Bol. Soc. Paran. Mat. (3s.) **v. 38** 7 (2020): 195–202.
©SPM –ISSN-2175-1188 on line ISSN-00378712 in press
SPM: www.spm.uem.br/bspm doi:10.5269/bspm.v38i7.45496

A Variation on Strongly Lacunary delta Ward Continuity in 2-normed Spaces

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ABSTRACT: A sequence (x_k) of points in a subset E of a 2-normed space X is called strongly lacunary δ-quasi-Cauchy, or N_θ-δ-quasi-Cauchy if (Δx_k) is N_θ-convergent to 0, that is $\lim_{r\to\infty} \frac{1}{h_r}\sum_{k\in I_r} ||\Delta^2 x_k, z|| = 0$ for every fixed $z \in X$. A function defined on a subset E of X is called strongly lacunary δ-ward continuous if it preserves N_θ-δ-quasi-Cauchy sequences, i.e. $(f(x_k))$ is an N_θ-δ-quasi-Cauchy sequence whenever (x_k) is. In this study we obtain some theorems related to strongly lacunary δ-quasi-Cauchy sequences.

Key Words: Strongly lacunary ward continuity, Quasi-Cauchy sequences, Continuity, 2-normed space.

Contents

1. Introduction

The concept of 2-normed spaces was introduced by S. Gähler in 1960's ([1], [2]). Since then a lot of interesting developments have occured in 2-normed spaces by many different authors, see for instance ([3,4,5,6,7,8,9,10,11,12,13]). Let X be a real vector space of dimension d, where $dimX > 1$. A 2-norm on X is a function $||.,.|| : X \times X \to \mathbb{R}^+$ which satisfies: (i) $\|x,y\| = 0 \Leftrightarrow x$ and y are linearly dependent, (ii) $\|x,y\| = \|y,x\|$, (iii) $\|\alpha x,y\| = |\alpha|\,\|x,y\|$ for all $\alpha \in \mathbb{R}$, (iv) $\|x,y+z\| \leq \|x,y\| + \|x,z\|$. Then $(X,||.,.||)$ is called a 2-normed space. Throughout this paper by X we will mean a 2-normed space with a 2-norm $\|.,.\|$. We note here that $\|.,.\|$ is a nonnegative real numbers and in a 2-normed linear space $(X,||.,.||)$, the 2-norm induces a topology which makes X a locally convex Hausdorff topological vector space. To get the topology first define for each $x \in X$ a seminorm p_z on X by $p_z(x) = ||x,z||$ for each $z \in X$. The set $\{p_z : z \in X\}$ forms a family of seminorms and the topology formed by this family of seminorms gives the required topology on X.

2010 *Mathematics Subject Classification:* 40A05, 40G15, 26A05, 26A15.
Submitted November 21, 2018. Published December 12, 2018

A sequence (α_k) of points in $\mathbb{R}$, the set of real numbers, is called statistically convergent to L, or st-convergent to L, if $\lim_{n\to\infty} \frac{1}{n}|\{k \leq n : |\alpha_k - L| \geq \varepsilon\}| = 0$ for every positive real number ε. This is denoted by $st - lim\ \alpha_k = L$ (see [14]).

A lacunary sequence $\theta = (k_r)$ is an increasing sequence of positive integers such that $k_0 = 0$ and $h_r = k_r - k_{r-1} \to \infty$ as $r \to \infty$. The intervals determined by θ will be denoted by $I_r = (k_{r-1}, k_r]$, the ratio k_r/k_{r-1} will be abbreviated by q_r, $q_1 = 0$ for convention, and we assume that $\lim inf_r\ q_r > 1$. In [15], the concept of a strongly lacunary convergent sequence of real numbers, or an N_θ convergent sequence, was defined by Freedman, Sember, and Raphael. A sequence (α_k) of points in $\mathbb{R}$ is called strongly lacunary convergent to a real number L or N_θ-convergent to an element L of $\mathbb{R}$ if $\lim_{r\to\infty} \frac{1}{h_r} \sum_{k\in I_r} |\alpha_k - L| = 0$, and it is denoted by $N_\theta - \lim \alpha_k = L$. Using the idea of Freedman, Sember, and Raphael; Fridy and Orhan introduced the concept of lacunary statistical convergence of a sequence of real numbers in [16,17]. A sequence (α_k) of points in $\mathbb{R}$ is called lacunary statistically convergent, or S_θ-convergent, to an element L of $\mathbb{R}$ if $\lim_{r\to\infty} \frac{1}{h_r}|\{k \in I_r : |\alpha_k - L| \geq \varepsilon\}| = 0$ for every positive real number ε, it is denoted by $S_\theta - \lim_{k\to\infty} \alpha_k = L$.

