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Hermite Transform for Distribution and Boehmian Space

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Abstract: Hermite transform involves weigth function and Hermite polynomial as its kernel is discussed. The Hermite transform and its basic properties are extended to the distribution spaces and to the space of integrable Boehmian.

Key Words: Hermite transform, distribution space, Boehmian space.

Contents

1. Introduction

Schwartz distribution spaces (also named as generalized functions) have been defined for many integral transforms such as Foureir, Laplace etc. The Boehmian spaces, which are the generaliztaion of distribution spaces [12,13], are also extended to integral transforms [1,2,10].

Integral transforms, having the kernel as the weight function have been introduced by many reseachers with their properties and applications enumerated in different areas. In the present paper, we investigate distribution spaces and Boehmian spaces for the Hermite transform. Debnath [3], first introduced the Hemite transform with its basic operational properties in [4].

The Hermite transform of a function $f(x)$, -$\infty < x < \infty$, is defined by the integral

$$H\{f(x)\} = f_H(n) = \int_{-\infty}^{\infty} f(x)\widetilde{H_n}(x)e^{-x^2}dx, \tag{1.1}$$

where $\widetilde{H_n}(x)$ is the Hermite polynomial of degree n, standardized in such a manner that

$$\widetilde{H_n}(x) = \left.\begin{matrix} \frac{H_{2k}(x)}{H_{2k}(0)}\ , \text{if } n = 2k, k \in N_0 \\ \frac{H_{2k+1}(x)}{DH_{2k+1}(0)}\ , \text{if } n = 2k+1, \end{matrix}\right\} \tag{1.2}$$

where $H_n(x) = (-1)^n e^{x^2} D^n(e^{-x^2})$, $n \in N_0, D = \frac{d}{dx}$.

The basic operational properties and the convolution for Hermite and generalized Hermite transform for Lebesgue spaces are known [4,5,6,7,8,9]. The inverse Hermite transform is given by

$$H^{-1}\{f_H(n)\} = f(x) = \sum_{n=0}^{\infty} (\delta_n)^{-1} f_H(n)\widetilde{H_n}(x), \tag{1.3}$$

where $\delta_n = 2^n n!\sqrt{\pi}$.

The function $f(x)$ is the expansion

$$f(x) = \sum_{n=0}^{\infty} a_n \widetilde{H_n}(x), \tag{1.4}$$

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where the coefficients a_n can be determined from the orthogonal relation of the Hermite polynomials $\widetilde{H}_n(x)$ as

$$\int_{-\infty}^{\infty} \widetilde{H_m}(x)\widetilde{H_n}(x)e^{-x^2}dx = \delta_{nm}\delta_n. \tag{1.5}$$

The relation between the Laguerre and Hermite polynomial is

$$\widetilde{H}_n(x) = \left.\begin{array}{c} R_k^{-1/2}(x^2);\ \text{if } n = 2k \\ xR_k^{1/2}(x^2);\ \text{if } n = 2k+1 \end{array}\right\} \tag{1.6}$$

where R_n^{α} are the Laguerre polynomails.

We consider the function $f(x)$ which are locally integrable (Lebesgue integrable) and of $O(e^{ax^2})$ for large $|x|, a < 1$. Denote $L_{1,\exp(\mathbb{R})} = L_{1,\exp}$ of measurable function, by the norm defined by

$$\|f\|_{1,\exp} = \int_{-\infty}^{\infty} |f(x)|\, e^{-x^2/2}dx. \tag{1.7}$$

From $\int_{-\infty}^{\infty} e^{-x^2/2}dx = \sqrt{\pi}$, it holds $\|1\|_{1,\exp} = 1$. Markett [11] extended this space to $L_{p,\exp}, 1 \le p < \infty$ for the measurable functions on $\mathbb{R}$ having the norm

$$\|f\|_{p,\exp} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left|f(x)e^{-x^2/2}\right|^p dx. \tag{1.8}$$

In what follow are some results

Theorem 1.1 : *Let $f \in L_{1,\exp}, k, n \in N_0$. Then $H\{f(x)\}$ is a linear transform and moreover*

(i) $|f_H(n)| \le c_n \|f\|_{1,\exp}$,

(ii) $H[\widetilde{H_k}](n) = \widetilde{h_n}\delta_{kn}$

with c_n and $\widetilde{h_n}$ are defined in [9, p.154].

