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A Sequence Involving an Extended Struve Function Via a Differential Operator

Gurmej Singh, Praveen Agarwal and Junesang Choi*

abstract: Various extensions of the Struve function have been presented and investigated. Here we aim to introduce an extended Struve function involving the k-gamma function. Then, by using a known differential operator, we introduce a sequence of functions associated with the above introduced extended Struve function and investigate its properties such as generating relations and a finite summation formula. The results presented here, being very general, are also pointed out to yield a number of relatively simple identities.

Key Words: Struve function, Extended Struve functions, k-gamma function, Differential operators, Generating relations, Finite summation formula.

Contents

1. Introduction and preliminaries

Due mainly to their significant importance and applications in various research fields, for example, engineering, mathematics and physics, diverse operational techniques and their extensions have been investigated (see, e.g., [11,12], [13], [14], [16], [17], [19,20,21], [29], [35], [37], [38]). A remarkably large number of sequence of functions involving a variety of special functions have been developed (see, e.g., [38]; see also [1,2,3,4,5,10,36]).

Here we aim to recall and investigate a new sequence of functions involving the generalized extended Struve function $\mathcal{W}^{\alpha,\mu}_{\mathtt{k},l,b,c,\xi}(z)$ (1.8) by using operational techniques. In particular, some generating relations and a finite summation formula associated with the newly introduced sequence (2.5) are presented. The results presented here, being very general, are also pointed out to yield a number of relatively simple identities.

* Corresponding author
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For our purpose, we begin by recalling the Struve function and its generalizations. The Struve function is defined by (see, e.g., [9, p. 675])

$$H_l(x) = \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{x}{2}\right)^{2n+l+1}}{\Gamma(n+l+\frac{3}{2})\Gamma(n+\frac{3}{2})} \quad (l \in \mathbb{C}) . \tag{1.1}$$

Here and in the following, let $\mathbb{C}$, $\mathbb{R}^+$ and $\mathbb{N}$ be the sets of complex numbers, positive real numbers, and positive integers, respectively, and let $\mathbb{N}_0 := \mathbb{N} \cup \{0\}$. The Struve function in (1.1) has been extended by many authors (see, e.g., [6], [7,8], [18], [22,26], [30], [31], [32], [33], [34])). Bhowmick [7] extended the Struve function in (1.1) as follows:

$$H_l^{\lambda}(x) = \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{x}{2}\right)^{2n+l+1}}{\Gamma(\lambda n+l+\frac{3}{2})\Gamma(n+\frac{3}{2})} \quad \left(l \in \mathbb{C};\, \lambda \in \mathbb{R}^+\right) . \tag{1.2}$$

Kanth [18] generalized the extended Struve function in (1.2)

$$H_l^{\lambda,\alpha}(x) = \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{x}{2}\right)^{2n+l+1}}{\Gamma(\lambda n+l+\frac{3}{2})\Gamma(\alpha n+\frac{3}{2})} \quad \left(l \in \mathbb{C};\, \alpha,\, \lambda \in \mathbb{R}^+\right) . \tag{1.3}$$

Singh [30] presented another extension of (1.2)

$$H_{l,\xi}^{\lambda}(x) = \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{x}{2}\right)^{2n+l+1}}{\Gamma(\lambda n+\frac{l}{\xi}+\frac{3}{2})\Gamma(n+\frac{3}{2})} \quad \left(l \in \mathbb{C};\, \lambda,\, \xi \in \mathbb{R}^+\right) . \tag{1.4}$$

Singh [31] gave the following extension (see also [22])

$$H_{l,\mu}^{\lambda,\alpha}(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{\Gamma(\alpha n+\mu)\Gamma(\lambda n+l+\frac{3}{2})} \left(\frac{x}{2}\right)^{2n+l+1} \quad \left(l,\, \mu \in \mathbb{C};\, \alpha,\, \lambda \in \mathbb{R}^+\right) . \tag{1.5}$$

Orhan and Yagmur [27,28] presented the following extension

$$\mathcal{H}_{l,b,c}(z) = \sum_{n=0}^{\infty} \frac{(-c)^n}{\Gamma(n+3/2)\Gamma(n+l+\frac{b}{2}+1)} \left(\frac{z}{2}\right)^{2n+l+1} \quad (l,\, b,\, c \in \mathbb{C}) . \tag{1.6}$$

