Bol. Soc. Paran. Mat.
©SPM –ISSN-2175-1188 on line
SPM: www.spm.uem.br/bspm

(3s.) **v. 2022 (40)** : 1–6.
ISSN-0037-8712 in press
doi:10.5269/bspm.51589

t-Extending Krasner Hypermodules

Burcu Nişancı Türkmen

Abstract: Let M be a hypermodule over a hyperring R such that the intersection of any two subhypermodules of M is a subhypermodule of M. We introduce the concept of a t-essential subhypermodule in M relative to an arbitrary subhypermodule T of M, which is called T-t-essential subhypermodule of M. Our aim in this work is to investigate properties of t-essential subhypermodules. We apply this concept to introduce t-extending hypermodules. Examples are provided to illustrate different concepts.

Key Words: Direct summand, t-extending hypermodule, t-essential subhypermodule.

Contents

1. Introduction

The categories of hypergroups, hypermodules and hyperrings have many important roles in hyperstructures. Some authors got many exiting results about these theories. Reader can see references [1], [6], [7], [8] and [10] to get some basic information about the categories of hypergroups, hyperrings and hypermodules. Also reference [13] can be suitable to get some information about theory of rings and modules.

We recall some definitions and theorems from above references which we need to develop our paper.

In this paper, we use $\circ : H \times H \longrightarrow P^*(H)$ instead of $. : H \times H \longrightarrow H$, where H is a non-empty set and $P^*(H)$ the set of all non-empty subsets of H. The map $\circ$ is called a *hyperoperation* on H. Therefore, we define $A \circ B = \bigcup_{a\in A,\, b\in B} a \circ b$, $x \circ A = \{x\} \circ A$ and $A \circ x = A \circ \{x\}$, for $x \in H$ and $A, B \in P^*(H)$. We say $(H, \circ)$ is a *semihypergroup* if for all x, y, z of H, we have $(x \circ y) \circ z = x \circ (y \circ z)$. A semihypergroup $(H, \circ)$ is called a *hypergroup* if for all $x \in H$, $x \circ H = H \circ x = H$ [5]. A non-empty subset K of a hypergroup $(H, \circ)$ is called *subhypergroup*, if for all $k \in K$, we have $k \circ K = K \circ k = K$. A hypergroup $(H, \circ)$ is called *commutative* if for all $x, y \in H$, then $x \circ y = y \circ x$. A commutative hypergroup $(H, \circ)$ is said to be *canonical*, if there exists a unique $0 \in H$, such that for all $x \in H$, $x \circ 0 = \{x\}$; for all $x \in H$, there exists a unique $x^{-1} \in H$, such that $0 \in x \circ x^{-1}$; if $x \in y \circ z$, then $y \in x \circ z^{-1}$ and $z \in y^{-1} \circ x$, for all $x, y, z \in H$ [5].

The triple $(R, \uplus, \circ)$ is called a *hyperring*, if $(R, \uplus)$ is a hypergroup, $(R, \circ)$ is a semihypergroup and $\circ$ is a distributive over $\uplus$ [7]. A non-empty subset I of a hyperring R is called a *hyperideal* if $(I, \uplus)$ is a subhypergroup of $(R, \uplus)$ and $r \circ x \cup x \circ r \subseteq I$ for all $x \in I$ and $r \in R$. A hyperring $(R, \uplus, \circ)$ is called *Krasner*, if $(R, \uplus)$ is a canonical hypergroup and $(R, \circ)$ is a semigroup such that 0 is zero element, i.e. for all $x \in R$, we have $x \circ 0 = 0 = 0 \circ x$ [7]. A non-empty subset I of a krasner hyperring $(R, \uplus, \circ)$ is called a *a right hyperideal* of R if $(I, \uplus)$ is a canonical subhypergroup of $(R, \uplus)$ and for every $a \in I$ and $r \in R$, $a \circ r \in I$ [11].

