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Characterization on Fuzzy Soft Ordered Banach Algebra

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ABSTRACT: In this paper, we define fuzzy soft ordered Banach algebra with fuzzy soft alfebra cone, and introduce the character on fuzzy soft ordered Banach algebra in both cases real and complex. Also, we deduce some of its basic properties and we define a new concept which is a maximal fuzzy soft algebra cone and showing that the set of all fuzzy character is isomorphism to the set of all maximal fuzzy soft algebra cone. We prove that the set of all real FS^x-characters is convex and extreme point, we applied Gelfand-Mazur theorem on fuzzy soft Banach algebra, we showed that FS^x-character (the set of all complex continuous FS^x- character) is fuzzy soft ordered Banach algebra. Also, any FS^x-OBA with inverse -closed FS^x-algebra cone $\breve{C}$ and a non-zero element in $\breve{A}$ has inverse we have it is an isomorphism to Banach space $Ch(\breve{C})$.

Key Words: Fuzzy soft sets, Fuzzy soft algebra cone, Fuzzy soft ordered Banach algebra, Maximal fuzzy soft ordered Banach algebra, Fuzzy soft Banach algebra.

Contents

1. Introduction

Lotif, Z. [5] introduced fuzzy sets in his famous paper. Which have found wide's applications in many fields including: computer science, economics, control engineering, robotics, etc. Bekir and Bur [1] introduced the notion of fuzzy topological vector space, later the concept of fuzzy norm was introduced by Reza and Ranjit [10] and define the fuzzy Banach spaces and it's quotient. Sadeqi and Amiripour [4] in 2007 gave a definition of fuzzy Banach algebra. Dmitir [3] initiated the theory of soft sets. Pabitr,... etc [9] initiated the theory of fuzzy soft sets, when the introduced a concept parametrized family of sets which applied by him in many fields including basic notions of soft normed space. P.k. Maji and A.R. Roy [8] defined basic notions of soft normed spaces. Reza and Ranjit [10] studied fuzzy normed linear spaces and later Sujoy, Pinaki and Syamal [11] found the concet of soft normed space. Tangaraj and Nirmal [13] introduced new notion yn fuzzy soft bormed spaces. Thakur and S. K. Samanta [14] introduced the definition of soft Banach algebra and studied some of its properties, and preceded that they introduce a new concept of convergence sequence of soft elements and they arrived to that finiteness of parameter sets is not necessary in many cases. Tudor and Ftavius [15] gave sturdiness fuzzy normed space. Zdzisław [17] intrudes the concept of rough sets. Boushra [2] formulated the concept of real character in ordered Banach algebra. Sonja [12] solved some spectral problems in ordered Banach spaces.

In this paper we will combine the concepts of functional analysis with other algebraic concepts in fuzzy soft theory by creating the construction of the space of fuzzy soft ordered Banach algebra with study and proof of many properties in this space. We define fuzzy soft ordered Banach algebra in both cases real and complex. Also, we deduce some of the basic properties and we define a new concept which is a maximal fuzzy soft algebra cone and showing the set of all fuzzy soft character is isomorphism to the set of all maximal fuzzy soft algebra cone. We prove that the set of all real FS^x-character is convex

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and extreme point, we applied Gelfand-Mazur theorem on fuzzy soft Banach algebra, we showed that FS^x-character (the set of all complex continuous FS^x-character) is fuzzy soft ordered Banach algebra. Any FS^x-OBA with inverse closed FS^x-algebra cone C and a non-zero element in A has inverse, we have it is an isomorphism to Banach space $Ch(\breve{C})$. The outline of this paper, section 2 we introduce the historical introduction and we will introduce many concepts which we need like fuzzy soft set, fuzzy soft normed space and many properties which we need. In section 3, we will define new many definitions like FS^x-algebra cone, FS^x-ordered Banach algebra, FS^x-Maximal algebra one and define ordered isomorphism between two FS^x-ordered Banach algebras. In section 4, we introduce many of results as proved that the set of real character in FS^x-ordered Banach algebra is convex and extreme point and proved that the set of all complex FS^x-characterization in FS^x-ordered Banach algebra $Ch(\breve{A})$ is isomorphism with FS^x-Maximal algebra cone. Finally we proved that there is a FS^x-character between $(\breve{A}, \breve{C})$ and $C_\circ(Ch(\breve{A})) = \{k : k : Ch(\breve{C}) \to \mathbb{C}, k \text{ continuous }\}$. In section 5, we clear the conclusion of our work.

2. Preliminaries

Definition 2.1 (6). *Let B is a universe and E be a set of parameter. Let $P(B)$ is the power set of B and X be a non-empty set of E. A pair (S, X) is called soft set over B, where S is a mapping given by $S : X \to P(B)$. That is, a soft set over B is a parametrized family of a subsets of the universe B. For e belong to X, $f(e)$ may be considered the set of $e-$approximate elements of the soft set (S, X).*

Definition 2.2 (8). *Let $(V, +, .)$ is a vector space over a field F and E be a set of parameter. $\breve{V}$ is said to be soft vector space over F if $\breve{V}(\beta)$ is a vector space of V, for all $\beta \in E$.*

Definition 2.3 (13). *Let $\beta(\mathbb{R})$ the collection of all non-empty bounded subsets of $\mathbb{R}$ and E be a set of parameters. Then a mapping $F : E \to \beta(\mathbb{R})$ is called a soft real set. It is denoted by (F, E). If specifically (F, E) is singleton soft set, then (F, E) is soft element. The set of all soft real numbers is denoted by $R(E)$ and the set of all non-negative soft real numbers by $R^\star(E)$.*

Definition 2.4 (8). *Let V be a vector space over a field F and E the parameter set is a real number set F. A soft set (S, X) is said to be soft vector in $\breve{V}$ if there is unique element $e \in E$, such that $S(\breve{e}) = \{a\}$ for some $a \in V$. Such that $S(\breve{e}) = \emptyset$, for all $\breve{e} \in E \backslash \{e\}$, the set of all soft vector spaces is denoted by $S_V(\breve{V})$.*

Definition 2.5 (8). *Let $S_V(\breve{V})$ be a soft vector space. The mapping $\|.\| : S_V(\breve{V}) \longrightarrow R^\star(E)$ is said to be soft norm on the soft vector space $\breve{V}$ is $\|.\|$ satisfies the following conditions: For all $\breve{a}_e, \breve{b}_e \in \breve{V}$*
(i) $\|\breve{a}_e\| = 0$, *for all* $\breve{a}_e \in \breve{V}$.
(ii) $\|\breve{a}_e\| = 0$, *if and only if* $\breve{a}_e = \emptyset$.
(iii) $\|\beta\breve{a}_e\| = |\beta|\|\breve{a}_e\|$, *for all* $\breve{a}_e \in \breve{V}$ *and all soft scalar* β.
(iv) $\|\breve{a}_e + \breve{b}_e\| \geqslant \|\breve{a}_e\| + \|\breve{b}_e\|$.
Then the soft vector space with soft norm $\|.\|$ on $\breve{V}$ is said to be a soft normed space and denoted by $(\breve{V}, \|.\|)$.

