Bol. Soc. Paran. Mat.
©SPM –ISSN-2175-1188 on line
SPM: www.spm.uem.br/bspm

(3s.) **v. 2023 (41)** : 1–11.
ISSN-0037-8712 in press
doi:10.5269/bspm.62650

A Mathematical Model and Optimal Control Analysis for Scholar Drop Out

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abstract: In this paper, we proposed and analyzed a non-linear mathematical model for scholar Drop out and we advanced an optimal control policy for this model by considering three variables namely the numbers of school-age children who are in school, school-age children who are out of school, and school-age children in non-formal education. The model is examined using the stability theory of differential equations. The optimal control analysis for the proposed scholar Drop out model is performed using Pontryagin's maximum principle. The conditions for optimal control of the problem with effective use of implemented policies to counter this scourge are derived and analyzed.

Key Words: Scholar drop out, mathematical model, equilibria, stability, optimal control.

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1. Introduction

Dropping out of school is a significant scale in developing countries, as mentioned by several UNESCO reports (2005, 2008, 2009, 2011, etc.). The literature review also indicates various consequences of the phenomenon on the social level, in particular the increase in the illiteracy rate, the achievement of universal primary education, as well as economically, including high unemployment rate. The majority of studies that have looked at this phenomenon are mainly of an economic nature, dealing little with the question from the point of view of families who are however the first to be affected by their child dropping out of school. In addition, most of these research papers are primarily quantitative. The figures obtained indicating the importance of risk factors, leaving in the shade an in-depth explanation of the phenomenon. At the theoretical level, the present study is informed by the social representations theory developed by Moscovici (1989), a theory that is mainly concerned with investigating the relationships that a social actor or group maintain with regard to social objects such as education. Dropping out of school deprives children of their right to equitable and quality education, as well as of their right to obtain a certification or diploma which is supposed to help them flourish, build their future and participate in the full development of their country. For exemple in Morocco, dropping out of school threatens thousands of students and forces them to leave school way before graduation time and in certain cases even before the completion of the cycles of compulsory education (primary and secondary college). Indeed, in 2018, around 431,876 students dropped out of public education without having obtained their certification, of which 78% were primary and college cycles, cycles which are supposed to retain children in class at least until the age of 15 [1]. Aware of the seriousness of this phenomenon and its harmful effects on students and their families and on the future of next generations, the 2015-2030 Strategic Vision called, in its lever 1, to implement the principle of equal access to education and training and in particular to ”continued

2010 *Mathematics Subject Classification:* 35B40, 35L70.
Submitted February 23, 2022. Published May 16, 2022

efforts targeting the fight against school dropouts and dropouts and to dry up their sources respectively ". In lever 3, promoting positive discrimination, it is even a question of giving priority in efforts to re-educte school dropouts in rural and peri-urban areas and deficit areas, taking into account the high abandonment rates which characterize these areas. However, it would not be fair to deny the efforts and devices set up by the state to reduce losses in the system of education and in particular in compulsory education cycles. With various social support programs, the public authorities are doing their best to reduce dropout by improving learning conditions for students from disadvantaged backgrounds. However, there are several efforts and devices set up by the Moroccan state to reduce losses in the system of education and in particular in compulsory education cycles. With various social support programs (e.g., canteens, scholarships, boarding schools, Tayssir), the public authorities are trying their best to reduce dropout by improving learning conditions for students from disadvantaged backgrounds. The search for solutions to two problems affecting the Moroccan education system, namely non-schooling and dropping out of school are at the origin of non-formal education. It was a question of ensuring alternative educational offers to children outside the school system, whatever the reason, aims to reintegrate young people aged 9 to 15 into formal education, training professional or prepare them for a more suitable integration into working life. In this paper we will deal with a social problem. We will model the problem of scholar drop out through non-linear ordinary differential equations, and will attempt to propose an optimal control policy for this model, a similar work is already being done for the social problem of unemployment [2]. The optimal control analysis for Scholar drop out model using Pontryagins maximum principle [3] is considered. The conditions were derived and analyzed for optimal control of the school dropout problem with effective use of the policies implemented by the government to reintegrate children who have dropped out of school into formal education and create new opportunities in non-formal education. We will consider the following three major variables in the above problem:

(1) S: the number of school-age children who are in school,

(2) D: the number of school-age children who are out of school,

(3) R: the number of school-age children in non-formal education.

