Bol. Soc. Paran. Mat.
©SPM –ISSN-2175-1188 ON LINE
SPM: www.spm.uem.br/bspm

(3s.) **v. 2022 (40)** : 1–9.
ISSN-0037-8712 IN PRESS
doi:10.5269/bspm.50973

# Mannheim Offsets of Ruled Surfaces under the 1-Parameter Motions*

Keziban Orbay and Tevfik Şahin

ABSTRACT: In this study, the situation of Mannheim offsets of ruled surfaces under the 1-parameter motions is investigated. Firstly, relationships between geodesic Frenet trihedrons of Mannheim offsets of ruled surfaces are obtained and the relationship between the curvatures of the surface pairs is examined. Also, change of integral invariants the surface pairs under the 1-parameter motions is studied. Finally, the relevant example is given for every Mannheim offsets of ruled surfaces.

Key Words: Mannheim curve, Mannheim offsets, Motion, Ruled surface.

## Contents



## 1. Introduction

Classical differential geometry is divided into two main sections: Curves and Surfaces. Properties of curves and surfaces are examined over a point in the neighborhood of this curve and surface components [3,6,11]. There are some special pairs of curves and pairs of surfaces. For example; pair of involute-evolute curves which is tangent vectors perpendicular to each other in the corresponding points [7], pair of Bertrand curves which is the principal normal vectors common [2] and pair of Mannheim curves which is the principal normal vector corresponds to binormal vector in the corresponding points [12,13,20] are pairs of curves recently studied. A pair of surfaces is defined using these pairs of curves [9,14,15].

Geometry is the theory of do not change under a transformation group. Kinematics is the geometry of transformation protecting distances. It is obtained a curve with the continuous movement of a point on a surface and a surface with a continuous movement in space of a curve. If a curve especially is a line, in this case, the obtained surface is called Ruled surface. Examine the properties of the surface to a period under a motion has attracted the attention of mathematicians. For example; the conoidal ruled surface which is a special surface was examined under the helical and homothetic motions [1,5].

Movement on each of the curves or surfaces are not of concern to the geometry only. It is used in physics, engineering, computer-aided design and manufacturing, robotics, and in many industrial applications. Wang, Liu, and Xiao studied kinematic differential geometry of a rigid body in spatial motion in three consecutive articles [17,18,19]. In these articles, the geometrical properties of a point trajectory and a line trajectory in spatial motion are searched by means of a new adjoint approach. The invariant of axoids are derived and their kinematic meanings are revealed. This new adjoint approach a curve adjoint to a ruled surface is expanded for another ruled surface adjoint to a ruled surface. Based on the results obtained from them, the kinematics conditions for the moving body to be connected with a binary link by a kinematic pair are discussed and described by the geometrical properties of the line trajectory and the constraint ruled surface. To explain the formation of a double curve, kinematic decomposition of the coupler plane is done by Lan, Huijun, and Liuming [10]. The analysis of coupler limit positions in a four-bar linkage, the formation of coupler curves are studied and the coupler plane is divided into four zones. Shapes and features of coupler curves in different zones are explained. Also,

* This study was supported by the Research Centre of Amasya University (Project No: FMB-BAP16-0212).
2010 *Mathematics Subject Classification:* 53A05, 53A17.
Submitted November 12, 2019. Published April 19, 2020

recently, Cui and Dai extend the adjoint approach to a general surface and investigate the kinematics of the relative motion of two rigid objects that maintain sliding-rolling contact [4].

In this study, the situation of Mannheim offsets of ruled surfaces under the 1-parameter motions is investigated. Relationships between geodesic Frenet trihedrons of Mannheim offsets of ruled surfaces are obtained and the relationship between the curvatures of the surface pairs is examined. Also, change of integral invariants the surface pairs under the 1-parameter motions is studied.

