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Soft $D-$ Metric Spaces

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abstract: The first aim to this paper is defining soft $D-$ metric spaces and giving some fundamental definitions. In addition to this, we prove fixed point theorem of soft continuous mappings on soft $D-$ metric spaces.

Key Words: Soft set, Generalized soft $D-$ metric space, Soft contractive mapping, Fixed point theorem.

Contents

1. Introduction

Metric space is one of the most important space in mathematic. There are various type of generalization of metric spaces. Bapure Dhage [7] in his PhD thesis [1992] introduced a new class of generalized metrics called $D-$metrics. In a subsequent series of papers Dhage attempted to develop topological structures in such spaces. Also he claimed that $D-$metrics provide a generalization of ordinary metric functions. Using the concept of $D-$metric, Y.J.Cho and R. Saadati [5] defined a $\Delta-$distance on a complete $D-$metric space which is a generalization of the concept of $\omega-$distance due to Kada, Suzuki and Takahashi [12]. Later S.V.R.Naidu et all. [16] researched topology of $D-$metric spaces.

Wide area of metric spaces provides a powerful tool to the study of optimization and approximation theory, variational inequalities and so many. After Molodtsov [15] initiated a novel concept of soft set theory as a new mathematical tool for dealing with uncertainties, applications of soft set theory in other disciplines and real life problems was progressing rapidly. The study of soft metric space which is based on soft point of soft sets was initiated by Das and Samanta [6]. Yazar et al. [19] examined some important properties of soft metric spaces and soft continuous mappings. They also proved some fixed point theorems of soft contractive mappings

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on soft metric spaces. Later Gunduz Aras at al. [9], [10] defined soft $S-$metric spaces and gave some fixed point theorems on this spaces.

Topological structures of soft set have been studied by some authors. M. Shabir and M. Naz [18] have initiated the study of soft topological spaces which were defined over an initial universe with a fixed set of parameters and showed that a soft topological space gave a parameterized family of topological spaces. Theoretical studies of soft topological spaces have also been researched by some authors in [3], [4], [8], [11], [14], [17], [21], [22] etc.

The purpose of this paper firstly is contributing for investigating on soft $D-$metric space which is based on soft point of soft sets. By using the concept of soft $D-$metric, we define a soft $\Delta-$ distance on a complete soft $D-$metric. Secondly, using the concept of soft $\Delta-$ distance, we give a fixed point theorem.

2. Preliminaries

In this section, we briefly recall some important basic definitions of soft set theory which serve a background to this paper. Throughout this paper, let X be an universe set, E be a non-empty set of all parameters, $P(X)$ be the power set of X.

Definition 2.1. *[15] A pair (F, E) is called a soft set over X, where F is a mapping given by $F : E \to P(X)$.*

In other words, the soft set is a parameterized family of subsets of the set X. For $a \in E$, $F(a)$ may be considered as the set of $a-$elements of the soft set (F, E), or as the set of $a-$approximate elements of the soft set.

Definition 2.2. *[1] Let (F, E) and (G, E) be two soft sets over X. (F, E) is called a soft subset of (G, E) if $F(a) \subset G(a)$, for all $a \in E$. This relationship is denoted by $(F, E)\widetilde{\subset}(G, E)$. Also (F, E) is called a soft super set of (G, E) if (G, E) is a soft subset of (F, E) and denoted by $(F, E)\widetilde{\supset}(G, E)$. Two soft sets (F, E) and (G, E) over X are called soft equal if (F, E) is a soft subset of (G, E) and (G, E) is a soft subset of (F, E).*

Definition 2.3. *[1] Let (F, E) and (G, E) be two soft sets over X. Then, soft union and soft intersection of (F, E) and (G, E) are defined by the soft sets (H, E) and (H^*, E), respectively,*

$$\begin{aligned} (H, E) &= (F, E)\widetilde{\cup}(G, E), \text{ where } H(a) = F(a) \cup G(a), \\ (H^*, E) &= (F, E)\widetilde{\cap}(G, E), \text{ where } H^*(a) = F(a) \cap G(a), \text{ for all } a \in E. \end{aligned}$$

Definition 2.4. *[13] A soft set (F, E) over X is said to be a null soft set denoted by Φ if for all $a \in E$, $F(a) = \varnothing$.*

Definition 2.5. *[13] A soft set (F, E) over X is said to be an absolute soft set denoted by $\widetilde{X}$ if for all $a \in E$, $F(a) = X$.*

Definition 2.6. *[18] The difference (H, E) of two soft sets (F, E) and (G, E) over X, denoted by $(F, E)\widetilde{\setminus}(G, E)$, is defined as $H(a) = F(a)\setminus G(a)$ for all $a \in E$.*