Let $(R, \uplus, \circ)$ be a hyperring and $(M, +)$ be a hypergroup. If there exists an external hyperoperation $. : M \times R \longrightarrow P^*(M)$ such that for all $a, b \in M$ and $r, s \in R$ we have $(a + b).r = (a.r) + (b.r)$, $a.(r \uplus s) = (a.r) + (a.s)$ and $a.(r \circ s) = (a.r).s$ then $(M, +, .)$ is called a *right hypermodule over* R [3]. Similarly, a left hypermodule over R can be defined. M is called a *hypermodule over* R, if it is a right and a left hypermodule over R. If $(M, +)$ is a canonical hypergroup and $(R, \uplus, \circ)$ is a Krasner hyperring, then M is said to be a *canonical R-hypermodule*. Moreover, M is called *Krasner R-hypermodule*, if it is a canonical R-hypermodule, where . is an external operation, that is $. : M \times R \longrightarrow M$ by $(m, r) \mapsto m.r \in M$, and

2010 *Mathematics Subject Classification:* 20N20, 16D10, 16N50.
Submitted December 21, 2019. Published May 24, 2020

$0.r = 0$. A non-empty subset N of an R-hypermodule M is called a *subhypermodule*, if N is a hypermodule over R. In this work, all R-hypermodules are right Krasner R-hypermodules unless otherwise stated.

Throughout this work, we admit that every hypermodule M is a Krasner R-hypermodule thereby $\{0\}$ is a subhypermodule of M. We denote a subhypermodule N of M by $N \leq M$. A subhypermodule N of M is called an *essential subhypermodule* of M if for every non-zero subhypermodule K of M, we have $N \cap K \neq \{0\}$, denoted by $N \trianglerighteq M$. Let M be an R-hypermodule and T a submodule of M. A subhypermodule N is called *T-t-essential* in M (denoted by $N \trianglerighteq_t^T M$) if $N \nsubseteq T$, $N \cap L \subseteq T$ implies $L \subseteq T$ and $L \subseteq Z_2(M)$ for every subhypermodule L of M, where $Z_2(M)$ is the second singular subhypermodule of M. Whenever $T = \cap_{K \leq M} K$ and $N \trianglerighteq_t^T M$, then we call N a *t-essential subhypermodule* of M and $N \trianglerighteq_t M$ shows that N is a t-essential subhypermodule of M. In the light of last remarks and comments, in what follows we start to study on t-essentially in R-hypermodules. It can be seen that for a Krasner R-hypermodule M and $K \leq M$, we can construct the quotient Krasner R-hypermodule M/K, endowed with $(x + K) \oplus (y + K) = \{\, t + K \mid t \in x + y \,\}$ and $(x + K) \odot r = x.r + K$, for all $x + K, y + K \in M/K$ and $r \in R$.

Let T be a subhypermodule of an R-hypermodule M. In [9], a subhypermodule N of M is called a *T-direct summand* provided there exists a subhypermodule K of M such that $M = N + K$ and $N \cap K \subseteq T$. If $T = \{0\}$, then N is a *direct summand* of M; this situation is denoted by $M = N \oplus K$. A subhypermodule N of M is called *closed* if, N is a closed subhypergroup $(M, +)$,that is, $x \in m + y$ $(x \in y + m)$ for all $x, y \in N$ and $m \in M$, implies that $m \in N$. If N is a closed subhypermodule of M, we denote it by $N \leq_c M$. It can be seen that a subhypergroup N of $(M, +)$ is closed if and only if $N + (M \setminus N) = M \setminus N$ [5].

2. t-Closed Subhypermodules and t-Extending Krasner Hypermodules

In this section, we define notions of t-closed subhypermodules and t-extending Krasner hypermodules, and we obtain some basic properties of t-extending Krasner hypermodules.

The singular subhypermodule $Z(M)$ of an R-hypermodule M is a set of $m \in M$ with $m.I = \{0\}$ for some essential right hyperideal of hyperring R. The second singular (Goldie torsion) subhypermodule $Z_2(M)$ is the subhypermodule of hypermodule M which is defined by $Z(M/Z(M)) = Z_2(M)/Z(M)$.