Definition 2.6 (15). *Let $\{\breve{x}_n\}$ be a sequence of soft elements of a soft normed space $(\breve{V}, \|.\|)$ such that $\{\breve{x}_n\}$ is said to be a Cauchy sequence if for every $\breve{\in}\breve{\geq}\breve{0}$, there is $k \in \mathbb{N}$ such that: $\|\breve{x}_i - \breve{x}_j\| \breve{<} \breve{\in}$, for all $i, j \geq k$. That is $\|\breve{x}_i - \breve{x}_j\| \to \breve{0}$ as $i, j \to \infty$.*

Definition 2.7 (15). *A soft normed space $(\breve{V}, \|.\|)$ is said that $\breve{V}$ is complete if every Cauchy sequence $\{\breve{x}_n\}$ in $\breve{V}$ converges to a soft element in $\breve{V}$. A complete soft normed space is called a soft Banach space.*

Definition 2.8. *The set (S, X) is said to be a soft Banach algebra if satisfies:*
(i) (S, X) *is a soft Banach space*
(ii) (S, X) *is a soft algebra, that is* (S, X) *satisfies: For all* $\breve{a}, \breve{b}, \breve{c} \in (S, X)$
(a) $(\breve{a}\breve{b})\breve{c} = \breve{a}(\breve{b}\breve{c})$.
(b) $\breve{a}(\breve{b} + \breve{c}) = \breve{a}\breve{b} + \breve{a}\breve{c}$, $(\breve{a} + \breve{b})\breve{c} = \breve{a}\breve{c} + \breve{b}\breve{c}$.
(c) $\beta(\breve{a}\breve{b}) = (\beta\breve{a})\breve{b} = (\breve{a}(\beta\breve{b}))$, $\beta \gneqq 0$.
(iii) A soft norm satisfies the inequality $\|\breve{a}\breve{b}\| \leq \|\breve{a}\|\|\breve{b}\|$ *and* $\breve{e}\breve{a} = \breve{a}\breve{e} = \breve{a}$, $\breve{e}$ *is a soft element with* $\|\breve{e}\| = 1$.

Definition 2.9 (6). *Let X be a non-empty set and $I = [0, 1]$ and $I^X : X \longrightarrow I$ such that $I^X = \{B :$ B is a function from X into $I\}$. A set $B \subset X$ is a fuzzy set in X such that $B : X \longleftrightarrow I$, that is $B \in I^X$ and let $b \in B$ and I^X is denote to the set of all fuzzy sets of X. A fuzzy set X in X is characteristic by a membership function and it is connect with every point $b \in B$ by the real number $F_B(b)$ in $I = [0, 1]$ such that $F_B(b)$ is the grad membership function for the fuzzy set B.*

Definition 2.10 (11). *Let V be a vector space and $*$ is a continuous $t-$norm and B is a fuzzy set on $V \times (0, \infty)$, then $(V, B, *)$ is said to be fuzzy normed space if satisfies the following conditions for every $a, b \in V$ and $t, k \in (0, \infty)$:*

(i) $N(a, t) \gneq 0$.
(ii) $N(a, t) = 1 \Leftrightarrow a = 0$.
(iii) $N(\beta a, t) = N\left(a, \frac{t}{|\beta|}\right)$ for all $\beta \neq 0$.
(iv) $N(a, t) * N(b, k) \leq N(a + b, t + k)$.
(v) $N(a, .) : (0, \infty) \longrightarrow [0, 1]$ is continuous.
(vi) $\lim_{t\to\infty} N(a, t) = 1$.

Definition 2.11 (16). *$(V, B, *)$ is said to be fuzzy Banach algebra space if satisfies:*
(i) V is an algebra.
(ii) $(V, B, *)$ is fuzzy vector space.
(iii) $N(ab, tk) \geq N(a, t) * N(b, k)$, for all $a, b \in V$ and $t, k \geq 0$. If $(V, B, *)$ is fuzzy complete space, then $(V, B, *)$ is fuzzy Banach algebra.

Definition 2.12 (17). *Let B be a linear algebra if $ab = ba$ for all $a, b \in V$ we say that V is a commutative linear algebra.*

Definition 2.13 (3). *Let $(V, +, .)$ be a linear algebra and $V_s \subseteq V$, V_s is a subalgebra if satisfies the same condition of linear algebra.*

Definition 2.14 (17). *We called a linear algebra V is unital, if it has a unit element e such that $e.a = a.e = a$ for all $a \in V$.*

Definition 2.15 (2). *Let B is a universe and E be a set of parameters and I^x is denote to the set of all fuzzy sets of X, a pair (IS^x, E) is called a fuzzy soft set over X. Where $F : E \longleftrightarrow I^X$ is a mapping from E into I^X, $F((e)) = \emptyset$ for all $e \in E - B$ such that $B \subseteq E$*

Example 2.16. *Let $X = \{r_1, r_2, r_3, r_4\}$ which is set of all cars purchase specifications. Take $E = \{e_1 =$ solid, $e_2 =$ cheep, $e_3 =$ collor full, $e_4 =$ modern$\}$.*
$F^x(e_1) = \{\frac{r_1}{0.5}, \frac{r_2}{0.1}, \frac{r_3}{0}, \frac{r_4}{0.9}\}$
$F^x(e_2) = \{\frac{r_1}{0.2}, \frac{r_2}{0.3}, \frac{r_3}{0.1}, \frac{r_4}{0}\}$
$F^x(e_3) = \{\frac{r_1}{1}, \frac{r_2}{0}, \frac{r_3}{0.8}, \frac{r_4}{0.5}\}$
$F^x(e_4) = \{\frac{r_1}{0.4}, \frac{r_2}{0.1}, \frac{r_3}{0.2}, \frac{r_4}{0.7}\}$
Them the set $\{F(e_i), i = 1, 2, 3, 4\}$ of I^x is fuzzy soft set (FS^x, E).