The paper is organized as follows. Section 2 describes the basic model, Section 3 describes the existence of equilibria, Section 4 describes the stability, Section 5 describes the formulation of mathematical model and the derivation of optimal control pair, and Section 6 deals with numerical simulation results. Conclusions are outlined in Section 7.

2. A Mathematical model for scholar drop out

Variables and parametres of the model are described in Table I. The assumptions we make are as follows:

(i)The school-age children who are out of school can enter in non-formal education program,

(ii) any child of school age who is not in school must go through non-formal education to return to school,

(iii) rate of migration of school-age children who are in school is assumed to be proportional to their number,

(iiii) rate of movement from children who are in non-formal education class to children who are in school class is jointly proportional to $R(t)$ and $D(t)$.

$\alpha_0 R(t)D(t)$ represent the number of school-age children return to school after going to non-formal education. Flow diagram of the model for scholar drop out problem considered in this study is given in fig1.

In view of the aforementioned considerations, the following system of equations that captures the problem of scholar drop out in a region is derived.

$$\begin{aligned} \frac{dD(t)}{dt} &= \Lambda - (\beta+\delta)\, D(t) + \alpha S(t) \\ \frac{dS(t)}{dt} &= \alpha_0 R(t)D(t) - (\beta+\alpha)\, S(t) \\ \frac{dR(t)}{dt} &= -\alpha_0 R(t)D(t) - \beta R(t) + \delta D(t) \end{aligned} \qquad (1)$$

tab 1. Variables and parameters.

Variables/parameters	Explanation
$S(t)$	The number of school-age children who are in school at time t
$D(t)$	The number of school-age children who are out of school at time t
$R(t)$	The number of school-age children in non-formal education at time t
Λ	The constant rate at which the number of the school-age children who are out of school is increasing continuously
α	Represents the rate at which the school-age children drop out of school
α_0	Represents the rate at which the school-age children who are out of school return to school
β	Represents the natural mortality rate of children at school age
δ	Represents the rate at which the school-age children who are out of school enter in non-formal education program

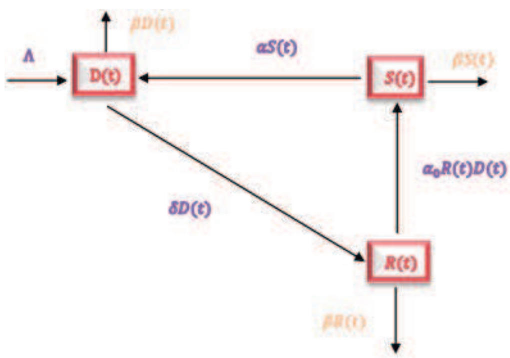

Figure 1: Model flow diagram with initial conditions $D(0) = D_0, S(0) = S_0$ and $R(0) = R_0$.

The feasible set of system (1) that attracts all solutions initiation in the interior of positive octant is given by

$$\Omega = \left\{ \begin{array}{c} (D(t), S(t), R(t)) : 0 \leq D(t) \leq D_0 + \frac{\Lambda}{\beta} + \frac{\alpha}{\beta} \|S\|_\infty , 0 \leq S(t) \leq S_0 + \alpha_0 \|R\|_\infty \|D\|_\infty , \\ 0 \leq R(t) \leq R_0 + \delta \|D\|_\infty \end{array} \right\}$$

Lemma 2.1. *The set*

$$\Omega = \left\{ \begin{array}{c} (D(t), S(t), R(t)) : 0 \leq D(t) \leq D_0 + \frac{\Lambda}{\beta} + \frac{\alpha}{\beta} \|S\|_\infty , 0 \leq S(t) \leq S_0 + \alpha_0 \|R\|_\infty \|D\|_\infty , \\ 0 \leq R(t) \leq R_0 + \delta \|D\|_\infty \end{array} \right\}$$

is a region of attraction for the model in system (1) and attracts all solutions initiation in the interior of positive octant.

