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# $g_{\Delta^*_\mu}-$Closed Sets in Generalized Topological Spaces

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ABSTRACT: In this paper, we introduce some new classes of generalized closed sets called $\Delta^*_\mu-g-$closed, $\Delta^*_\mu-g_\mu-$closed and $g_{\Delta^*_\mu}-$closed sets, which are related to the classes of $g_\mu-$closed sets, $g-\lambda_\mu-$closed sets and $\lambda_\mu-g-$closed sets. We investigate their properties as well as the relations among these classes of generalized closed sets.

Key Words: Generalized topology, $\lambda_\mu-$closed, $\Delta^*_\mu-$closed, $\Delta^*_\mu-g-$closed, $\Delta^*_\mu-g_\mu-$closed, $g_{\Delta^*_\mu}-$closed sets.

## Contents



## 1. Introduction

In 1997, *Á.Császár* [2] introduced the concept of a generalization of topological spaces, which is called a generalized topological space. A subset $\mu$ of $exp(X)$ is called a generalized topology [4] on $X$ if $\emptyset\in\mu$ and $\mu$ is closed under arbitrary union. Elements of $\mu$ are called $\mu-$open sets. The complement of a $\mu-$open set is said to be $\mu-$closed. A set $X$ with a GT $\mu$ on it is called a generalized topological space (briefly GTS) and is denoted by $(X,\mu)$. For a subset $A$ of $X$, we denote by $c_\mu(A)$ the intersection of all $\mu-$closed sets containing $A$ and by $i_\mu(A)$ the union of all $\mu-$open sets contained in $A$. Then $c_\mu(A)$ is the smallest $\mu-$closed set containing $A$ and $i_\mu(A)$ is the largest $\mu-$open set contained in $A$. A point $x\in X$ is called a $\mu-$cluster point of $A$ if for every $U\in\mu$ with $x\in U$ we have $A\cap U\neq\emptyset$. $c_\mu(A)$ is the set of all $\mu-$cluster points of $A$ [4]. A GTS $(X,\mu)$ is called a quasi-topological space [3] if $\mu$ is closed under finite intersections. A subset $A$ of $X$ is said to be $\pi-$regular [5] (resp. $\sigma-$regular) if $A=i_\mu c_\mu(A)$ (resp. $A=c_\mu i_\mu(A)$).

**Definition 1.1.** [6] *If $(X,\mu)$ is a GTS and $A\subseteq X$, then the set $\wedge_\mu(A)$ is defined as follows:*

$$\wedge_\mu(A)=\begin{cases}\cap\{G:A\subseteq G,G\in\mu\} & \text{if there exists } G\in\mu \text{ such that } A\subseteq G;\\ X & \text{otherwise.}\end{cases}$$

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**Definition 1.2.** [6] *In a GTS $(X,\mu)$, a subset $B$ is called a $\wedge_\mu-$set if $B = \wedge_\mu(B)$.*

**Definition 1.3.** [1] *A subset $A$ of a GTS $(X,\mu)$ is called a $\lambda_\mu-$closed set if $A = T \cap C$, where $T$ is a $\wedge_\mu-$set and $C$ is a $\mu-$closed set. The complement of a $\lambda_\mu-$closed set is called a $\lambda_\mu-$open set. We set $\lambda_\mu O(X,\mu) = \{U : U$ is $\lambda_\mu-$open in $(X,\mu)\}$*

**Definition 1.4.** [10] *Let $(X,\mu)$ be a GTS. A subset $A$ of $X$ is called a ${}^*\wedge_\mu-$set if $A = {}^*\wedge_\mu(A)$, where ${}^*\wedge_\mu(A) = \cap\{U : A \subset U, U \in \lambda_\mu O(X,\mu)\}$.*

**Definition 1.5.** [9] *Let $(X,\mu)$ be a GTS. A subset $A$ of $X$ is called a $\Delta_\mu-$set if $\wedge_\mu(A) = {}^*\wedge_\mu(A)$.*

**Definition 1.6.** [9] *A subset of a GTS $(X,\mu)$ is called a $\Delta_\mu^*-$closed set if $A = T \cap F$, where $T$ is a $\Delta_\mu-$set and $F$ is a $\mu-$closed set. The complement of a $\Delta_\mu^*-$closed set is said to be $\Delta_\mu^*-$open.*