In recent years many kinds of continuities were introduced and investigated ([18,19,20]). A sequence (α_k) of points in $\mathbb{R}$ is called strongly lacunary quasi-Cauchy if $N_\theta - lim\Delta\alpha_k = 0$, where $\Delta\alpha_k = \alpha_{k+1} - \alpha_k$ for each positive integer k ([21,22,23,27,24,25,26]). The set of strongly lacunary quasi-Cauchy sequences in $\mathbb{R}$ will be denoted by ΔN_θ. A function defined on a subset A of $\mathbb{R}$ is called strongly lacunary ward continuous or N_θ-ward continuous if it preserves N_θ-quasi-Cauchy sequences of points in A, i.e. $(f(\alpha_k))$ is N_θ-quasi-Cauchy whenever (α_k) is an N_θ-quasi-Cauchy sequence of points in A. Recently, the concept of the ward continuity in 2-normed spaces was investigated in [28,29,30].

The purpose of this paper is to introduce the concept of strongly lacunary delta ward continuity in 2-normed spaces and prove some related theorems.

2. Strongly Lacunary δ-ward continuity

A sequence (x_k) of points in X is said to be convergent to an element $l \in X$ if $\lim_{k\to\infty} ||x_k - l, z|| = 0$ for every $z \in X$. This is denoted by $limx_k = l$ or $\lim_{k\to\infty} ||x_k, z|| = ||l, z||$. A sequence (x_k) of points in a 2-normed space $(X, ||., .||)$ is called quasi-Cauchy if $\lim_{k\to\infty} ||\Delta x_k, z|| = 0$ for every $z \in X$ where $\Delta x_k = x_{k+1} - x_k$ for every $k \in \mathbb{N}$ [28].

A sequence (x_k) of points in a subset E of a 2-normed space X is called strongly lacunary quasi-Cauchy, or N_θ-quasi-Cauchy if (Δx_k) is N_θ-convergent to 0, that is

$$\lim_{r\to\infty} \frac{1}{h_r} \sum_{k\in I_r} ||\Delta x_k, z|| = 0$$

for every fixed $z \in X$ and it is denoted by $N_\theta - \lim_{k\to\infty} ||\Delta x_k, z|| = 0$. A function defined on a subset E of X is called strongly lacunary ward continuous if it preserves N_θ-quasi-Cauchy sequences, i.e. $(f(x_k))$ is an N_θ-quasi-Cauchy sequence whenever (x_k) is [30].

Now we introduce strongly lacunary delta quasi-Cauchy sequence in 2-normed space X in the following.

Definition 2.1. *A sequence (x_k) of points in a subset E of X is called strongly lacunary delta quasi Cauchy, or N_θ-δ quasi Cauchy if the sequence (Δx_k) is an N_θ quasi Cauchy sequence, that is*

$$\lim_{r\to\infty} \frac{1}{h_r} \sum_{k\in I_r} ||\Delta^2 x_k, z|| = 0$$

for each positive integer k and for any fixed $z \in X$.

Throughout this paper, N_θ, $\Delta N_\theta(x)$ and $\Delta^2 N_\theta(x)$ will denote the set of strongly lacunary convergent sequences, strongly lacunary quasi-Cauchy sequence and strongly lacunary delta quasi-Cauchy sequence in X respectively.