Nisar et al. [23] gave the following generalization

$$ {}_a\mathcal{W}_{l,b,c,\xi}^{\alpha,\mu}(z) = \sum_{n=0}^{\infty} \frac{(-c)^n}{\Gamma(\alpha n+\mu)\Gamma(an+\frac{l}{\xi}+\frac{b+2}{2})} \left(\frac{z}{2}\right)^{2n+l+1} \tag{1.7}$$

$$\left(\mu,\, l,\, b,\, c \in \mathbb{C};\, \alpha,\, \xi \in \mathbb{R}^+;\, a \in \mathbb{N}\right) .$$

We recall the following extended Struve function (see [25]; see also [24])

$$\mathcal{W}_{\mathtt{k},l,b,c,\xi}^{\alpha,\mu}(z) := \sum_{n=0}^{\infty} \frac{(-c)^{\alpha n}}{\Gamma_{\mathtt{k}}(\alpha n+\mu)\Gamma_{\mathtt{k}}\left(\alpha n+\frac{l}{\xi}+\frac{b+2}{2}\right)} \left(\frac{z}{2}\right)^{2\alpha n+l+1} \tag{1.8}$$

$$\left(c,\, z \in \mathbb{C};\, \mathtt{k},\, \alpha,\, \xi \in \mathbb{R}^{+};\, \Re(\mu) > 0,\, \Re(l/\xi + b/2 + 1) > 0\right), \tag{1.9}$$

where $\Gamma_{\mathtt{k}}$ denotes the $\mathtt{k}$-gamma function defined by (see [15])

$$\Gamma_{\mathtt{k}}(x) := \int_0^\infty t^{x-1}\, \mathrm{e}^{-\frac{t^{\mathtt{k}}}{\mathtt{k}}}\, dt \quad \left(\Re(x) > 0;\, \mathtt{k} \in \mathbb{R}^{+}\right). \tag{1.10}$$

Then, by using operational techniques, we introduce a sequence of operators (2.5) involving the extended Struve function (1.8) and investigate its generating relations and finite summation formulas. We further present some graphical interpretations for the sequence.

2. Certain formulas involving diverse differential operators

Here we recall some differential operators and introduce a sequence of operators involving the extended Struve function (1.8). Here and in the following, $p_r(x)$ denotes a polynomial in x of degree $r \in \mathbb{N}_0$. We recall the following differential operators

$$D := \frac{d}{dx}, \quad T_s := x\,(s + xD)\,, \quad T_x^{a,s} := x^a\,(s + xD)\,, \tag{2.1}$$

where $a,\, s \in \mathbb{C}$ are constants and x is a variable.

Mittal [19] gave the Rodrigues type formula for the generalized Lagurre polynomials

$$T_{rn}^{(\alpha)}\,(x) := \frac{1}{n!} x^{-\alpha} \exp\left(p_r\,(x)\right) D^n \left[x^{\alpha+n} \exp\left(-p_r\,(x)\right)\right]. \tag{2.2}$$

Mittal [20] also proved the following relation for (2.2)

$$T_{rn}^{(\alpha+s-1)}\,(x) = \frac{1}{n!} x^{-\alpha-n} \exp\left(p_r\,(x)\right) \left(T_s\right)^n \left[x^{\alpha} \exp\left(-p_r\,(x)\right)\right]. \tag{2.3}$$

Srivastava and Singh [38] introduced and investigated a sequence of operators $V_n^{(\alpha)}\,(x; a, r, s)$ defined by

$$V_n^{(\alpha)}\,(x; a, r, s) := \frac{x^{-\alpha}}{n!} \exp\left\{p_r\,(x)\right\} \left(T_x^{a,s}\right)^n \left[x^{\alpha} \exp\left\{-p_r\,(x)\right\}\right]. \tag{2.4}$$

By modifying the sequence in (2.4), we introduce a sequence of operators involving the extended Struve function (1.8)

$$\begin{aligned} V_n^{(\lambda,\mu;\mathtt{k},l,b,c,\xi;\alpha)}\,(x; a, r, s) \quad := \quad & \frac{x^{-\alpha}}{n!} \mathcal{W}^{\lambda,\mu}_{\mathtt{k},l,b,c,\xi} \left[p_r(x)\right] \left(T_x^{a,s}\right)^n \\ & \times \left\{x^{\alpha} \mathcal{W}^{\lambda,\mu}_{\mathtt{k},l,b,c,\xi} \left[-p_r(x)\right]\right\}, \end{aligned} \tag{2.5}$$

where notations and conditions are found and modified from (1.9), (2.1) and (2.4).