Definition 2.1. *Let M be an R-hypermodule and T a submodule of M. A subhypermodule N is called* T-t-essential *in M (denoted by $N \trianglerighteq_t^T M$) if $N \nsubseteq T$, $N \cap L \subseteq T$ implies $L \subseteq T$ and $L \subseteq Z_2(M)$ for every subhypermodule L of M, where $Z_2(M)$ is the second singular subhypermodule of M. Whenever $T = \cap_{K \leq M} K$ and $N \trianglerighteq_t^T M$, then we call N a* t-essential subhypermodule *of M and will be denoted by $N \trianglerighteq_t M$. Note that for Krasner R-hypermodule M, we have $\cap_{K \leq M} K = \{0\}$.*

Let M and N be two R-hypermodules. Recall from [9] that the function $f : M \longrightarrow N$ is called *a homomorphism* if $f(x + y) \subseteq f(x) + f(y)$ and $f(x.r) = f(x).r$ for all $x, y \in M$ and $r \in R$. Also, f is said to be a *strong homomorphism* if $f(x + y) = f(x) + f(y)$ and $f(x.r) = f(x).r$ for all $x, y \in M$ and $r \in R$. Note that in this case, $f(0_M) = 0_N$. If a strong homomorphism f is one-to-one and surjective function, it is called a *strong isomorphism*.

We call *singular subhypermodule* $Z(M)$ of a hypermodule M is the set of $m \in M$ with $ann(m) \trianglerighteq R_R$, or equivalently, $m \circ I = 0$ for some essential right hyperideal I of R. *The second singular (or Goldie torsion) subhypermodule* $Z_2(M)$ is the subhypermodule of M which is defined by $Z(M/Z(M)) = Z_2(M)/Z(M)$. We call a hypermodule M *Z_2-torsion* if $Z_2(M) = M$. Clearly, every singular hypermodule is Z_2-torsion.

Proposition 2.2. *The following statements are equivalent for a subhypermodule N of an R-hypermodule M:*

(1) N is t-essential subhypermodule of M;

(2) $(N + Z_2(M))/Z_2(M)$ is an essential subhypermodule of $M/Z_2(M)$;

(3) $N + Z_2(M)$ is an essential subhypermodule of M;

(4) M/N is Z_2-torsion.

Proof. (1) $\Rightarrow$ (2) There exists a subhypermodule K of M such that $N \oplus K$ is an essential subhypermodule of M. By hypothesis, $K \leq Z_2(M)$ therefore; $N + Z_2(M)$ is an essential subhypermodule of M, and since $Z_2(M)$ is a closed subhypermodule of M, we conclude that $(N + Z_2(M))/Z_2(M)$ is an essential subhypermodule of $M/Z_2(M)$.

(2) $\Rightarrow$ (3) This is obvious.

(3) $\Rightarrow$ (4) By hypothesis $M/(N + Z_2(M))$ is singular, and hence Z_2-torsion. On the other hand, there exists a strong isomorphism $(N + Z_2(M))/N$ to a factor of $Z_2(M)$, thus it is Z_2-torsion.Therefore, from the strong isomorphism $(M/N)/[(N + Z_2(M))/N] \cong M/(N + Z_2(M))$, we obtain that M/N is Z_2-torsion.

(4) $\Rightarrow$ (1) Since M/N is Z_2-torsion, $(M/N)/(Z(M/N))$ is singular. Additionally there exists a strong isomorphism from $(M/N)/(Z(M/N))$ to M/N^*, where $N^*/N = Z(M/N)$. Thus M/N^* is singular. Let $N \cap K \leq Z_2(M)$ for some subhypermodule K of M, and $k \in K$. As M/N^* is singular, there exists an essential right hyperideal I of R such that $k.I \leq N^*$. Then for every $x \in I$, there exists an essential right hyperideal J of R such that $(k.x).J \leq N$. Hence $(k.x).J \leq N \cap K \leq Z_2(M)$, and so $k.x + Z_2(M) \in Z(M/Z_2(M)) = \{0\}$. Thus $k.I \leq Z_2(M)$, and this implies that $k + Z_2(M) \in Z(M/Z_2(M)) = \{0\}$. It means that $k \in Z_2(M)$. Consequently, $K \leq Z_2(M)$. □

By Proposition 2.2, we obtain that every essential subhypermodule of an R-hypermodule M is t-essential.

