces introduced by Šerstnev [5], and studied by Muštari [6], Radu [7], Cheng and Mordeson [8], Rano and Bag [9] and others, some mathematician defined another version of fuzzy normed (k-normed) space as the following definition.

Definition 2.1 ([10,11,12]). Suppose that S is a linear space (or real vector space). Let the fuzzy subset ν of $S^p \times (0, +\infty)$ (dimension $S = d \geq k$) satisfy

(ν1) $\nu(s_1, \ldots, s_k, \tau) = 0$, for $\tau \leq 0$;

(ν2) $\nu(s_1, \ldots, s_k, \tau) = 1$ for $\tau \geq 0$ if and only if $s_1, \ldots, s_n$ are linearly dependent;

(ν3) $\nu(s_1, \ldots, s_k, \tau)$ is invariant under any permutation of $s_1, \ldots, s_k \in S$;

(ν4) $\nu(\alpha s_1, \ldots, s_k, \tau) = \nu(s_1, \ldots, s_k, \frac{\tau}{|\alpha|})$ if $\alpha \neq 0$;

(ν5) $\nu(s_0 + s_1, s_2, \ldots, s_k, \tau + \varsigma) \geq \bigwedge(\nu(s_0, s_2, \ldots, s_k, \varsigma), \nu(s_1, s_2, \ldots, s_k, \tau))$;

(ν6) $\nu(s_1, \ldots, s_k, .) : (0, +\infty) \to [0, 1]$ is left continuous;

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($\nu 7$) $\lim_{t\to+\infty}\nu(s_1,\ldots,s_k,\tau)=1$.

Then the triple (S,ν) is said to be a fuzzy k-normed linear space or in short f-k-NLS.

For more details see [13,14,15,16,17,18,19].

Example 2.2. *Let $(S,\|.,\ldots,.\|)$ be an k-normed space. Define the fuzzy set ν as $\nu(s_1,\ldots,s_k,\tau)=\exp(-\|s_1,\ldots,s_k\|/\tau)$ for $s_1,\ldots,s_k\in S$ and $\tau\in(0,+\infty)$. So, (S,ν) is an f-k-NLS.*

Let (Γ,Σ,ξ) be a probability measure space. Assume that $(T,\mathfrak{B}_T)$ and $(S,\mathfrak{B}_S)$ are Borel measureable spaces, in which T and S are complete f-k-NLS. A mapping $F:\Gamma\times T\to S$ is said to be a random operator if $\{\gamma:\ F(\gamma,t)\in B\}\in\Sigma$ for all t in T and $B\in\mathfrak{B}_S$. Also, F is random operator, if $F(\gamma,t)=s(\gamma)$ be a S-valued random variable for every t in T. A random operator $F:\Gamma\times T\to S$ is called *linear* if $F(\gamma,\alpha t_1+\beta t_2)=\alpha F(\gamma,t_1)+\beta F(\gamma,t_2)$ almost every where for each t_1,t_2 in T and α,β are scalers, and *bounded* if there exists a nonnegative real-valued random variable $M(\gamma)$ such that

$$\nu(F(\gamma,t_1)-F(\gamma,s_1),\ldots,F(\gamma,t_k)-F(\gamma,s_k),M(\gamma)\tau)\geq\nu(t_1-s_1,\ldots,t_k-s_k,\tau),$$

almost every where for each t_i,s_i $(i=1,\ldots,k)$ in T and $\tau>0$.

Recently, some authors have published some papers on approximation of functional equations in several spaces by the direct technique and the fixed point technique, for example, fuzzy Menger normed algebras [21], fuzzy metric spaces [22,23,24], fuzzy normed spaces [25], non-Archimedean spaces [26], random multi-normed space [27], see also [28]. Also, we suggest some papers some ones can prove their stability with fuzzy control functions [29]–[41].

Note that, a $[0,\infty]$-valued metric is called a generalized metric.