We collect some results involving operators $T_x^{a,s}$ (see, e.g., [21,29,38]) in the following lemma. Here and in the following, $(\lambda)_n$ denotes the Pochhammer symbol (see, e.g., [36, p. 4]).

Lemma 2.1. *Let β, $s \in \mathbb{C}$ and $a \in \mathbb{C} \setminus \{0\}$. Then each of the following identities holds.*

$$\left(T_x^{a,s}\right)^n \left(x^\beta\right) = (a\, x^a)^n \left(\frac{s+\beta}{a}\right)_n x^\beta \quad (n \in \mathbb{N}_0)\,. \tag{2.6}$$

$$\sum_{n=0}^{\infty} \frac{t^n}{n!} \left(T_x^{a,s}\right)^n \left(x^\beta p_r(x)\right) = x^\beta \left(1 - a x^a t\right)^{-\frac{\beta+s}{a}} p_r\left(x\left(1 - a x^a t\right)^{-1/a}\right), \tag{2.7}$$

or, equivalently,

$$\exp\left(t T_x^{a,s}\right) \left(x^\beta p_r(x)\right) = x^\beta \left(1 - a x^a t\right)^{-\frac{\beta+s}{a}} p_r\left(x\left(1 - a x^a t\right)^{-1/a}\right). \tag{2.8}$$

$$\sum_{n=0}^{\infty} \frac{t^n}{n!} \left(T_x^{a,s}\right)^n \left(x^{\alpha - an} p_r(x)\right) = x^\alpha \left(1 + at\right)^{-1+\frac{\alpha+s}{a}} p_r\left(x\left(1 + at\right)^{1/a}\right), \tag{2.9}$$

or, equivalently,

$$\exp\left(t T_x^{a,s}\right) \left(x^{\alpha - an} p_r(x)\right) = x^\alpha \left(1 + at\right)^{-1+\frac{\alpha+s}{a}} p_r\left(x\left(1 + at\right)^{1/a}\right). \tag{2.10}$$

$$\left(T_x^{a,s}\right)^n (xuv) = x \sum_{m=0}^{\infty} \binom{n}{m} \left(T_x^{a,s}\right)^{n-m} (v) \left(T_x^{a,1}\right)^m (u)\,. \tag{2.11}$$

Proof: We find

$$\begin{aligned}\left(T_x^{a,s}\right)^n \left(x^\beta\right) &= (s+\beta)(s+\beta+a)(s+\beta+2a)\cdots\{s+\beta+(n-1)a\}\, x^{na+\beta} \\ &= a^n \frac{s+\beta}{a}\left(\frac{s+\beta}{a}+1\right)\left(\frac{s+\beta}{a}+2\right)\cdots\left\{\frac{s+\beta}{a}+(n-1)\right\} x^{na+\beta},\end{aligned}$$

which is the right sided expression of (2.6). By using (2.6), (2.7) and (2.9) can be proved. We omit the details.

□

3. Generating relations

Here we establish two generating relations for the sequence in (2.5) in the following theorem.

Theorem 3.1. *Each of the following generating relations holds.*

$$\begin{aligned}&\sum_{n=0}^{\infty} V_n^{(\lambda,\mu;\mathtt{k},l,b,c,\xi;\alpha)}(x; a, r, s)\, x^{-an} t^n \\ &\quad = (1 - at)^{-\frac{\alpha+s}{a}}\, \mathcal{W}_{\mathtt{k},l,b,c,\xi}^{\lambda,\mu}\left[p_r(x)\right] \mathcal{W}_{\mathtt{k},l,b,c,\xi}^{\lambda,\mu}\left[-p_r\left(x\left(1 - at\right)^{-1/a}\right)\right]\end{aligned} \tag{3.1}$$

and

$$\begin{aligned}&\sum_{n=0}^{\infty} V_n^{(\lambda,\mu;\mathtt{k},l,b,c,\xi;\alpha-an)}(x;a,r,s)\, x^{-an} t^n \\ &\quad = (1+at)^{-1+\frac{\alpha+s}{a}}\, \mathcal{W}^{\lambda,\mu}_{\mathtt{k},l,b,c,\xi}\,[p_r(x)]\, \mathcal{W}^{\lambda,\mu}_{\mathtt{k},l,b,c,\xi}\left[-p_r\left(x\,(1+at)^{1/a}\right)\right],\end{aligned} \tag{3.2}$$

where notations and conditions are found and modified from (1.9) *and* (2.5).