Definition 2.17 (10). *Let (FS^x, E), (GS^x, E) be two fuzzy soft sets. Then*
(i) We say that the pair (FS^x, E) is a fuzzy soft subset of (GS^x, E) if $F^x(e) \subseteq G^x(e)$, for every $e \in E$. Symbolically, we write $(FS^x, E) \bar{=} (GS^x, E)$ if $(FS^x, E) \bar{\subseteqq} (GS^x, E)$ and $(GS^x, E) \bar{\subseteqq} (FS^x, E)$, the pairs (FS^x, E) and (GS^x, E) are said to be fuzzy soft equal. Symbolically, we write $(FS^x, E) = (GS^x, E)$.
(ii) Null fuzzy set denoted by $\bar{\emptyset}$ if for all $e \in E$, $FS^x(e) = \bar{\emptyset}$.
(iii) Absolute fuzzy soft set denoted by $\check{X}$, if for all $e \in E$, $F^x(e) = \check{1}$.
(iv) The complement of fuzzy soft set (FS^x, E), denoted by $(FS^x, E)^c$, is defined by $(FS^x, E)^c = ((FS^x)^c, E)$, $(FS^x)^c : E \to I^x$ is a function given by $(FS^x)^c(e) = 1 - FS^x(e)$, for all $e \in E$. $(FS^x)^c$ is called the fuzzy soft complement function of F. Clearly, $((FS^x)^x c)^c = FS^x$ and $((FS^x, E)^c)^{\mathbf{c}} = (FS^x, E)$. Clearly $\check{X}^c = \breve{\emptyset}$ and $\breve{\emptyset}^c = \check{X}$.
(v) The difference (HS^x, E) of two fuzzy soft sets (FS^x, E) and (GS^x, E) denoted by $(HS^x, E) = (FS^x, E)/(GS^x, E)$, is denoted by $HS^x(e) = FS^x(e)/GS^x(e)$, for all $e \in E$.

Definition 2.18 (10). *Let two fuzzy soft sets (FS^x, E), (GS^x, E) over a common universe X, we say that (FS^x, E) is a fuzzy soft subset of (GS^x, E) if:*
(i) $A \subset B$
(ii) $\forall e \in A$, $F(e)$ is fuzzy subset of $G(e)$.

Definition 2.19 (10). *Two soft sets (F, A) and (G, B) over a common universe X are said to be fuzzy soft equal if (F, A) is fuzzy soft subset of (G, B) and (G, B) is a fuzzy soft subset of (F, B).*

Definition 2.20. *Union of two fuzzy soft sets (FS^x, A) and (GS^x, B) over a common universe X is a soft set (HS^x, C) where $C = A \cup B$ and which is defined as follows: $HS^x(e) = FS^x(e)$, $e \in A - B = GS^x(e)$, $e \in B - A = FS^x(e) \cup GS^x(e)$, $e \in A \cap B$, for all $e \in C$ and write $(HS^x, E) = (FS^x, E)\breve{\cup}\ (GS^x, E)$.*

Definition 2.21 (10). *Let $FS^x(\breve{V})$ be a fuzzy soft on a vector space $\breve{V}$. A fuzzy soft subset on $FS^x(\breve{V}) \times \mathbb{R}^\star$ is said to be fuzzy soft norm on the fuzzy soft vector space $\breve{V}$ if N satisfies the following conditions:*
for all $\breve{a}, \breve{b} \breve{\in}\ FS^x(\breve{V})$ and $t, k \in \mathbb{R}^\star(E)$
(i) $N(\breve{a}, t) = 0$, for all $t \breve{\leq} \breve{0}$.
(ii) $N(\breve{a}, t) = 1$ iff $\breve{a} = \breve{0}$, for all $t \breve{>} \breve{0}$.
(iii) $N(\beta\breve{a}, t) = N(\breve{a}, \frac{t}{|\beta|})$ if $\beta \neq \breve{0}$,
(iv) $N\left(\breve{a} \centerdot \breve{b}, t \cdot k\right) \breve{\geqslant} N(\breve{a}, t) \centerdot N\left(\breve{b}, k\right)$.
(v) $N(\breve{a}, .)$ is continuous non-decreasing function of $F^{\mathfrak{X}}$.
That is $FS^x(\breve{a}, .)$:$\mathbb{R}^\star(E) \to [0, 1]$ and $\lim_{t\to\infty} FS^x(\breve{a}, t) = 1$. Then $(\breve{V}, N, .)$ is a fuzzy soft normed space and if $\breve{V}$ is complete said that it is fuzzy soft Banach space.

Definition 2.22 (17). *Let V be a linear space and $C \subset V$, C is called convex set if for $tC_(1 - t)C \subset C$, for all $0 \leq t \leq 1$.*

Definition 2.23 (7). *If V is a linear space and $K \subseteq X$, such K is convex and $x \in K$, then x is extreme point, that mean if $x = ta + (1 - t)b$, such that a, $b \in K$ and $0 < t < 1$ then $a = b$. And we denote to the set of extreme point by $e(K)$.*

Theorem 2.24. *(Gelfand-Mazur)[17] Let A is Banach algebra which is every non-zero element is invertible. Then there is a unique isomorphism from A in to $\mathbb{C}$.*

3. Fuzzy Soft Ordered Banach Algebras

In this section, we define many basic definitions like, fuzzy soft algebra cone, fuzzy soft ordered Banach algebra, and maximal fuzzy soft ordered Banach algebra.

Definition 3.1 (Fuzzy Soft Linear Algebra). *Let $\breve{V}$ be a vector space with respect to $F = \mathbb{C}$ or $\mathbb{R}$. $\breve{V}$ is called fuzzy soft linear algebra, if there exists an operation which is called multiplication satisfying that for all $\breve{a}$, $\breve{b}$, $\breve{c} \breve{\in} \breve{V}$ and $\beta \in F$.*
(i) $\beta\left(\breve{a} \centerdot \breve{b}\right) = (\beta\breve{a}) \centerdot \breve{b} = \breve{a} \centerdot (\beta\breve{b})$.
(ii)$\left(\breve{a} \centerdot \breve{b}\right) \centerdot \breve{c} = \breve{a} \centerdot \left(\ \breve{b} \centerdot \breve{c}\right)$.
(iii)$(\breve{a} + \breve{b}) \centerdot \breve{z} = (\breve{a} \centerdot \breve{z} + \breve{b} \centerdot \breve{z})$, and $\breve{a} \centerdot \left(\breve{b} + \breve{z}\right) = \breve{a} \centerdot \breve{b} + \breve{a} \centerdot \breve{z}$.
We say that $\breve{e} \in \breve{V}$ is identity element if satisfy $\breve{e} \centerdot \breve{a} = \breve{a} \centerdot \breve{e} = \breve{a}$ For all $\breve{a} \in \breve{V}$.