Consider $\mathbf{R^2}$ as a 2-normed space with the 2-norm $\|.,.\|$ defined by $||\mathbf{a}, \mathbf{b}|| = |a_1 b_2 - a_2 b_1|$ where $\mathbf{a} = (a_1, a_2)$, $\mathbf{b} = (b_1, b_2) \in \mathbf{R^2}$. The sequence

$$(x_n) = \begin{cases} (0, \sqrt{n}) & n = k^2 \\ (0,0) & otherwise \end{cases}$$

is an N_θ-δ quasi Cauchy sequence in $\mathbf{R^2}$ with this 2-norm. This sequence strongly lacunary converges to the point $(0, 0)$. However this sequence is not convergent at all. Thus the set of convergent sequence is a proper subset of strongly lacunary delta quasi-cauchy sequence. So it is obvious that every N_θ quasi-Cauchy sequence is also N_θ-δ quasi-Cauchy, but the converse is not always true. For example, consider $\mathbf{R^2}$ as a 2-normed space with the previously given 2-norm. The sequence

$$(x_n) = \begin{cases} (n, n) & n = k^2 \\ (0,0) & otherwise \end{cases}$$

is an N_θ-δ quasi Cauchy sequence in $\mathbf{R^2}$ with this 2-norm. But this sequence is not N_θ quasi-Cauchy sequence. Moreover the subsequence of the N_θ-δ quasi-Cauchy sequence need not to be a N_θ-δ quasi-Cauchy. Now we introduce the concept of N_θ-δ ward compactness of a subset of X.

Definition 2.2. *If any sequence of points in a subset E has an N_θ-δ quasi-Cauchy subsequence, then E is called strongly lacunary delta ward compact, or $N_\theta - \delta$ ward compact.*

Any finite subset of X is δ-ward compact. A union of two δ-ward compact subsets of X is δ-ward compact and also the intersection of any δ-ward compact subsets of X is δ-ward compact. Any ward compact subset of X is strongly δ-ward compact.

In the following we introduce a definition of N_θ-δ ward continuity in X.

Definition 2.3. *A real valued function f defined on a subset E of X is called N_θ-δ ward continuous if it preserves N_θ-δ quasi-Cauchy sequences of points in E, in other words; $(\Delta f(x_k))$ is a N_θ-δ quasi-Cauchy sequence whenever (Δx_k) is a N_θ-δ quasi-Cauchy sequence of points in E.*

The set of N_θ-δ ward continuous functions on E will be denoted by $\Delta^2 N_\theta(E)$.

Proposition 2.4. *The set of N_θ-δ ward continuous functions is a vector space.*

Proof. Firstly we prove that the sum of two N_θ-δ ward continuous functions is N_θ-δ ward continuous,i.e. if $f, g \in \Delta^2 N_\theta(E)$, then $f + g \in \Delta^2 N_\theta(E)$. Consider f, g be N_θ-δ ward continuous functions on a subset E of X. Let $\varepsilon > 0$ be given and (x_k) be an N_θ-δ quasi Cauchy sequence of points in E. Since f and g are N_θ-δ ward continuous functions then $(f(x_k))$ and $(g(x_k))$ are also N_θ-δ quasi-Cauchy sequences. That is

$$\lim_{r\to\infty} \frac{1}{h_r} \sum_{k\in I_r} ||\Delta^2 f(x_k), z|| = 0 \ \text{ and } \ \lim_{r\to\infty} \frac{1}{h_r} \sum_{k\in I_r} ||\Delta^2 g(x_k), z|| = 0.$$

We have

$$\sum_{k\in I_r} ||\Delta^2 (f+g)(x_k), z|| \le \sum_{k\in I_r} ||\Delta^2 f(x_k), z|| + \sum_{k\in I_r} ||\Delta^2 g(x_k), z||.$$

Hence $\lim_{r\to\infty} \frac{1}{h_r} \sum_{k\in I_r} ||\Delta^2 (f(x_k) + g(x_k)), z|| = 0$.

The product of the N_θ-δ ward continuous function f and any constant real number α is also the N_θ-δ ward continuous function. That is, let f be the N_θ-δ ward continuous function on E and for any $\alpha \in R$ and $z \in X$

$$\sum_{k\in I_r} ||\Delta^2 \alpha f(x_k), z|| = \sum_{k\in I_r} |\alpha| ||\Delta^2 f(x_k), z|| = |\alpha| \sum_{k\in I_r} ||\Delta^2 f(x_k), z||.$$

$$\lim_{r\to\infty} \frac{1}{h_r} \sum_{k\in I_r} ||\Delta^2 \alpha f(x_k), z|| = |\alpha| \lim_{r\to\infty} \frac{1}{h_r} \sum_{k\in I_r} ||\Delta^2 f(x_k), z|| = 0.$$

So the set of N_θ-δ ward continuous functions is a vector space. □

If a function f is N_θ-δ ward continuous on a subset E of the 2-normed space X, then it is N_θ ward continuous on E. But the converse of the statement is not true. Here is the proof of the statement.