**Definition 1.7.** *A subset $A$ of GTS $(X,\mu)$ is said to be $g_\mu-$closed [11] (resp. $g-\lambda_\mu-$closed [8], $\lambda_\mu-g-$closed [8]) if $c_\mu(A) \subseteq U$ (resp. $c_{\lambda_\mu}(A) \subseteq U$, $c_\mu(A) \subseteq U$) whenever $A \subseteq U$ and $U$ is $\mu-$open (resp. $U$ is $\mu-$open, $U$ is $\lambda_\mu-$open) in $(X,\mu)$.*

**Lemma 1.1.** [7] *For a GTS $(X,\mu)$ and $S, T \subset X$, the following properties hold:*
*(i) $i_\mu(S \cap T) \subseteq i_\mu(S) \cap i_\mu(T)$.*
*(ii) $c_\mu(S) \cup c_\mu(T) \subseteq c_\mu(S \cup T)$.*

**Remark 1.8.** [7] *In general, for subsets $S$ and $T$ of a GTS $(X,\mu)$, $i_\mu(S \cap T) \supseteq i_\mu(S) \cap i_\mu(T)$ is not true.*

**Lemma 1.2.** [5] *Let $(X,\mu)$ be a quasi-topological space. Then $c_\mu(A \cup B) = c_\mu(A) \cup c_\mu(B)$ for every $A$ and $B$ of $X$.*

**Lemma 1.3.** [1,6,9] *For a subset of a GTS $(X,\mu)$, the following implication hold:*

$$\begin{array}{ccccc} \mu-\text{open} & \Rightarrow & \wedge_\mu-\text{set} & \Rightarrow & \Delta_\mu-\text{set} \\ & & \Downarrow & & \Downarrow \\ \mu-\text{closed} \Rightarrow & & \lambda_\mu-\text{closed} & \Rightarrow & \Delta_\mu^*-\text{closed} \end{array}$$

For $A \subseteq X$, we denote by $c_{\Delta_\mu^*}(A)$ [9] (resp. $c_{\lambda_\mu}(A)$ [1]) the intersection of all $\Delta_\mu^*-$closed (resp. $\lambda_\mu-$closed) subsets of $X$ containing $A$. Then we have

$$c_{\Delta_\mu^*}(A) \subseteq c_{\lambda_\mu}(A) \subseteq c_\mu(A)$$

for every $A \subseteq X$.

The purpose of this present paper is to define some new classes of generalized closed sets called $\Delta_\mu^* - g-$closed, $\Delta_\mu^* - g_\mu-$closed and $g_{\Delta_\mu^*}-$closed and to obtain some basic properties of these closed sets. Further, we establish the relation between these classes of sets.

## 2. $\Delta_\mu^* - g-$closed sets

In this section, we introduce the notion of $\Delta_\mu^* - g-$closed sets and discuss its properties.

**Definition 2.1.** *Let $(X,\mu)$ be a GTS. A subset $A$ of $X$ is called a $\Delta_\mu^* - g-$closed set if $c_\mu(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is a $\Delta_\mu^*-$open set in $X$. The complement of a $\Delta_\mu^* - g-$closed set is called a $\Delta_\mu^* - g-$open set.*

**Theorem 2.2.** *Every $\mu-$closed set is a $\Delta_\mu^* - g-$closed set.*

**Proof:** Let $A$ be a $\mu$-closed set and $U$ be any $\Delta_\mu^*-$open set containing $A$. Since $A$ is $\mu$-closed, we have $c_\mu(A) = A$. Therefore $c_\mu(A) \subseteq U$. Thus $A$ is $\Delta_\mu^* - g-$closed.

Example 2.3 shows that the converse of the above theorem is not true.

**Example 2.3.** *Let $X = \{a,b,c,d\}$ and $\mu = \{\emptyset, \{a\}, \{b\}, \{a,b\}, X\}$. Then $\{c\}$ is $\Delta_\mu^* - g-$closed but not $\mu-$closed.*

Theorem 2.4 shows that every $\Delta_\mu^* - g-$closed set is a $g_\mu-$closed set (a $g - \lambda_\mu-$closed set, a $\lambda_\mu - g-$closed set) and Example 2.5 shows that converses are not true.