Definition 2.3. *Let M be an R-hypermodule. The subhypermodule L of a hypermodule M is called* t-closed *and write $L \leq_{tc} M$ if $L \trianglelefteq_t L' \leq M$ implies that $L' = L$.*

It is clear that every t-closed subhypermodule is a closed subhypermodule and if L is a subhypermodule of a nonsingular R-hypermodule M, then L is t-closed in M if and only if L is closed in M.

Lemma 2.4. *Let M be an R-hypermodule.*

(1) If $L \leq_{tc} M$, then $Z_2(M) \leq L$.

(2) $\{0\} \leq_{tc} M$ if and only if M is nonsingular.

(3) If $N \leq L$, then $L \leq_{tc} M$ if and only if $L/N \leq_{tc} M/N$.

Proof. (1) Since there exists a strong isomorphism from $(L + Z_2(M))/L$ to $Z_2(M)/(L \cap Z_2(M))$ such that $(L + Z_2(M))/L$ is Z_2-torsion by Proposition 2.2, $L \trianglelefteq_t L + Z_2(M)$. Thus $L = L + Z_2(M)$, and so $Z_2(M) \leq L$.

(2) ($\Rightarrow$) Follows from part (1).

($\Leftarrow$) It is clear.

(3) This follows from Proposition 2.2 (4). □

Recall from [9, Definition 2.16 (1)] that for subhypermodules T, K, N of an R-hypermodule M; N is called a *T-complement* of K in M, when N is maximal with respect to $K \cap N \subseteq T$ (note that by Zorn's lemma such a subhypermodule does exist). For the case $T = \{0\}$, it has been prefered to say that N is a complement of K.

Proposition 2.5. *Let N be a subhypermodule of a hypermodule M. The following statements are equivalent:*

(1) There exists a subhypermodule K such that N is maximal with respect to the property that $N \cap K$ is Z_2-torsion;

(2) N is t-closed in M;

(3) N contains $Z_2(M)$ and $N/Z_2(M)$ is a closed subhypermodule of $M/Z_2(M)$;

(4) N contains $Z_2(M)$ and N is a closed subhypermodule of M;

(5) N is a complement to a nonsingular subhypermodule of M;

(6) M/N *is nonsingular.*

Proof. (1) $\Longrightarrow$ (2) Assume that the first condition is provided and $N \trianglerighteq_t N^{'} \leq M$ for some $N^{'}$. Then $N \cap (N^{'} \cap K) \leq Z_2(M)$ implies that $N^{'} \cap K \leq Z_2(M)$. Hence $N = N^{'}$.

(2) $\Longrightarrow$ (3) By Lemma 2.4, N contains $Z_2(M)$. Let $N/Z_2(M) \trianglerighteq N^{'}/Z_2(M)$. By Proposition 2.2 (2), we obtain that $N \trianglerighteq_t N^{'}$. Hence $N = N^{'}$.

(3) $\Longrightarrow$ (4) Let $N \trianglerighteq N^{'} \leq M$. Since every essential subhypermodule is t-essential, by Proposition 2.2 $N/Z_2(M) \trianglerighteq N^{'}/Z_2(M)$. Hence $N = N^{'}$.

(4) $\Longrightarrow$ (5) Since N is closed, it has been easily available that $N = L \cap M$ for some direct summand L of the injective hull $E(M)$, say $E(M) = L \oplus Y$, and let $K = M \cap Y$ by [12, Proposition 6.32]. Then we have $N \cap K = \{0\}$. Thus $Z_2(K) = Z_2(M) \cap K \leq N \cap K = \{0\}$, and hence K is nonsingular. It is easily shown that N is a complement to K by means of [12, Proposition 6.32 ((3) $\Longrightarrow$ (1))]. The proof is completed.

