Bol. Soc. Paran. Mat.
©SPM –ISSN-2175-1188 on line
SPM: www.spm.uem.br/bspm

(3s.) **v. 2022 (40)** : 1–6.
ISSN-0037-8712 in press
doi:10.5269/bspm.42922

Topological Rank of (-1, 1) Metabelian Algebras*

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ABSTRACT: In 1981, Pchelintsev developed the idea for arranging non-nilpotent subvarieties in a given variety by using topological rank for spechtian varieties of algebra as a fixed tool. In this paper we show that for a given topological rank over a field of 2, 3 - torsion free of $(-1, 1)$ metabelian algebra solvable of index 2 that are Lie-nilpotent of step not more than p is equal to P.

Key Words: -1, 1) algebra, Metabelian algebra, Topological rank, Lie-nilpotent algebra, Superalgebra.

Contents

1. Introduction

Belov [5] devoted his study to the construction of infinitely based varieties of associative algebras over an infinite field of arbitrary positive torsions. Il'tyakov [4] proved that a finitely generated alternative PI-algebra over a field of zero torsion a particular, finite dimensional. Then the variety var A, generated by the algebra A, is specht (i.e., each sub-variety is defined by a finite set of identities)

Pchelintsev [10] studied the right alternative metabelian (solvable of index 2) Grassmann algebra of rank 1 and 2. Any identity in a nonnilpotent 2,3-torsion free $(-1, 1)$ metabelian ring of degree greater than or equal to 6 is a consequence of four defining identity of M where M is the metabelian $(-1, 1)$ ring (See [8]).

Platonoa, [12] studied the non-nilpotent subvarieties of the variety of two-step solvable algebras of type (γ, δ) an additive basis of a free metabelian (γ, δ)- algebra and constructed to proved that any identity in a non-nilpotent metabelian (γ, δ) - algebra of degree at least C is a consequence of four defing relations. Badeev [1] provided a chain $\vartheta_1 \subset ... \subset \vartheta_n \subset ... \subset \vartheta$ of varieties of commutative alternative nil-algebras over a field of 3-torsion similarly topological rank of γ_n is linear function on n and topological rank $\vartheta = \mathcal{N}_0$.

In 1985, Isaev [7]shown that non-finitely based varieties of right alternative metabelian algebras can even be produced by limited dimensional algebras. The Specht properties for fixed varieties of right alternative algebras were also learned in [8]-[11]. From 1976, it is familiar [2] that the variety of every right alternative metabelian algebras over an arbitrary field is not Spechtian. So, we can say that the different types of algebra is called spechtian, if it's each subvariety is limited intervals.

The paper is divided into 4 sections. In Section 2, we give some initial results about the free $(-1, 1)$ algebra $F_{(-1,1)^{<p>}}[Z]$ on a finite set Z of produces over F. Section 3, describes to the relations of the free algebras $(-1, 1)^{<p>}$ [Z]. In Section 4, we build a system of linear produces for the space of multilinear polynomials in $F_(-1, 1)^{(<p>)}[Z]$ of enough high degree and obtain the upper bound for topological rank of 2, 3-torsionfree $(-1, 1)^{<p>} \leq P$ by calculating the values of topological ranks of some subvarieties in 2, 3-torsionfree $(-1, 1)^{<p>}$ of special type.

* The project is partially supported by UGC New Delhi Grant No. F.No. 42-17 (2013).
2010 *Mathematics Subject Classification:* 17A30.
Submitted May 24, 2018. Published September 23, 2018