Proof. As $\dot{D} = \Lambda - (\beta + \delta) D(t) + \alpha S(t)$ then $\frac{d}{dt} \left(D(t) e^{\beta t} \right) \leq \Lambda e^{\beta t} + \alpha S(t) e^{\beta t}$

Hence $D(t) e^{\beta t} - D_0 \leq \frac{\Lambda}{\beta} \left(e^{\beta t} - 1 \right) + \alpha \int_0^t S(u) e^{\beta u} du$

so $D(t) \leq D_0 e^{-\beta t} + \frac{\Lambda}{\beta} - \frac{\Lambda}{\beta} e^{-\beta t} + \alpha e^{-\beta t} \int_0^t S(u) e^{\beta u} du$

$\Rightarrow D(t) \leq D_0 + \frac{\Lambda}{\beta} + \alpha e^{-\beta t} \int_0^t S(u) e^{\beta u} du$

$\Rightarrow D(t) \leq D_0 + \frac{\Lambda}{\beta} + \frac{\alpha}{\beta} \|S\|_\infty e^{-\beta t} \left(e^{\beta t} - 1 \right)$

Hence $D(t) \leq D_0 + \frac{\Lambda}{\beta} + \frac{\alpha}{\beta} \|S\|_\infty \left(1 - e^{-\beta t} \right)$

Or $0 \leq 1 - e^{-\beta t} \leq 1$

So $D(t) \leq D_0 + \frac{\Lambda}{\beta} + \frac{\alpha}{\beta} \|S\|_\infty$

by Analogy, we prove that $S(t) \leq S_0 + \alpha_0 \|R\|_\infty \|D\|_\infty$ and $R(t) \leq R_0 + \delta \|D\|_\infty$ □

2.1. Equilibrium analysis

The model system (1) has one non negative equilibrium point $E_0(D^*, S^*, R^*)$ obtained by solving the following set of algebraic equations.

$$\begin{array}{ll} \Lambda - (\beta+\delta)\, D(t) + \alpha S(t) = 0 & (2) \\ \alpha_0 R(t) D(t) - (\beta+\alpha)\, S(t) = 0 & (3) \\ -\alpha_0 R(t) D(t) - \beta R(t) + \delta D(t) = 0 & (4) \end{array}$$

From Eq. (2), we get $D = \frac{1}{\beta+\delta}(\Lambda + \alpha S)$

Taking an addition of equation (3) and (4) $\delta D - \beta R - (\beta+\alpha)\, S = 0$

by replacing this value of D we get $R = \frac{1}{\beta}[\frac{\delta}{\beta+\delta}\Lambda + (\frac{\delta\alpha}{\beta+\delta} - (\beta+\alpha))S]$

Put values of D and R in (3) we get, $-A_0 S^2 - A_1 S + A_2 = 0$ (*)

Where,

$$A_0 = \frac{\alpha_0 \alpha}{\beta(\beta+\delta)^2}(\beta^2 + (\alpha+\delta)\,\beta)$$

$$A_1 = \frac{1}{\beta(\beta+\delta)^2}\left[\Lambda\alpha_0\left(\beta^2 + (\alpha+\delta)\,\beta - \delta\alpha\right) + \beta\,(\beta+\alpha)\,(\beta+\delta)^2\right]$$

$$A_2 = \frac{\alpha_0 \delta \Lambda^2}{\beta(\beta+\delta)^2}$$

we can easily prove that $\beta^2 + (\alpha+\delta)\,\beta - \delta\alpha \geq 0$ if $\beta \geq \frac{\sqrt{(\alpha+\delta)^2 + 4\alpha\delta} - (\alpha+\delta)}{2}$

Since $A_i, i = 0, 1, 2$ all are positive and number of changes in signs of equation (*) is only one. So, by Descart's rule equation (*) has only one positive solution say S^*. So, we get the non-negative equilibrium point of model with coordinates:

$$D^* = \frac{1}{\beta+\delta}(\Lambda + \alpha S^*)$$

$$R^* = \frac{1}{\beta}[\frac{\delta}{\beta+\delta}\Lambda + (\frac{\delta\alpha}{\beta+\delta} - (\beta+\alpha))S^*]$$

So, $E_0(D^*, S^*, R^*)$ is required non negative solution of the Model.