Definition 2.7. *[18] The complement of a soft set* (F, E)*, denoted by* $(F, E)^c = (F^c, E)$*, where* $F^c : E \to P(X)$ *is a mapping given by* $F^c(a) = X \backslash F(a)$*, for all* $a \in E$*. Here* F^c *is called the soft complement function of* F*.*

It is easy to see that $(\Phi)^c = \widetilde{X}$ *and* $\left(\widetilde{X}\right)^c = \Phi$*.*

Definition 2.8. *[18] Let* $\widetilde{\tau}$ *be the collection of soft sets over* X*, then* $\widetilde{\tau}$ *is called a soft topology on* X *if the following conditions are satisfied:*

1) $\Phi, \widetilde{X}$ *belong to* $\widetilde{\tau}$*;*

2) the union of any number of soft sets in $\widetilde{\tau}$ *belongs to* $\widetilde{\tau}$*;*

3) the intersection of any two soft sets in $\widetilde{\tau}$ *belongs to* $\widetilde{\tau}$*.*

The triplet $(X, \widetilde{\tau}, E)$ *is called a soft topological space over* X*. Then members of* $\widetilde{\tau}$ *are said to be soft open sets in* X*.*

Definition 2.9. *[18] Let* $(X, \widetilde{\tau}, E)$ *be a soft topological space over* X*. A soft set* (F, E) *over* X *is said to be a soft closed set in* X*, if its complement* $(F, E)^c$ *belongs to* $\widetilde{\tau}$*.*

Proposition 2.1. *[18] Let* $(X, \widetilde{\tau}, E)$ *be a soft topological space over* X*. Then the family* $\widetilde{\tau}_a = \{F(a) : (F, E) \in \widetilde{\tau}\}$ *for each* $a \in E$*, defines a topology on* X*.*

Definition 2.10. *[18] Let* $(X, \widetilde{\tau}, E)$ *be a soft topological space over* X *and* (F, E) *be a soft set over* X*. Then the soft closure of* (F, E)*, denoted by* $\overline{(F, E)}$ *, is the intersection of all soft closed super sets of* (F, E)*. Clearly* $\overline{(F, E)}$ *is the smallest soft closed set over* X *which contains* (F, E)*.*

Definition 2.11. *([2], [6]) Let* (F, E) *be a soft set over* X*. The soft set* (F, E) *is called a soft point, denoted by* (x_a, E)*, if for the element* $a \in E$*,* $F(a) = \{x\}$ *and* $F(a') = \varnothing$ *for all* $a' \in E - \{a\}$ *(briefly denoted by* x_a*).*

It is obvious that each soft set can be expressed as union of all soft points belonging to it. For this reason, to give the family of all soft sets on X *it is sufficient to give only soft points on* X*.*

Definition 2.12. *[2] Two soft points* x_a *and* y_b *over a common universe* X*, we say that the soft points are different if* $x \neq y$ *or* $a \neq b$*.*

Definition 2.13. *[2] The soft point* x_a *is said to be belonging to the soft set* (F, E)*, denoted by* $x_a \widetilde{\in} (F, E)$*, if* $x_a(a) \in F(a)$*, i.e.,* $\{x\} \subseteq F(a)$*.*

Definition 2.14. *[6] Let* $\mathbb{R}$ *be the set of all real numbers,* $B(\mathbb{R})$ *be the collection of all non-empty bounded subsets of* $\mathbb{R}$ *and* E *be taken as a set of parameters. Then a mapping* $F : E \to B(\mathbb{R})$ *is called a soft real set. It is denoted by* (F, E)*. If a soft real set is a singleton soft set, it will be called a soft real number and denoted* $\widetilde{r}, \widetilde{s}$ *etc. Here* $\widetilde{r}, \widetilde{s}$ *will denote a particular type of soft real numbers such that* $\widetilde{r}(a) = r$*, for all* $a \in E$*. For instance,* $\widetilde{0}$ *and* $\widetilde{1}$ *are the soft real numbers where* $\widetilde{0}(a) = 0$*,* $\widetilde{1}(a) = 1$ *for all* $a \in E$ *respectively.*

Definition 2.15. *[6] Let* $\widetilde{r}, \widetilde{s}$ *be two soft real numbers, then the following statement are hold:*

(i) $\widetilde{r}\widetilde{\leq}\widetilde{s}$, *if* $\widetilde{r}(a) \leq \widetilde{s}(a)$, *for all* $a \in E$,
(ii) $\widetilde{r}\widetilde{\geq}\widetilde{s}$, *if* $\widetilde{r}(a) \geq \widetilde{s}(a)$, *for all* $a \in E$,
(iii) $\widetilde{r}\widetilde{<}\widetilde{s}$, *if* $\widetilde{r}(a) < \widetilde{s}(a)$, *for all* $a \in E$,
(iv) $\widetilde{r}\widetilde{>}\widetilde{s}$, *if* $\widetilde{r}(a) > \widetilde{s}(a)$, *for all* $a \in E$.