Theorem 2.3 ([42,43])**.** *Consider a complete generalized metric space (T,δ) and a strictly contractive function $\Lambda:T\to T$ with Lipschitz constant $\beta<1$. So, for every given element $t\in T$, either*

$$\delta(\Lambda^n t,\Lambda^{n+1}t)=\infty$$

for each $n\in\mathbb{N}$ or there is $n_0\in\mathbb{N}$ such that

(1) $\delta(\Lambda^n t,\Lambda^{n+1}t)<\infty,\qquad \forall n\geq n_0$;
(2) *the fixed point s^* of Λ is the convergent point of sequence $\{\Lambda^n t\}$;*
(3) *in the set $V=\{s\in T\mid \delta(\Lambda^{n_0}t,s)<\infty\}$, s^* is the unique fixed point of Λ;*
(4) $(1-\beta)\delta(s,s^*)\leq\delta(s,\Lambda s)$ *for every $s\in V$.*

3. Main results

In the main theorem, first, we introduce a fuzzy controller function, this function make stable the random operator F. Next by fixed point technique we get a linear approximation from mentioned random operator.

Theorem 3.1. *Let (Γ,Σ,ξ) be a probability measure space. Let T be a vector space and S be a f-k-NLS. Suppose that $F:\Gamma\times T\to S$ is an odd random operator holds the next inequality*

$$\begin{aligned}&\nu(F(\gamma,t_1)+F(\gamma,s_1)+F(\gamma,p_1)+F(\gamma,r_1),\cdots,\\&F(\gamma,t_k)+F(\gamma,s_k)+F(\gamma,p_k)+F(\gamma,r_k),\tau)\\&\quad\geq\bigwedge\{\nu(F(\gamma,t_1+s_1+p_1+r_1),\cdots,F(\gamma,t_k+s_k+p_k+r_k),\tau)\\&,\varphi(t_1,\cdots,t_k,s_1,\cdots,s_k,p_1,\cdots,p_k,r_1,\cdots,r_k,\tau)\}\end{aligned}\tag{3.1}$$

for each $t_i,s_i,p_i,r_i\in T$ $(i=1,\cdots,k)$, $\gamma\in\Gamma,\tau>0$, in which $\varphi:T^{4n}\times(0,\infty)\to J$ is called fuzzy controller function. If there exists $0<\beta<3$ such that

$$\begin{aligned}&\varphi(t_1,\cdots,t_k,s_1,\cdots,s_k,p_1,\cdots,p_k,r_1,\cdots,r_k,\tau)\\\geq\ &\varphi\left(3t_1,\cdots,3t_k,3s_1,\cdots,3s_k,3p_1,\cdots,3p_k,3r_1,\cdots,3r_k,\frac{3\tau}{\beta}\right)\end{aligned}\tag{3.2}$$

$t_i, s_i, p_i, r_i \in T$ $(i = 1, \cdots, k)$. *So, there is a unique additive random operator* $Q : \Gamma \times T \to S$ *satisfying*

$$\begin{aligned} &\nu(F(\gamma,t_1)-Q(\gamma,t_1),...,F(\gamma,t_k)-Q(\gamma,t_k),\tau) \\ \geq\ &\varphi\left(t_1,\cdots,t_k,t_1,\cdots,t_k,t_1,\cdots,t_k,-3t_1,\cdots,-3t_k,\frac{3-\beta}{\beta}\tau\right) \end{aligned} \tag{3.3}$$

for each $t_i \in T$ $(i = 1, \cdots, k)$, $\gamma \in \Gamma$ *and* $\tau > 0$.

Proof. Consider the set $B := \{K : \Gamma \times T \to S\}$ and the generalized metric on B as,

$$\begin{aligned} &\delta(K,H) = \inf\{\lambda \in \mathbb{R}_+ : \nu(K(\gamma,t_1)-H(\gamma,t_1),...,K(\gamma,t_k)-H(\gamma,t_k),\tau) \\ &\geq \varphi(t_1,\cdots,t_k,t_1,\cdots,t_k,t_1,\cdots,t_k,-3t_1,\cdots,-3t_k,\frac{\tau}{\lambda}), \forall t_i \in T\ (i=1,\cdots,k), \gamma\in\Gamma\}. \end{aligned}$$