(5) $\Longrightarrow$ (1) It is clear.

(2) $\Longleftrightarrow$ (6) By Lemma 2.4, a subhypermodule N of M is t-closed if and only if the zero subhypermodule $\{0\} = N/N$ of M/N is t-closed if and only if M/N is nonsingular. □

Corollary 2.6. *Let M be an R-hypermodule. Then*

(1) *$Z_2(M)$ is t-closed in M.*

(2) *If f is a strong endomorphism of M and N is a t-closed subhypermodule of M, then $f^{-1}(N)$ is t-closed in M.*

Proof. (1) Since $M/Z_2(M)$ is nonsingular, $Z_2(M)$ is t-closed in M by Proposition 2.5.

(2) There exists a natural embedding of $M/f^{-1}(N)$ into the nonsingular hypermodule M/N. Hence $M/f^{-1}(N)$ is nonsingular, and thus by Proposition 2.5, $f^{-1}(N)$ is t-closed in M. □

Corollary 2.7. *Let N be a subhypermodule of an R-hypermodule M.*

(1) *If $N \leq_{tc} M$, then $N = Z_2(M)$ if and only if N is Z_2-torsion if and only if there exists a t-essential subhypermodule K of M for which $N \cap K \leq Z_2(M)$.*

(2) *Let $N \leq T \leq M$. If $N \leq_{tc} M$, then $N \leq_{tc} T$.*

(3) *If $N \leq_{tc} T$ and $T \leq_{tc} M$, then $N \leq_{tc} M$.*

Proof. (1) By Lemma 2.4 (1) it suffices to show that if $N = Z_2(M)$, then there is a t-essential subhypermodule K of M for which $N \cap K \leq Z_2(M)$. Then there exists a subhypermodule K of M such that N is maximal with respect to the property that $N \cap K$ is Z_2-torsion by Proposition 2.5. Let $K \cap T \leq Z_2(M)$. By Zorn's Lemma, T can be enlarged into a t-closed subhypermodule $N^{'}$ such that $K \cap N^{'} \leq Z_2(M)$. In addition, by Lemma 2.4 (1), $N = Z_2(M) \leq N^{'}$. Hence $N^{'} = N = Z_2(M)$. Then $T \leq Z_2(M)$ and so K is t-essential.

(2) and (3) follow from Proposition 2.5 ((2) $\Longleftrightarrow$ (6)). □

We have in general

$$N \leq M,\, N^{'} \leq_c M \nRightarrow N \cap N^{'} \leq_c N,$$

$$N \leq_c M,\, N^{'} \leq_c M \nRightarrow N \cap N^{'} \leq_c M;$$

for an R-hypermodule M. But above conditions are always true if we replace $\leq_c$ by $\leq_{tc}$.

Proposition 2.8. *Let M be a Krasner R-hypermodule. Then:*

(1) *If $N \leq M$, then $N^{'} \leq_{tc} M$, then $N \cap N^{'} \leq_{tc} N$;*

(2) *If $N \leq_{tc} M$ and $N^{'} \leq_{tc} M$, then $N \cap N^{'} \leq_{tc} M$.*

Moreover, an arbitrary intersection of t-closed subhypermodules is t-closed.

Proof. (1) Let $N \cap N^{'} \trianglerighteq_t K \leq N$. Then $K/(N \cap N^{'})$ is $Z_2(M)$-torsion by Proposition 2.2. Thus, $K/(K \cap N^{'})$ is Z_2-torsion, and so $(K + N^{'})/N^{'}$ is Z_2-torsion. Hence $N^{'} \trianglerighteq_t K + N^{'}$ and, as $N^{'}$ is t-closed in M, we conclude that $N^{'} = K + N^{'}$. Then we have $K \leq N^{'}$. Finally $K \leq N \cap N^{'}$ and so $K = N \cap N^{'}$.

