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On Compositional Dynamics on Hardy Space

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ABSTRACT: In this work, we examine super-recurrence and super-rigidity of composition operators acting on $H(\mathbb{D})$ the space of holomorphic functions on the unit disk $\mathbb{D}$ and on $H^2(\mathbb{D})$ the Hardy-Hilbert space. We characterize the symbols that generate super-recurrent and super-rigid composition operators acting on $H(\mathbb{D})$ and $H^2(\mathbb{D})$.

Key Words: Hypercyclicity, supercyclicity, transitivity, recurrence, super-recurrence.

Contents

1. Introduction and preliminaries

Throughout this paper, $\mathbb{C}$ will represent the complex plane, $\mathbb{C}^*$ the punctured plane $\mathbb{C}\backslash\{0\}$, and $\hat{\mathbb{C}} = \mathbb{C} \cup \{\infty\}$ will be the one-point compactification of $\mathbb{C}$. Moreover, we will write $\mathbb{D}$ for the open unit disk of $\mathbb{C}$ and $\mathbb{T}$ for its boundary. Finally, we denote by $\mathbb{N}$ the set of positive integers.

Let X be a topological vector space over the field $\mathbb{K}$ ($\mathbb{R}$ or $\mathbb{C}$). Let $\mathcal{B}(X)$ be the set of operators (continuous linear self-maps acting on X).

Hypercyclicity and supercyclicity are among the most studied notions in Linear Dynamics.

An operator $T \in \mathcal{B}(X)$ is called *hypercyclic* if we can find some vector x whose orbit under T, that is

$$\text{Orb}(x, T) := \{T^n x : n \in \mathbb{N}\},$$

is dense in X. The vector x is called a *hypercyclic vector* for T.
On a separable Fréchet space X, Birkhoff proved in [14] that an operator $T \in \mathcal{B}(X)$ is hypercyclic if and only if for each pair (U, V) of nonempty open subsets of X there exists $n \in \mathbb{N}$ such that

$$T^n(U) \cap V \neq \emptyset.$$

An operator which satisfies the latter property is said to be *topological transitive*, see [8,31].

The operator T is called *supercyclic* if there exists some vector x whose projective orbit under T, that is

$$\mathbb{K}.\text{Orb}(x, T) := \{\lambda T^n x : \lambda \in \mathbb{K},\, n \in \mathbb{N}\},$$

is dense in X, see [33]. Such a vector x is called a *supercyclic vector* for T.
Again, if we assume that the space X is a separable Fréchet space, then T is supercyclic if and only if for two of nonempty open subsets U and V of X there exist some scalar $\lambda \in \mathbb{K}$ and some positive integer n such that

$$\lambda T^n(U) \cap V \neq \emptyset.$$

For more information about these two classes of operators, the reader may be referred to K.G. Grosse-Erdmann and A. Peris's book [31] and F. Bayart and É. Matheron's book [8], and the survey article [29] by K.G. Grosse-Erdmann. In [2,4,5,6] it was studied the dynamics of a set of operators instead of a single operator.

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Another important notion in Linear Dynamics is that of recurrence. This notion goes back to Poincaré [36]. Since then, it has been studied by many authors: Gottschalk and Hedlund in [27], and also Furstenberg in [22]. However, a fundamental systematic study of recurrent operators was done until 2014 in [18] by Costakis, Manoussos, and Parissis.

An operator $T \in \mathcal{B}(X)$ is called *recurrent* if for each nonempty open subset U of X, there exists some positive integer n such that

$$T^n(U) \cap U \neq \emptyset.$$

A vector $x \in X$ is said to be a *recurrent vector* for T if there exists an increasing sequence (n_k) of positive integers such that

$$T^{n_k}x \longrightarrow x \text{ as } k \longrightarrow \infty.$$

The set of all recurrent vectors for T will be denoted by $\text{Rec}(T)$:

$$\text{Rec}(T) := \{x \in X : \; x \in \overline{\text{Orb}}(Tx, T)\}.$$

The study of the recurrent behaviour of operators has become an active, exciting area in mathematics in the last few decades.

For more information about this class of operators, see [1,15,17,19,24,28,32,38].