Now, we show that (B, δ) is complete. Consider the Cauchy sequence $\{H_\ell\}$ in (B, δ). Let $\epsilon > 0$. So, there is an integer $\ell_0 > 0$ such that $\delta(H_\jmath, H_\ell) < \epsilon$ for all $\jmath, \ell \geq \ell_0$. Then, there is $\lambda \in (0, \epsilon)$ such that

$$\begin{aligned} &\nu(H_\jmath(\gamma,t_1)-H_\ell(\gamma,t_1),...,H_\jmath(\gamma,t_k)-H_\ell(\gamma,t_k),\tau) \\ &\geq \varphi\left(t_1,\cdots,t_k,t_1,\cdots,t_k,t_1,\cdots,t_k,-3t_1,\cdots,-3t_k,\frac{\tau}{\lambda}\right) \\ &\geq \varphi\left(t_1,\cdots,t_k,t_1,\cdots,t_k,t_1,\cdots,t_k,-3t_1,\cdots,-3t_k,\frac{\tau}{\epsilon}\right) \end{aligned} \tag{3.4}$$

for each $\jmath, \ell \geq \ell_0$ and $t_i \in T$ $(i = 1, \cdots, k)$. Since S is complete, $\{H_\ell(\gamma, t_i)\}$ converges for each $t_i \in T$ $(i = 1, \cdots, k)$. Thus, a random operator $H : \Gamma \times T \to S$ can be defined by

$$H(\gamma,t) := \lim_{\ell\to\infty} H_\ell(\gamma,t) \tag{3.5}$$

for all $t \in T$. Letting $\ell \to \infty$ in (3.4), we have

$$\begin{aligned} &\jmath \geq \ell_0 \Longrightarrow \nu(H_\jmath(\gamma,t_1)-H(\gamma,t_1),...,H_\jmath(\gamma,t_k)-H(\gamma,t_k),\tau) \\ &\geq \varphi\left(t_1,\cdots,t_k,t_1,\cdots,t_k,t_1,\cdots,t_k,-3t_1,\cdots,-3t_k,\frac{\tau}{\epsilon}\right) \\ &\Longrightarrow \epsilon \in \{\lambda\in\mathbb{R}_+ : \nu(K(\gamma,t_1)-H(\gamma,t_1),...,K(\gamma,t_k)-H(\gamma,t_k),\tau) \\ &\geq \varphi\left(t_1,\cdots,t_k,t_1,\cdots,t_k,t_1,\cdots,t_k,-3t_1,\cdots,-3t_k,\frac{\tau}{\lambda}\right), \forall t_i\in T\ (i=1,\cdots,k), \gamma\in\Gamma\} \\ &\Longrightarrow \delta(H_\jmath,H) \leq \epsilon \end{aligned}$$

for all $t_i \in T$ $(i = 1, \cdots, k), \gamma \in \Gamma, \tau > 0$. So, $\{H_\jmath\}$ converges to H in (B, δ). Hence (B, δ) is complete.

Now, we define the linear mapping $\Lambda : B \to B$ as,

$$\Lambda(K(\gamma,t)) := 3K\left(\gamma,\frac{t}{3}\right) \tag{3.6}$$

for all $t \in T$ and $\gamma \in \Gamma$. For $K, H \in B$, let $\lambda_{K,H} \geq 0$ with $\delta(K, H) < \lambda_{K,H}$. Then,

$$\begin{aligned} &\nu(K(\gamma,t_1)-H(\gamma,t_1),..,K(\gamma,t_k)-H(\gamma,t_k),\tau) \\ \geq\ &\varphi\left(t_1,\cdots,t_k,t_1,\cdots,t_k,t_1,\cdots,t_k,-3t_1,\cdots,-3t_k,\frac{\tau}{\lambda_{K,H}}\right). \end{aligned}$$

On the other hand,

$$\begin{aligned} &\nu(\Lambda(K(\gamma,t_1)-\Lambda(H(\gamma,t_1)),...,\Lambda(K(\gamma,t_k)-\Lambda(H(\gamma,t_k)),\tau) = \\ &\nu(3K\left(\gamma,\frac{t_1}{3}\right)-3H\left(\gamma,\frac{t_1}{3}\right),...,3K\left(\gamma,\frac{t_k}{3}\right)-3H\left(\gamma,\frac{t_k}{3}\right),\tau) \\ &\geq \varphi\left(\frac{t_1}{3},\cdots,\frac{t_k}{3},\frac{t_1}{3},\cdots,\frac{t_k}{3},\frac{t_1}{3},\cdots,\frac{t_k}{3},-t_1,\cdots,-t_k,\frac{\tau}{\lambda_{K,H}}\right), \\ &\geq \varphi\left(t_1,\cdots,t_k,t_1,\cdots,t_k,t_1,\cdots,t_k,-3t_1,\cdots,-3t_k,\frac{3\tau}{\beta\lambda_{K,H}}\right) \end{aligned}$$