2.2. Stability analysis

To check the local stability of equilibrium point $E_0(D^*, S^*, R^*)$ we calculate the variational matrix M of the model system (1) corresponding to $E_0(D^*, S^*, R^*)$

$$M = \begin{bmatrix} -(\beta+\delta & \alpha & 0 \\ \alpha_0 R^* & -(\alpha+\beta) & \alpha_0 D^* \\ -\alpha_0 R^* + \delta & 0 & -\alpha_0 D^* - \beta \end{bmatrix}$$

The characteristic equation of the above matrix M is $\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0$ (5)

where

$$a_1 = \alpha_0 D^* + 3\beta + \alpha + \delta$$

$$a_2 = (\alpha_0 D^* + \beta)(2\beta + \alpha + \delta) + (\alpha + \beta)(\delta + \beta) - \alpha\alpha_0 R^*$$

$$a_3 = (\alpha_0 D^* + \beta)\left(\beta^2 + (\alpha + \delta)\beta - \alpha\alpha_0\right) + \alpha\alpha_0 D^* R^*$$

Since, a_1, a_2, a_3 are all positive and a little algebraic manipulation yields that $a_1 a_2 - a_3 > 0$. So, by Routh Hurwitz criteria all roots of equation (5) are negative or having a negative real part. Therefore equilibrium point $E_0(D^*, S^*, R^*)$ is locally asymptotically stable.

3. A mathematical model for scholar drop out with control variables

3.1. The mathematical model

The dropping out situation has prompted educational decision-makers in Morocco to implement projects and programs to counter this scourge. among these programs there is one based on conditional cash transfer called Tayssir and was launched in October 2008. This program is managed by the Moroccan Association for Support to Schooling (AMAS) in partnership with the Ministry of Education and the Higher Education Council. It "consists in bringing a financial contribution to poor families in order to act on abandonment school by neutralizing certain factors affecting the demand for education " (Permanent Mission of the Kingdom of Morocco in Geneva, 2009, p. 1). Thus, with its various partners, AMAS provides grants to the families concerned. Those scholarships are given according to the child's educational level and the number children going to school for a family; this money transfer is conditional on the presence of the child at school. Tayssir's goal is to reduce wastage by encouraging families to continue sending their children to school and, above all, to support them throughout their primary school career.The search for solutions to two problems affecting the Moroccan education system, namely non-schooling and dropping out, is at the origin of non-formal education, one of its objectives is to reintegrate young people aged 9 to 15 into formal education.

we can have a optimal control of the school dropout problem. For effective control to be achievable in finite time, time dependent controls [4] need to be considered. When the control is time dependent, the Pontryagin's maximum principle can be used to determine the conditions for effective control of scholar drop out in finite time. The following control variables are introduced in the model described in Section 5.

- Control $u_1(t) \in [0, 1]$ is the implemented policy of government based on conditional cash transfer called Tayssir.
- Control $u_2(t) \in [0, 1]$ is the implemented policy of government to integrate young people aged 9 to 15 into non-formal education.

Introducing the controls based on conditional cash transfer called Tayssir and on integrate young people aged 9 to 15 into non-formal education, the model described in (1) becomes

$$\begin{aligned} \frac{dD(t)}{dt} &= \Lambda - (\beta + \delta) D(t) + \alpha u_1 S(t) - u_2 D(t) \\ \frac{dS(t)}{dt} &= \alpha_0 R(t) D(t) - \beta S(t) - \alpha u_1 S(t) \\ \frac{dR(t)}{dt} &= -\alpha_0 R(t) D(t) - \beta R(t) + \delta D(t) + u_2 D(t) \end{aligned} \tag{6}$$