Corollary 1.1 [9] : *Let $f(x)$ is a polynomial of degree m, then $f_H(n) = 0$, for $n > m$, iff $f(x) = 0$ (a.e.).*

Theorem 1.2[9] : *Let f, g be continuous and bounded and let $f_H(n) = g_H(n)$ for every $n \in N_0$. Then $f(x) = g(x)$ (a.e.), i.e. $f = g$.*

Theorem 1.3: (Differentiation) [5,9] : *Let $f'(x)$ is continuous and $f''(x)$ is bounded and locally integrable in $-\infty < x < \infty$ (in other words, let $f, f' \in L_{1,\exp}$ and f'' is differentiable a.e. on $\mathbb{R}$), and if $H\{f(x)\} = f_H(n)$, then*

$$H\{R[f(x)]\} = -2nf_H(n), \tag{1.9}$$

where $R[f(x)]$ is the differential form given by

$$R[f(x)] = \exp(x^2)\frac{d}{dx}\left[\exp(-x^2)\frac{df}{dx}\right]. \tag{1.10}$$

Theorem 1.4 [5,9]: *If $f(x)$ is bounded and locally integrable in $-\infty < x < \infty$ ($f \in L_{1,\exp}$) and $f_H(0) = 0$, then $H\{f(x)\} = f_H(n)$ exists for each constant C,*

$$H^{-1}\left[-\frac{f_H(n)}{2n}\right] = R^{-1}[F(x)], \tag{1.11}$$

where R^{-1} is the inverse of the differential operator R and n is a positive integer.

Theorem 1.5 [5] : *If $f(x)$ has bounded derivatives of order m and if $H\{f(x)\} = f_H(x)$ exists, then*

$$H\{f^{(m)}(x)\} = f_H(n+m). \tag{1.12}$$

The product formula for Hermite functions is proved [7] by using suitable spaces, and by studying convolution for the generalized Hermite transform. Debnath [4] defined convolution of Hermite transform of odd functions. Dimovski and Kalla [6] extended the convolution for Hermite transform (1.1) for the odd and even functions for a suitably chosen Lebesgue space. The details for the convolution of Hermite transform can be referred to [5,6,7,8,9]. In general, if $f(x)$ and $g(x)$ are two arbitrarty functions defined on $\mathbb{R}$, then the convolution is given by

$$f * g = f_o \overset{o}{*} g_o + f_e \overset{e}{*} g_e \tag{1.13}$$

where f and g are expressed as sums of even and odd functions, i.e. $f(x) = f_o(x) + f_e(x)$, $g(x) = g_o(x) + g_e(x)$, and $f_e(x) = \frac{1}{2}[f(x) + f(-x)]$. Then the result [6] follows as

Thorem 1.6 [9] : *Let f, g are locally integrable on $\mathbb{R}$ and of order $O(e^{ax})$ as $|x| \to \infty$, with $a < 1$. Then there exists the Hermite transform (1.1) of $f * g$, defined by (1.13) as*

$$H[f * g] = H[f]H[g]. \tag{1.14}$$

Using the relation connecting Laguerre and Hermite polynomials, the product formula with the norm estimated on Lebesgue space as symmetry [cf. [11]], assist to define the generalized translation operator as under

Definition 1.1 [9] : *Let $f \in L_{1,\exp}$. Then the generalized translation operator T_x is defined by*

$$\begin{aligned}(T_x f)(y) &= \frac{1}{4}\{[f(x+y) + f(x-y)]e^{-xy} + [f(x-y) + f(y-x)]e^{xy}\} \\ &\quad + \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(z)k(x,y,z)e^{-z^2} dz, \end{aligned} \tag{1.15}$$

where $k(x, y, z)$ is the kernel.

Proposition 1.1 [9] : *Let $f, \sqrt{|x|}\, f \in L_{1,\exp}$. Then*

(i) $\|T_x f\|_{1,\exp} \le M e^{x^2/2}\{\|f\|_{1,\exp} + \sqrt{|x|}\, \|\sqrt{y} f(y)\|_{1,\exp}\}$

(ii) $(T_x f)(y) = (T_y f)(x)$

(iii) $(T_x \widetilde{H}_n)(y) = \widetilde{H}_n(x)\widetilde{H}_n(y)$.