for all $t_i \in T$ $(i = 1, \cdots, k), \gamma \in \Gamma, \tau > 0$. This means $\delta(\Lambda K, \Lambda H) \leq \frac{\beta}{3}\lambda_{K,H}$. So, $\delta(\Lambda K, \Lambda H) \leq \frac{\beta}{3}\delta(K, H)$ for each $K, H \in B$. Then, Λ is a strictly contractive on B with the Lipschitz constant $0 < \frac{\beta}{3} < 1$.

Putting $s_1, \cdots, s_k = p_1, \cdots, p_k = t_1, \cdots, t_k$ and $r_1, \cdots, r_k = -3t_1, \cdots, -3t_k$ in (3.1), we get

$$\begin{aligned}
&\nu(3F(\gamma, t_1) - F(\gamma, 3t_1), ..., 3F(\gamma, t_k) - F(\gamma, 3t_k), \tau) \\
&\geq \varphi(t_1, \cdots, t_k, t_1, \cdots, t_k, t_1, \cdots, t_k, -3t_1, \cdots, -3t_k, \tau)
\end{aligned} \tag{3.7}$$

for all $t_i \in T$ $(i = 1, \cdots, k), \gamma \in \Gamma$ and $\tau > 0$. So

$$\begin{aligned}
&\nu\left(F(\gamma, t_1) - 3F\left(\gamma, \frac{t_1}{3}\right), ..., F(\gamma, t_k) - 3F\left(\gamma, \frac{t_k}{3}\right), \tau\right) \\
&\geq \varphi\left(\frac{t_1}{3}, \cdots, \frac{t_k}{3}, \frac{t_1}{3}, \cdots, \frac{t_k}{3}, \frac{t_1}{3}, \cdots, \frac{t_k}{3}, -t_1, \cdots, -t_k, \tau\right) \\
&\geq \varphi\left(t_1, \cdots, t_k, t_1, \cdots, t_k, t_1, \cdots, t_k, -3t_1, \cdots, -3t_k, \frac{3\tau}{\beta}\right)
\end{aligned}$$

for all $t_i \in T$ $(i = 1, \cdots, k), \gamma \in \Gamma, \tau > 0$. So,

$$\delta(F, \Lambda F) \leq \frac{\beta}{3} < 1.$$

Now, Theorem 2.3 implies that the sequence $\{\Lambda^n F\}$ converges to an unique additive fixed point Q of Λ, i.e.,

$$Q(\gamma, t) := \lim_{n\to\infty} 3^n F\left(\gamma, \frac{t}{3^n}\right)$$

and $Q(\gamma, 3t) = 3Q(\gamma, t)$ for each $t \in T$. Moreover,

$$\delta(Q, F) \leq \frac{1}{1 - \frac{\beta}{3}}\delta(\Lambda F, F) \leq \frac{\beta}{3 - \beta},$$

so, (3.3) holds. □

In the next corollary we get a better error estimation of main result of [44,45].

Corollary 3.2. *Let (Γ, Σ, ξ) be a probability measure space, let T be a k-normed space and S a f-k-NLS. Suppose that $F : \Gamma \times T \to S$ is an odd random operator and there is a constant $\imath \geq 0$ and $\jmath \in \mathbb{R}$ such that $4\jmath \neq 1$ and F holds in the nest inequality*

$$\begin{aligned}
&\nu(F(\gamma, t_1) + F(\gamma, s_1) + F(\gamma, p_1) + F(\gamma, r_1), \cdots, \\
&F(\gamma, t_k) + F(\gamma, s_k) + F(\gamma, p_k) + F(\gamma, r_k), \tau) \\
&\quad \geq \bigwedge\{\nu(F(\gamma, t_1 + s_1 + p_1 + r_1), \cdots, F(\gamma, t_k + s_k + p_k + r_k), \tau) \\
&, \exp(-\frac{\imath(||t_1, \cdots, t_k||^{\jmath}||s_1, \cdots, s_k||^{\jmath}||p_1, \cdots, p_k||^{\jmath}||r_1, \cdots, r_k||^{\jmath})}{\tau})\}
\end{aligned} \tag{3.8}$$

