



Einstein Operations on Fuzzy Soft Multisets and Decision Making

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ABSTRACT: In this paper, we define Einstein product and Einstein sum of fuzzy soft multisets (FSM-sets) and using these products, we introduce an adjustable approach to FSM-set based decision-making, for solving decision-making in an uncertain situation. The feasibility of our proposed FSM-set based decision-making procedure in practical application is shown by some numerical examples.

Key Words: Soft set, Fuzzy set, Decision-making, FSM-set, Einstein product, Einstein sum.

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1. Introduction

The present reality is full of indeterminacy, inaccuracy and vagueness. In fact, a large portion of the issues we dealt with are vague instead of exact. Facing such a variety of uncertain data, classical methods are not generally effective, the reason is that different types of uncertainties present in these problems. Fuzzy sets and soft sets are effective mathematical tools for modelling different types of instability and often useful approaches to describe uncertainties. These theories have been applied in many areas, such as economics, information sciences, intelligent systems, machine learning, cybernetics, the smoothness of functions, game theory, operations research, measurement theory, probability theory and so on.

Since the idea of fuzzy set was started by Zadeh [43], many new methodologies and theories treating imprecision and uncertainty have been proposed, such as the Fuzzy ideals in right regular LA-semigroups was published by Khan et al. [25], A new gener-alized intuitionistic fuzzy set introduced by Jamkhaneh and Nadarajah [24] and so on [[17], [39], [41], [45]]. Fuzzy sets have applications in assorted sorts of territories, for instance in information bases, pattern recognition, neural systems, fuzzy modelling, medicine, economy, multicriteria decision making (see [41], [47]).

Theory of soft set has a rich potential for the application in various directions, some of which are accounted for by Molodtsov [30] in his work. Later on Maji et al. [28] characterized some new definitions on soft sets and Ali et al. [2] introduced some new mathematical operations on soft sets. Joining soft sets [30] with fuzzy sets [43], Maji et al. [27] characterized fuzzy soft sets (FS-sets), which are rich potential for dealing with decision-making. By utilizing these definitions, the uses of soft set theory have been concentrated progressively. Feng [22] presented the use of level soft sets in decision-making in view of fuzzy soft sets. Later on, more broad properties and applications of soft set theory have been examined by Maji, Feng and others, for instance, see [[15], [21], [23], [29], [34], [42], [44]]. Correspondingly, fuzzy soft comprehensively connected [[16], [26], [38]].

Alkhazaleh and others [[1], [5], [7], [8], [33], [40]] as a modification of soft set, introduced the meaning of the soft multiset and studied its essential operations on soft multiset. Alkhazaleh and Salleh

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[3] introduced the idea of FSM-set as a speculation of soft multiset and concentrated on the utilization of FSM-set based decision-making issues using Roy-Maji Algorithm [37]. Recently, Mukherjee and Das [31] called attention to that Alkhezaleh-Salleh technique [3] is not adequate to understand FSM-set based decision-making issues and they acquainted another algorithm with FSM-set based decision-making issues using Feng's algorithm [22]. Also, some other articles were committed to this point, for instance [[32], [34], [35], [36]].

In fact, every one of these ideas having a decent application in different controls and genuine issues are currently getting force. However, it is seen that every one of these theories has their own troubles, that is the reason in this paper, we define Einstein product and Einstein sum of FSM-sets and using these products, we introduce an adjustable approach to FSM-set based decision-making, for solving decision-making in an uncertain situation. Firstly, we briefly review some definitions and results helpful in our further thought (Section 2). In section 3, we define Einstein product and Einstein sum of FSM-sets. Finally, using these products, we introduce an adjustable approach to FSM-set based decision-making and the feasibility of our proposed FSM-set based decision-making procedure in practical application is shown by some numerical models (Section 4).

2. Preliminary Notes

In this paper, let U be a starting universe, E be an arrangement of parameters and $P(U)$ mean the power set of U with $A \subseteq E$.