On the whole, pre-selected objective involves minimization of the number of children who have dropped out of school at minimum cost of policymaking based on conditional cash transfer called Tayssir and on integrate young people aged 9 to 15 into non-formal education. The objective function J is given by

$$J(u_1(t),u_2(t)) = \int_0^{t_f} \left[AD(t) + \frac{B}{2}u_1^2(t) + \frac{C}{2}u_2^2(t)\right] dt \qquad (7)$$

where t_f is the final time and coefficients A, B and C are balancing cost factors. The terms $\frac{B}{2}u_1^2(t)$ and $\frac{C}{2}u_2^2(t)$ are the costs associated with implemented policies of government based on conditional cash transfer called Tayssir and on integrate young people aged 9 to 15 into non-formal education. A quadratic costs on the controls with the given objective function $J(u_1(t),u_2(t))$ is chosen [4,5]. The goal is to minimize the number of children who have dropped out of school, while minimizing the costs of controls $u_1(t)$ and $u_2(t)$ such that

$$J(u_1^*(t),u_2^*(t)) = \min\left\{J(u_1(t),u_2(t))/u_1(t),u_2(t) \in w\right\} \qquad (8)$$

where $w = (u_1(t),u_2(t))$ such that u_1,u_2 measurable with $0 \le u_1(t) \le 1, 0 \le u_2(t) \le 1$, for $t \in [0,t_f]$, is the control set. The necessary conditions that an optimal control problem must satisfy are obtained using Pontryagin's maximum principle [6]. This principle converts equations (7) and (8) into a problem of minimizing pointwise Hamiltonian H with respect to $u_1(t)$ and $u_2(t)$.

$$\begin{aligned} H = AD(t) + \frac{B}{2}u_1^2(t) + \frac{C}{2}u_2^2(t) \quad & +\lambda_D[\Lambda - (\beta+\delta)\,D(t) + \alpha u_1 S(t) - u_2 D(t)] \\ & +\lambda_S[\alpha_0 R(t)D(t) - \beta S(t) - \alpha u_1 S(t)] \\ & +\lambda_R[-\alpha_0 R(t)D(t) - \beta R(t) + \delta D(t) + u_2 D(t)] \end{aligned} \qquad (9)$$

where λ_D, λ_S and λ_R are the associated adjoint variables or co-state variables for the states D, S and R , respectively. By applying Pontryagian's maximum principle [6] and the existing result for optimal control from [3], the system of equations is obtained, taking the appropriate partial derivatives of the Hamiltonian (9) with respect to the associated state variables.

Theorem 3.1. *Given the optimal controls $u_1(t)$ and $u_2(t)$ and solutions $D^*(t), S^*(t)$ and $R^*(t)$ of the corresponding state system (6) that minimizes $J(u_1(t),u_2(t))$ over w, there exist adjoint variables λ_D, λ_S, and λ_R satisfying*

$$-\frac{d\lambda_i}{dt} = \frac{\partial H}{\partial i} \qquad (10)$$

with transversality condition

$$\lambda_D(t_f) = \lambda_S(t_f) = \lambda_R(t_f) = 0 \text{ where } i = D,S,R \qquad (11)$$

Further,

$$\begin{aligned} u_1^*(t) &= \min\left\{1, \max\left\{0, \frac{(\lambda_S-\lambda_D)\alpha S(t)}{B}\right\}\right\} \\ \text{and } u_2^*(t) &= \min\left\{1, \max\left\{0, \frac{(\lambda_D-\lambda_R)D(t)}{C}\right\}\right\} \end{aligned} \qquad (12)$$

Proof. The existence of an optimal control follows from corollary 4.1 of Fleming and Rishel [3], because the integrand J is a convex function of $u_1(t)$ and $u_2(t)$. A priori boundedness of the state solutions and also the state system satisfies the Lipschitz property with respective to the state variables. The differential equations governing the adjoint variables are obtained by differentiating the Hamiltonian function and evaluating at the optimal control pair. Then the adjoint system can be written as

$$\begin{aligned}
\frac{d\lambda_D}{dt} &= -\frac{\partial H}{\partial D(t)} \\
&= -A + \alpha_0 R(t)(\lambda_R - \lambda_S) + \lambda_D(\beta+\delta) + (\lambda_D - \lambda_R)u_2(t) - \lambda_R\delta \\
\frac{d\lambda_S}{dt} &= -\frac{\partial H}{\partial S(t)} \\
&= \alpha u_1(t)(\lambda_s - \lambda_D) + \lambda_s\beta \\
\frac{d\lambda_R}{dt} &= -\frac{\partial H}{\partial R(t)} \\
&= \alpha_0 D(t)(\lambda_R - \lambda_S) + \lambda_R\beta
\end{aligned}$$