## 2. Preliminaries

It has been defined as a ruled surface as a surface generated by the motion of a straight line, its generating line, generator, or ruling. There are $\infty^1$ generators on a ruled surface. Let $e = e(s)$ be the unit vector in the direction of the generating line passing through a point of an arbitrary nonisotropic curve $\alpha$, which is called the base curve, lying on the surface, of which the equation is $\alpha = \alpha(s)$. The ruled surface is given by a parametric representation

$$\phi(s, v) = \alpha(s) + ve(s) \tag{2.1}$$

the vector $e = e(s)$, drawn through the center 0 of the unit sphere, describe the director cone of the surface. When $\alpha$ is a constant the surface is a cone, when $e$ is a constant, the surface is a cylinder [16]. The vector $e$ traces a curve on the surface of unit sphere $S^2$ called spherical indicatrix of the ruled surface [15]. The orthonormal system $\{e, t, g\}$ is called the geodesic Frenet trihedron of the ruled surface $\phi$ such that $t = \frac{e_s}{\|e_s\|}$ and $g = \frac{e \times e_s}{\|e_s\|}$ are the central normal and the asymptotic normal direction of $\phi$, respectively. For the geodesic Frenet vectors $e$, $t$ and $g$, we can write

$$\begin{aligned} e_q &= t \\ t_q &= \gamma g - e \\ g_q &= -\gamma t \end{aligned} \tag{2.2}$$

where $q$ and $\gamma$ are the arc-length of spherical indicatrix $(e)$ and the geodesic curvature of (e) with respect to $S^2$, respectively [15].

The striction point on a ruled surface $\phi$ is the foot of the common normal between two consecutive generators. The set of striction points defines the striction curve given by

$$c(s) = \alpha(s) - \frac{\langle \alpha_s, e_s \rangle}{\langle e_s, e_s \rangle} e_s \tag{2.3}$$

If consecutive generators of a ruled surface intersect, then the surface is said to be developable. The spherical indicatrix $e$, of a developable surface is tangent of its striction curve [15].

The distribution parameter of the ruled surface $\phi$ is defined by

$$P_e = \frac{\det(\alpha_s, e, e_s)}{\|e_s\|^2}. \tag{2.4}$$

The ruled surface is developable if and only if $P_e = 0$ [15].

In this paper, the striction curve of the ruled surface $\phi$ will be taken as the base curve. In this case, for the parametric equation of $\phi$, we can write

$$\phi(s, v) = c(s) + ve(s) \tag{2.5}$$

The ruled surface $\phi^*$ is said to be Mannheim offset of the ruled surface $\phi$ if there exists a one to one correspondence between their generators such that the asymptotic normal of $\phi$ is the central normal of $\phi^*$. In this case, $(\phi, \phi^*)$ is called a pair of Mannheim ruled surface [14]. Let $\{e, t, g\}$ and $\{e^*, t^*, g^*\}$ be two geodesic Frenet triples at the point $c(s)$ and $c^*(s)$ of the striction curves $(c)$ and $(c^*)$ of the ruled surface $\phi$ and $\phi^*$, respectively. Let parametric representations of the ruled surface $\phi$ and $\phi^*$ be given by

$$\left.\begin{aligned} \phi(s, v) &= c(s) + ve(s), & \|e(s)\| &= 1 \\ \phi^*(s, v) &= c^*(s) + ve^*(s), & \|e^*(s)\| &= 1 \end{aligned}\right\} \tag{2.6}$$

respectively.

If $\phi^*$ is a Mannheim Offset of $\phi$, there is the equation

$$\begin{bmatrix} e^* \\ t^* \\ g^* \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ 0 & 0 & 1 \\ \sin\theta & -\cos\theta & 0 \end{bmatrix} \begin{bmatrix} e \\ t \\ g \end{bmatrix} \tag{2.7}$$

between the geodesic Frenet triples $\{e,\, t,\, g\}$ and $\{e^*,\, t^*,\, g^*\}$, where $\theta$ is the angle between corresponding generators [14].

## 3. Mannheim Offsets under the 1-Parameter Motions

Let the ruled surface $\bar{\phi}$ roll on the ruled surface $\phi$ along curves $(c)$ and $(\bar{c})$ whose arc-length parameter $s$ and $\bar{s}$ under 1- parameter spatial motion, respectively. While $(\phi,\, \phi^*)$ is a pair of Mannheim ruled surface, the pair of ruled surface $\left(\bar{\phi},\, \bar{\phi}^*\right)$ also continues to be a pair of Mannheim ruled surface during movement.