2. Preliminaries

Let F be a field over 2, 3 - torsion free (-1, 1) metabelian algebra defined by the identities

$$(a,b,c)+(a,c,b)=0, \tag{2.1}$$

$$(a,b,c)+(b,c,a)+(c,a,b)=0, \tag{2.2}$$

$$(ab)(ct)=0 \tag{2.3}$$

where $(a,b,c)=(ab)c-a(bc)$ is the associator of the variables a,b,c. By $(-1,1)^{<p>}$ we denote the subvariety of 2,3- torsion free ring $(-1,1)$ distinguished by the identity

$$[[...[z_1,z_2],...,z_p],z_{p+1}]=0 \tag{2.4}$$

of Lie-nilpotentency of step P, where $[a,b]=ab-ba$ is the commutator of a,b. Throughout the paper, F is a 2, 3-torsionfree field; all vector spaces are examined over a field F ; $Z=\{z_1,z_2,...\}$ is a complete set; $\mathfrak{u}=F_{(-1,1)}[Z]$ is a 2, 3-torsionfree $(-1,1)$ algebra on the set Z of produces;L_a and R_a are, consequently, the operators of left and right multiplication by the element a ; the associative algebra, $H_a=R_a-L_a;\mathfrak{u}^*$ produced by the right operator R_a and left operator L_a , for $a\in\mathfrak{u}$, transforming $\mathfrak{u}^2$ and by the identical mapping id; Var A is the variety produced by an algebra A.

Recall [2, 10] that $\mathfrak{u}^*$ satisfies the relations

$$R_a^2=0 \tag{2.5}$$

$$[R_aR_b,L_c]=0, \tag{2.6}$$

$$[R_a,L_b]=-L_aL_b. \tag{2.7}$$

We begin by proving the following Lemma.

Lemma 2.1. *The operator R_aR_b lies in the center of $\mathfrak{u}^*$.*

Proof. From relations (2.5) and (2.6) the lemma is proved. □

Lemma 2.2. *The algebra $\mathfrak{u}^*$ makes the relation*

$$3R_aR_b+H_aH_b=2[R_a,H_b]+H_aR_b+H_bR_a. \tag{2.8}$$

Proof. Using equation (2.7), we have $H_aH_b=(R_a-L_a)(R_b-L_b)=R_aR_b-R_aL_b-L_aR_b+L_aL_b=R_aR_b-[R_a,L_b]-L_aR_b-L_bR_a-[R_a,L_b]=R_aR_b-2[R_a,L_b]-L_aR_b-L_bR_a$. From above relation with equations (2.5) and (2.7), we get $3R_aR_b+H_aH_b=4R_aR_b+2[R_a,L_b]-L_aR_b-L_bR_a=2[R_a,L_b]+2[R_a,L_b]+4R_aR_b-2[R_a,L_b]-L_aR_b-L_bR_a=R_aR_b+R_bR_a+2[R_a,H_b]+H_aR_b+H_bR_a=2[R_a,H_b]+H_aR_b+H_bR_a$. □

From now, the notation of both R and H are represented as a usual notation with symbol T. The notation $w=T_a,....,T_b$ indicates, every notation of w could be equivalent to R or H independently. By assuming all notations in some word w are equal to every notation, the usage of operator symbols,

$$T(i_1...i_n)=\begin{cases}R_{a_{i_1}...R_{i_{1_n}}}, & \text{if } T=R,\\ H_{a_{i_1}...H_{i_{1_n}}}, & \text{if } T=H.\end{cases}$$

and $T(\phi)$ =id.

Lemma 2.3. *The algebra $\mathfrak{u}^*$ is extended by the operators $H(i_1,i_2,...,i_n)R(j_1,...,j_m)$.*

Proof. By assuming I be a linear extent of all operators $H(i_1,i_2,...,i_n)R(j_1,...,j_m)$. It suffices to show the inclusions $R(k)I\subseteq I\ andIH(K)\subseteq I$. We note that equation (2.8) yields $R(i)H(j)\in I$. Consequently the inclusion $R(k)I\subseteq I$ can be easily shown by induction on the length of the operator $H(i_1,i_2,...,i_n)$. Simultaneously, Lemma 2.1 implies $IH(K)\subseteq I$.
Assume L be a linear extent in $\mathfrak{u}^*$ of all operators as $L_{x_i}w,w\in\mathfrak{u}^*$. But from equation (2.7), one can show the congruence by generalizing n and representing L as an ideal of $\mathfrak{u}^*$.

$$H(1,...,n)\equiv R(1,...,n)(modL),n\in N. \tag{2.9}$$

□

3. Relations of the free $(-1,1)^{<p>}$-algebra

Let $\mathfrak{u}_p = F_{(-1,1)^{<p>}}[Z]$ be the free 2, 3 - torsion free on the set Z of generators. The following was indicated by Lemma 2.3 .