Theorem 2.5. *If a function f is N_θ-δ ward continuous on a subset E of the 2-normed space X, then it is N_θ ward continuous on E.*

Proof. Assume that f is N_θ-δ ward continuous. To prove that f is N_θ ward continuous, take any N_θ-quasi Cauchy sequence (x_n) of points in E. We are going to show that $(f(x_n))$ is an N_θ-quasi Cauchy sequence. Now define the sequence

$$(\xi_n) = (x_1, x_1, x_2, x_2, \ldots x_n, x_n, \ldots).$$

Then (ξ_n) is also N_θ-quasi Cauchy therefore (ξ_n) is N_θ-δ quasi Cauchy. As f is N_θ-δ ward continuous $(f(\xi_n))$ is N_θ-δ quasi-Cauchy.

Then

$$(f(\xi_n)) = (f(x_1), f(x_1), f(x_2), f(x_2), ..., f(x_n), f(x_n), ...).$$

Therefore it is obvious that the sequence $(f(\xi_n))$ is an N_θ-quasi-Cauchy sequence on E. That is for every $z \in X$,

$$\lim_{r\to\infty} \frac{1}{h_r} \sum_{k\in I_r} ||\Delta f(\xi_k), z|| = 0.$$

So the proof of the theorem is completed. □

Theorem 2.6. *If a function f is N_θ-δ ward continuous on a subset E of X, then it is N_θ-sequentially continuous on E.*

Proof. Although the proof could be seen by using Theorem 10 in [30], we give a direct proof for completeness. Assume that f is N_θ-δ ward continuous function on a subset E of X. To prove that f is N_θ-sequentially continuous, take any N_θ-convergent sequence (x_n) of points in E with $N_\theta - \lim_{n\to\infty} ||x_n, z|| = ||\ell, z||$ or $\lim_{r\to\infty} \frac{1}{h_r} \sum_{k\in I_r} ||x_n - \ell, z|| = 0$. We are going to show that $(f(x_n))$ is an N_θ-convergent sequence. Now define the sequence

$$(\xi_n) = (x_1, \ell, x_2, \ell, \ldots x_n, \ell, \ldots).$$

Then (ξ_n) is also N_θ-convergent therefore (ξ_n) is N_θ-δ convergent. As f is N_θ-δ ward continuous $(f(\xi_n))$ is N_θ-δ convergent.

Then

$$(f(\xi_n)) = (f(x_1), f(\ell), f(x_2), f(\ell), ..., f(x_n), f(\ell), ...).$$

Therefore the sequence $(f(\xi_n))$ is N_θ-convergent sequence. That is, for every $z \in X$,

$$\lim_{r\to\infty} \frac{1}{h_r} \sum_{k\in I_r} ||f(\xi_k) - f(\ell), z|| = 0.$$

So the proof of the theorem is completed. □

Theorem 2.7. *The function f is uniformly continuous on a subset E of X. If (x_n) is any quasi-Cauchy sequence of points in E, then the sequence $(f(x_n))$ is a N_θ-δ quasi-Cauchy.*

Proof. Let (x_n) is any quasi-Cauchy sequence of points in E. If a function f is uniformly continuous on a subset E of X, for every $x, y, z \in X$ and $\varepsilon > 0$ there exists a $\delta > 0$ such that $||f(x) - f(y), z|| < \varepsilon$ whenever $||x - y, z|| < \delta$. For this δ there exists an $n_0 \in \mathbf{N}$ such that $||\Delta x_k, z|| = ||x_{k+1} - x_k, z|| < \delta$ for all $k \geq n_0$ and for $z \in X$. For $k \geq n_0$, uniformly continuity implies that $||f(x_{k+1}) - f(x_k), z|| < \frac{\varepsilon}{2}$ for $z \in X$. Thus

$$\lim_{r\to\infty} \frac{1}{h_r} \sum_{k\in I_r} ||\Delta^2 f(x_k), z|| = \lim_{r\to\infty} \frac{1}{h_r} \sum_{k\in I_r} ||f(x_{k+2}) - 2f(x_{k+1}) + f(x_k), z||$$

$$\leq \lim_{r\to\infty} \frac{1}{h_r} \sum_{k\in I_r} ||f(x_{k+2}) - f(x_{k+1}), z|| + \lim_{r\to\infty} \frac{1}{h_r} \sum_{k\in I_r} ||f(x_{k+1}) - f(x_k), z|| = 0$$