A more robust notion than recurrence is that of rigidity. This notion goes back to Furstenberg and Weiss [23] in the ergodic theoretic setting. In Topological Dynamics, the notions of rigidity and uniform rigidity have been introduced and studied by Glasner and Maon [25]. In Linear Dynamics, the rigidity and uniform rigidity have been studied in [20,21] by Eisner and Grivaux.

An operator $T \in \mathcal{B}(X)$ is called *rigid* (resp. *uniformly rigid*) if there exists a strictly increasing sequence of positive integers (n_k) such that

$$T^{n_k}x \longrightarrow x \text{ as } k \longrightarrow \infty, \text{ for all } x \in X$$

$$(\text{resp. } \|T^{n_k} - I\| = \sup_{\|x\| \leq 1} \|T^{n_k}x - x\| \longrightarrow 0 \text{ as } k \longrightarrow \infty).$$

Motivated by the relationship between hypercyclicity and supercyclicity, a new class of operators has been introduced in [3]. This class called the class of *super-recurrent* operators. We say that T is super-recurrent if, for each nonempty open subset U of X, one can find a scalar λ and a positive integer n such that

$$\lambda T^n(U) \cap U \neq \emptyset.$$

A vector x is called a super-recurrent vector for T provided that there exist an increasing sequence (n_k) of positive integers and a sequence (λ_k) of scalars such that

$$\lambda_k T^{n_k}x \longrightarrow x \text{ as } k \longrightarrow \infty.$$

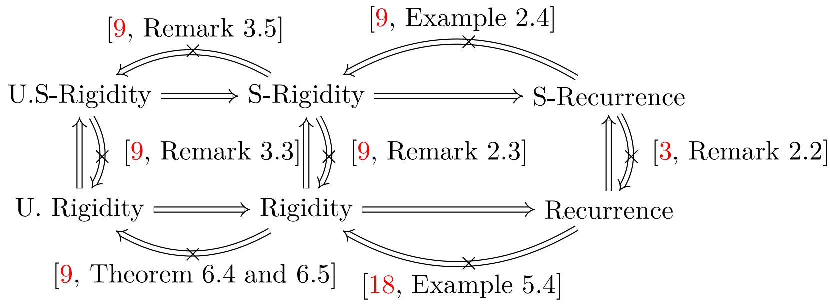
$\text{SRec}(T)$ will denotes the set of all super-recurrent vectors for T.

Furthermore, taking into consideration the relationship between the notions of recurrence, rigidity, and uniform rigidity, in [9] it was introduced the concepts of super-rigid and uniformly super-rigid operators. We say that T is *super-rigid* (resp. *uniformly super-rigid*) if we can provide the existence of a strictly increasing sequence (n_k) of positive integers and a sequence $(\lambda_k)_k$ of scalars such that

$$\lambda_k T^{n_k}x \longrightarrow x \text{ as } k \longrightarrow \infty, \text{ for all } x \in X$$

$$(\text{resp.} \quad \|\lambda_k T^{n_k} - I\| = \sup_{\|x\| \leq 1} \|\lambda_k T^{n_k}x - x\| \longrightarrow 0 \text{ as } k \longrightarrow \infty).$$

We have the following diagram of the relationship between recurrence, super-recurrence, and their deviations.



The study of composition operators has gained interest due to its importance as a good supply of examples for describing and generalizing results to other operators.

Recall that if $E \subset \mathbb{C}$ and $\phi : E \longrightarrow E$, then the composition operator induced by ϕ is defined by

$$C_\phi f = f \circ \phi,$$

for every f a function acting of E in $\mathbb{C}$. The map ϕ is called the symbol of the operator C_ϕ, see [37].

Different authors have studied the linear dynamics of composition operators, see [7,10,11,12,16,18,26, 30].

The super-recurrence, super-rigidity, and uniform super-rigidity of composition operators acting on $C([0,1])$ the space of continuous functions on $[0,1]$, on $H(\mathbb{C})$ the space of entire functions, and on $H(\mathbb{C}^*)$ the space of holomorphic functions on the punctured plan $\mathbb{C}^*$, were studied in [34]. Here, in this paper, we aim to study super-recurrence and super-rigidity of composition operators induced by linear fractional maps on the spaces $H(\mathbb{D})$ and $H^2(\mathbb{D})$.