Definition 2.1. [43] A fuzzy set X on U is a set having the form $X = \{(u, \mu_X(u)) : u \in U\}$, where the function $\mu_X : U \rightarrow [0, 1]$ is called the membership function and $\mu_X(u)$ represents the degree of membership of each element $u \in U$. We denote the class of all fuzzy sets on U by $FS(U)$.

Definition 2.2. [47] Einstein product t is a two placed function $t : [0, 1] \times [0, 1] \rightarrow [0, 1]$ and defined as $t(\mu_X(u), \mu_Y(u)) = \frac{\mu_X(u) \cdot \mu_Y(u)}{2 - [\mu_X(u) + \mu_Y(u) - \mu_X(u) \cdot \mu_Y(u)]}$

Definition 2.3. [47] Einstein sum s is a two placed function $s : [0, 1] \times [0, 1] \rightarrow [0, 1]$ and defined as $s(\mu_X(u), \mu_Y(u)) = \frac{\mu_X(u) + \mu_Y(u)}{1 + \mu_X(u) \cdot \mu_Y(u)}$

Definition 2.4. [30] A couple (F, A) is known as soft set on U , where $F : A \rightarrow P(U)$ is a mapping.

Definition 2.5. [3] Let $\{U_i : i \in I\}$ be a collection of universes such that $\bigcap_{i \in I} U_i = \phi$ and let $\{E_{U_i} : i \in I\}$ be a collection of sets of parameters. Let $U = \prod_{i \in I} FS(U_i)$ where $FS(U_i)$ denotes the set of all fuzzy subsets of U_i , $E = \prod_{i \in I} E_{U_i}$ and $A \subseteq E$. A pair (F, A) is called a fuzzy soft multi set over U , where F is a mapping given by $F : A \rightarrow U$.

For any $e \in A$, $F(e)$ is referred to as the collection of fuzzy approximate value set of the parameter e and it is actually a collection of fuzzy set on U , it can be written as $F(e) = (\{\frac{u}{\mu_{F(e)}(u)} : u \in U_i\} : i \in I)$, where $\mu_{F(e)}(u)$ is the fuzzy membership value of that object u holds on parameter e .

Simply, we denote the sets of all FSM-sets over U by $FSMS(U, A)$, where the parameter set A is fixed.

To illustrate this let us consider the following example:

Example 1 Let us consider three universes $U_1 = \{h_1, h_2, h_3, h_4\}$, $U_2 = \{c_1, c_2, c_3\}$ and $U_3 = \{v_1, v_2, v_3\}$ which are the collections of houses, autos and inns respectively. Assume Mr. X has a financial plan to purchase a house, an auto and rent a venue to hold a wedding festival. Suppose a FSM-set (F, A) which depicts houses, autos and inns that Mr. X is considering for settlement buy, transportation buy, and a venue to hold a wedding festival, separately. Let $\{E_{U_1}, E_{U_2}, E_{U_3}\}$ be a collection of sets of decision parameters related to the above universes, where

$$\begin{aligned} E_{U_1} &= \{e_{U_1,1} = \text{expensive}, e_{U_1,2} = \text{cheap}, e_{U_1,3} = \text{wooden}\}, \\ E_{U_2} &= \{e_{U_2,1} = \text{expensive}, e_{U_2,2} = \text{cheap}, e_{U_2,3} = \text{sporty}\}, \\ E_{U_3} &= \{e_{U_3,1} = \text{expensive}, e_{U_3,2} = \text{cheap}, e_{U_3,3} = \text{in Kuala Lumpur}\}. \end{aligned}$$

Let $U = \prod_{i=1}^3 P(U_i)$, $E = \prod_{i=1}^3 E_{U_i}$ and $A \subseteq E$, such that

$A = \{a_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}), a_2 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,1}), a_3 = (e_{U_1,2}, e_{U_2,3}, e_{U_3,1}), a_4 = (e_{U_1,3}, e_{U_2,3}, e_{U_3,1}), a_5 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,2}), a_6 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,2})\}$.