Therefore $(f(x_n))$ is a N_θ-δ quasi-Cauchy sequence. This completes the proof of this theorem. □

Theorem 2.8. *If (f_n) is a sequence of N_θ-δ ward continuous functions on a subset E of X and (f_n) is uniformly convergent to a function f, then f is also N_θ-δ ward continuous on E.*

Proof. Let (f_n) be uniformly convergent to a function f and let ε be any positive real number. There exists a number $n_0 \in \mathbb{N}$ such that for every $n \geq n_0$, $||f_n(x) - f(x), z|| \leq \frac{\varepsilon}{4}$ for all $x, z \in E$. Take any N_θ-δ-quasi-Cauchy sequence of points in E. If (f_n) is a sequence of N_θ-δ ward continuous functions on a subset E then $\lim_{r\to\infty} \frac{1}{h_r} \sum_{k\in I_r} ||\Delta^2 f_n(x_k), z|| = 0$ for any $z \in E$.

Our aim is to show that f is also N_θ-δ ward continuous function on E, i.e. $\forall z \in E$

$$\lim_{r\to\infty} \frac{1}{h_r} \sum_{k\in I_r} ||\Delta^2 f(x_k), z|| = 0.$$

By using the property $||x + y, z|| \leq ||x, z|| + ||y, z||$, we have

$$\lim_{r\to\infty} \frac{1}{h_r} \sum_{k\in I_r} ||\Delta^2 f(x_k), z|| = \lim_{r\to\infty} \frac{1}{h_r} \sum_{k\in I_r} ||f(x_{k+2}) - 2f(x_{k+1}) + f(x_k), z||$$

$$= \lim_{r\to\infty} \frac{1}{h_r} \sum_{k\in I_r} ||f(x_{k+2}) - f_{n_0}(x_{k+2}) + f_{n_0}(x_{k+2}) - 2f_{n_0}(x_{k+1})$$

$$+ f_{n_0}(x_k) + 2f_{n_0}(x_{k+1}) - f_{n_0}(x_k) - 2f(x_{k+1}) + f(x_k), z||$$

$$\leq \lim_{r\to\infty} \frac{1}{h_r} \sum_{k\in I_r} ||f(x_{k+2}) - f_{n_0}(x_{k+2}), z||$$

$$+ \lim_{r\to\infty} \frac{1}{h_r} \sum_{k\in I_r} ||f_{n_0}(x_{k+2}) - 2f_{n_0}(x_{k+1}) + f_{n_0}(x_k), z||$$

$$+ \lim_{r\to\infty} \frac{1}{h_r} \sum_{k\in I_r} ||2(f_{n_0}(x_{k+1}) - f(x_{k+1})), z||$$

$$+ \lim_{r\to\infty} \frac{1}{h_r} \sum_{k\in I_r} ||f_{n_0}(x_k) - f(x_k), z|| = 0 + 0 + 0 + 0 = 0$$

This result completes the proof of the theorem. So the uniform convergence preserves the property of the N_θ-δ ward continuity for the functions in X. □

3. Conclusion

In this study, strongly lacunary delta quasi-Cauchyness in 2-normed space is introduced and investigated. We have proved that any strongly delta ward continuous function is strongly ward continuous and strongly sequentially continuous and uniformly limit of strongly delta ward continuous function is strongly delta ward continuous.

4. Acknowledgements

The author acknowledges that the statements of some results in this paper were presented at the 2nd International Conference of Mathematical Sciences, ICMS 2018, Maltepe University, Istanbul, Turkey, and have been appeared in the American Institute of Physics Conference Proceedings of the 2nd International Conference of Mathematical Sciences, ICMS 2018, Maltepe University, Istanbul, Turkey ([31]).

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