**Theorem 2.4.** *Let$(X,\mu)$ be a GTS. Then the following hold:*
*(i) Every $\Delta_\mu^* - g-$closed set is a $g_\mu-$closed set.*
*(ii) Every $g_\mu-$closed set is a $g - \lambda_\mu-$closed set.*
*(iii) Every $\Delta_\mu^* - g-$closed set is a $\lambda_\mu - g-$closed set.*
*(iv) Every $\lambda_\mu - g-$closed set is $g - \lambda_\mu-$closed.*

**Proof:** (i) Let $A$ be a $\Delta_\mu^* - g-$closed set and $U$ be any $\mu-$open set containing $A$ in$(X,\mu)$. Since every $\mu-$open set is $\Delta_\mu^*-$open, we have $U$ is $\Delta_\mu^*-$open. Since $A$ is $\Delta_\mu^* - g-$closed, $c_\mu(A) \subseteq U$. Therefore $A$ is $g_\mu-$closed.
(ii) Let $A$ be a $g_\mu-$closed set and $U$ be any $\mu-$open set containing $A$ in$(X,\mu)$. Since $A$ is $g_\mu-$closed, $c_\mu(A) \subseteq U$. Since $c_{\lambda_\mu}(A) \subseteq c_\mu(A)$, $c_{\lambda_\mu}(A) \subseteq U$ and hence $A$ is $g - \lambda_\mu-$closed.
(iii) Let $A$ be a $\Delta_\mu^* - g-$closed set and $U$ be a $\lambda_\mu-$open set containing $A$ in$(X,\mu)$. Since every $\lambda_\mu-$open set is $\Delta_\mu^*-$open and $A$ is $\Delta_\mu^* - g-$closed, then $c_\mu(A) \subseteq U$. Therefore $A$ is $\lambda_\mu - g-$closed.
(iv) Suppose that $A$ is a $\lambda_\mu - g-$closed set. Let $A \subseteq U$ and $U$ be $\mu-$open. Then $U$ is $\lambda_\mu-$open and $A$ is $\lambda_\mu - g-$closed. Therefore, $c_\mu(A) \subseteq U$and hence $c_{\lambda_\mu}(A) \subseteq c_\mu(A) \subseteq U$. Hence $A$ is $g - \lambda_\mu-$closed.

Form Theorem 2.4, we have the following diagram:

DIAGRAM I

$$\begin{array}{ccc} \Delta_\mu^* - g\text{-closed} & \Rightarrow & g_\mu\text{-closed} \\ \Downarrow & & \Downarrow \\ \lambda_\mu - g\text{-closed} & \Rightarrow & g - \lambda_\mu\text{-closed} \end{array}$$

**Example 2.5.** *Let $X = \{a, b, c\}$ and $\mu = \{\emptyset, \{a, b\}, \{b, c\}, X\}$. Then $\{a, c\}$ is both $g_\mu-$closed and $\lambda_\mu-g-$closed but not $\Delta^*_\mu-g-$closed. Further $\{b, c\}$ is $g-\lambda_\mu-$closed but neither $\lambda_\mu - g-$closed nor $g_\mu-$closed.*

Theorem 2.6 gives a characterization of $\Delta^*_\mu - g-$closed sets.

**Theorem 2.6.** *Let $(X, \mu)$ be a GTS. A subset $A$ of $X$ is a $\Delta^*_\mu - g-$closed set if and only if $F \subseteq c_\mu(A)\backslash A$ and $F$ is $\Delta^*_\mu-$closed implies that $F$ is empty.*

**Proof:** Let $A$ be $\Delta^*_\mu - g-$closed. Suppose that $F$ is a subset of $c_\mu(A)\backslash A$ and $F$ is $\Delta^*_\mu-$closed. Then $A \subseteq X\backslash F$ and $X\backslash F$ is $\Delta^*_\mu-$open. Since $A$ is $\Delta^*_\mu - g-$closed, we have $c_\mu(A) \subseteq X\backslash F$. Consequently $F \subseteq X\backslash c_\mu(A)$. Hence $F$ is empty.
Conversely, Suppose $A \subseteq U$, where $U$ is $\Delta^*_\mu-$open. If $c_\mu(A) \nsubseteq U$, then $c_\mu(A) \cap (X - U)$ is a non-empty $\Delta^*_\mu-$closed subset of $c_\mu(A)\backslash A$. Therefore $A$ is $\Delta^*_\mu - g-$closed.