Definition 1.2 [9]: *Let T_x be translation operator. Then the convolution for Hermite transform (HT), $f * g$ is defined by*

$$(f * g)(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(y)(T_x g)(y)e^{-y^2} dy \tag{1.16}$$

provided the integral exists.

Theorem 1.7 [9]: Convolution Theorem : *Let $\sqrt{|x|}\, f, \sqrt{|x|}\, g \in L_{1,\exp}$. Then $f * g \in L_{1,\exp}$ and it holds that*

(i) $H[f * g] = H[f]H[g]$,

(ii) $f * g = g * f$

The convolution appears to be commutative and associative. The convolution theorem of Hermite transform is proved by using (1.16) and Proposition 1.1.

The proofs of the Theorems 1.1 to 1.7 can be seen in [4,9]. The product and convolution formulae of generalized Hermite transform is also proved for $L_{p,\mu}-$ spaces, refer [8].

2. Distribution Spaces for Hermite Transform

In this section, the testing function space and its dual, which is known as distribution spaces, are employed on the Hermite transform. Notations, terminologies and definitions used, follow [15].

The testing function space, which is denoted by $\mathcal{D}$, consists of all complex valued functions $\varphi(t)$ that are infinitely smooth and vanishes outside some finite interval.

A sequence of testing functions $\{\varphi_v(t)\}_{v=1}^{\infty}$ is said to converge in $\mathcal{D}$ if $\varphi_v(t)$ are in $\mathcal{D}$, if they are all zero outside some fixed finite interval I, and if for every fixed non - negative integer k the sequence $\{\varphi_v^{(k)}(t)\}_{v=1}^{\infty}$ converges uniformly for $-\infty < t < \infty$.

The functionals that assign a complex number to every member of $\mathcal{D}$ is denoted as $\langle f, \varphi \rangle$ possess two properties, namely linearity and continuity.

Linearity : The functional f on $\mathcal{D}$ is said to be linear if testing functions φ_1 and φ_2 and any complex number α form relations

$$\left.\begin{array}{c} \langle f, \varphi_1 + \varphi_2 \rangle = \langle f, \varphi_1 \rangle + \langle f, \varphi_2 \rangle \\ \langle f, \alpha\varphi_1 \rangle = \alpha \langle f, \varphi_1 \rangle \end{array}\right\} \quad . \tag{2.1}$$

Continuity : A functional f on $\mathcal{D}$ is said to be continuous if for any sequence of testing functions $\{\varphi_v(t)\}_{v=1}^{\infty}$ that converges to $\varphi(t)$ in $\mathcal{D}$. In other words, if $f(t)$ is a locally integrable function, then the distribution f the convergent integral ($f(t)$ in $\mathcal{D}'$), defined by

$$\langle f, \phi \rangle = \langle f(t), \phi(t) \rangle \triangleq \int_{-\infty}^{\infty} f(t)\phi(t)dt \qquad ,$$

shows that the function $f(t)$ generates a distribution, or we say, $f(t)$ is in $\mathcal{D}'$ and $\phi(t)$ belongs to $\mathcal{D}$, where $\mathcal{D}$ and $\mathcal{D}'$ denote, respectively, testing function space and its dual .

In other words, a continuous linear functional on the space $\mathcal{D}$ is a distribution. The space of all such distributions is denoted by $\mathcal{D}'$ ($\mathcal{D}'$ is called the dual (or conjugate) space of $\mathcal{D}$).

2.1 Distributional Hermite Transform

Let $f(x)$ is locally integrable function that is absolutely continuous (or integrable) over $-\infty < x < \infty$ whose norm is

$$\|f\| = \int_{-\infty}^{\infty} |f(x)|\, e^{-x^2/2} dx.$$

The Hermite transform as defined in (1.1), is bounded and uniformly continuous. Then $f(x)$ generates a regular distriution in $\mathcal{D}'$ which is written in the form

$$H\{f(x)\} = f_H(n) = \left\langle f(x), e^{-x^2} \widetilde{H_n}(x)) \right\rangle , \tag{2.11}$$

where $f(x) \in \mathcal{D}'$ and $e^{-x^2} \widetilde{H_n}(x) \in \mathcal{D}$. This is said to be the distributional Hermite transform.