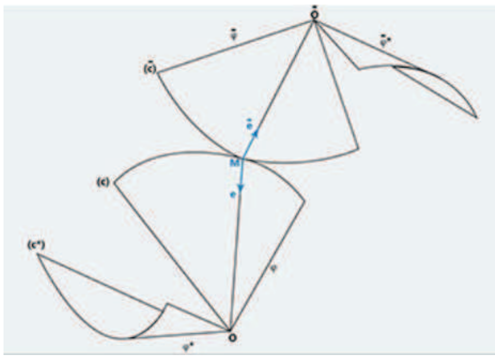


Figure 1: The motion of the ruled surfaces along curves $(c)$ and $(\bar{c})$

Let $\{e,\, t,\, g\}$ and $\{\bar{e},\, \bar{t},\, \bar{g}\}$ be geodesic Frenet triples at the common point M of the striction curves $(c)$ and $(\bar{c})$ of the ruled surface $\phi$ and $\bar{\phi}$, respectively. Where $t = -\bar{t}$ and there is like an angle $\beta$ between $e$ and $\bar{e}$. In this case, we can write

$$\begin{aligned} \bar{e} &= e\cos\beta + g\sin\beta \\ \bar{t} &= -t \\ \bar{g} &= -e\sin\beta + g\cos\beta \end{aligned}$$

or

$$\begin{bmatrix} \bar{e} \\ \bar{t} \\ \bar{g} \end{bmatrix} = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & -1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \begin{bmatrix} e \\ t \\ g \end{bmatrix}. \tag{3.1}$$

While there is the equation

$$\begin{bmatrix} e^* \\ t^* \\ g^* \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ 0 & 0 & 1 \\ \sin\theta & -\cos\theta & 0 \end{bmatrix} \begin{bmatrix} e \\ t \\ g \end{bmatrix}$$

between the geodesic Frenet triples $\{e,\, t,\, g\,\}$ and $\{e^*,\, t^*,\, g^*\}$ of the pair of Mannheim ruled surface$(\phi,\, \phi^*)$, there is the equation

$$\begin{bmatrix} \bar{e}^* \\ \bar{t}^* \\ \bar{g}^* \end{bmatrix} = \begin{bmatrix} \cos\mu & \sin\mu & 0 \\ 0 & 0 & 1 \\ \sin\mu & -\cos\mu & 0 \end{bmatrix} \begin{bmatrix} \bar{e} \\ \bar{t} \\ \bar{g} \end{bmatrix}.$$

between the geodesic Frenet triples $\{\bar{e},\,\bar{t},\,\bar{g}\}$ and $\{\bar{e}^*,\,\bar{t}^*,\,\bar{g}^*\}$ of the pair of Mannheim ruled surface $\left(\bar{\phi},\,\bar{\phi}^*\right)$ too, where $\mu$ is the angle between $\bar{e}$ and $\bar{e}^*$.

If the necessary actions are done, we obtain

$$\begin{bmatrix} \bar{e}^* \\ \bar{t}^* \\ \bar{g}^* \end{bmatrix} = \begin{bmatrix} \cos\mu\cos\beta & -\sin\mu & \cos\mu\sin\beta \\ -\sin\beta & 0 & \cos\beta \\ \sin\mu\cos\beta & \cos\mu & \sin\mu\sin\beta \end{bmatrix} \begin{bmatrix} e \\ t \\ g \end{bmatrix}$$

between the geodesic Frenct triples of the ruled surface $\varphi$ and $\bar{\varphi}^*$,