Recall that linear fractional maps are those of the form

$$\phi(z) = \frac{az+b}{cz+d},\ z \in \mathbb{D},$$

for some $a, b, c, d \in \mathbb{C}$ satisfying $ad - bc \neq 0$. The latter condition is necessary and sufficient for f to be nonconstant. The automorphisms of $\mathbb{D}$ are the linear fractional transformations of the form

$$\phi(z) = b\frac{a-z}{1-\overline{a}z}, \quad |a| < 1,\ |b| = 1,$$

see [31, Proposition 4.36]. We classify linear fractional maps as:

1. Linear fractional maps without a fixed point in $\mathbb{D}$.
   (a) parabolic linear fractional maps: those have a unique, attractive fixed point on $\mathbb{T}$.
   (b) hyperbolic maps with an attractive fixed point on $\mathbb{T}$: those having an attractive fixed point $\alpha \in \mathbb{T}$ and a second fixed point $\beta \in \hat{\mathbb{C}} \backslash \mathbb{D}$. The linear fractional map is a hyperbolic automorphism of $\mathbb{D}$ if and only if both fixed points are on $\mathbb{T}$.
2. Linear fractional maps having a fixed point in $\mathbb{D}$. Here there are two cases:
   (a) either the interior fixed point is attractive, or
   (b) the map is an elliptic automorphism: The automorphisms of $\mathbb{D}$ having a fixed point $\alpha \in \mathbb{D}$ and the second fixed point $\beta \in \hat{\mathbb{C}} \backslash \overline{\mathbb{D}}$.

For more information about these notions and classification, we refer the reader to [37].

The organization of this paper is as follows. In Section 2, we study the super-recurrence and the super-rigidity on the space $H(\mathbb{D})$ (the space of holomorphic functions on the disk $\mathbb{D}$). We prove that both concepts super-recurrence and recurrence of the composition operator C_ϕ are equivalent to the fact

that the symbol ϕ is either univalent without fixed point in $\mathbb{D}$ or ϕ is an elliptic automorphism. As a result, we show that in the case of composition operator generated by a linear fractional map ϕ, these two notions are equivalents to the fact that ϕ is either parabolic or hyperbolic with no fixed point in $\mathbb{D}$, or an elliptic automorphism.
In the case of super-rigidity, we prove that the super-rigidity; and the rigidity of C_ϕ are both equivalent to the fact that the symbol ϕ is an elliptic automorphism.

In Section 3, we carry out the characterization of composition operator action on the Hardy space. We show that the super-recurrence and the recurrence are equal, and both equivalents to that the map ϕ is either hyperbolic with no fixed point in $\mathbb{D}$, a parabolic automorphism, or an elliptic automorphism. Furthermore, we prove that we have equivalence between rigidity and super-rigidity and that they are equivalents to that ϕ is an elliptic automorphism.

2. Super-recurrence on $H(\mathbb{D})$

Let $H(\mathbb{D})$ be the space of holomorphic functions on the disk $\mathbb{D}$ endowed with the topology of uniform convergence on compact subsets of $\mathbb{D}$, which is a Fréchet space. If ϕ: $\mathbb{D} \longrightarrow \mathbb{D}$ is a holomorphic map, then C_ϕ is a well defined continuous linear operator.

The following theorem shows that the hypercyclicity and the supercyclicity of C_ϕ on simply connected domains different from $\mathbb{C}$ are equivalent.

Theorem 2.1. *[12, Theorem 3.3 and Corollary 3.4] Let $G \neq \mathbb{C}$ be a simply connected domain, and ϕ an automorphism of G and let $C_\phi : H(G) \to H(G)$ be the composition operator with symbol ϕ. Then the following properties are equivalent:*

1. *$(\phi^n(z_0))$ approximates the boundary of G for some (all, resp.) $z_0 \in G$;*
2. *ϕ has no fixed point in G.*
3. *C_ϕ is hypercyclic on $H(G)$;*
4. *C_ϕ is supercyclic on $H(G)$.*

In the case of recurrence, we have the following theorem.

Theorem 2.2. *[18, Theorem 6.9] Let $\phi : \mathbb{D} \to \mathbb{D}$ be a holomorphic function and let $C_\phi : H(\mathbb{D}) \to H(\mathbb{D})$ be the composition operator with symbol ϕ. Then the following assertions are equivalent:*

1. *C_ϕ is recurrent;*
2. *ϕ is either univalent and has no fixed point in $\mathbb{D}$ or ϕ is an elliptic automorphism.*

In the case of linear fractional maps, we have the following corollary.