Since a regular distribution determines the function generating it almost everywhere, we may extend the uniqueness for the Hermite transform in the following:

Corollary 2.11 : *If the locally integrable functions $f(x)$ and $g(x)$ are absolutely integrable over $-\infty < x < \infty$ (that is, $f, g \in \mathcal{D}'$) and if their $f_H(n)$ and $g_H(n)$ are equal everywhere, then $f = g$ almost everywhere.*

Proof : Let f and g assign the same value to each $\varphi \in \mathcal{D}$. Indeed, by definition of distribution spaces choosing $\varphi \in \mathcal{D}$, inversse Hermite transform (1.3) can be written as

$$\left\langle \sum_{n=0}^{\infty} (\delta_n)^{-1} f_H(n) \widetilde{H_n}(x), \varphi \right\rangle = \langle f(x), \varphi(x) \rangle ,$$

which shows that $f \in \mathcal{D}'$. Then, by invoking above equation and by Corollary 1.1, under the same conditions, we readily get the uniqness theorem (Theorem 1.2), $\langle f, \varphi \rangle = \langle g, \varphi \rangle$ that is, $f_H(n) = g_H(n)$

Therefore φ is dense in $\mathcal{D}$, and $f, g \in \mathcal{D}'$.

Theorem 2.11 : *Let f , $g \in \mathcal{D}'$ (f and g are distributions) and f_H and g_H be their Hermite transform. Then*

$$H\{f * g\} = f_H(n)\ g_H(n) = H[f]H[g].$$

Proof : Let $\varphi \in \mathcal{D}$ and f , $g \in \mathcal{D}'$.*Then the convolution is*

$$\langle f * g, \varphi \rangle = \langle f(x), \langle g(y), \varphi(x+y) \rangle \rangle \quad .$$

This makes sense as $\langle g(y), \varphi(x+y) \rangle \in \mathcal{D}, f * g \in \mathcal{D}'$. Since $\varphi \in \mathcal{D} \subset L_{1,\exp}$, using (1.15) and (1.16), the convolution theorem of Hermite transform in the sense of distribution space holds, that is,

$$\langle H\{f * g\}, \varphi\rangle = \langle H[f]H[g], \varphi\rangle \, .$$

The convolution of Hermite transform that is commutative and associative (as mentioned above), will also be applicable for distribution space as $f * g \in \mathcal{D}'$.

The Hermite transform $f_H(n)$ is defined on distribution spaces when the functional $f \in \mathcal{D}'$. Then the basic operational formula such as differentation (1.9), (1.10); inverse Hermite transform (1.11), $m-$th order of derivative of Hermite transform, and the product formula will be easily applicale for the distribution spaces $\mathcal{D}'$.

3. Hermite Transform for Integrable Boehmians

In this section, we define integrable Boehmians and employ it on the Hermite transform.

Construction of Boehmians is given in [12,13,14]. We consider a special case of Boehmian space. Consider the space L_1 of complex valued Lebesgue integrable functions on the real line $\mathbb{R}$, having the norm $L_1(\|f\| = \int |f(x)|\, dx)$. If $f, g \in L_1$. Then the convolution product

$$(f * g)(x) = \int_{\mathbb{R}} f(u)g(x-u)du$$

is an element of L_1 and $\|f * g\| \leq \|f\| \, \|g\|$.

A sequence of continuous real functions $\delta_n \in L_1$ will be called a delta sequence if

(i) $\int_{\mathbb{R}} \delta_n(x)dx = 1$, $\forall n \in N$

(ii) $||\delta_n|| < M$, for some $M \in R$ and all $n \in N$, and

(iii) $\lim\limits_{n\to\infty} \int_{|x|>\epsilon} |\delta_n(x)|\, dx = 0$, for each $\epsilon > 0$.

If (ζ_n) and (ψ_n) are delta sequences, then we have $(\zeta_n * \psi_n)$. If $f \in L_1$ and (δ_n) is a delta sequence, then $\|f * \delta_n - f\| \to 0$ as $n \to \infty$. Delta sequences are called approximate identities or summability kernels.

A pair of the sequences ($f_i, \zeta_i)$ is called a quotient of sequence, denoted by f_i/ζ_i if $f_i \in L_1(i = 1, 2, \ldots), (\zeta_i)$ is a delta sequence, whereas two quotients of sequences f_i/ζ_i and g_i/ψ_i are equivalent if $f_i * \psi_i = g_i * \zeta_i$, for $i \in N$. The equivalence class of quotient of sequences will be called an integrable Boehmians, which will be denoted by B_{L_1}.