$$\begin{bmatrix} \bar{e}^* \\ \bar{t}^* \\ \bar{g}^* \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\beta\cos\mu - \sin\theta\sin\mu & \cos\mu\sin\beta & \cos\beta\cos\mu\sin\theta + \cos\theta\sin\mu \\ -\cos\theta\sin\beta & \cos\beta & -\sin\theta\sin\beta \\ \cos\theta\cos\beta\sin\mu + \sin\theta\cos\mu & \sin\mu\sin\beta & \cos\beta\sin\theta\sin\mu - \cos\theta\cos\mu \end{bmatrix} \begin{bmatrix} e^* \\ t^* \\ g^* \end{bmatrix}$$

between the geodesic Frenet triples of the ruled surface $\varphi^*$ and $\bar{\varphi}^*$, and

$$\begin{bmatrix} e^* \\ t^* \\ g^* \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\beta & -\sin\theta & -\cos\theta\sin\beta \\ \sin\beta & 0 & \cos\beta \\ \sin\theta\cos\beta & \cos\theta & -\sin\theta\sin\beta \end{bmatrix} \begin{bmatrix} \bar{e} \\ \bar{t} \\ \bar{g} \end{bmatrix}$$

between the geodesic Frenet triples of the ruled surface $\varphi^*$ and $\bar{\varphi}$.

If the derivative of the geodesic Frenet vectors of the ruled surface $\varphi$ in the equations $e = e$, $t = \frac{e_s}{\|e_s\|}$ and $g = \frac{e \times e_s}{\|e_s\|}$ with respect to the arc-length parameter $s$ of the curve $(c)$ is taken the derivative formulae is given as follows [8]:

$$\begin{aligned} e_s &= q_s t \\ t_s &= -q_s e + \gamma q_s g \\ g_s &= -\gamma q_s t. \end{aligned} \tag{3.2}$$

Similarly, if the derivative of the geodesic Frenet vectors of the ruled surface $\bar{\varphi}$ in the equations $\bar{e} = \bar{e}$, $\bar{t} = \frac{\bar{e}_{\bar{s}}}{\|\bar{e}_{\bar{s}}\|}$ and $\bar{g} = \frac{\bar{e} \times \bar{e}_{\bar{s}}}{\|\bar{e}_{\bar{s}}\|}$ with respect to the arc-length parameter $\bar{s}$ of the curve $(\bar{c})$ is taken, we can give the following equations:

$$\begin{aligned} \bar{e}_{\bar{s}} &= \bar{q}_{\bar{s}} \bar{t} \\ \bar{t}_{\bar{s}} &= -\bar{q}_{\bar{s}} \bar{e} + \bar{\gamma}\,\bar{q}_{\bar{s}} \bar{g} \\ \bar{g}_{\bar{s}} &= -\bar{\gamma}\,\bar{q}_{\bar{s}} \bar{t} \end{aligned} \tag{3.3}$$

where $\bar{q}$ and $\bar{\gamma}$ are the arc-length of spherical indicatrix $(\bar{e})$ and the geodesic curvature of $(\bar{e})$ with respect to $\bar{S}^2$, respectively. Taking the derivative of (3.1) with respect to $s$ and using (3.2), (3.1) and (3.3), respectively, we have

$$\begin{aligned} \bar{q}_s &= -q_s\cos\beta + \gamma q_s\sin\beta \\ \beta_{\bar{s}} &= 0 \\ \bar{\gamma}\,\bar{q}_s &= -q_s\sin\beta - \gamma\, q_s\cos\beta. \end{aligned} \tag{3.4}$$

**Theorem 3.1.** *There is the equation*

$$\bar{\gamma} - \gamma - (1 + \gamma\bar{\gamma})\tan\beta = 0 \tag{3.5}$$

*between the geodesic curvature of the curve* $(c)$ *and* $(\bar{c})$*.*

*Proof.* Reorganizing the equations first and third of (3.4), we get

$$-\,\bar{\gamma}\,\bar{q}_q = \sin\beta + \gamma\,\cos\beta \tag{3.6}$$

and

$$\bar{q}_q = -\cos\beta + \gamma\sin\beta \tag{3.7}$$

respectively. Considering together of (3.6) and (3.7), we obtain

$$\begin{aligned}
(\cos\beta - \gamma\sin\beta)\,\bar{\gamma} &= \sin\beta + \gamma\cos\beta \\
(1-\gamma\tan\beta)\bar{\gamma} &= \tan\beta + \gamma \\
\bar{\gamma} - \gamma - \gamma\bar{\gamma}\tan\beta - \tan\beta &= 0 \\
\bar{\gamma} - \gamma - (1+\gamma\bar{\gamma})\tan\beta &= 0.
\end{aligned}$$