Therefore,

$$\begin{aligned}
\frac{d\lambda_D}{dt} &= -A + \alpha_0 R(t)(\lambda_R - \lambda_S) + \lambda_D(\beta+\delta) + (\lambda_D - \lambda_R)u_2(t) - \lambda_R\delta \\
\frac{d\lambda_S}{dt} &= \alpha u_1(t)(\lambda_s - \lambda_D) + \lambda_s\beta \qquad (13)\\
\frac{d\lambda_R}{dt} &= \alpha_0 D(t)(\lambda_R - \lambda_S) + \lambda_R\beta
\end{aligned}$$

with transversality conditions $\lambda_D(t_f) = \lambda_S(t_f) = \lambda_R(t_f) = 0$.

Because of the a priori boundedness of the state system, adjoint system, and the resulting Lipschitz structure of the ordinary differential equations, the uniqueness of the optimal control for small t_f is obtained. The uniqueness of the optimal control follows from the uniqueness of optimality system, which consists of (10) and (11) with characterization (12). There is a restriction on the length of time interval in order to guarantee the uniqueness of the optimality system. This smallness restriction of the length on the time is due to the opposite time operations of (10) and (11). The state problem has initial values, whereas the adjoint problem has final values. This restriction is very common in control problems [4, 7, 8].

In order to minimize Hamiltonian H with respect to the controls at the optimal controls, H is differentiated with respect to $u_1(t)$ and $u_2(t)$. on the set w, and equating to zero, the following solutions are obtained.

Now, $\frac{dH}{u_1(t)} = 0$ gives $u_1^*(t) = \frac{(\lambda_S - \lambda_D)\alpha S(t)}{B}$

and $\frac{dH}{u_2(t)} = 0$ gives $u_2^*(t) = \frac{(\lambda_D - \lambda_R)D(t)}{C}$

Then by standard control arguments involving the bands on the controls, it is concluded for $u_1(t)$:

$$u_1^*(t) = \begin{cases} 0, & \frac{(\lambda_S-\lambda_D)\alpha S(t)}{B} \le 0 \\ \frac{(\lambda_S-\lambda_D)\alpha S(t)}{B}, & 0 < \frac{(\lambda_S-\lambda_D)\alpha S(t)}{B} < 1 \\ 1, & \frac{(\lambda_S-\lambda_D)\alpha S(t)}{B} \ge 1 \end{cases}$$

In compact form,

$u_1^*(t) = \min\left\{1, \max\left\{0, \frac{D(t)(\lambda_D - \lambda_R)}{B}\right\}\right\}$

Similarly, for $u_2(t)$ in compact form, there is

$u_2^*(t) = \min\left\{1, \max\left\{0, \frac{(\lambda_D - \lambda_R)D(t)}{C}\right\}\right\}$ □

3.2. Numerical simulations

This section discusses the numerical simulations of the optimality system and the corresponding results of varying the optimal controls u_1 and u_2.according to official data in Morocco ("Ministry of National Education", and "the High Planning Commission") [9], $\alpha = 0.021, \beta = 0.05$, the parameter values are given in tab 2. Numerical solutions to the optimality system composing the state equation (6) and adjoint equation (13) are carried out in MATLAB R2018a using parameters from the Table II together with the following weight factors $A = 20, B = 10$ and $C = 10$ and initial conditions $S(0) = 10000, D(0) = 1000$ and $R(0) = 100$ (the choice of these values is approximate to reality).