Suppose Mr. X wants to choose objects from the sets of given objects with respect to the sets of choice parameters. Let the resultant fuzzy soft multi set be (F, A) as in Table 1.

Table 1: FSM-set (F, A)

U_i		a_1	a_2	a_3	a_4	a_5	a_6
U_1	h_1	0.3	0.8	0.7	0.8	0.3	0.7
	h_2	0.4	0.9	0.8	0.9	0.4	0.6
	h_3	0.9	0.3	0.1	0.3	0.9	0.9
	h_4	0.7	0.8	0	0.8	0.7	0.5
U_2	c_1	0.8	0.8	0.6	0.8	0.9	0.6
	c_2	0.6	0.8	0.8	0.8	1	0.8
	c_3	0.6	0.5	0.3	0.5	0.9	0.3
U_3	v_1	0.9	0.9	0.5	0.9	0.8	0.9
	v_2	0.7	0.7	0.5	0.7	0.5	0.8
	v_3	0.9	0.9	0.7	0.9	0.4	1

Definition 2.6. [3] For any FSM-set (F, A) , a pair $(e_{U_i,j}, F_{e_{U_i,j}})$ is called a U_i -fuzzy soft multi set part (U_i -FSMS-part) for all $e_{U_i,j} \in a$ and $F_{e_{U_i,j}} \subseteq F(A)$ is a fuzzy approximate value set, where $a \in A$, $i, j \in I$.

To illustrate this let us consider the following example:

Example 2 For the FSM-set (F, A) as in Table 1, the U_1 – FSMS-part, U_2 – FSMS-part and U_3 – FSMS-part are represented as in Table 2, Table 3 and Table 4 respectively.

Table 2: U_1 -FSMS-part of (F, A)

U_1	a_1	a_2	a_3	a_4	a_5	a_6
h_1	0.3	0.8	0.7	0.8	0.3	0.7
h_2	0.4	0.9	0.8	0.9	0.4	0.6
h_3	0.9	0.3	0.1	0.3	0.9	0.9
h_4	0.7	0.8	0	0.8	0.7	0.5

Table 3: U_2 -FSMS-part of (F, A)

U_2	a_1	a_2	a_3	a_4	a_5	a_6
c_1	0.8	0.8	0.6	0.8	0.9	0.6
c_2	0.6	0.8	0.8	0.8	1	0.8
c_3	0.6	0.5	0.3	0.5	0.9	0.3

Table 4: U_3 -FSMS-part of (F, A)

U_3	a_1	a_2	a_3	a_4	a_5	a_6
v_1	0.9	0.9	0.5	0.9	0.8	0.9
v_2	0.7	0.7	0.5	0.7	0.5	0.8
v_3	0.9	0.9	0.7	0.9	0.4	1

3. t-norms and t-conorms products on FSM-sets

In this section, we define the t-norm and t-conorm products on FSM-sets.

Definition 3.1. Let $(F, A), (G, A) \in FSMS(U, A)$. Then the t -norm product of (F, A) and (G, A) , denoted by $(F, A) \otimes (G, A)$ is a FSM-set (H, A) and defined as for all $e \in A$, $H(e) = (\{\frac{u}{\mu_{H(e)}(u)} : u \in U_i\} : i \in I)$, where $\mu_{H(e)}(u) = \frac{\mu_{F(e)}(u) \cdot \mu_{G(e)}(u)}{2 - [\mu_{F(e)}(u) + \mu_{G(e)}(u) - \mu_{F(e)}(u) \cdot \mu_{G(e)}(u)]}$.

Definition 3.2. Let $(F, A), (G, A) \in FSMS(U, A)$. Then the t -conorm product of (F, A) and (G, A) , denoted by $(F, A) \oplus (G, A)$ is a FSM-set (H, A) and defined as for all $e \in A$, $H(e) = (\{\frac{u}{\mu_{H(e)}(u)} : u \in U_i\} : i \in I)$, where $\mu_{H(e)}(u) = \frac{\mu_{F(e)}(u) + \mu_{G(e)}(u)}{1 + \mu_{F(e)}(u) \cdot \mu_{G(e)}(u)}$.