for all $t_i, s_i, p_i, r_i \in T, \gamma \in \Gamma, \tau > 0$. So, there is a unique additive random operator $Q : \Gamma \times T \to S$ such that

$$\begin{aligned}
&\nu(F(\gamma, t_1) - Q(\gamma, t_1), ..., F(\gamma, t_k) - Q(\gamma, t_k), \tau) \\
\geq\ &\exp(-\frac{3^{-3\jmath}\imath||t_1, \cdots, t_k||^{4\jmath}}{|81^{\jmath} - 3|\tau})
\end{aligned} \tag{3.9}$$

for all $t_i \in T$ $(i = 1, \cdots, k), \gamma \in \Gamma$.

Proof. Take the fuzzy controller function φ as,

$$\varphi(t_1, \cdots, t_k, s_1, \cdots, s_k, p_1, \cdots, p_k, r_1, \cdots, r_k, \tau)$$
$$:= \exp(-\frac{\imath||t_1, \cdots, t_k||^\jmath||s_1, \cdots, s_k||^\jmath||p_1, \cdots, p_k||^\jmath||r_1, \cdots, r_k||^\jmath}{\tau})$$

for all $t_i, s_i, p_i, r_i \in T$ $(i = 1, \cdots, k), \tau > 0$ and $\beta = 81^{-\jmath}$. Then, the random operator F will be stabled. By Theorem 3.1 we get an approximation of F by unique additive random operator Q showed in (3.9). □

In the next Corollary we change the fuzzy controller function to get another estimation.

Corollary 3.3. *Let (Γ, Σ, ξ) be a probability measure space, let T be a k-normed space and S a f-k-NLS. Suppose that $F : \Gamma \times T \to S$ is an odd random operator and there is a constant $\imath \geq 0$ and $\jmath \in \mathbb{R}$ such that $4\jmath \neq 1$ and F holds in the nest inequality*

$$\begin{aligned}
&\nu(F(\gamma, t_1) + F(\gamma, s_1) + F(\gamma, p_1) + F(\gamma, r_1), \cdots, \\
&F(\gamma, t_k) + F(\gamma, s_k) + F(\gamma, p_k) + F(\gamma, r_k), \tau) \\
&\quad \geq \bigwedge\{\nu(F(\gamma, t_1 + s_1 + p_1 + r_1), \cdots, F(\gamma, t_k + s_k + p_k + r_k), \tau) \\
&, \exp(-\frac{\imath(||t_1, \cdots, t_k||^\jmath + ||s_1, \cdots, s_k||^\jmath + ||p_1, \cdots, p_k||^\jmath + ||r_1, \cdots, r_k||^\jmath)}{\tau})\}
\end{aligned} \tag{3.10}$$

for all $t_i, s_i, p_i, r_i \in T$ $(i = 1, \cdots, k), \gamma \in \Gamma$. So, there exists a unique additive operator $Q : \Gamma \times T \to S$ such that

$$\begin{aligned}
&\nu(F(\gamma, t_1) - Q(\gamma, t_1), ..., F(\gamma, t_k) - Q(\gamma, t_k), \tau) \\
\geq\ &\exp(-\frac{(3^{1-4\jmath} + 3^{-3\jmath})\imath||t_1, \cdots, t_k||^q}{|81^\jmath - 3|\tau})
\end{aligned} \tag{3.11}$$

or every $t_i \in T$ $(i = 1, \cdots, k), \gamma \in \Gamma$ and $\tau > 0$.

Proof. Take the fuzzy controller function φ as,

$$\varphi(t_1, \cdots, t_k, s_1, \cdots, s_k, p_1, \cdots, p_k, r_1, \cdots, r_k, \tau)$$
$$:= \exp(-\frac{\imath(||t_1, \cdots, t_k||^\jmath + ||s_1, \cdots, s_k||^\jmath + ||p_1, \cdots, p_k||^\jmath + ||r_1, \cdots, r_k||^\jmath)}{\tau})$$

for all $t_i, s_i, p_i, r_i \in T$ $(i = 1, \cdots, k)$, $\tau > 0$ and $\beta = 81^{-\jmath}$. Then, the random operator F will be stabled. By Theorem 3.1 we get an approximation of F by unique additive random operator Q showed in (3.11). □

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