Corollary 2.3. *[18, Corollary 6.10] Let ϕ be a linear fractional map on $\mathbb{D}$ and let $C_\phi : H(\mathbb{D}) \to H(\mathbb{D})$ be the composition operator with symbol ϕ. The following assertions are equivalent:*

1. *C_ϕ is recurrent;*
2. *ϕ is either parabolic, or hyperbolic without fixed point in the unit open disk, or an elliptic automorphism.*

Now we turn into the case of super-recurrent. In the following theorem, we prove that the recurrence and super-recurrence of composition operators acting on $H(\mathbb{D})$ are equivalent to the fact that ϕ be either univalent with no fixed point in $\mathbb{D}$ or an elliptic automorphism.

Theorem 2.4. *Let $\phi : \mathbb{D} \to \mathbb{D}$ be a holomorphic function and let $C_\phi : H(\mathbb{D}) \to H(\mathbb{D})$ be the composition operator with symbol ϕ. Then the following assertions are equivalent:*

1. *C_ϕ is super-recurrent;*

2. *C_ϕ is recurrent;*

3. *ϕ is either univalent and has no fixed point in $\mathbb{D}$ or ϕ is an elliptic automorphism.*

Proof: The equivalence between (2) and (3) are due to Theorem 2.2 and The implication (3) $\Rightarrow$ (1) is trivial. So to complete the proof, we need only to show that (1) $\Rightarrow$ (3).
(1) $\Rightarrow$ (3): Assume that C_ϕ is super-recurrent. First, we will show that ϕ is univalent; that is holomorphic and one-to-one.
We prove it by contradiction. Assume that ϕ is not one-to-one. Then there exist two elements x_1, x_2 of $\mathbb{D}$ such that

$$x_1 \neq x_2 \quad \text{and} \quad \phi(x_1) = \phi(x_2).$$

Let f be an arbitrary super-recurrent vector for C_ϕ. Then there exist a sequence (λ_k) of numbers and a strictly increasing sequence (n_k) of positive integers such that

$$\lambda_k C_\phi^{n_k}(f) \longrightarrow f, \quad \text{as} \quad k \longrightarrow \infty.$$

Hence,

$$\lambda_k f\left(\phi^{n_k}(x_1)\right) \longrightarrow f(x_1) \quad \text{and} \quad \lambda_k f\left(\phi^{n_k}(x_2)\right) \longrightarrow f(x_2), \quad \text{as} \quad k \longrightarrow \infty.$$

Since $\phi(x_1) = \phi(x_2)$, it follows that $f(x_1) = f(x_2)$. Then

$$\mathrm{SRec}(C_\phi) \subset \{f \in H(\mathbb{D}) : f(x_1) = f(x_2)\}.$$

Thus by [3, Theorem 3.8], we have that

$$H(\mathbb{D}) = \overline{\mathrm{SRec}(C_\phi)} \subset \{f \in H(\mathbb{D}) : f(x_1) = f(x_2)\},$$

which is a contradiction. This means that ϕ is one-to-one. Hence ϕ is univalent. We have then the following two cases:

Case 1: If ϕ has no fixed point in $\mathbb{D}$, then there is nothing to prove.

Case 2: If ϕ has an interior fixed point $p \in \mathbb{D}$. In this case, we also have two cases:

(a) If ϕ is an automorphism of the disk the it is necessarily an elliptic automorphism; see [37].

(b) If ϕ is not an elliptic automorphism, then the Denjoy-Wolff Iteration Theorem [37, Proposition 1, Chapter 5], implies that $(\phi^n)_{n\in\mathbb{N}}$ converges to p uniformly on compact subsets of $\mathbb{D}$. Thus, the only super-recurrent vectors of C_ϕ are constant functions. Hence, C_ϕ cannot be super-recurrent in this case.

□

In the case of linear fractional maps, we have the following corollary.

Corollary 2.5. *Let ϕ be a linear fractional map on $\mathbb{D}$ and let $C_\phi : H(\mathbb{D}) \to H(\mathbb{D})$ be the composition operator with symbol ϕ. The following assertions are equivalent:*

1. *C_ϕ is super-recurrent;*

2. *C_ϕ is recurrent;*

3. *ϕ is either parabolic, or hyperbolic with no fixed point in $\mathbb{D}$, or an elliptic automorphism.*

In the case of rigidity, we have the following theorem.