Proposition 3.3. If $(F, A), (G, A), (H, A) \in FSMS(U, A)$, then

Commutative laws

$$\begin{aligned} [i] \quad (F, A) \otimes (G, A) &= (G, A) \otimes (F, A) \\ [ii] \quad (F, A) \oplus (G, A) &= (G, A) \oplus (F, A) \end{aligned}$$

Associative laws

$$\begin{aligned} [i] \quad (F, A) \otimes ((G, A) \otimes (H, A)) &= ((F, A) \otimes (G, A)) \otimes (H, A) \\ [ii] \quad (F, A) \oplus ((G, A) \oplus (H, A)) &= ((F, A) \oplus (G, A)) \oplus (H, A) \end{aligned}$$

Proof. The proofs can be easily obtained using the Definition 3.1 and Definition 3.2. □

Proposition 3.4. If $(F, A), (G, A) \in FSMS(U, A)$, then

$$\begin{aligned} [i] \quad (F, A)_\phi \otimes (G, A) &= (F, A)_\phi \\ [ii] \quad (F, A)_\phi \oplus (G, A) &= (G, A) \\ [iii] \quad (F, A)_U \otimes (G, A) &= (G, A) \\ [iv] \quad (F, A)_U \oplus (G, A) &= (F, A)_U \end{aligned}$$

Proof. The proofs can be easily obtained using the Definition 3.1 and Definition 3.2. □

Now, we define a soft fuzzification operator on FSM-set.

Definition 3.5. Let $(F, A) \in FSMS(U, A)$ and $r \in [0, 1]$. Then a soft fuzzification operator with respect to $r \in [0, 1]$, denoted by S_r and defined as $S_r(F, A) = \{\frac{u}{\mu_{S_r(F,A)}(u)} : u \in \bigcup_{e \in A} [F(e)]_r, r \in [0, 1]\}$, where

$$\mu_{S_r(F,A)}(u) = \frac{1}{|A|} \sum_{e \in A} \mu_{F(e)}(u) \chi_e(u),$$

where

$$\chi_e(u) = \begin{cases} 1, & \text{if } u \in [F(e)]_r \\ 0, & \text{if } u \notin [F(e)]_r \end{cases}$$

and $[F(e)]_r = \{u \in U_i : \mu_{F(e)}(u) \geq r, i \in I\}$, $r \in [0, 1]$.

4. Application of FSM-sets in decision-making

In this section, an adjustable approach to FSM-set based decision-making is presented, for solving decision-making in an uncertain situation.

The steps of our algorithm are listed bellow

Algorithm 1

- Step 1. Input the FSM-sets $(F, A), (G, A) \in FSMS(U, A)$, which observations by two experts.
- Step 2. Compute the resultant FSM-set (H, A) using t -norm or t -conorm products.
- Step 3. Input a fixed $r \in [0, 1]$.
- Step 4. Compute the $S_r(H, A)$, using the soft fuzzification operator S_r .
- Step 5. For each $i \in I$, the decision D_i is to select u from U_i , if the corresponding membership value $\mu_{S_r(H,A)}(u)$ is maximized.
- Step 6. If u has more than one value, then the investor may be chosen any one of u .
- Step 7. The final optimal decision is $(D_i : i \in I)$.

Remark 4.1. *On the off chance that there is an excess of ideal decisions in Step 7, we may do a re-versal to the second step and third step, change the operation or the value of $r \in [0, 1]$, that he once utilized in order to confirm the last ideal choice, particularly when there are too much "optimal decisions" to be selected.*