**Theorem 2.7.** *If $A$ is a $\Delta^*_\mu - g-$closed set in a GTS $(X, \mu)$, then $c_\mu(A)\backslash A$ does not contain any non-empty $\lambda_\mu-$closed ($\mu-$open / $\mu-$closed) subset of $X$.*

**Proof:** Suppose $c_\mu(A)\backslash A$ contains a non-empty $\lambda_\mu-$closed ($\mu-$open /$\mu-$closed) subset of $X$. Since every $\lambda_\mu-$closed ($\mu-$open /$\mu-$closed) set is $\Delta^*_\mu-$closed, a non-empty $\Delta^*_\mu-$closed set is contained in $c_\mu(A)\backslash A$, which is contrary to Theorem 2.6.

Example 2.8 shows that the converse of the above theorem is not true.

**Example 2.8.** *Let $X = \{a, b, c, d\}$ and $\mu = \{\emptyset, \{a\}, \{a, d, c\}, \{b, c, d\}, X\}$. If $A = \{a, b, d\}$, then $c_\mu(A)\backslash A = \{c\}$, which does not contain any nonempty $\lambda_\mu-$closed ( $\mu-$open / $\mu-$ closed ) sets but $A$ is not a $\Delta^*_\mu - g-$closed set.*

**Theorem 2.9.** *Let $(X, \mu)$ be a quasi-topological space. Then $A \cup B$ is a $\Delta^*_\mu - g-$closed set whenever $A$ and $B$ are $\Delta^*_\mu - g-$closed sets.*

**Proof:** Let $U$ be a $\Delta^*_\mu-$open set such that $A \cup B \subseteq U$. Then $A \subseteq U$ and $B \subseteq U$. Since $A$ and $B$ are $\Delta^*_\mu - g-$closed, we have $c_\mu(A) \subseteq U$ and $c_\mu(B) \subseteq U$. Hence by Lemma1.2 $c_\mu(A \cup B) = c_\mu(A) \cup c_\mu(B) \subseteq U$ and the proof follows.

**Example 2.10.** *Let $X = \{a, b, c\}$ and $\mu = \{\emptyset, \{a, b\}, \{b, c\}, X\}$. Then $\mu$ is a GT but not a quasi-topology. If $A = \{a\}$ and $B = \{c\}$, then $A$ and $B$ are $\Delta^*_\mu-g-$closed sets but their union is not a $\Delta^*_\mu - g-$closed set.*

**Example 2.11.** *Let $X = \{a, b, c, d\}$ and $\mu = \{\emptyset, \{a\}, \{c\}, \{a, c\}, X\}$. If $A = \{b, d\}$ and $B = \{a, c, d\}$, then $A$ and $B$ are $\Delta^*_\mu - g-$closed sets but $A \cap B = \{d\}$ is not a $\Delta^*_\mu - g-$closed set.*

**Theorem 2.12.** *Let $(X, \mu)$ be a GTS. If $A$ is $\Delta^*_\mu-$open and $\Delta^*_\mu - g-$closed, then $A$ is $\mu-$closed.*

**Proof:** Since $A$ is $\Delta^*_\mu-$open and $\Delta^*_\mu - g-$closed, $c_\mu(A) \subseteq A$ and hence $A$ is $\mu-$closed.

## 3. $\Delta_\mu^* - g_\mu-$ closed sets

In this section, we introduce the concept of $\Delta_\mu^* - g_\mu-$closed sets and study its properties.