Theorem 2.6. *[18, Theorem 6.11] Let $\phi : \mathbb{D} \to \mathbb{D}$ be a holomorphic function and let $C_\phi : H(\mathbb{D}) \to H(\mathbb{D})$ be the composition operator with symbol ϕ. Then the following assertions are equivalent:*

1. *C_ϕ is rigid;*

2. *ϕ is an elliptic automorphism.*

We now characterize the super-rigidity of composition operators on $H(\mathbb{D})$. In particular, we prove that the rigidity and super-rigidity of a composition operator on $H(\mathbb{D})$ are equivalent. We need to recall the following stronger notion than hypercyclicity to show that.

Definition 2.7. *[13, Definition 2.1]. Let X be a Fréchet space, T be an operator acting on X, and $(n_k)_{k\geq 0}$ be a strictly increasing sequence of positive integers. The operator T is called hereditarily hypercyclic with respect to $(n_k)_{k\geq 0}$ if for each subsequence $(n_{k_j})_{j\geq 0}$ of $(n_k)_{k\geq 0}$, the family $\{T^{n_{k_j}}\}$ is hypercyclic; that is, there exists some $x \in X$ such that $\{T^{n_{k_j}}x : j \geq 0\}$ is dense in X. If T is hereditarily hypercyclic with respect to the whole sequence of natural numbers, we will say that T is hereditarily hypercyclic.*

Remark 2.8. *If T is a hereditarily hypercyclic operator, it is recurrent and thus super-recurrent. However, a hereditarily hypercyclic operator can never be rigid ([18, Definition 1.5]) or even super-rigid. Indeed, let $(\lambda_k)_{k\in\mathbb{N}}$ be a sequence of scalars and $(n_k)_{k\in\mathbb{N}}$ be a sequence of integers. Then there is a sequence $(\mu_k)_{k\in\mathbb{N}}$ such that $|\frac{\lambda_k}{\mu_k}| = 1$ for every $k \in \mathbb{N}$. Then the sequence $\{\frac{\lambda_k}{\mu_k}T^k : k \in \mathbb{N}\}$ is hereditary hypercyclic. Thus, there is some $x_0 \in X$ such that*

$$\frac{\lambda_k}{\mu_k}T^{n_k}x_0 \not\to x_0.$$

Hence,

$$\lambda_k T^{n_k}x_0 \not\to x_0.$$

Theorem 2.9. *Let $\phi : \mathbb{D} \to \mathbb{D}$ be holomorphic and let $C_\phi : H(\mathbb{D}) \to H(\mathbb{D})$ be the composition operator with symbol ϕ. The following assertions are equivalent*:

1. *C_ϕ is super-rigid;*
2. *C_ϕ is rigid;*
3. *ϕ is an elliptic automorphism.*

Proof: The equivalence between (2) and (3) are due to Theorem 2.6, the implication (3) $\Rightarrow$ (1) is obvious. So to complete the proof of the theorem, we need only to show that (1) $\Rightarrow$ (3).

(1) $\Rightarrow$ (3): Suppose that C_ϕ is super-rigid. Then, Theorem 2.4 implies that either ϕ has no fixed point in $\mathbb{D}$ or that it is an elliptic automorphism. If ϕ has no fixed point in $\mathbb{D}$, then it follows as in the proof of [18, Theorem 6.10] that for every compact set $K \subset \mathbb{D}$ there exists a positive integer n_0 such that

$$\phi(K) \cap K = \emptyset, \text{ for all } n \geq n_0.$$

This implies that C_ϕ is hereditarily hypercyclic; see [30, Theorem 3.2]. In this case, C_ϕ cannot be super-rigid. Thus ϕ is an elliptic automorphism. □

3. Super-recurrence on $H^2(\mathbb{D})$

Let $H^2(\mathbb{D})$ be the Hardy space, that is, the space of holomorphic functions $f : \mathbb{D} \to \mathbb{C}$ satisfying

$$\|f\|_{H^2(\mathbb{D})} = \sup_{0<r<1}\left(\frac{1}{2\pi}\int_0^{2\pi}|f(re^{it})|^2dt\right)^{1/2}$$

is finite. If $\phi : \mathbb{D} \to \mathbb{D}$ is a holomorphic map, then C_ϕ is a well defined a bounded linear operator, which is a consequence of Littlewood's Principle ([35], [37, Chap. 1]).