(2) Let N_i be a t-closed subhypermodule of M for any i in an index set I. It is clear that there is a strong monomorphism from $M/\cap_i N_i$ to $\prod_i M/N_i$. Then M/N_i is nonsingular by Proposition 2.5 (6). Thus $\prod_i M/N_i$ is nonsingular; hence, $M/\cap_i N_i$ is nonsingular, and so $\cap_i N_i$ is t-closed in M. □

Motivated by the definition of an extending hypermodule, we define the following notion.

Definition 2.9. *An R-hypermodule M is called* t-extending *if every t-closed subhypermodule of M is a direct summand of M.*

It is clear that every Z_2-torsion hypermodule is t-extending by Lemma 2.4. Moreover, every extending hypermodule is t-extending since every t-closed subhypermodule is closed by Proposition 2.5. The following Theorem gives several equivalent conditions for an hypermodule to be t-extending.

Theorem 2.10. *The following statements are equivalent for an R-hypermodule:*

(1) M is t-extending;

(2) For every subhypermodule N of M, K is a direct summand of M where $K/N = Z_2(M/N)$;

(3) $M = Z_2(M) \oplus M^{'}$ where $M^{'}$ is a (nonsingular) extending hypermodule;

(4) Every subhypermodule of M which contains $Z_2(M)$ is essential in a direct summand of M;

(5) Every subhypermodule of M is t-essential in a direct summand of M;

(6) For every subhypermodule K of M, there exists a decomposition $M/K = N/K \oplus N^{'}/K$ such that N is a direct summand of M and $N^{'} \trianglerighteq_t M$.

Proof. (1) $\Longrightarrow$ (2) Since there exists a strong isomorphism from M/K to $(M/N)/(Z_2(M/N))$, the factor hypermodule M/K is nonsingular. Then K is a t-closed subhypermodule of M by Proposition 2.5. Thus K is a direct summand of M.

(2) $\Longrightarrow$ (3) Since $M/Z_2(M)$ is nonsingular, (2) implies that $Z_2(M)$ is a direct summand of M, say $M = Z_2(M) \oplus M^{'}$. Let N be a closed subhypermodule of $M^{'}$. Since $M^{'}$ is nonsingular, $M^{'}/N$ is nonsingular by Proposition 2.5. Then $M/(Z_2(M) \oplus N)$ is nonsingular, and so $Z_2(M) \oplus N$ is a direct summand of M by (2). This implies that N is a direct summand of $M^{'}$, and hence $M^{'}$ is extending.

(3) $\Longrightarrow$ (4) Let K be a subhypermodule of M which contains $Z_2(M)$. It is clear that $K = Z_2(M) \oplus (K \cap M^{'})$. There exists a direct summand L of $M^{'}$ such that $K \cap M^{'}$ is essential in L. Thus K is essential in $Z_2(M) \oplus L$ which is a direct summand of M.

(4) $\Longrightarrow$ (5) Suppose that K is a subhypermodule of M. There exists a direct summand N of M such that $K + Z_2(M)$ is essential in N. By Proposition 2.2, $K \trianglerighteq_t N$.

(5) $\Longrightarrow$ (6) Let K be a subhypermodule of M. There exist a decomposition $M = N \oplus L$ such that $K \trianglerighteq_t N$. Then $M/K = N/K \oplus (L + K)/K$. It is clear that there exist strong isomorphisms from $M/(L + K)$ to $(M/K)/[(L + K)/K]$ and N/K. Therefore N/K is Z_2-torsion. So $L + K \trianglerighteq_t M$.

(6) $\Longrightarrow$ (1) Let L be a t-closed subhypermodule of M. There exists a decomposition $M/L = N/L \oplus N^{'}/L$, where N is a direct summand of M and $N^{'} \trianglerighteq_t M$. Then we have $L \trianglerighteq_t N$ by Proposition 2.2 (4). Since L is t-closed, we conclude that $L = N$ is a direct summand of M. □

Example 2.11. (1) *Since every torsion-free finitely generated $\mathbb{Z}$-hypermodule is extending, every finitely generated $\mathbb{Z}$-hypermodule is t-extending by Theorem 2.10.*

(2) *For an arbitrary R-hypermodule M, the R-hypermodule $M/Z_2(M)$ is nonsingular, and so $E(M/Z_2(M))$ is nonsingular extending. Thus $E(M/Z_2(M)) \oplus Z_2(M)$ is t-extending.*

We call a subhypermodule N of a hypermodule M *fully invariant* if $f(N) \leq N$ for every strong endomorphism f of M.