**Corollary 3.2.** *If the curve $(c)$ is a geodesic, the curve $(\bar{c})$ is a circle.*

**Corollary 3.3.** *The angle between the directrixes of ruled surfaces $\varphi$ and $\overline{\varphi}$ corresponds to the angle between the directions of the slopes $\gamma$ and $\overline{\gamma}$ directions.*

**Theorem 3.4.** *Let the ruled surface $\bar{\varphi}$ rolls on the ruled surface $\varphi$ along curves $(c)$ and $(\bar{c})$ under 1-parameter spatial motion. While the curve $(c)$ is a geodesic, the curve $(\bar{c})$ is a geodesic if and only if $q = |\bar{q}|$.*

*Proof.* Because $(c)$ is a geodesic, using (3.5), we obtain

$$\begin{aligned}
\bar{\gamma} &= \tan\beta \\
\bar{\gamma}^2 &= \frac{1}{\cos^2\beta} - 1.
\end{aligned}$$

After that, considering (3.7), we reach

$$\bar{\gamma}^2 = (q_{\bar{q}})^2 - 1.$$

So,

$$\begin{aligned}
(\bar{c})\text{ is a geodesic} \Leftrightarrow &\quad \bar{\gamma} = 0 \\
\Leftrightarrow &\quad (q_{\bar{q}})^2 = 1 \\
\Leftrightarrow &\quad q = |\bar{q}|\,.
\end{aligned}$$

**Theorem 3.5.** *Let the ruled surface $\bar{\varphi}$ rolls on the ruled surface $\varphi$ under 1- parameter spatial motion. There is the following relation between distribution parameters $P_e$ and $\bar{P}_{\bar{e}}$ of the Mannheim ruled surfaces $\varphi$ and $\bar{\varphi}$*

$$\frac{P_e}{\bar{P}_{\bar{e}}} = \frac{R_s}{\bar{R}_s}\,|\bar{q}_q|\,, \tag{3.8}$$

*where $R$ and $\bar{R}$ are distance between corresponding striction points of the pair of Mannheim ruled surface $(\varphi,\,\varphi^*)$ and $(\bar{\varphi},\,\bar{\varphi}^*)$, respectively.*

*Proof.* The relation

$$\|e_s\|\,P_e + R_s = 0 \tag{3.9}$$

was indicate between distance $R$ between corresponding striction points $c(s)$ and $c^*(s)$ of the pair of Mannheim ruled surface $(\varphi,\,\varphi^*)$ and distribution parameter $P_e$ of ruled surface $\varphi$ by Orbay et al [14].

Similarly, there is the relation

$$\|\bar{e}_{\bar{s}}\|\,\bar{P}_{\bar{e}} + \bar{R}_{\bar{s}} = 0 \tag{3.10}$$

between distance $\bar{R}$ between corresponding striction points $\bar{c}(\bar{s})$ and $\bar{c}^*(\bar{s})$ of the pair of Mannheim ruled surface $(\bar{\varphi},\,\bar{\varphi}^*)$ and distribution parameter $\bar{P}_{\bar{e}}$ of ruled surface $\bar{\varphi}$. Using (3.2) and (3.3) in (3.9) and (3.10), we can write

$$|q_s|\,P_e + R_s = 0 \text{ and } |\bar{q}_{\bar{s}}|\,\bar{P}_{\bar{e}} + \bar{R}_{\bar{s}} = 0$$