Lemma 3.1. *The linear extent of all operators of degree $d \geq P in \mathfrak{u}_p^*$ is extended by the operators $H(i_1, ..., i_n)R(j_1, .., j_{d-n}), n < P$. The word "polynomial" defined as homogeneous polynomial of degree greater than or equal to two.*

Definition: Assume $\approx$ be a symmetric relation on the set of polynomials of $\mathfrak{u}$ similarly $f_0 \approx f_1$ if $f_i \approx f_{1-i}R(j_1, ..., j_{2k}), i \in \{0, 1\}$, and f_{1-i} doesn't depend on the variables $x_{j_1} \dots, xj_{2k}$. With the like symbol $\approx$ we indicate the generated relation on $\mathfrak{u}^* : \xi \approx \eta$ for $\xi, \eta \in \mathfrak{u}^*$ if $(x_i x_j)\ \xi \approx (x_i x_j)\eta$ and ξ, η do not depend on x_i, x_j .

Lemma 3.2. *The algebra $\mathfrak{u}_p$ makes the relation*

$$a^3 \approx 0 \tag{3.1}$$

Proof. By equations (2.5), (2.1) and (2.3), we have $2ba^3 = b(a \circ a^2) = b(a.a^2) + b(a^2.a) = (ba)a^2 + b(a^2.a) = b(a^2.a) = ((ba).a)a = (ba).a^2 = 0$.

Therefore, $a^3 L = 0$. By applying equations (2.9), for even $n \geq P$, we obtain $a^3 \approx a^3 R(1, ..., n) = a^3 H(1, ..., n) = 0$. Hence, We can conclude that nearly every polynomial of algebra $\mathfrak{u}_P$ that is operators of $\mathfrak{u}_p^*$ satisfies some condition $\mathfrak{v}$. For a natural $n, \mathfrak{v}$ sustain for every polynomial of degree more than n. □

Lemma 3.3. *If $f \approx 0$ for $f \in \mathfrak{u}_p$, then almost every operator of $\mathfrak{u}_p^*$ annihilate f.*

Proof. Let $fR(j_1, ..., j_{2k}) = 0$, where f doesn't depend on $a_{j_1} ... a_{j_{2k}}$. In sight of Lemma 3.1, the degree $d \geq p + 2k$ of each operator word $\xi \in \mathfrak{u}_p^*$ can be defined as $\xi = nR(j_1, ..., j_{2K}), \eta \in \mathfrak{u}_p^*$. Therefore, using Lemma 2.1, we have $f_\xi = fR(j_1, ..., j_{2K})\eta = 0$. □

Lemma 3.4. *Almost all operators of $\mathfrak{u}_p^*$ are skew-symmetric with respect to all their variables.*

Proof. Let $w \in u_p^2$. By equation (2.3) and equation (2.5), the partial linearization (see [9, chap. 1]) of equation (3.1) represented in the form of $(wa)a + (aw)a + a^2 w = (aw)a = 0$, whence, $H_a R_a = (R_a - L_a)R_a = -L_a R_a \approx 0$. Hence in aspect of Lemma 3.1, it is used to evaluate $H_a H_a \approx (H_a H_a)R_b R_c = H_a(H_a R_b)R_c \approx -H_a H_b R_a R_c \approx -H_a R_a R_c H_b \approx 0$.
□

Lemma 3.5. *The algebra $\mathfrak{u}_p$ suffices the relation*

$$(ab)T_a T_b \approx 0. \tag{3.2}$$

Proof. Because of Lemma 2.1, 3.4, it suffices to verify that $(ab)R_a R_b \approx 0$. Using equations (2.5), (2.1) and Lemma 3.4, we have $(ab)R_a R_b = -(ab)R_b R_a = -((ab)b)a = -b^2 L_a R_a \approx 0$. □