The space B_{L_1} is a convolution algebra when the multiplication by scalar, addition, and convolution are defined as

(i) $\lambda[f_i/\zeta_i] = [\lambda f_i/\zeta_i]$.

(ii) $[f_i/\zeta_i] + [g_i/\psi_i] = \left[\frac{f_i*\psi_i + g_i*\zeta_i}{\psi_i*\zeta_i}\right]$,

(iii) $[f_i/\zeta_i] * [g_i/\psi_i] = \left[\frac{f_i*g_i}{\psi_i*\zeta_i}\right]$.

A function $f \in L_1$ can be identified with Boehmian $\left[\frac{f_i*\delta_i}{\delta_i}\right]$, where (δ_i) is any delta sequence. It is convenient to treat L_1 or $L_{1,\exp}$ (as considered in previous section) as a subspace of B_{L_1}. If $F = [f_i/\delta_i]$, then $F * \delta_i = f_i$ and hence $F * \delta_i \in B_{L_1}$, for every $n \in N$.

There are two types of convergence, $\delta-$ convergence and Δ - convergence for Boehmian space. Similarly, for convergence in B_{L_1}, we have

(1) A sequence of Boehmians (F_i) is δ - convergent to a Boehmain F in B_{L_1}, denoted by $F_i \underset{\rightarrow}{\delta} F$, if there exists a delta sequence (δ_i) such that $(F_i * \delta_k), (F * \delta_k) \in L_1$ for every $n, k \in N$ and $\|(F_i - F) * \delta_k\| \to 0$, for each $k \in N$.

(2) A sequence of Boehmians (F_i) in B_{L_1} is said to be Δ - convergent, denoted by $F_n \underset{\rightarrow}{\Delta} F$ if there exists a $(\delta_i) \in \Delta$ such that $(F_i - F) * \delta_k \in L_1$ and $(F_i - F) * \delta_k \to 0$ as $n \to \infty$ in L_1.

Let $F = [f_i/\delta_i] \in B_{L_1}$. Then for each $n \in N$, we have $f_1 * \delta_i = f_i * \delta_1$. Since $\int \delta_i(x)dx = 1$ for each $i \in N$, we have $\int f_1(x)dx = \int_{\mathbb{R}}(f_1 * \delta_i)(x)dx = \int_{\mathbb{R}}(f_i * \delta_1)(x)dx = \int_{\mathbb{R}} f_i(x)dx$. This property allows to define the integral of Boehmian: If $F = [f_i/\delta_i] \in B_{L_1}$, then $\int F = \int f_1$. For a function from L_1 (or $L_{1,\exp}$) this integral is the same as the Lebesgue integral.

Lemma 3.1 : *If* $[f_i/\delta_i] \in B_{L_1}$, *then the sequence*

$$H\{f_i(x)\} = [f_H(n)]_i = \int_{-\infty}^{\infty} f_i(x)\widetilde{H_n}(x)e^{-x^2}dx$$

converges uniformly on each compact set in $(-\infty, \infty)$.

Proof. If (δ_i) is a delta sequence, then $H[\delta_i]$ converges uniformly on each compact set to a constant function. Hence, for each compact set K, $H(\delta_k) > 0$ on K for almost all $k \in K$ and

$$H(f_i) = H(f_i)\frac{H(\delta_k)}{H(\delta_k)}.$$

Using definition of Boehmian space and that of the convolution of Hermite transform, we write right side of above relation

$$= \frac{H(f_i * \delta_k)}{H(\delta_k)} = \frac{H(f_k * \delta_i)}{H(\delta_k)}$$

that is,

$$H(f_i) = \frac{H(f_k)}{H(\delta_k)}H(\delta_i) \quad \text{on } K.$$

Thus, the lemma is proved.

Theorem 3.1 : *If* $[f_i/\delta_i] \in B_{L_1}$, *then the sequnece* $H(f_i)$ *converges in* $\mathcal{D}'$. *Moreover, if* $[f_i/\delta_i] = [g_i/\delta_i]$, *then* $H(f_i)$ *and* $H(g_i)$ *converge to the same limit of Hermite transform for an integrable Boehmian.*

Proof : Let $[f_i/\delta_i], [g_i/\delta_i] \in B_{L_1}$ such that $[f_i/\delta_i] = [g_i/\delta_i]$ or $f_i * \delta_i = g_i * \delta_i$. Applying Hermite convolution to both sides of the above relation, we have

$$\begin{aligned} H(f_i * \delta_i) &= H(g_i * \delta_i) \\ H(f_i)H(\delta_i) &= H(g_i)H(\delta_i) \\ H[f_i/\delta_i] &= H[g_i/\delta_i] \end{aligned}$$

Hence $H[f_i/\delta_i] = H[g_i/\delta_i]$ is in B_{L_1}.