The algorithm is the forward–backward scheme, starting with an initial guess for the optimal controls u_1 and u_2. The state variables are then solved forward in time from the dynamics (6) using a Runge–Kutta method of the fourth order. Then, those state variables and initial guess for the controls u_1 and u_2 are used to solve the adjoint equation (10) backward in time with given final conditions (9), again employing a fourth-order Runge–Kutta method. The controls u_1 and u_2 are updated and used to solve the state and then the adjoint system. This iterative process terminates when current state, adjoint, and control values converge sufficiently.

tab 2. Parameter values used in the optimal control simulation.

parameters	Baseline value	Reference
Λ	500	Assumed
α_0	0.000009	Assumed
α	0.021	official data in Morocco
β	0.05	official data in Morocco
δ	0.7	Assumed

Numerical simulations are investigated when both controls on reintegrate children who have dropped out of school into formal education and on creating new opportunities in non-formal education are optimized. It is also compared with the model when controls are not used. When both controls are optimized, the control profile is shown in Figure 2. It is observed that control u_1 dropped gradually from the lower bound to the upper bound after time $t = 2$ units and the control u_2 dropped gradually from the lower bound to the upper bound after time $t = 3$ units.

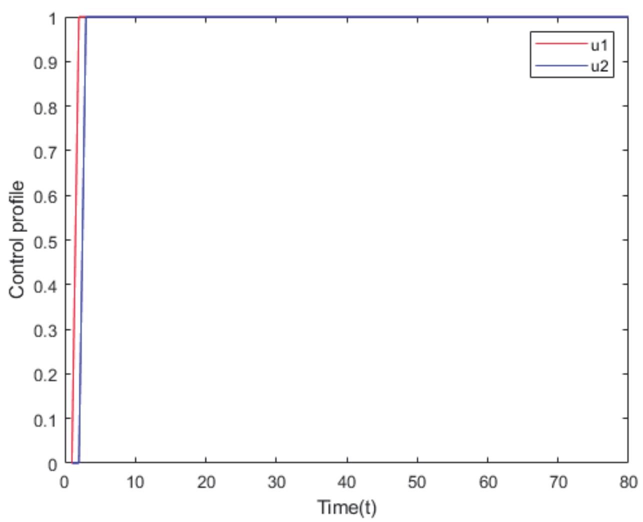

Figure 2: Simulation of the model showing the control profile of the intervention strategy

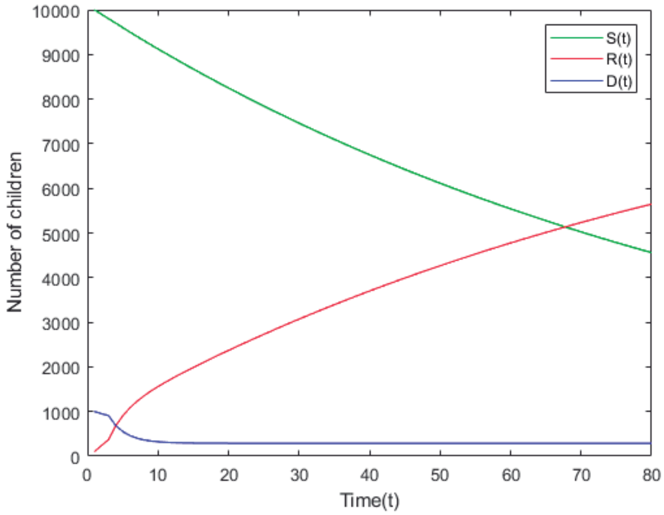

Figure 3: Time series plot of the scholar Drop out model when control strategy is in use

Figure 3 shows the time series plot of the scholar Drop out model when both control strategies are in use. With this strategy to optimize objective function J, in Figure 4, it is observed that this control strategy results in a significant increase in the number of school-age children who are out of school compared with the case of without control.