Lemma 3.6. *The algebra $\mathfrak{u}_p^*$ suffices the relations*

$$3R_a R_b - 2[R_a, H_b] + H_a H_b \approx 0, \tag{3.3}$$

$$[R_a, H_b H_c] = 0. \tag{3.4}$$

Proof. Applying Lemma 3.4 in the equation (2.8) we obtain $3R_a R_b - 2[R_a, H_b] + H_a H_b \approx 0$. Now computing this equation towards left and using Jacobi identity we see that $3[R_a, R_b R_c] - 2[R_a, [R_b, H_b]] + [R_a, H_b H_c] \approx 0$ that is $[R_a, H_b H_c] \approx 2[R_a, [R_a, H_c]] \approx [R_a, R_b, H_c] - [R_a, [R_a, H_c]] = [H_c, [R_b, R_a]] = 0$.
□

Definition: Assume I be the ideal of $\mathfrak{u}_p^*$. For $\xi, \eta \in \mathfrak{u}_p^*$ we write $\xi \cong \eta (modI)$ if there is $a\theta \in I$ such that $\xi - \eta \approx \theta$. Let $H_n(n < p)$ be the ideal of $\mathfrak{u}_p^*$ generated by all the elements $H(i_1, ..., i_n)$.

Lemma 3.7. *The algebra $\mathfrak{u}_p^*$ suffices the relation*

$$H(1, ..., 2t) \cong 0(mod\mathbb{H}_{2t+1}) \tag{3.5}$$

Proof. We set $\eta = H(1......2t)$. By using equations (3.3) and (3.4), we have $3\eta \approx 3\eta R_a R_b \cong 2\eta R_a H_b \cong 2R_a \eta H_b \cong 0(mod\mathbb{H}_{2t+1})$ □

4. Upper bound for the topological rank of $(-1, 1)^{<P>}$

Definition: An n-allotted variety $(1 \leq n \leq p)$ is a subvariety V of $(-1, 1)^{<p>}$ such that the free V - algebra on the set $X$of generators satisfies the relation

$$\psi(x_1, x_2, ...x_{n+1}) \tag{4.1}$$

where

$$\psi(x_1, x_2...x_{n+1}) = \begin{cases} [[....[x_1, x_2]....x_n], x_{n+1}], & \text{if n is even,} \\ [[....[x_1 x_2, x_3], ...x_n], x_{n+1}], & \text{if n is odd.} \end{cases}$$

Assume A be the free V- algebra on the set Z of produces and v be an n - allotted variety $(n \geq 2)$ and $\mathcal{P}_{d,n}(d \geq 3)$ be the subspace of multilinear polynomials in $\mathcal{A}$ on the variables $z_1, ..., z_d$. We eliminated the indices of variable at the operator symbols and they are arranged in ascending order in order to stop complex formulas while writing down the polynomials of $\mathcal{P}_{d,n}$. For instance, notation $w = (a_2, a_5)H^2R^3$ means the monomial $w = (a_2, a_5)H(1, 3)R(4, 6, 7)$.

Definition: Systematic words are the polynomials of $\mathcal{P}_{d,n}$ of the following types:
1. $(a_1 \circ a_i)H^{2j}R^{d-2j-2}$
2.$[a_1, a_i]H^{2j}R^{d-2j-2}$
3.$[a_2, a_3]H^{2j}R^{d-2j-2}$
4.$[a_1, a_2]H^{2k-1}R^{d-2k-1}$
where $i = 2, 3, ..., d_i; j = 0, 1, ..., t - 1; k = 1, 2, ..., n - t - 1; t = [\frac{n}{2}]$.