Definition 3.1 : *The Hermite transform of an integrable Boehmian* $F = [f_i/\delta_i]$ *is defined as the limit of* $H(f_i)$ *in the space of continuous functions in* L_1. *Thus, the Hermite transform of an integrable Boehmian is a continuous function.*

Theorem 3.2 : *Let* F *and* $G \in B_{L_1}$. *Then*

(1) $H(\lambda F) = \lambda H(F)$ *(for any complex* λ*) and* $H(F + G) = H(F) + H(G)$ *and*

(2) $H(F * G) = H(F)H(G)$.

Proof : Let $F = [f_i/\varphi_i]$ and $\lambda F = [\lambda f_i/\varphi_i]$. Since $f_i \in L_1 \Longrightarrow \lambda f_i \in L_1$and $\varphi_i \in \mathcal{D}$

$$H(\lambda F) = [H(\lambda f_i)/H(\varphi_i)] = \lambda H[f_i/\varphi_i] = \lambda(HF)$$

Therefore, $\lambda(HF) \in B_{L_1}$.

If $F = [f_i/\varphi_i]$ and $G = [g_i/\psi_i]$ in B_{L_1}, then $H(F) = H[f_i/\varphi_i]$ and $H(G) = H[g_i/\varphi_i]$ are in B_{L_1}.Now

$$F + G = \left[\frac{(f_i * \psi_i) + (g_i * \varphi_i)}{(\varphi_i * \psi_i)}\right].$$

Applying convolution of Hermite transform to both the sides of above relation, we get

$$\begin{aligned} H(F+G) &= \frac{H(f_i * \psi_i) + H(g_i * \varphi_i)}{H(\varphi_i * \psi_i)} \\ &= \frac{H(f_i)H(\psi_i) + H(g_i)H(\varphi_i)}{H(\varphi_i)H(\psi_i)} \\ &= H\left(\frac{f_i}{\varphi_i}\right) + H\left(\frac{g_i}{\varphi_i}\right) = HF + HG \end{aligned}$$

These properties show that the Hermite transform for integrable Boehmian is linear.

Proof of (2) is straight forward conclusion from the property of convolution for the Hermite transform in L_1 and distribution space, as given the previous sections.

Theorem 3.3 : *The Hermite transform* (HF) *is continuous with respect to* δ *- convergence and* Δ *- convergence, and that* $HF_i \to HF$ *in* B_{L_1} *iff* $F_i \to F$ *in* B_{L_1}.

Proof : The proof suffice in showing that $F = 0 \Longrightarrow H(F) = 0$ (as in Corollary 1.1). lim $F_i \underset{\rightarrow}{\delta} F$ implies $HF_i \to HF$ uniformly on each compact set. Let (δ_i) be a delta sequence such that $F_i * \delta_k, F * \delta_k \in L_1$ for all $i, k \in N$ and $\|(F_i - F) * \delta_k\| \to 0$ as $i \to \infty$ for all $k \in N$, and let K be a compact set in $\mathbb{R}$. Then $H(\delta_k) \to 0$ on K for some $k \in N$.

Since $H(\delta_k)$ is a continuous function, therefore $H(F_i)H(\delta_k) \to H(F)H(\delta_k)$ uniformly on K.

But $H(F_i)H(\delta_k) - H(F)H(\delta_k) = H[(F_i - F) * (\delta_k)]$ and $\|(F_i - F) * (\delta_k)\| \to 0$ as $n \to \infty$.Thus, $HF_i \to HF$ in B_{L_1}.

The basic properties of Hermite transform such as differentiation, integration, and inversion which are employed for the Lebesgue spaces, can be proved for the distribution spaces and for the Boehmian spaces as integrable Boehmian.

The distribution (also named as generalized functions) spaces are applied to Hermite polynomails. There are, arguably, many other types of distribution spaces and Boehmian spaces which are yet to be ventenured with regard to the Hermite transform.

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