respectively. So, we have

$$\begin{aligned}
\frac{P_e}{\bar{P}_{\bar{e}}} &= \frac{R_s}{\bar{R}_{\bar{s}}}\frac{|\bar{q}_{\bar{s}}|}{|q_s|} \\
&= \frac{R_s}{\bar{R}_{\bar{s}}}\left|\frac{-\frac{q_s}{\bar{s}_s}\cos\beta + \frac{q_s}{\bar{s}_s}\gamma\sin\beta}{q_s}\right| \\
&= \frac{R_s}{\bar{R}_{\bar{s}}}\left|s_{\bar{s}}\left(-\cos\beta + \gamma\sin\beta\right)\right| \\
&= \frac{R_s}{\bar{R}_s}\left|-\cos\beta + \gamma\sin\beta\right|.
\end{aligned}$$

From (3.7), we get $\frac{P_e}{\bar{P}_{\bar{e}}} = \frac{R_s}{\bar{R}_{\bar{s}}}\left|\bar{q}_q\right|$ .

**Example 3.6.** *The ruled surface*

$$\varphi^*(s,v) = \Big(\cos(s) - \frac{\sqrt{2}}{2}\sin(s)s - (1+\frac{\sqrt{2}}{2})v\cos(s)\sin(s),\ \sin(s) - \frac{\sqrt{2}}{2}\cos(s)s + \frac{\sqrt{2}}{2}v\cos^2(s) - v\sin^2(s),\ \frac{\sqrt{2}}{2}s + \frac{\sqrt{2}}{2}v\cos(s)\Big)$$

*is a Mannheim offset of the ruled surface*

$$\varphi(s,v) = (\cos(s) - \frac{\sqrt{2}}{2}v\sin(s),\ \sin(s) + \frac{\sqrt{2}}{2}v\cos(s),\ \frac{\sqrt{2}}{2}v)$$

*for $R = R(s) = s$.*

*The ruled surface $\bar{\varphi}(\bar{s},\bar{v})$ rolls on the ruled surface $\varphi(s,v)$ along curves*

$$c(s) = \big(\cos(s), \sin(s), 0\big) \text{ and } \bar{c}(\bar{s}) = \big(\cos(s)\cos(\bar{s}), -\sin(s), -\cos(s)\sin(\bar{s})\big)$$

*under the 1- parameter spatial motion. Because of $\bar{\varphi}(\bar{s},\bar{v}) = \bar{c}(\bar{s}) + \bar{v}\bar{e}(\bar{s})$, we get*

$$\begin{aligned}
\bar{\varphi}(\bar{s},\bar{v}) = \Big(&\cos(s)\cos(\bar{s}) - \frac{\sqrt{2}}{2}\sin(s)\cos(\bar{s})\bar{v} + \frac{\sqrt{2}}{2}\sin(s)\sin(\bar{s})\bar{v}, \\
&-\sin(s) + \frac{\sqrt{2}}{2}\cos(s)\cos(\bar{s})\bar{v} + \frac{\sqrt{2}}{2}\cos(s)\sin(\bar{s})\bar{v}, \\
&-\cos(s)\sin(\bar{s}) + \frac{\sqrt{2}}{2}\cos(\bar{s})\bar{v} + \frac{\sqrt{2}}{2}\sin(\bar{s})\bar{v}\Big).
\end{aligned}$$

*If we take $\bar{R} = \bar{R}(\bar{s}) = s$, we have the following Mannheim offset of the ruled surface $\bar{\varphi}(\bar{s},\bar{v})$*