**Definition 3.1.** *Let $(X, \mu)$ be a GTS. A subset $A$ of $X$ is called a $\Delta_\mu^* - g_\mu-$closed set if $c_{\lambda_\mu}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is a $\Delta_\mu^*-$open set in $X$. The complement of a $\Delta_\mu^* - g_\mu-$closed set is called a $\Delta_\mu^* - g_\mu-$open set.*

**Theorem 3.2.** *For a GTS $(X, \mu)$, every $\lambda_\mu-$closed set is $\Delta_\mu^* - g_\mu-$closed.*

**Proof:** Let $A$ be a $\lambda_\mu-$closed set and $U$ be any $\Delta_\mu^*-$open set containing $A$. Since $A$ is $\lambda_\mu-$closed, we have $c_{\lambda_\mu}(A) = A$. Therefore $c_{\lambda_\mu}(A) \subseteq U$and hence $A$ is $\Delta_\mu^* - g_\mu-$closed.

**Corollary 3.3.** *For a GTS $(X, \mu)$, the following hold:*
*(i) Every $\mu-$closed set is $\Delta_\mu^* - g_\mu-$closed.*
*(ii) Every $\mu-$open set is $\Delta_\mu^* - g_\mu-$closed.*

Example 3.4 shows that the converse of the above theorem is not true.

**Example 3.4.** *Let $X = \{a, b, c, d\}$ and $\mu = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$. If $A = \{c\}$, then $A$ is $\Delta_\mu^* - g_\mu-$closed but not $\lambda_\mu-$closed ($\mu-$closed, $\mu-$open).*

**Theorem 3.5.** *Let $(X, \mu)$ be a GTS and $A \subseteq X$. If $A$ is a $\Delta_\mu^* - g_\mu-$closed set, then $A$ is a $g - \lambda_\mu-$closed set.*

**Proof:** Let $U$ be a $\mu-$open set containing $A$ in $(X, \mu)$. Since every $\mu$- open set is $\Delta_\mu^*-$open and $A$ is $\Delta_\mu^* - g_\mu-$closed, $c_{\lambda_\mu}(A) \subseteq U$. Therefore $A$ is $g - \lambda_\mu-$closed.

Theorem 3.6 shows that the relation between $\Delta_\mu^* - g-$closed set and $\Delta_\mu^* - g_\mu-$closed set.

**Theorem 3.6.** *In a GTS $(X, \mu)$, every $\Delta_\mu^* - g-$closed set is $\Delta_\mu^* - g_\mu-$closed.*

**Proof:** Let $A$ be a $\Delta_\mu^* - g-$closed set and $U$ be a $\Delta_\mu^*-$open set containing $A$ in $(X, \mu)$. Then $c_\mu(A) \subseteq U$. Since $c_{\lambda_\mu}(A) \subseteq c_\mu(A)$, we have $c_{\lambda_\mu}(A) \subseteq U$. Therefore $A$ is $\Delta_\mu^* - g_\mu-$closed.

**Remark 3.7.** *$\Delta_\mu^* - g_\mu-$closed sets and $g_\mu-$closed (resp. $\lambda_\mu - g-$closed )sets are independent of each other.*

**Example 3.8.** *Let $X = \{a, b, c, d\}$ and $\mu = \{\emptyset, \{a, b\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}, X\}$. Then $\{a, b, d\}$ is $g_\mu-$closed but not $\Delta_\mu^* - g_\mu-$closed and $\{c\}$ is $\Delta_\mu^* - g_\mu-$closed but not $g_\mu-$closed.*

**Example 3.9.** *Let $X = \{a, b, c\}$ and $\mu = \{\emptyset, \{a\}\}$. Then $\{b\}$ is $\lambda_\mu - g-$closed but not $\Delta_\mu^* - g_\mu-$closed and $\{a\}$ is $\Delta_\mu^* - g_\mu-$closed but not $\lambda_\mu - g-$closed.*

**Remark 3.10.** *By Theorems 3.5 and 3.6, the following diagram holds:*

*DIAGRAM II*

$$\Delta_\mu^* - g-\text{closed} \Rightarrow \Delta_\mu^* - g_\mu-\text{closed} \Rightarrow g - \lambda_\mu-\text{closed}$$

*The converses of all implications in DIAGRAM II are not true.*

Theorem 3.11 gives a characterization of $\Delta_\mu^* - g_\mu-$closed sets.