Proof: We prove (3.1). From (2.5), we have

$$\begin{aligned}&\sum_{n=0}^{\infty} V_n^{(\lambda,\mu;\mathtt{k},l,b,c,\xi;\alpha)}(x;a,r,s)\, t^n \\ &\quad = x^{-\alpha}\mathcal{W}^{\lambda,\mu}_{\mathtt{k},l,b,c,\xi}\,[p_r(x)]\exp\left(tT_x^{a,s}\right)\left\{x^{\alpha}\mathcal{W}^{\lambda,\mu}_{\mathtt{k},l,b,c,\xi}\,[-p_r(x)]\right\}.\end{aligned} \tag{3.3}$$

Applying (2.8) to the right side of (3.3), we obtain

$$\begin{aligned}&\sum_{n=0}^{\infty} V_n^{(\lambda,\mu;\mathtt{k},l,b,c,\xi;\alpha)}(x;a,r,s)\, t^n \\ &\quad = (1-ax^a t)^{-\frac{\alpha+s}{a}}\, \mathcal{W}^{\lambda,\mu}_{\mathtt{k},l,b,c,\xi}\,[p_r(x)]\, \mathcal{W}^{\lambda,\mu}_{\mathtt{k},l,b,c,\xi}\left[-p_r\left(x(1-ax^a t)^{-1/a}\right)\right],\end{aligned} \tag{3.4}$$

which, upon replacing t by $x^{-a}t$, yields (3.1).

We first replace α by $\alpha - an$. Then, similarly as in the proof of (3.1) with the aid of (2.10), we obtain (3.2).

□

4. Finite Summation Formulas

Here we present two summation formulas for the sequence in (2.5) in the following theorem.

Theorem 4.1. *Let $n \in \mathbb{N}_0$ and $\alpha,\ \beta \in \mathbb{C}$. Then each of the following summation formulas holds.*

$$\begin{aligned}&V_n^{(\lambda,\mu;\mathtt{k},l,b,c,\xi;\alpha)}(x;a,r,s) \\ &\quad = \sum_{m=0}^{n} \frac{(ax^a)^m}{m!}\left(\frac{\alpha-\beta}{a}\right)_m V_{n-m}^{(\lambda,\mu;\mathtt{k},l,b,c,\xi;\beta)}(x;a,r,s),\end{aligned} \tag{4.1}$$

and

$$V_n^{(\lambda,\mu;\mathtt{k},l,b,c,\xi;\alpha)}(x;a,r,s) = \sum_{m=0}^{n} \frac{(ax^a)^m}{m!}\left(\frac{\alpha}{a}\right)_m V_{n-m}^{(\lambda,\mu;\mathtt{k},l,b,c,\xi;0)}(x;a,r,s), \tag{4.2}$$

where notations and conditions are found and modified from (1.9) *and* (2.5).

Proof: Let

$$\mathcal{L} := \sum_{n=0}^{\infty} V_n^{(\lambda,\mu;\mathtt{k},l,b,c,\xi;\alpha)}(x;a,r,s)\, t^n.$$

Using the generalized binomial theorem, we have

$$\begin{aligned}(1-ax^a t)^{-\frac{\alpha+s}{a}} &= \mathcal{L},(1-ax^a t)^{-\frac{\alpha-\beta}{a}} \\ &= (1-ax^a t)^{-\frac{\beta+s}{a}} \sum_{m=0}^{\infty} \left(\frac{\alpha-\beta}{a}\right)_m \frac{(ax^a t)^m}{m!}.\end{aligned} \tag{4.3}$$

Applying (4.3) to the right side of (3.4), we get

$$\begin{aligned}\mathcal{L} =& (1-ax^a t)^{-\frac{\beta+s}{a}}\, \mathcal{W}_{\mathtt{k},l,b,c,\xi}^{\lambda,\mu}[p_r(x)] \sum_{m=0}^{\infty} \left(\frac{\alpha-\beta}{a}\right)_m \frac{(ax^a t)^m}{m!} \\ &\times \mathcal{W}_{\mathtt{k},l,b,c,\xi}^{\lambda,\mu}\left[-p_r\left(x(1-ax^a t)^{-1/a}\right)\right].\end{aligned} \tag{4.4}$$