Proposition 2.12. *Let M be a t-extending hypermodule.*

(1) Every strong homomorphic image of M is t-extending. In particular, every direct summand of M is t-extending.

(2) Every fully invariant subhypermodule of M is t-extending.

Proof. (1) Let K be a subhypermodule of M and $L/K \leq M/K$. Since M is t-extending, there exists a direct summand N of M such that $L \trianglerighteq_t N$. Therefore, N/K is a direct summand of M/K such that $L/K \trianglerighteq_t N/K$ by Proposition 2.2 (4). Thus M/K is t-extending by Theorem 2.10 (5).

(2) Let L be a fully invariant subhypermodule of M and K be a subhypermodule of L. There exists a decomposition $M = N \oplus N^{'}$ such that $K \trianglerighteq_t N$. Since L is fully invariant, $L = (N \cap L) \oplus (N^{'} \cap L)$. However, $K \trianglerighteq_t N \cap L$ since $(N \cap L)/K \leq N/K$ is Z_2-torsion. Thus L is t-extending. □

The following example shows that the notions of extending and t-extending are not the same. Moreover, this example shows that a t-extending hypermodule not be Z_2-torsion.

Example 2.13. *For an arbitrary $\mathbb{Z}$-hypermodule M, since $\mathbb{Z}_2 \oplus \mathbb{Z}_8$ is not extending, the $\mathbb{Z}$-hypermodule $E(M) \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_8$ is t-extending but not extending.*

Acknowledgments

The author would like to thank the referees for careful reading of the paper and valuable suggestions.

References

1. Ameri, R., *On categories of hypergroups and hypermodules*, Journal of Discrete Mathematical Sciences 6(2-3), 121-132, (2003).
2. Anderson, F.W., Fuller, K.R., *Rings and Categories of Modules*, Springer-Verlag, New York (1992).
3. Anvariyeh, S.M., Davvaz, B., *Strongly transitive geometric spaces associated to hypermodules*, Journal of Algebra 322, 1340-1359, (2009).
4. Asgari, S.H., Haghany, A., *t-extending Modules and t-Baer Modules*, Communications in Algebra 39, 1605-1623, (2011).
5. Corsini, P., *Prolegomena of Hypergroup Theory*, 2nd ed. Tricesimo Italy, Aviani editore Italy, (1993).
6. Davvaz, B., *Remarks on weak hypermodules*, Bull. Korean Math. Soc. 36(3), 599-608, (1999).
7. Davvaz, B., Fotea, V.L., *Hyperring Theory and Applications*, Palm Harbor, FL, USA International Academic Press, (2007).
8. Fotea, V.L., *uzzy hypermodules*, Computers and Mathematics with Applications 57, 466-475, (2009).
9. Hamzekolaee, A.R.M., Norouzi, M., *A hyperstructural approach to essentially*, Communications In Algebra 46(11), 4954-4964, (2018).
10. Krasner, M., *A class of hyperrings and hyperfields*, IJMMS 6(2), 307-311, (1999).
11. Omidi, S., Davvaz, B., *Hyperideal theory in ordered Krasner hyperrings*, An. Şt. Univ. Ovidius Constanta 27(1), 193-210, (2019).
12. Lam, T.Y., *Lectures on Modules and Rings*, Graduate Texts in Mathematics. 189, New York, Springer-Verlag, (1998).
13. Wisbauer, R., *Foundations of module and ring theory*, Gordon and Breach, (1991).

Burcu Nişancı Türkmen,
Department of Mathematics,
Amasya University, Faculty of Art and Science,
Turkey.
E-mail address: burcunisancie@hotmail.com