Definition 3.2 (Fuzzy Soft Banach Algebra). *Let $FS^x(\breve{V})$ be a fuzzy soft on a vector space $\breve{V}$, which is called Fuzzy Soft Banach Algebra if satisfies that:*
(i) Fuzzy soft Banach space.
(ii) Fuzzy soft algebra.
(iii) $N(\breve{a} \centerdot \breve{b}, t \cdot k) \geq \min\{N(\breve{a}, t), N(\breve{b}, k)\}$ for all $\breve{a}, \breve{b} \breve{\in} FS^x$, for all $t, k \in \mathbb{R}^*(E)$.
(iv) If $\breve{e} \in\ FS^x$ then $N(\breve{e}, t) = \breve{1}$.

Definition 3.3 (Fuzzy Soft Algebra Cone)**.** *Let $\breve{A}$ be a real or complex Banach algebra with identity and $\emptyset \neq \breve{C} \subseteq \breve{A}$. We call $\breve{C}$ a Fuzzy Soft Cone if it satisfies the following:*
for all $\breve{a}, \breve{b} \in \breve{C}, \breve{a} + \breve{b} \breve{\in} \breve{C}$, for all $\breve{a}$, $\breve{b}$ $\breve{\in} \breve{C}$, $\beta \breve{a}$ $\breve{\epsilon}$ $\breve{C}$, for all $\breve{a} \breve{\epsilon}$ $\breve{C}$ and $\beta \geq 0$.

In addition, if $\breve{C}$ satisfies $\breve{C} \cap -\breve{C} = \{\breve{0}\}$, then $\breve{C}$ will be called a proper cone induced an ordering ($\preccurlyeq$) on A by $\breve{a} \preccurlyeq \breve{b}$,if and only if $\breve{b} - \breve{a} \breve{\in} \breve{C}$ for all $\breve{a}$, $\breve{b}$ $\breve{\in} \breve{A}$. We say that $\breve{C}$ is $FS-$ algebra cone if it is satisfies the following:
(i) $\breve{a} \bullet \breve{b} \breve{\in} \breve{C}$, for all $\breve{a}$, $\breve{b} \in \breve{C}$.
(ii) $\breve{e} \breve{\in} \breve{C}$.

Definition 3.4 (Fuzzy Soft Ordered Banach Algebras)**.** *Let $\breve{A}$ be a complex ordered Banach algebra with identity generated by $\breve{C}$, and F^A the set of all fuzzy soft sets of ordered Banach algebra A, $\breve{A} = (F^A, E)$ is called fuzzy soft ordered Banach algebra $FS^x - OBA$ if $\breve{A}$ is ordered by a relation $\breve{\preccurlyeq}$ such that for every $\breve{a}, \breve{b}, \breve{c}$ $\breve{\in} \breve{A}$ and $\beta \geq 0$:*
(1) $\breve{a}$, $\breve{b} \breve{\succcurlyeq} \breve{0} \Longrightarrow \breve{a} \breve{+} \breve{b} \breve{\succcurlyeq} \breve{0}$.
(2) $\breve{a}$, $\breve{\succcurlyeq} \breve{0}$, $\breve{e} \geq 0 \Longrightarrow \breve{e} \centerdot \breve{a} \breve{\succcurlyeq} \breve{0}$.
(3) $\breve{a}, \breve{b} \breve{\succcurlyeq} 0 \Longrightarrow \breve{a} \centerdot \breve{b} \breve{\succcurlyeq} 0$.

So if $\breve{A}$ is ordered by an FS^x-algebra cone C, we will obtain $(\breve{A}, \breve{C})$ is a $FS^x - OBA$.

Definition 3.5 (Maximal ***FS*^{x}-Algebra Cone**)**.** *Let $(\breve{A}$, $\breve{C})$ be a $FS^x - OBA$ with fuzzy soft algebra cone $\breve{C}$ and $\breve{M}$ is FS^x- sub algebra cone of $(\breve{A}$, $\breve{C})$ is called Maximal FS^x- algebra cone in $(\breve{A}$, $\breve{C})$ if for each $\breve{a}, \breve{b} \breve{\in}$ $\breve{M}$ satisfies $\breve{a} \centerdot$ $\breve{b} = \breve{b} \centerdot$ $\breve{a}$ and $\breve{M}$ is not a proper subset of another commutative subset of $\breve{C}$.*

Definition 3.6 ((***FS*$^x-$ Character**))**.** *Let $\breve{A}$ be $FS^x - OBA$, for all $\breve{a}, \breve{b}$ $\in \breve{A}$ and $\beta \in F$, we call the functional $\breve{f}$ is FS^x- Character if it satisfies:*
(1) $\breve{f}$ is Linear that is $\breve{f}(\breve{a} + \breve{b}) = \breve{f}(\breve{a}) + \breve{f}(\breve{b})$ and $\beta \breve{f}(\breve{a}) = \breve{f}(\beta$ $\breve{a})$.
(2) $\breve{f}$ is Multiplicative that is $\breve{f}(\breve{a} \centerdot \breve{b})$ $=$ $\breve{f}(\breve{a}) \cdot \breve{f}(\breve{b})$.
(3) $\breve{f}$ (e) $= 1$.

Definition 3.7. *Let $(\breve{A}$, $\breve{C})$,$(\breve{B}$, $\breve{C})$ be $FS^x - OBA$, we say that $\breve{A}$ is fuzzy soft isomorphism to $\breve{B}$ if there exist a homomorphism bijective from $(\breve{A}$, $\breve{C})$ to $(\breve{B}$, $\breve{C})$.*

Definition 3.8. *Let $(\breve{A}$, $\breve{C})$ $(\breve{B}$, $\breve{C})$ be a $FS^x - OBA$ we say that $(\breve{A}$, $\breve{C})$ is fuzzy soft ordered-isomorphism to $(\breve{B}$, $\breve{C})$ if there exists a bijective FS^x-character from $(\breve{A}$, $\breve{C})$ to $(\breve{B}$, $\breve{C})$.*

Definition 3.9 (Inverse-closed ***FS*$^x-$algebra cone**)**.** *Let $\breve{C}$ be a FS^x- algebra cone we say that it is inverse-closed FS^x-algebra cone if $\breve{a}$ $\breve{\epsilon}\breve{A}$ and $\breve{a}$ is invertible then, $\breve{a}^{-1}$ $\breve{\in}\breve{C}$.*

4. Main Results

In this section, we introduced many theorem and properties about fuzzy soft ordered Banach algebra like, convex and extreme point, and character on fuzzy soft ordered Banach algebra.