**Theorem 3.11.** *Let $(X, \mu)$ be a GTS. A subset $A$ of $X$ is a $\Delta_\mu^* - g_\mu-$closed set if and only if $F \subseteq c_{\lambda_\mu}(A) \backslash A$ and $F$ is $\Delta_\mu^*-$closed implies that $F$ is empty.*

**Proof:** The proof is similar to Theorem 2.6.

**Theorem 3.12.** *If $A$ is a $\Delta_\mu^* - g_\mu-$ closed set in a GTS $(X, \mu)$, then $c_{\lambda_\mu}(A) \backslash A$ does not contain any non-empty $\lambda_\mu-$closed ($\mu-$open / $\mu-$closed) subset of $X$.*

**Proof:** The proof is similar to Theorem 2.7.
Example 3.13 shows that the converse of Theorem 3.12 is not true.

**Example 3.13.** *Let $X = \{a, b, c\}$ and $\mu = \{\emptyset, \{a, b\}, X\}$. If $A = \{b, c\}$, $c_{\lambda_\mu}(A) \backslash A = \{a\}$, which does not contain any non-empty $\lambda_\mu-$closed (resp. $\mu-$closed, $\mu-$open) sets but $A$ is not $\Delta_\mu^* - g_\mu-$closed.*

**Theorem 3.14.** *Let $(X, \mu)$ be a GTS and $A$ and $B$ be subsets of $X$. If $A \subseteq B \subseteq c_{\lambda_\mu}(A)$ and $A$ is a $\Delta_\mu^* - g_\mu-$closed set, then $B$ is $\Delta_\mu^* - g_\mu-$closed.*

**Proof:** If $F$ is a $\Delta_\mu^*-$closed set such that $F \subseteq c_{\lambda_\mu}(B) \backslash B$, then $F \subseteq c_{\lambda_\mu}(A) \backslash A$. By Theorem 3.11, $F = \emptyset$ and so $B$ is $\Delta_\mu^* - g_\mu-$closed.

**Theorem 3.15.** *Let $A$ be a $\Delta_\mu^* - g_\mu-$closed set in a quasi-topological space $(X, \mu)$. Then the following hold:*
*(i) If $A$ is a $\pi-$regular set, then $i_\pi(A)$ and $c_\sigma(A)$ are $\Delta_\mu^* - g_\mu-$closed sets.*
*(ii) If $A$ is a $\sigma-$regular set, then $c_\pi(A)$ and $i_\sigma(A)$ are $\Delta_\mu^* - g_\mu-$closed sets.*

**Proof:** (i) Since $A$ is a $\pi-$regular set, $c_\sigma(A) = A \cup i_\mu c_\mu(A) = A$ and $i_\pi(A) = A \cap i_\mu c_\mu(A) = A$. Thus $i_\pi(A)$ and $c_\sigma(A)$ are $\Delta_\mu^* - g_\mu-$closed sets.
(ii) Since $A$ is a $\sigma-$regular set, $c_\pi(A) = A$ and $i_\sigma(A) = A$. Thus $c_\pi(A)$ and $i_\sigma(A)$ are $\Delta_\mu^* - g_\mu-$closed sets.

**Remark 3.16.** *The union ( resp. intersection ) of two $\Delta_\mu^* - g_\mu-$closed sets need not be a $\Delta_\mu^* - g_\mu-$closed set.*

**Example 3.17.** *Let $X = \{a, b, c, d\}$ and $\mu = \{\emptyset, \{c\}, \{a, b, c\}, \{b, c, d\}, X\}$. Then $\{a\}$ and $\{c\}$ are $\Delta_\mu^* - g_\mu-$closed sets but their union is not a $\Delta_\mu^* - g_\mu-$closed set. Further $\{a, b, c\}$ and $\{a, c, d\}$ are $\Delta_\mu^* - g_\mu-$closed sets but their intersection is not a $\Delta_\mu^* - g_\mu-$closed set.*

## 4. $g_{\Delta_\mu^*}-$ closed sets

In this section, we introduce the notion of $g_{\Delta_\mu^*}-$closed sets and discuss its properties.