Example 3 Let us consider three universes $U_1 = \{h_1, h_2, h_3, h_4\}$, $U_2 = \{c_1, c_2, c_3\}$ and $U_3 = \{v_1, v_2, v_3\}$ which are the collections of houses, autos and inns respectively. Let $\{E_{U_1}, E_{U_2}, E_{U_3}\}$ be a collection of sets of decision parameters related to the above universes, where

$$E_{U_1} = \{e_{U_1,1} = \text{expensive}, e_{U_1,2} = \text{cheap}, e_{U_1,3} = \text{wooden}\},$$

$$E_{U_2} = \{e_{U_2,1} = \text{expensive}, e_{U_2,2} = \text{cheap}, e_{U_2,3} = \text{sporty}\},$$

$E_{U_3} = \{e_{U_3,1} = \text{expensive}, e_{U_3,2} = \text{cheap}, e_{U_3,3} = \text{in Kuala Lumpur}\}$ be a collection of sets of choice parameters identified with the above universes. Also, let $U = \prod_{i=1}^3 P(U_i)$, $E = \prod_{i=1}^3 E_{U_i}$ and $A \subseteq E$, such that

$$A = \{a_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}), a_2 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,1}), a_3 = (e_{U_1,2}, e_{U_2,3}, e_{U_3,1}), a_4 = (e_{U_1,3}, e_{U_2,3}, e_{U_3,1}), a_5 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,2}), a_6 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,2})\}.$$

Assume that Mr. X has a financial plan to purchase a house, an auto and rent a venue to hold a wedding festival. Also, suppose that there be two observations (F, A) and (G, A) by two experts as in Table 5 and Table 6 respectively, which depicts houses, autos and inns that Mr. X is considering for settlement buy, transportation buy, and a venue to hold a wedding festival, separately. Then we select a house, an auto and an inn on the basic of the sets of members parameters by using the soft fuzzification operator.

Table 5: FSM-set (F, A)

U_i		a_1	a_2	a_3	a_4	a_5	a_6
U_1	h_1	0.3	0.8	0.7	0.8	0.3	0.7
	h_2	0.4	0.9	0.8	0.9	0.4	0.6
	h_3	0.9	0.3	0.1	0.3	0.9	0.9
	h_4	0.7	0.8	0	0.8	0.7	0.5
U_2	c_1	0.8	0.8	0.6	0.8	0.9	0.6
	c_2	0.6	0.8	0.8	0.8	1	0.8
	c_3	0.6	0.5	0.3	0.5	0.9	0.3
U_3	v_1	0.9	0.9	0.5	0.9	0.8	0.9
	v_2	0.7	0.7	0.5	0.7	0.5	0.8
	v_3	0.9	0.9	0.7	0.9	0.4	1

Table 6: FSM-set (G, A)

U_i		a_1	a_2	a_3	a_4	a_5	a_6
U_1	h_1	0.7	0.7	0.8	0.7	0.7	0.3
	h_2	0.6	0.8	0.9	0.8	0.6	0.4
	h_3	0.9	1	0.3	1	0.9	0.9
	h_4	0.5	0	0.8	0	0.5	0.7
U_2	c_1	0.8	0.5	0.9	0.5	0.9	0.9
	c_2	0.5	0.5	0.7	0.5	0.8	0.7
	c_3	0.4	0.7	0.9	0.7	1	0.9
U_3	v_1	0.6	0.6	0.9	0.6	0.8	0.6
	v_2	0.8	0.8	0.7	0.8	0.8	0.8
	v_3	0.3	0.3	0.9	0.3	0.5	0.3

We consider the resultant FSM -set $(H, A) = (F, A) \otimes (G, A)$ using t -norm product as in Table 7. Now, we chose $r = 0.5$, then we find $S_r(H, A)$ as in Table 8.