Theorem 4.1. *Let $(\breve{A}$, $\breve{C})$ be a $FS^x - OBA$ with FS^x- Algebra cone $\breve{C}$, then $\mathcal{M}_A$ is convex.*

Proof. Let $\breve{f}_1, \breve{f}_2 \breve{\in} \mathcal{M}_{\breve{A}}$ and $\beta \in [0, 1]$, $\breve{a}, \breve{b} \breve{\in} (\breve{A}, \breve{C})$:
(i) $\left(\beta \breve{f}_1 \breve{+} (1 - \beta)\, \breve{f}_2\right) \left(\breve{a} \breve{+} \breve{b}\right) = \beta \breve{f}_1 \left(\breve{a} \breve{+} \breve{b}\right) \breve{+} (1 - \beta)\, \breve{f}_2 \left(\breve{a} \breve{+} \breve{b}\right)$
$\beta \breve{f}_1(\breve{a}) + \beta \breve{f}_1(\breve{b}) \breve{+} (1 - \beta) \breve{f}_2(\breve{a}) \breve{+} (1 - \beta) \breve{f}_2(\breve{b})$.
(ii) $(\beta \breve{f}_1 + (1 - \beta) \breve{f}_2)(\alpha \breve{a}) = \beta \breve{f}_1(\alpha \breve{a}) + (1 - \beta) \breve{f}_2(\alpha \breve{a})$.

Since $\breve{f}_1, \breve{f}_2 \breve{\in} \mathcal{M}_{\breve{A}}$ then we obtain $[\alpha \beta \breve{f}_1(\breve{a}) \breve{+} \alpha (1 - \beta) \breve{f}_2(\breve{a}] = \alpha [\beta \breve{f}_1(\breve{a}) \breve{+} (1 - \beta) \breve{f}_2(\breve{a}] = \alpha [\beta \breve{f}_1 \breve{+} (1 - \beta) \breve{f}_2](\breve{a})$.
(iii) To prove $\left(\beta \breve{f}_1 \breve{+} (1 - \beta)\, \breve{f}_2\right) \left(\breve{a} \breve{.} \breve{b}\right) = \left(\beta \breve{f}_1 \breve{+} (1 - \beta)\, \breve{f}_2\right) (\breve{a}) \breve{\cdot} \left(\beta \breve{f}_1 \breve{+} (1 - \beta)\, f_2\right) (\breve{b})$

Let $t \in [0, 1]$, $\breve{a} > 0$ and $\breve{f}_1, \breve{f}_2 \in \mathcal{M}_\lambda$. Let $t = \frac{i}{2^n}$ for some $i, n \in \mathbb{N}$ such that $|\beta - t|$ $< \in$ and $t \breve{f}_1 + (1 - t) \breve{f}_2 \in \mathcal{M}$, let $\{f_\lambda\}_{\lambda \in \Lambda}$ be a net in $\mathcal{M}_\lambda$ satisfy $f_\lambda \to \beta$, $\lambda \in \Lambda$ and $f_\lambda \breve{f}_1(\breve{a}.\breve{b}) \breve{+} (1 - f_\lambda) \breve{f}_2(\breve{a}.\breve{b}) = (f_\lambda \breve{f}_1 \breve{+} (1 - f_\lambda) \breve{f}_2)(\breve{a}).(f_\lambda \breve{f}_1 + (1 - f_\lambda) \breve{f}_2)(\breve{b})$ for every $\breve{a}, \breve{b} \in \breve{A}$.

Hence, $(\beta\breve{f}_1(\breve{a}\dot{\times}\breve{b})\breve{+}(1-\beta)\breve{f}_2(\breve{a}\dot{\times}\breve{b}) = (\beta\breve{f}_1\breve{+}(1-\beta)\breve{f}_2)(\breve{a})\breve{\cdot}(\beta\breve{f}_1\breve{+}(1-\beta)\breve{f}_2)(\breve{b})$, for every $\breve{a},\breve{b} \in (\breve{A},\breve{C})$. Then, from (i), (ii), (iii) we obtain $\beta\breve{f}_1\breve{+}(1-\beta)\breve{f}_2\breve{\in}\mathcal{M}_\lambda$.

$(\beta\breve{f}_1\breve{+}(1-\beta)\breve{f}_2)(\breve{e}) = \beta\breve{f}_1(\breve{e} + (1-\beta)\breve{f}_2(\breve{e})$. □

Proposition 4.2. *Let $(\breve{A},\breve{C})$ be a $FS^x - OBA$ with FS^x-Algebra cone $\breve{C}$ and let $\mathcal{F}$ the set of extreme points of $\mathcal{M}_{\breve{A}}$. Then every FS^x- character in $\mathcal{M}_{\breve{A}}$ is extreme point.*

Proof. Let $\breve{f}$ be a character and $\breve{f} = t\ \breve{f}_1 + (1-t)\breve{f}_2$ such that $\breve{f}_1,\breve{f}_2\breve{\in}\mathcal{M}_{\breve{A}}$

To prove that $\breve{f}_1 = \breve{f}_2$ Assume that $\breve{f}_1 \neq \breve{f}_2$,there exist $\breve{a},\breve{b}\breve{\in}\ \breve{A}$ such that $\breve{f}_1(\breve{a}) \neq \breve{f}_2\left(\breve{b}\right)$. Then $t\ \breve{f}_1(\breve{a}) \neq t\breve{f}_2\left(\breve{b}\right)$ and $(1-t)\ \breve{f}_1(\breve{a}) \neq (1-t)\breve{f}_2\left(\breve{b}\right)$, and since $\mathcal{M}_{\breve{A}}$ is FS^x- character implies that $t\ \breve{f}_1(\breve{a})\breve{\cdot}\ \breve{f}_1\left(\breve{b}\right)\breve{+}(1-t)\ \breve{f}_1(\breve{a})\breve{\cdot}\ \breve{f}_1\left(\breve{b}\right) \neq t\breve{f}_2(\breve{a})\breve{\cdot}\breve{f}_2\left(\breve{b}\right)\breve{+}(1-t)\breve{f}_2(\breve{a})\breve{\cdot}\breve{f}_2\left(\breve{b}\right)$ for some $\breve{b} \neq \breve{0}$,

$$t\ \breve{f}_1(\breve{a})\breve{\cdot}\ \breve{f}_1\left(\breve{b}\right)\breve{+}(1-t)\ \breve{f}_2(\breve{a})\breve{\cdot}\breve{f}_2\left(\breve{b}\right) \neq \left(t\ \ \breve{f}_1(\breve{a})\breve{+}(1-t)\ \breve{f}_2(\breve{a})\right)\breve{\cdot}(t\ \breve{f}_1\left(\breve{b}\right)\breve{+}(1-t)\ \breve{f}_2\left(\breve{b}\right))$$