The hypercyclicity and the supercyclicity of the composition operators on $H^2(\mathbb{D})$ are equivalent, as shown in the following theorem.

Theorem 3.1. *[16] Let ϕ be a linear fractional map of $\mathbb{D}$ and let $C_\phi : H^2(\mathbb{D}) \to H^2(\mathbb{D})$ be the composition operator with symbol ϕ. The following assertions are equivalent*:

1. *C_ϕ is hypercyclic;*
2. *C_ϕ is supercyclic;*
3. *ϕ is either hyperbolic without fixed point on $\mathbb{D}$, or parabolic automorphism.*

In the case of recurrence, the authors in [18] gave the following characterization of composition operators acting on $H^2(\mathbb{D})$.

Theorem 3.2. *[18, Theorem 6.12] Let ϕ be a linear fractional map of $\mathbb{D}$ and let $C_\phi : H^2(\mathbb{D}) \to H^2(\mathbb{D})$ be the composition operator with symbol ϕ. The following assertions are equivalent*:

1. *C_ϕ is recurrent;*
2. *ϕ is either hyperbolic with no fixed point in $\mathbb{D}$, or a parabolic automorphism, or an elliptic automorphism.*

In the following theorem, we prove that the equivalence between the hypercyclicity and the supercyclicity of composition operators acting on $\mathbb{D}$ remains steadfast in the cases of recurrence and super-recurrence.

Theorem 3.3. *Let ϕ be a linear fractional map of $\mathbb{D}$ and let $C_\phi : H^2(\mathbb{D}) \to H^2(\mathbb{D})$ be the composition operator with symbol ϕ. The following assertions are equivalent*:

1. *C_ϕ is super-recurrent;*
2. *C_ϕ is recurrent;*
3. *ϕ is either hyperbolic with no fixed point in $\mathbb{D}$, or a parabolic automorphism, or an elliptic automorphism.*

Proof: The equivalence between (2) and (3) are due to Theorem 3.2 and The implication (3) $\Rightarrow$ (1) is trivial. So to complete the proof, we need only to show that (1) $\Rightarrow$ (3).
(1) $\Rightarrow$ (3): Suppose that C_ϕ is super-recurrent. Then as the proof of Theorem 2.4, ϕ is either parabolic or hyperbolic with no fixed point in $\mathbb{D}$ or an elliptic automorphism. If ϕ is a parabolic non-automorphism, then by [37, The Linear Fractional Hypercyclicity Theorem, p.114], only constant functions can be super-recurrent vectors. Therefore C_ϕ is not super-recurrent in this case. □

Theorem 3.4. *[18, Theorem 6.13] Let ϕ be a linear fractional map of $\mathbb{D}$ and let $C_\phi : H^2(\mathbb{D}) \to H^2(\mathbb{D})$ be the composition operator with symbol ϕ.*

1. *C_ϕ is rigid;*
2. *ϕ is an elliptic automorphism.*

To characterize the super-rigidity of composition operators acting on $H^2(\mathbb{D})$, we prove that the rigidity and the super-rigidity of these operators are equivalent.

Theorem 3.5. *Let ϕ be a linear fractional map of $\mathbb{D}$ and let $C_\phi : H^2(\mathbb{D}) \to H^2(\mathbb{D})$ be the composition operator with symbol ϕ.*

1. *C_ϕ is super-rigid;*
2. *C_ϕ is rigid;*
3. *ϕ is an elliptic automorphism.*

Proof: The equivalence between (2) and (3) are due to Theorem 3.4 and The implication (3) $\Rightarrow$ (1) is trivial. So to complete the proof, we need only to show that (1) $\Rightarrow$ (3).
(1) $\Rightarrow$ (3): Suppose that C_ϕ is super-rigid, then C_ϕ is super-recurrent. Thus, by Theorem 3.3 either ϕ is hyperbolic with no fixed point in $\mathbb{D}$, or a parabolic automorphism, or an elliptic automorphism. If ϕ is hyperbolic with no fixed point in $\mathbb{D}$ or a parabolic automorphism, then C_ϕ is hereditarily hypercyclic; see [8]. Thus C_ϕ cannot be super-rigid. □

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