Table 7: Table for $(H, A) = (F, A) \otimes (G, A)$

U_i		a_1	a_2	a_3	a_4	a_5	a_6
U_1	h_1	0.174	0.528	0.528	0.528	0.174	0.174
	h_2	0.194	0.59	0.59	0.59	0.194	0.194
	h_3	0.802	0.3	0.3	0.3	0.802	0.802
	h_4	0.304	0	0	0	0.304	0.304
U_2	c_1	0.615	0.364	0.519	0.364	0.802	0.519
	c_2	0.25	0.264	0.528	0.364	0.8	0.528
	c_3	0.194	0.304	0.252	0.304	0.9	0.252
U_3	v_1	0.519	0.519	0.429	0.519	0.615	0.519
	v_2	0.528	0.528	0.304	0.528	0.364	0.615
	v_3	0.258	0.252	0.612	0.252	0.154	0.3

From the Table 8, we see that for the $U_1 - FSM$ -part of (H, A) , house h_3 has the largest membership value $\mu_{S_{0.5}(H,A)}(h_3) = 0.401$; hence house h_3 is the best suits for the requirement for settlement buy. For the $U_2 - FSM$ -part of (H, A) , auto c_2 has the largest membership value $\mu_{S_{0.5}(H,A)}(c_2) = 0.195$; hence auto c_2 is the best suits for the requirement for transportation buy. Also, for the $U_3 - FSM$ -part of (H, A) , inn v_2 has the largest membership value $\mu_{S_{0.5}(H,A)}(v_2) = 0.178$; hence v_2 is the best suits for the requirement for a venue to hold a wedding festival. Thus the final optimal decision for Mr. X is (h_3, c_2, v_2) .

Table 8: Table for $S_r(H, A)$, $r=0.5$

U_i		a_1	a_2	a_3	a_4	a_5	a_6	$\mu_{S_{0.5}(H,A)}(u)$
U_1	h_1	0.174	0.528	0.528	0.528	0.174	0.174	0.264
	h_2	0.194	0.59	0.59	0.59	0.194	0.194	0.295
	h_3	0.802	0.3	0.3	0.3	0.802	0.802	0.401
	h_4	0.304	0	0	0	0.304	0.304	0
U_2	c_1	0.615	0.364	0.519	0.364	0.802	0.519	0.409
	c_2	0.25	0.264	0.528	0.364	0.8	0.528	0.309
	c_3	0.194	0.304	0.252	0.304	0.9	0.252	0.15
U_3	v_1	0.519	0.519	0.429	0.519	0.615	0.519	0.449
	v_2	0.528	0.528	0.304	0.528	0.364	0.615	0.367
	v_3	0.258	0.252	0.612	0.252	0.154	0.3	0.102

Example 4 If we consider the resultant FSM -set $(P, A) = (F, A) \oplus (G, A)$ using t -conorm product as in Table 9 and we chose $r = 0.95$, then we find $S_r(P, A)$ as in Table 10. Then from the Table 10, we see that for the $U_1 - FSM$ -part of (P, A) house h_3 has the largest membership value $\mu_{S_{0.95}(P,A)}(h_3) = 0.896$; hence house h_3 is the best suits for the requirement for settlement buy. For the $U_2 - FSM$ -part of (P, A) auto c_1 has the largest membership value $\mu_{S_{0.95}(P,A)}(c_1) = 0.802$; hence auto c_1 is the best suits for the requirement for transportation buy. Also, for the $U_3 - FSM$ -part of (P, A) inn v_1 has the largest membership value $\mu_{S_{0.95}(P,A)}(v_1) = 0.832$; hence v_1 is the best suits for the requirement for a venue to hold a wedding festival. Therefore the final optimal decision for Mr. X is (h_3, c_1, v_1) .

Table 9: FSM-set $(P, A) = (F, A) \oplus (G, A)$

U_i		a_1	a_2	a_3	a_4	a_5	a_6
U_1	h_1	0.826	0.962	0.962	0.962	0.826	0.826
	h_2	0.806	0.988	0.988	0.988	0.806	0.806
	h_3	0.994	1	1	1	0.994	0.994
	h_4	0.889	0.8	0.8	0.8	0.889	0.889
U_2	c_1	0.976	0.929	0.974	0.929	0.994	0.974
	c_2	0.846	0.929	0.962	0.929	1	0.962
	c_3	0.806	0.889	0.945	0.889	1	0.945
U_3	v_1	0.974	0.974	0.966	0.974	0.976	0.974
	v_2	0.962	0.962	0.889	0.962	0.929	0.976
	v_3	0.945	0.945	0.982	0.945	0.75	1