Then $\breve{f}\left(\breve{a}\ \breve{\cdot}\ \breve{b}\right) \neq \breve{f}(\breve{a})\breve{\cdot}\breve{f}\left(\breve{b}\right)$ which is impossible because of $\breve{f}$ FS^x- character. □

Theorem 4.3. *Let $(\breve{A},\breve{C})$ be a $FS^x - OBA$ with FS^x-Algebra cone $\breve{C}$ and $\breve{\mathcal{M}}$ be a maximal FS^x- Algebra cone. Then there is isomorphism from the set of all complex FS^x-characters $Ch\left(\breve{A}\right)$ to the set of all maximal algebra cones $(\breve{\mathcal{M}}(\breve{A}))$.*

Proof. Let $\breve{\gamma}:\ Ch\left(\breve{A}\right) \to \breve{\mathcal{M}}(\breve{A})$ defined by $\breve{\gamma}(\breve{f}) = Ker\breve{f}$

To prove that $\breve{\gamma}$ is isomorphism, first we prove that $\breve{\gamma}$ is homomorphism

Let $\breve{f}_1,\breve{f}_2\breve{\in}\ Ch\left(\breve{A}\right)$, $\beta \geq 0$.

(i) $\breve{\gamma}\left(\breve{f}_1\breve{+}\breve{f}_2\right) = Ker\left(\breve{f}_1\breve{+}\breve{f}_2\right) = Ker\breve{f}_1\breve{+}Ker\breve{f}_2 = \breve{\gamma}(\breve{f}_1)\breve{+}\breve{\gamma}(\breve{f}_2)$.

(ii) $\beta(\breve{\gamma}(\breve{f}_1)) = \beta Ker(\breve{f}_1) = Ker\beta(\breve{f}_1) = \breve{\gamma}(\beta\breve{f}_1)$.

(iii) $\breve{\gamma}(\breve{f}_1\breve{\cdot}\breve{f}_2) = Ker(\breve{f}_1\breve{\cdot}\breve{f}_2) = Ker\breve{f}_1\breve{\cdot}Ker\breve{f}_2 = \breve{\gamma}(\breve{f}_1)\breve{\cdot}(\breve{f}_2)$.

To prove that $\breve{\gamma}$ is bijective. Let $\breve{f}_1,\breve{f}_2\breve{\in}\ Ch\left(\breve{A}\right)$. Suppose that $Ker\breve{f}_1 = Ker\breve{f}_2$. The functional $\breve{f}_1 : \breve{A}\backslash Ker\breve{f}_1 \longleftrightarrow \mathbb{C}$ defined by $\breve{f}(\breve{a}\breve{+}\breve{f}_1) = \breve{f}_1(\breve{a})$, $\breve{f}_2 : \breve{A}\backslash Ker\breve{f}_2 \longleftrightarrow \mathbb{C}$ defined by $\breve{f}_2(\breve{a}\breve{+}Ker\breve{f}_2) = \breve{f}_2(\breve{a})$.

To prove that $\breve{f}_1(\breve{a}) = \breve{f}_2(\breve{a})$ since $\breve{f}_1(\breve{a}) - \breve{f}_2(\breve{a}) = \breve{f}_1\left(\breve{a}\breve{+}Ker\breve{f}_1\right) - \breve{f}_2\left(\breve{a}\breve{+}Ker\breve{f}_2\right)$,that is $Ker\breve{f}_1 = Ker\breve{f}_2$ and since $\breve{f}_1$ is a unique by Gelfand-Mazur. Then $\breve{f}_1(\breve{a}) - \breve{f}_2(\breve{a}) = 0$, that is $\breve{f}_1(\breve{a}) = \breve{f}_2(\breve{a})$ for all $\breve{a}\breve{\epsilon}\breve{A}$. Then, $\breve{f}_1 = \breve{f}_2$. So $\breve{\gamma}$ is injective.

Suppose $\breve{\mathcal{M}}\ \breve{\in}\breve{\mathcal{M}}(\breve{A})$, then there is an isomorphism $\breve{\psi} : \breve{A}/\breve{\mathcal{M}} \to \mathbb{C}$ defined by $\breve{\psi}(\breve{a}\ \breve{+}\breve{\mathcal{M}}) = \breve{f}(\breve{a})$, such that $\breve{f}$ is character from $\breve{A}$ into $\mathbb{C}$ and $\breve{\mathcal{M}}$ is a maximal FS^x- Algebra cone such that $Ker\ \breve{f} = \breve{\mathcal{M}}$ that is $\breve{\psi}(\breve{a}\ \breve{+}Ker\ \breve{f}) = \breve{\psi}(\breve{a})$. Then $\breve{f}\breve{\epsilon}(Ch(\breve{A}))$. Hence $\breve{\gamma}\left(\breve{f}\right) = ker\breve{f}\ = \breve{\mathcal{M}}$. □

Theorem 4.4. *Let $(\breve{A},\ \breve{C})$ be FS^x-OBA with FS^x- Algebra cone $\breve{C}$. Then $(\breve{A},\ \breve{C})$ is a FS^x- character to $C_o(Ch(\breve{C}))$ such that $C_o(Ch(\breve{C})) = \{\breve{k}:Ch(\breve{C}) \to \mathbb{C},\ k continuous\ \}$.*

Proof. Let $L{:}\breve{A} \longrightarrow C_o\ (Ch(\breve{C}))$ defined by $L(\breve{a}) = k(\breve{f}_{\breve{a}})$ such that $\breve{k}(\breve{f}_{\breve{a}}) = \breve{f}(\breve{a})$ for all $\breve{a}\breve{\in}(\breve{A},\breve{C})$ such that $\breve{f}$ is every FS^x-character from closed algebra $\breve{C}$ into $\mathbb{C}$. Let $\breve{a},\breve{b}\breve{\in}\breve{C}$, $\alpha \in \mathbb{C}$.