$$\begin{aligned}
\bar{\varphi}^*(\bar{s},\bar{v}) = \Big(&\cos(s)\cos(\bar{s}) + \frac{\sqrt{2}}{2}\sin(\bar{s})\sin(s)s + \frac{\sqrt{2}}{2}\cos(\bar{s})\sin(s)s \\
&- \frac{\sqrt{2}}{2}\sin(s)\cos^2(\bar{s})\bar{v} + \frac{\sqrt{2}}{2}\sin(s)\sin(\bar{s})\cos(\bar{s})\bar{v} + \cos(s)\sin(\bar{s})\bar{v}, \\
&\sin(s) - \frac{\sqrt{2}}{2}\sin(\bar{s})\cos(s)s + \frac{\sqrt{2}}{2}\cos(\bar{s})\cos(s)s \\
&+ \frac{\sqrt{2}}{2}\cos(s)\cos^2(\bar{s})\bar{v} - \frac{\sqrt{2}}{2}\cos(s)\sin(\bar{s})\cos(\bar{s})\bar{v} + \sin(s)\sin(\bar{s})\bar{v}, \\
&-\cos(s)\sin(\bar{s}) + \frac{\sqrt{2}}{2}\cos(\bar{s})s - \frac{\sqrt{2}}{2}\sin(\bar{s})s + \frac{\sqrt{2}}{2}\cos^2(\bar{s})\bar{v} + \frac{\sqrt{2}}{2}\sin(\bar{s})\cos(\bar{s})\bar{v}\Big).
\end{aligned}$$

*We can see the ruled surfaces $\varphi(s,v)$, $\varphi^*(s,v)$, $\bar{\varphi}(s,v)$ and $\bar{\varphi}^*(s,v)$ as follows:*

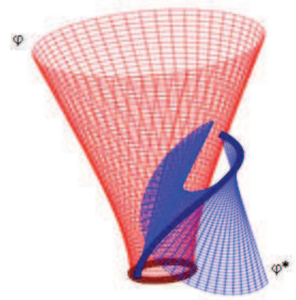


Figure 2: The ruled surfaces $\varphi(s,v)$ and $\varphi^*(s,v)$ under 1- parameter spatial motion

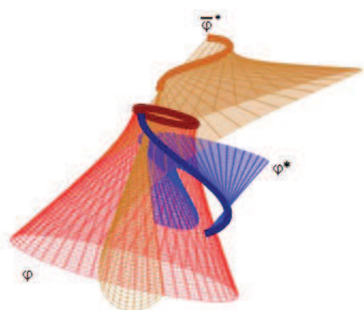


Figure 3: The ruled surfaces $\varphi(s,v)$, $\varphi^*(s,v)$ and $\bar{\varphi}^*(s,v)$

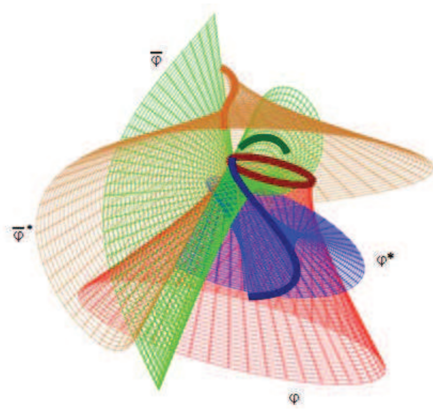


Figure 4: The ruled surfaces $\varphi(s,v)$, $\bar{\varphi}(s,v)$, $\varphi^*(s,v)$ and $\bar{\varphi}^*(s,v)$

## 4. Conclusions

In this study, it is investigated that relationship between the geodesic curvatures of these curves when the couple of Mannheim ruled surfaces roll on along the striction curves under the 1-parameter motions. If one of the curves is a geodesic, the other would be a circle. In order that while one of the curves is geodesic, the other of the curve to be the geodesic too, the absolute value of the arc-lengths of spherical indicatrix should be equal. Moreover, if the ratio of distribution parameters of the pair the

Mannheim ruled surfaces and the ratio of the distance between corresponding striction points of the pair of Mannheim ruled surface is proportional, it must be the arc-length of spherical indicatrix. With these data, considering the couple of Mannheim ruled surfaces and using the relations obtained under the 1-parameter motions, new surfaces can be formed. The physical relationships of these surfaces can be examined. Studies on industrial use can be made. It can be used to obtain artistic products and architectural designs.

## Acknowledgments

The authors would like to thank the editor and referee for the valuable comments that improved the first version of the paper.

## References

1. K. Akdemir, M. Çalışkan and N. Kuruoğlu, *On the pair of konoidal ruled surfaces under the homothetic motions of the Euclidean space* $E^n$, Pure and Applied Mathematics Sciences, 4 (1997), 39-43.