Table 10: Table for $S_r(P, A)$, $r=0.95$

U_i		a_1	a_2	a_3	a_4	a_5	a_6	$\mu_{S_{0.95}(P,A)}(u)$
U_1	h_1	0.826	0.962	0.962	0.962	0.826	0.826	0.481
	h_2	0.806	0.988	0.988	0.988	0.806	0.806	0.494
	h_3	0.994	1	1	1	0.994	0.994	0.997
	h_4	0.889	0.8	0.8	0.8	0.889	0.889	0
U_2	c_1	0.976	0.929	0.974	0.929	0.994	0.974	0.653
	c_2	0.846	0.929	0.962	0.929	1	0.962	0.487
	c_3	0.806	0.889	0.945	0.889	1	0.945	0.167
U_3	v_1	0.974	0.974	0.966	0.974	0.976	0.974	0.973
	v_2	0.962	0.962	0.889	0.962	0.929	0.976	0.644
	v_3	0.945	0.945	0.982	0.945	0.75	1	0.330

Remark 4.2. From the Example 3, we see that applying $AND - t$ -norm product and for $r = 0.5$, we have house h_3 , auto c_2 and inn v_2 are the best suits for the requirement for settlement buy, transportation buy and also, for a venue to hold a wedding festival respectively. In Example 4, we see that applying the $OR - t$ -conorm product and choosing $r = 0.95$, we have house h_3 , auto c_1 and inn v_1 are the best suits for the requirement for settlement buy, transportation buy and also, for a venue to hold a wedding festival respectively. Thus, we can see that by using $AND - t$ -norm and $OR - t$ -conorm product, the final optimal decision for Mr. X is not same; the reason is that the way our selection is not the same. In general, by applying $AND - t$ -norms product, the membership grade value of each element in universal set is somewhat littler than by applying $OR - t$ -conorm product.

5. Advantages

By using Algorithm1, we might acquire the less option of objects, this can help us settle on the decision all the more effortlessly. Then again, by using Algorithm1, we get more far reaching data; this will help the decision of leaders. However, we also see that by using Algorithm 1, we can acquire the arrangement

of options of items is a empty set, this is terrible for our decision. What's more, we additionally take note of that the estimation of r is vital to acquire a better decision. In the formula of Definition 3.3, if we pick the estimation of r is too little, we might get a considerable measure of different options for pick, infrequently it is terrible for our decision, the reason is that the decision-maker has a tendency to look over less options. Sometimes the more choices, more inconvenience to pick. Hence, the choices we pick ought not all that much. Then again, on the off chance that we pick the estimation of r is too substantial, we might acquire less options, some of the time we might get the arrangement of options of objects is an empty set, this implies our decisions are failed, we require from the new pick the estimation of r with the goal that we can pick.

The right way is: most importantly the judges as per the choices real situation make the relating judgment and after that give their individual estimation of r , where $r \in [0, 1]$. The more noteworthy the estimation of r is given, the all the more requesting in the interest of the judges. At long last, by the recipe of Definition 3.3 to ascertain, one can decide the last option. In the event that the computed estimation of r is too substantial, one might get the arrangement of choices of items is a void set, for this situation, the judges ought to by altering their individual given in the calculated esteemed of r , keeping in mind the end goal to show signs of improvement result.

6. Conclusion and future work

In this paper, we characterize t -norms and t -conorm products on FSM -sets and applying these products we introduce an adjustable approach to FSM -set based decision-making, for solving decision-making in an uncertain situation. In our examples, we see that by applying the t -norms and t -conorm products, the advantages are not same, we are currently real application, pick what sort of strategy as indicated by the circumstances, with the goal that we can make a well decision. By applying these products, we can see that it can be connected to numerous fields that contain questionable ties. We trust this investigation along this field can be preceded. In the future, the methodology ought to be more extensive to tackle related issues, for example, computer science, software engineering, current life state and so on.

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