**Definition 4.1.** *Let $(X, \mu)$ be a GTS. A subset $A$ of $X$ is called a $g_{\Delta_\mu^*}-$closed set if $c_{\Delta_\mu^*}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is a $\Delta_\mu^*-$open set in $X$. The complement of a $g_{^*\lambda_\mu}-$closed set is called a $g_{\Delta_\mu^*}-$open set.*

**Theorem 4.2.** *For a GTS $(X, \mu)$, every $\Delta_\mu^*-$closed set is $g_{\Delta_\mu^*}-$closed.*

**Proof:** Let $A$ be a $\Delta_\mu^*-$closed set and $U$ be any $\Delta_\mu^*-$open set containing $A$. Since $A$ is $\Delta_\mu^*-$closed, $c_{\Delta_\mu^*}(A) = A$. Therefore $c_{\Delta_\mu^*}(A) \subseteq U$ and hence $A$ is $g_{\Delta_\mu^*}-$closed.

**Corollary 4.3.** *For a GTS $(X, \mu)$, the following hold:*
*(i) Every $\lambda_\mu-$closed set is $g_{\Delta_\mu^*}-$closed.*
*(ii) Every $\mu-$closed set is $g_{\Delta_\mu^*}-$closed.*
*(iii) Every $\mu-$ open set is $g_{\Delta_\mu^*}-$closed.*

Example 4.4 shows that the converses of Theorem 4.2 and Corollary 4.3 are not true.

**Example 4.4.** *Let $X = \{a, b, c, d\}$ and $\mu = \{\emptyset, \{c\}, \{a, b, c\}, \{b, c, d\}, X\}$. Then $\{b, c\}$ is a $g_{\Delta_\mu^*}-$closed set but it is not $\Delta_\mu^*-$ closed (resp. $\lambda_\mu-$closed, $\mu-$closed, $\mu-$open).*

**Remark 4.5.** *$g_{\Delta_\mu^*}-$closed sets and $\lambda_\mu - g-$closed (resp. $g_\mu-$closed) sets are independent of each other.*

**Example 4.6.** *Let $X = \{a, b, c\}$ and $\mu = \{\emptyset, \{a\}\}$. Then $\{b, c\}$ is $\lambda_\mu-g-$closed but not $g_{\Delta_\mu^*}-$closed and $\{a\}$ is $g_{\Delta_\mu^*}-$closed but neither $\lambda_\mu - g-$closed not $g_\mu-$closed.*

**Example 4.7.** *Let $X = \{a, b, c, d\}$ and $\mu = \{\emptyset, \{a, b\}, \{c, d\}, \{b, c, d\}, X\}$. Then $\{a, c\}$ is $g_\mu-$closed but not $g_{\Delta_\mu^*}-$closed.*

Theorem 4.8 shows the relation between $g_{\Delta_\mu^*}-$closed set and $\Delta_\mu^* - g_\mu-$closed set.

**Theorem 4.8.** *For a GTS $(X, \mu)$, every $\Delta_\mu^* - g_\mu-$closed set is $g_{\Delta_\mu^*}-$closed.*

**Proof:** Let $A$ be a $\Delta_\mu^* - g_\mu-$closed set and $U$ be a $\Delta_\mu^*-$open set containing $A$ in$(X, \mu)$. Then $c_{\lambda_\mu}(A) \subseteq U$. Since $c_{\Delta_\mu^*}(A) \subseteq c_{\lambda_\mu}(A)$, we have $c_{\Delta_\mu^*}(A) \subseteq U$. Therefore $A$ is $g_{\Delta_\mu^*}-$closed.

Example 4.9 shows that the converse of Theorem 4.8 is not true.

**Example 4.9.** *Let $X = \{a, b, c\}$ and $\mu = \{\emptyset, \{a, b\}, X\}$. Then $\{a\}$ is $g_{\Delta_\mu^*}-$closed but not $\Delta_\mu^* - g_\mu-$closed.*

**Remark 4.10.** *By Theorems 3.6 and 4.8, the following diagram holds:*

*DIAGRAM III*

$$\Delta_\mu^* - g-closed \Rightarrow \Delta_\mu^* - g_\mu-closed \Rightarrow g_{\Delta_\mu^*}-closed$$

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