2. J. Bertrand, *Memoire sur la teorie des courbes a double courbure*, ComptesRendus Des Seances De I'Academie Des Sciences, 31 (1850), 623-624.

3. M.P. Carmo, *Differential geometry of curves and surfaces*, Prentice-Hall, Englewood Cliffs, 1976.

4. L. Cui and J.S. Dai, *From sliding-rolling loci to instantaneous kinematics: An adjoint approach*, Mechanism and Machine Theory. 85 (2015), 161-171.

5. M. Çalışkan and N. Kuruoğlu, *On the pair of konoidalaxoids*, Pure and Applied Mathematica Sciences. 36 (1992), 7-13.

6. A. Gray, *Modern differential geometry of curves and surfaces with mathematica*, CRC Press İnc., 1996.

7. C. Huyghens, *Horologium oscillatoriumsive de motupendulorumadhorologiaaptato demonstrations geometricae, Paris; English translation: The pendulum clock or geometrical demontrations concerning the motion of pendula as applied to clocks*, Iowa State University Press, 1673, available online at http://historical.library.cornell.edu/kmoddl/toc_huygens1.html.

8. E. Kasap and N. Kuruoğlu, *On the some new characteristic properties of the pair of the Bertrand ruled surfaces*, Pure Appl. Math. Sci. 53 (2001), 73-79.

9. E. Kasap, S. Yüce and N. Kuruoğlu, *The Involute-Evolute offsets of ruled surfaces*, Trans.A. 33 (2009), 195-201.

10. Z. Lan, Z. Huijun and L. Liuming, *Kinematic decomposition of coupler plane and the study on the formation and distribution of coupler curves*, Mechanism and Machine Theory. 37 (2002), 115-126.

11. M. Lipschutz, *Schaum's outline of differential geometry*, Schaum's Outlines, 1969.

12. H. Liu and F. Wang, *Mannheim partner curves in 3-space*, Journal of Geometry. 88 (2008), 120-126.

13. K. Orbay, and E. Kasap, *On Mannheim partner curves in* $E^3$, International Journal of Physical Sciences. 4 (2009), 261-264.

14. K. Orbay, E. Kasap, and İ. Aydemir, *Mannheim offsets of ruled surfaces*, Mathematical Problems in Engineering. 2009(160917) (2009), 1-9.

15. B. Ravani, and T.S. Ku, *Bertrand offsets of ruled and developable surfaces*, Computer-Aided Design. 23 (1991), 145-152.

16. D.J. Struik, *Lectures on classical differential geometry*, Dover Publications, 1961.

17. D.L. Wang, J. Liu, and D.Z. Xiao, *Kinematic differential geometry of a rigid body in spatial motion-I. A new adjoint approach and instantaneous properties of a point trajectory in spatial kinematics*, Mechanism and Machine Theory. 32 (1997a), 419-432.

18. D.L. Wang, J. Liu, and D.Z. Xiao, *Kinematic differential geometry of a rigid body in spatial motion-II. A new adjoint approach and instantaneous properties of a line trajectory in spatial kinematics*, Mechanism and Machine Theory. 32 (1997b), 433-444.

19. D.L. Wang, J. Liu, and D.Z. Xiao, *Kinematic differential geometry of a rigid body in spatial motion-III. Distribution of characteristic lines in the moving body in spatial motion*, Mechanism and Machine Theory. 32 (1997c), 445-457.

20. F. Wang and H. Liu, *Mannheim partner curve in 3-Euclidean space*, Mathematics in Practice and Theory. 37 (2007), 141-143.

*Keziban Orbay,*
*Department of Mathematics,*
*Faculty of Education,*
*Amasya University,*
*05100, Amasya, Turkey.*
*E-mail address:* keziban.orbay@amasya.edu.tr

*and*

*Tevfik Şahin,*
*Department of Mathematics,*
*Arts and Science Faculty,*
*Amasya University,*
*05189, Amasya, Turkey.*
*E-mail address:* tevfik.sahin@amasya.edu.tr, tevfiksah@gmail.com