(i) $L(\breve{a} + \breve{b}) = \breve{k}(\breve{f}_{\breve{a}+\breve{b}})$ such that $\breve{k}(\breve{f}_{\breve{a}\breve{+}\breve{b}}) = \breve{f}(\breve{a}\breve{+}\breve{b}) = \breve{f}(\breve{a})\breve{+}\breve{f}(\breve{b}) = \breve{k}(\breve{f}_{\breve{a}}) + \breve{k}(\breve{f}_{\breve{b}}) = \breve{L}(\breve{a}) + \breve{L}(\breve{b})$.

(ii) $\alpha(L(\breve{a}) = \alpha k(\breve{f}_{\breve{a}})$ such that $\alpha\breve{k}(\breve{f}_{\breve{a}}) = \alpha\breve{f}(\breve{a}) = \breve{f}(\alpha\breve{a}) = \breve{k}(\breve{f}_{\breve{a}}) = L(\alpha\breve{a})$.

(iii) $L(\breve{a}\breve{\cdot}\breve{b}) = \breve{k}(\breve{f}_{\breve{a}\breve{\cdot}\breve{b}})$ such that $\breve{k}(\breve{f}_{\breve{a}\breve{\cdot}\breve{b}}) = \breve{f}(\breve{a}\breve{\cdot}\breve{b}) = \breve{k}\breve{f}(\breve{a})\breve{\cdot}\breve{k}\breve{f}(\breve{b}) = \breve{k}(\breve{f}_{\breve{a}})\breve{\cdot}\breve{k}(\breve{f}_{\breve{b}}) = L(\breve{a})\breve{\cdot}L(\breve{b})$.

(iv) $L(\breve{e}) = \breve{k}(\breve{f}_{\breve{e}})$ such that $\breve{k}(\breve{f}_{\breve{e}}) = \breve{f}(\breve{e}) = \breve{e}$. Then L is FS^x-character. □

Theorem 4.5. *Let $(\breve{A},\breve{C})$ be a FS^x-OBA with inverse –closed FS^x- algebra cone $\breve{C}$. If a non-zero element in $\breve{A}$ has inverse. Then, $(\breve{A},\breve{C})$ is an isomorphism to Banach space $Ch\left(\breve{C}\right)$.*

Proof. Let $\breve{\phi}:(\breve{A},\breve{C})\longrightarrow Ch\left(\breve{C}\right)$ define by $\breve{\phi}(\breve{a})=\breve{f}_{\breve{a}}$ such that $\breve{f}_{\breve{a}}(\breve{x})=\breve{a}.\breve{x}$ for all $\breve{x}\breve{\in}\breve{C}$, for all $\breve{a}\breve{\in}\breve{A}$ and $\breve{f}:\breve{C}\longrightarrow\mathbb{C}$ is a FS^x-character.

To prove that $Ch\left(\breve{C}\right)$ is Banach space. First, we prove that $Ch\left(\breve{C}\right)$ is a closed subspace of the set of all continuous functions on $(\breve{A},\breve{C})$: Let $\breve{f}_1,\ \breve{f}_2\ \breve{\in}Ch\left(\breve{C}\right)$. To show that:

(1) $\breve{f}_1\breve{+}\breve{f}_2$ is a FS^x- character
(i) $(\breve{f}_1\breve{+}\breve{f}_2)(\breve{a}+\breve{b})$
$=\breve{f}_1(\breve{a}+\breve{b})\breve{+}\breve{f}_2(\breve{a}+\breve{b})$
$=\breve{f}_1(\breve{a})+\breve{f}_2(\breve{b})+\breve{f}_1(\breve{a})\breve{+}\breve{f}_2(\breve{b})=(\breve{f}_1\breve{+}\breve{f}_2)(\breve{a})+(\breve{f}_1\breve{+}\breve{f}_2)(\breve{b})$.

(ii)$\left(\breve{f}_1\breve{+}\ \breve{f}_2\right)\left(\breve{a}\,.\,\breve{b}\right)\ =\ (\breve{f}_1\breve{+}\ \breve{f}_2)(\frac{1}{2})\ \left[\left(\breve{a}+\breve{b}\right)^2-\breve{a}^2-\breve{b}^2\right])=\frac{1}{2}\left[\ (\breve{f}_1\breve{+}\breve{f}_2)\left(\breve{a}+\breve{b}\right)^2-(\breve{f}_1\breve{+}\breve{f}_2)(\breve{a}^2)-\right.$ $(\breve{f}_1\breve{+}\ \breve{f}_2)(\breve{b}^2)]=(\frac{1}{2}.2)\ \ (\breve{f}_1+\breve{f}_2)\,(\breve{a})\,.\,(\breve{f}_1\breve{+}\ \breve{f}_2)(\breve{b})=(\breve{f}_1\breve{+}\ \breve{f}_2)\,(\breve{a})\,.(\breve{f}_1\breve{+}\ \breve{f}_2)(\breve{b})$.
(iii) $\beta\ \ (\breve{f}_1\breve{+}\ \breve{f}_2)\,(\breve{a})\ \ =\beta\ (\ \breve{f}_1\,(\breve{a})\,\breve{+}\ \ \breve{f}_2(\breve{a}))=\ \breve{f}_1\,(\beta\breve{a})\,\breve{+}\ \ \breve{f}_2\,(\beta\breve{a})\ \ =\ \ (\ \breve{f}_1\breve{+}\ \ \breve{f}_2)(\ \beta\breve{a})$. For all $\breve{a},\ \breve{b}\breve{\in}\breve{A}$ and $\beta\in\mathbb{C}$.
(2) $\beta\ \breve{f}_1$ is FS^x- character
(i) $\beta\ \ \breve{f}_1\left(\breve{a}+\breve{b}\right)=\beta\ (\ \breve{f}_1(\breve{a})+\ \ \breve{f}_1(\breve{b}))=\beta\ \ \breve{f}_1(\breve{a})+\ \breve{f}_1(\breve{b})$
(ii) $\breve{f}_1\left(\breve{a}\breve{b}\right)\qquad=\beta(\ \breve{f}_1\,(\breve{a})\,.\,\breve{f}_1\left(\breve{b}\right))$
(iii) $\mu(\beta)\breve{f}_1\,(\breve{a})\ \ =\beta\ \ (\breve{f}_1\,(\mu\breve{a})),\ \beta\ ,\mu\ \in\mathbb{C}$. Then $Ch\left(\breve{C}\right)$ is subspace

Now we will prove that $Ch\left(\breve{C}\right)$ is closed. We have to prove that $Ch\left(\breve{C}\right)=\overline{Ch\left(\breve{C}\right)}$. Let $\breve{f}\breve{\epsilon}\overline{Ch\left(\breve{C}\right)}$. Then, there exists a sequence $\left\{\breve{f}_\imath\right\}$ *in* $\overline{Ch\left(\breve{C}\right)}$ such that $\lim_{\imath\to\infty}\breve{f}_\imath\ \ =\breve{f}$.