Using (2.8) in (4.4), we obtain

$$\begin{aligned}\mathcal{L} =& x^{-\beta}\, \mathcal{W}_{\mathtt{k},l,b,c,\xi}^{\lambda,\mu}[p_r(x)] \sum_{m=0}^{\infty} \left(\frac{\alpha-\beta}{a}\right)_m \frac{(ax^a t)^m}{m!} \\ &\times \exp\left(t\, T_x^{a,s}\right)\left\{x^{\beta}\, \mathcal{W}_{\mathtt{k},l,b,c,\xi}^{\lambda,\mu}[-p_r(x)]\right\}.\end{aligned}$$

Then we have

$$\begin{aligned}\mathcal{L} =& x^{-\beta}\, \mathcal{W}_{\mathtt{k},l,b,c,\xi}^{\lambda,\mu}[p_r(x)] \\ &\times \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} \left(\frac{\alpha-\beta}{a}\right)_m \frac{(ax^a t)^m}{m!n!} \left(T_x^{a,s}\right)^n \left\{x^{\beta}\, \mathcal{W}_{\mathtt{k},l,b,c,\xi}^{\lambda,\mu}[-p_r(x)]\right\} t^{m+n} \\ =& x^{-\beta}\, \mathcal{W}_{\mathtt{k},l,b,c,\xi}^{\lambda,\mu}[p_r(x)] \\ &\times \sum_{n=0}^{\infty}\sum_{m=0}^{n} \left(\frac{\alpha-\beta}{a}\right)_m \frac{(ax^a t)^m}{m!(n-m)!} \left(T_x^{a,s}\right)^{n-m} \left\{x^{\beta}\, \mathcal{W}_{\mathtt{k},l,b,c,\xi}^{\lambda,\mu}[-p_r(x)]\right\} t^{n}.\end{aligned} \tag{4.5}$$

In view of (2.5), we find

$$\begin{aligned}& x^{-\beta}\, \mathcal{W}_{\mathtt{k},l,b,c,\xi}^{\lambda,\mu}[p_r(x)] \left(T_x^{a,s}\right)^{n-m} \left\{x^{\beta}\, \mathcal{W}_{\mathtt{k},l,b,c,\xi}^{\lambda,\mu}[-p_r(x)]\right\} \\ &\quad = (n-m)!\, V_{n-m}^{(\lambda,\mu;k,l,b,c,\xi;\beta)}(x;a,r,s).\end{aligned} \tag{4.6}$$

Applying (4.6) to the second identity of (4.5), we get

$$\begin{aligned}& \sum_{n=0}^{\infty} V_n^{(\lambda,\mu;\mathtt{k},l,b,c,\xi;\alpha)}(x;a,r,s)\, t^n \\ &\quad = \sum_{n=0}^{\infty}\sum_{m=0}^{n} \frac{(ax^a)^m}{m!} \left(\frac{\alpha-\beta}{a}\right)_m V_{n-m}^{(\lambda,\mu;\mathtt{k},l,b,c,\xi;\beta)}(x;a,r,s)\, t^n,\end{aligned}$$

which, upon equating the coefficients of t^n, yields the desired identity (4.1).

The identity (4.2) is a special case of (4.1) when $\beta = 0$. The identity (4.2) can be proved by using (2.11). □

5. Concluding remarks

The results presented here, being very general, can yield a number of identities involving relatively simple sequences reduced from the sequence (2.5). For example, the results in Theorems 3.1 and 4.1 when $\mathtt{k} = 1$ give the corresponding identities involving the familiar gamma function.

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Gurmej Singh,
Department of Mathematics,
Mata Sahib Kaur Girls College, Talwandi Sabo, Bathinda 151103, India.

Research Scholar, Department of Mathematics,
Singhania University, Pacheri Bari,
Jhunjhunu,
India.
E-mail address: gurmejsandhu11@gmail.com

and

Praveen Agarwal,
Department of Mathematics,
Anand International College of Engineering,
Jaipur 303012,
India.
E-mail address: goyal.praveen2011@gmail.com

and

Junesang Choi,
Department of Mathematics,
Dongguk University, Gyeongju 38066,
Republic of Korea.
E-mail address: junesang@mail.dongguk.ac.kr