Lemma 4.1. *Approximately every polynomial of $\bigcup_{d=3}^{\infty} \mathcal{P}_{d,n}$is linear combinations of regular words.*

Proof. By Lemma 3.4, there is a degree d of similarly all monomial $(a_1, a_2)T_3, ..., T_d \in \mathcal{P}_{d,n}$ is skew-symmetric w. r. t. $a_3, ..., a_d$. Therefore, in view of relation (3.5) and Lemma 3.1,$\mathcal{P}_{d,n}$ can be spanned by polynomials

$$(x_i \circ x_j)H^k R^{d-k-2}, [x_i, x_j]H^k R^{d-k-2},$$

where $x \circ y = xy + yx, 1 \leq i < j \leq d$, and $k = 0, 1, ..., 2t - 1$.

$$(a \circ b)T_c + (b \circ c)T_a + (c \circ a)T_b \approx 0,$$

Linearizing equation (3.1), we obtain

$$[a, b]T_c T_t + [a, b]T_c T_b + [c, b]T_a T_t + [c, t]T_a T_b \approx 0 \ .$$

By using these relations, it is not difficult to show that $\mathcal{P}_{d,n}$ can be spanned by the polynomials:
1')$(a_i \circ a_j)H^k R^{d-k-2}$,
2')$[a_1, a_i]H^k R^{d-k-2}$,
3') $[a_2, a_3]H^k R^{d-k-2}$,
where $i = 2, 3, ..., d$ and $k = 1, 2, ..., 2t - 1$.
Linear extent of all systematic words of type 1) -3) are denoted by w. For even k the polynomials of type 1') -3') lie in w. Let us validate for odd k, prove the polynomials of types 1') -3') can be represented as linear combinations of systematic words.
Because of equation (2.1), we have

$$(a \circ b)H_c = (ac + ba)(R_c - L_c) = (a \circ b)c - c(a \circ b) = (a \circ b)c - (ca)b - (cb)a.$$

Hence, in view of equation (3.4), all polynomials of type 1') lie in w. In addition, using Lemmas 2.1, 3.4, the partial linearization

$$(ab)b + (ba)b + b^2 a \approx 0$$

of (3.1), identity (2.1) and relation (3.3), we get

$$[a, b]H_b \approx [a, b]H_c R_c R_u \approx [a, b]R_c R_u H_b = [a, b]R_b R_c H_u$$
$$= b^2(2L_a + R_a)R_c H_u = (b^2 R_a + b^2 L_a)H_c H_u = (b^2 R_a + (ab)R_b).$$

In view of equation (3.4), for odd k the secured relation indicates that the polynomials of types 2'), 3') are skew-symmetric modulo W in respect of every variable. Therefore all polynomials of type 2'), 3') is proportional modulo W to a systematic word of type 4). □

Lemma 4.2. *For all n - allotted variety V ($n \geq 2$) there is a punctured neighbourhood $\bigcup_d^0(V)$ similarly all variety of $\bigcup_d^0(V)$ is (n - 1) - allotted.*

Proof. Because of the restriction 2 torsionfree F, relation (3.1) and Lemma 3.4 implies that for some punctured neighbourhood of V all varieties can be described by a system of identities where every polynomial begins from some enough high degree are multilinear. According to Lemma 4.1, select punctured neighbourhood $\bigcup_d^0(V)$ for all varieties of $m \in \bigcup_d^0(V)$ fulfils an identity $f = 0$, since f is a nontrivial linear combination of regular words of $\mathcal{P}_{d,n}$. Let A be the free $\mathfrak{m}$ – algebra on the set Z of produces. By rewriting the relation (4.1) shortly as $A^2 H^{2t} \approx 0$ $n = 2t + 1$ and as $AH^{2t} \approx 0$ if $n = 2t$. Firstly we examine the case $n = 2t + 1$ and we rpove that relation $A^2 H^{2t} \approx 0$ and identity $f = 0$ imply $AH^{2t} \approx 0$. From Lemma 4.1, f can be defined as

$$f = \sum_{i=2}^{d} \sum_{j=0}^{t-1} (\alpha_{2j}^{(i)}(a_1 \circ a_i)H^{2j} R^{d-2j-2} + \alpha_{2j+1}^{(i)}[a_1, a_i]H^{2j} R^{d-2j-2}) (\text{mod } w_{3,4}),$$