To prove $\breve{f}\breve{\epsilon}Ch\left(\breve{C}\right)$.

Let $\breve{a},\ \breve{b}\breve{\in}\ \breve{C}$ so, $\breve{f}\left(\breve{a}+\breve{b}\right)=\lim_{\imath\to\infty}\breve{f}_\imath\ \ (\breve{a}+\breve{b})=\lim_{\imath\to\infty}\breve{f}_\imath\ \ (\breve{a})+\lim_{\imath\to\infty}\breve{f}_\imath\ \ (\breve{b})=\breve{f}\,(\breve{a})+\breve{f}(\breve{b})$.

Let $\beta\in\ \mathbb{C},\ \breve{a}\breve{\in}\breve{C}\ \breve{f}\,(\beta\breve{a})=\lim_{\imath\to\infty}\breve{f}_\imath\ \ (\beta\breve{a})=\lim_{\imath\to\infty}\beta\breve{f}_\imath\ (\breve{a})=\beta\lim_{\imath\to\infty}\breve{f}_\imath\ \ (\breve{a})=\beta\breve{f}\,(\breve{a})$.

Let $\breve{b}\breve{\in}\breve{C},\ \breve{f}\left(\breve{a}.\breve{b}\right)=\lim_{\imath\to\infty}\breve{f}_\imath\ \ (\breve{a}).\breve{f}_\imath\ (\breve{b})=\lim_{\imath\to\infty}\breve{f}_\imath\ \ (\breve{a})\,.\lim_{\imath\to\infty}\breve{f}_\imath\ \ (\breve{b})=\ \breve{f}\,(\breve{a})\,.\breve{f}(\breve{b})$.

Then $\breve{f}\in Ch\left(\breve{C}\right)$, we obtain $\overline{Ch\left(\breve{C}\right)}\subseteq Ch\left(\breve{C}\right)$, since $Ch\left(\breve{C}\right)\subseteq\overline{Ch\left(\breve{C}\right)}$ so $Ch\left(\breve{C}\right)$ is closed.

Then $Ch\left(\breve{C}\right)$ is Banach space.

To show that $\breve{\phi}$ is an isomorphism.

First to prove that $\breve{\phi}$ is a homomorphism Let $\breve{a},\breve{b}\breve{\in}\breve{A}$ and $\acute{\breve{x}}\breve{\in}\breve{C}$.

1. $\breve{\phi}(\breve{a}+\breve{b})=\breve{f}_{\breve{a}+\breve{b}}$ such that $f_{\breve{a}+\breve{b}}(\breve{x})=(\breve{a}+\breve{b}(\breve{x})=\breve{a}\breve{x}+\breve{b}\breve{x}=\breve{f}_{\breve{a}}(\breve{x})+\breve{f}_{\breve{b}}(\breve{x}$.
2. $\breve{f}(\alpha\breve{a})=\breve{f}_{\alpha\breve{a}}$ such that $\breve{f}_{\alpha\breve{a}}(\acute{x})=\alpha(\breve{a}\,.\,\acute{\breve{x}})=\alpha\breve{f}_{\breve{a}}(\acute{\breve{x}})=\alpha\breve{\phi}(\breve{a})$.
3. $\breve{\phi}(\breve{a}\,.\,\breve{b})=\breve{f}_{\breve{a}.\breve{v}}$ such that $\breve{f}_{\breve{a}.\breve{b}}(\acute{\breve{x}})=(\breve{a}\,.\,\breve{b})\acute{\breve{x}}=\breve{a}\,.\,(\breve{b}\,.\,\acute{\breve{x}})=\breve{a}\,.\,\breve{f}_{\breve{b}}\acute{\breve{x}}=\breve{f}_{\breve{a}}\,.\,\breve{f}_{\breve{b}}\acute{\breve{x}}$.

Let $(\breve{a},\breve{b})\breve{\in}(\breve{A},\breve{C})$ such that $\breve{a}\neq\breve{b}$ since $\acute{\breve{x}}$ invertible, so $\breve{a}\,.\,\acute{\breve{x}}\neq\breve{b}\,.\,\acute{\breve{x}}$ for each $\acute{\breve{x}}\breve{\in}\breve{C}$.

Implies that $\breve{f}_{\breve{a}}(\acute{\breve{x}})\neq\breve{f}_{\breve{a}}(\acute{\breve{x}})$, we obtain $\breve{\phi}\,(\breve{a})\neq\ \breve{\phi}(\breve{b})$. Then $\breve{\phi}$ is injective. Suppose $\breve{f}_{\breve{a}}\ \breve{\in}Ch\left(\breve{C}\right)$, Since $\breve{f}$ is a FS^x- character functional. Then we can define $\breve{f}_{\breve{a}}$ by $\breve{f}_{\breve{a}}(\breve{x})=\breve{a}\ .\ \breve{x}$ for all $\breve{x}\ \breve{\in}\breve{C}$. Then there exist $\breve{a}\ \breve{\in}\breve{A}$ such that $\breve{\phi}(\breve{a})=\breve{f}_{\breve{a}}$ such that $\breve{f}_{\breve{a}}(\breve{x})\ =\breve{a}.\ \breve{x}$.

Hence $(\breve{A},\breve{C})$ is an isomorphism to Banach space $Ch\left(\breve{C}\right)$. □

5. Conclusion

In fuzzy soft Banach space, we needed define soft Banach algebra [14] and fuzzy Banach space [10], in our paper, we define a new concepts called fuzzy soft algebra cone, fuzzy soft ordered Banach algebra and maximal fuzzy soft algebra cone, we then proved many properties about real and complex FS^x- character like the convexity and the extreme point to the FS^x- character in real numbers and shown that there is isomorphism from the set of all complex FS^x- character to the set of all maximal algebra cone, more than we arrived to that the fuzzy soft ordered Banach algebra is a FS^x- character with the set of all continuous FS^x- characterization which is a set of functions from fuzzy soft algebra cone in to the set of complex numbers.

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