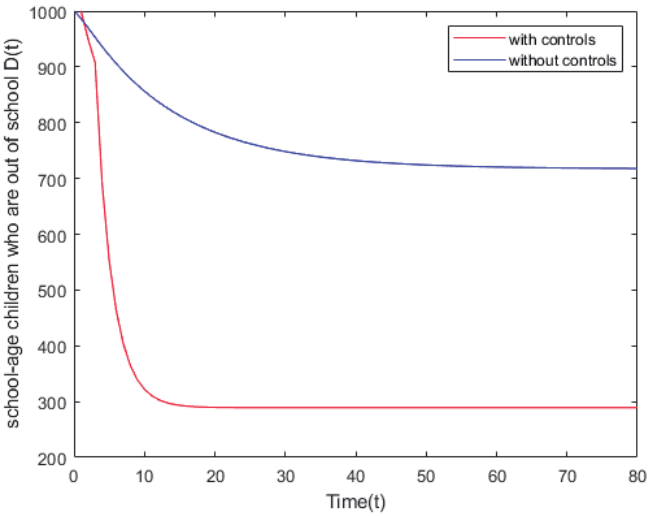

Figure 4: Simulation of the model showing the effect of control profile on D

Similarly, it can be observed from Figure 5 that control strategy results in a significant decrease in the number of school-age children who are in school compared with the case without control, and also in Figure 6, it is observed that control strategy results in a significant increase in the number of school-age children in non-formal education compared with the case without control.

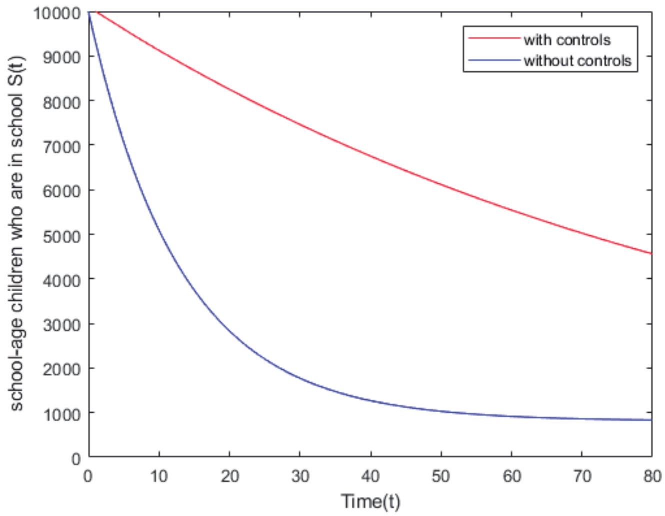

Figure 5: Simulation of the model showing the effect of control profile on S

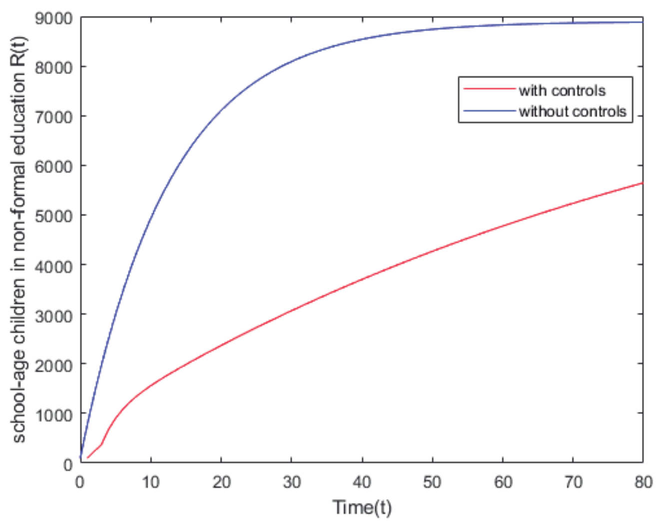

Figure 6: Simulation of the model showing the effect of control profile on R

4. Conclusion

With some simplifications, we built a mathematical model for a complex social situation. This is because the purpose of the model is the description and prediction of the essential patterns of the process and not the achievement of its complete analysis [5]. In this paper, optimal control analysis for scholar Drop out model is performed using Pontryagins maximum principle. The conditions for optimal control of the scholar Drop out problem were derived and analyzed with the effective use of government through the implemented policies to reintegrate children who have dropped out of school into formal education and to create new opportunities in non-formal education, it is observed from the numerical results that this control strategy results in a significant increase in the number of school-age children who are out of school and decrease in the number of school-age children who are in school compared with the case of without control. It is concluded that the successful policy of a government to create new opportunities in non-formal education has significant impact in reducing scholar drop out. Control programs that follow these strategies can effectively reduce the scholar drop out problem in the society. We can extend the study of this social phenomenon of school dropout in spatiotemporal model, similar works has been done in the field of biomathematics [10,11,12,13,14,15,16].

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