where $w_{3,4}$ is the linear extent of systematic words of types 3), 4). By fixing $i \geq 4$ and a minimal index l in case Then by the substituting $a_i := xH^{2t-1}$, for $x \in A$, using the identity $R_a + L_b = 2R_b - H_b$ and relation (3.4), we get $xH^{2t} \approx 0$. Alternatively, writing f as,

$$f \equiv \textstyle\sum_{k=0}^{t-1} g_k + \sum_{k=1}^{t} h_k$$

where

$$g_0 = (\alpha_0[a_1, a_2]a_3 + \beta_0[a_3, a_1]a_2 + \gamma_0[a_2, a_3]a_1)R^{d-3},$$
$$h_t = \xi_t[a_1, a_2]H^{2t-1}R^{d-2k-1},$$

and

$$g_k = (\alpha_k[[a_1, a_2], a_3] + \beta_k[[a_3, a_1]a_2] + \gamma_k[a_2, a_3]a_1)H^{2k-1}R^{d-2k-2},$$
$$h_k = \delta_k(a_1 \circ a_2)H^{2k}R^{d-2k-2} + \varepsilon_k(a_1 \circ a_3)H^{2k-1}R^{d-2k-2} + \zeta_k[a_1, a_2]H^{2k-1}R^{d-2k-1},$$

for $k = 1, ..., t - 1$. If at least one of the coefficient $\alpha_0, \beta_0, \delta_0$ is not zero, then by three consecutive substitutions $a_i = aH^{2t-1} (i = 1, 2, 3)$, we have

$$\begin{cases} (\alpha_0 + \beta_0)aH^{2t} \approx 0, \\ (\alpha_0 + \gamma_0)aH^{2t} \approx 0, \\ (\beta_0 + \gamma_0)aH^{2t} \approx 0. \end{cases}$$

Therefore in view of restriction 2- torsion free F, we get either $aH^{2t} \approx 0$ or $g_0 = 0$. Further, if $\varepsilon_1 \neq 0$, then by the substitution $a_3 := xH^{2t-2}$, we have $xH^{2t} \approx 0$. Alternatively, if at least one of the co-efficient δ_1 or ξ_1 is not zero, by two consecutive substitutions $a_i = xH^{2t-2} (i = 1, 2)$, we obtain

$$\begin{cases} (2\delta_1 + \zeta_1)aH^{2t} \approx 0, \\ (2\delta_1 - \zeta_1)aH^{2t} \approx 0. \end{cases}$$

Thus we have either $aH^{2t} \approx 0$ or $h_1 = 0$ and therefore, f as

$$f = \sum_{k=1}^{t-1} g_k + \sum_{k=2}^{t} h_k$$

Therefore by the same arguments as above, we obtain either $xH^{2t} \approx 0$ or $g_1 = h_2 = ... = g_{t-2} = h_{t-1} = g_{t-1} = 0$ and $f = h_t = \zeta_t[a_1, a_2]H^{2t-1}R^{d-2t-1}$. At this instance, the conclusion of the Lemma indicates $\zeta_t \neq 0$ and, therefore, $AH^{2t} \approx 0$. Let us examine the instance $n = 2t$. We require to show the relation $AH^{2t} \approx 0$ and identity $f = 0$ imply $A^2H^{2t-2} \approx 0$. The systematic word of type 4) with respect to the index $k = t$ vanishes, in contrast the case $n = 2t + 1$. All the other systematic words are equal. Hence by the above alike arguments, decreasing per unit the power $P(t)$ for all substitution $a_i := xH^{p(t)}$ and suppose $x \in A^2$, one can prove that $A^2H^{2t-1} \approx 0$. By (3.5), the obtained relation yields $A^2H^{2t-2} \approx 0$. □

Theorem 4.3. *The topological rank of the variety* $(-1, 1)^{<p>}$ *is equal to* P *for all natural* P.

Proof. As mentioned earlier, all 1- assigned variety has the topological rank 1. Thus Lemma 4.2 mean that the topological rank of every n –assigned variety is not greater than n. In particular topological rank $(-1, 1)^{<p>} \leq P$. □

Acknowledgments

This work is partially supported by UGC New Delhi Grant No. F.No. 42-17 (2013).

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