Definition 2.16. *[7] Let X be a non-empty set. A function $D : X^3 \to [0,\infty)$ is called a $D-$metric if the following conditions are satisfied:*
(1) $D(x,y,z) \geq 0$ for all $x,y,z \in X$ and equality holds if and only if $x = y = z$,
(2) $D(x,y,z) = D(x,z,y) = D(y,x,z) = \ldots$
(3) $D(x,y,z) \leq D(x,y,u) + D(x,u,z) + D(u,y,z)$, for all $x,y,z,u \in X$.
Then the pair (X,D) is called a $D-$ metric space.

3. Soft $D-$ Metric Spaces

In this section, we introduce the definition of soft $D-$ metric spaces, soft $\Delta-$distance function, from the family of all soft points of a soft set to the set of all non-negative soft real numbers. Later, we study some important results of them. Also, we give some important concepts such as converge, Cauchy sequence, soft complete on soft $D-$ metric spaces. Let $\widetilde{X}$ be the absolute soft set, E be a non-empty set of parameters and $SP(\widetilde{X})$ be the collection of all soft points of $\widetilde{X}$. Let $\mathbb{R}(E)^*$ denote the set of all non-negative soft real numbers.

Definition 3.1. *A mapping $D : SP(\widetilde{X}) \times SP(\widetilde{X}) \times SP(\widetilde{X}) \to \mathbb{R}(E)^*$ is called a soft $D-$ metric on the soft set $\widetilde{X}$ that D satisfies the following conditions, for each soft points $x_a, y_b, z_c, u_d \in SP(\widetilde{X})$,*
D1) $D(x_a,y_b,z_c) \geq \widetilde{0}$ and equality holds if and only if $x_a = y_b = z_c$.(coincidence)
D2) $D(x_a,y_b,z_c) = D(y_b,x_a,z_c) = D(x_a,z_c,y_b) = \ldots$(symmetry)
D3)$D(x_a,y_b,z_c) \leq D(x_a,y_b,u_d) + D(x_a,u_d,z_c) + D(u_d,y_b,z_c)$.
Then the soft set $\widetilde{X}$ with a soft $D-$ metric is called a soft $D-$ metric space and denoted by $\left(\widetilde{X},D,E\right)$.

Example 3.2. *Let X be a non-emty set and E be the non-emty set of parameters. If we define a mapping*

$$D : SP(\widetilde{X}) \times SP(\widetilde{X}) \times SP(\widetilde{X}) \to \mathbb{R}(E)^*$$

by,

$$D(x_a,y_b,z_c) = \begin{cases} \widetilde{0}, & \text{all of } x_a, y_b, z_c \text{ are equal,} \\ \widetilde{1}, & \text{otherwise} \end{cases}$$

for all $x_a, y_b, z_c \in SP(\widetilde{X})$. Then D is a soft $D-$metric on $\widetilde{X}$.

Example 3.3. *Let X be a non-empty set and $E \subset \mathbb{R}$ be a non-emty set of parameters. Let (X,d^*) be an ordinary metric on X. Therefore $d^*_s(x_a,y_b) = |a-b| + d^*(x,y)$ is a soft metric. Then let us define a mapping*

$$D : SP(\widetilde{X}) \times SP(\widetilde{X}) \times SP(\widetilde{X}) \to \mathbb{R}(E)^*$$

by,

$$D(x_a, y_b, z_c) = \{d_s^*(x_a, y_b) + d_s^*(y_b, z_c) + d_s^*(x_a, z_c)\}$$

for all $x_a, y_b, z_c \in SP(\widetilde{X})$.It is clear that D is a soft $D-$metric on $\widetilde{X}$. For this, let us only verify D3) for soft $D-$metric.

$$\begin{aligned} D(x_a, y_b, z_c) &= d_s^*(x_a, y_b) + d_s^*(y_b, z_c) + d_s^*(x_a, z_c) \\ &= \left[\begin{array}{c} |a-b| + d^*(x, y) + |b-c| \\ +d^*(y, z) + |a-c| + d^*(x, z) \end{array}\right] \\ &\leq \left[\begin{array}{c} |a-c| + |c-b| + d^*(x, z) \\ +d^*(z, y) + |b-d| + |d-c| \\ +d^*(y, u) + d^*(u, z) + |a-b| \\ +|b-c| + d^*(x, u) + d^*(u, z) \end{array}\right] \\ &\leq D(x_a, y_b, u_d) + D(x_a, u_d, z_c) + D(u_d, y_b, z_c). \end{aligned}$$

Thus D is a soft $D-$ metric on $\widetilde{X}$.

Remark 3.4. *If $\left(\widetilde{X}, D, E\right)$ is a soft $D-$ metric space, then (X, D_a) is a $D-$metric space for each $a \in E$. Here D_a stands for the $D-$metric for only parameter a. It is clear that every soft $D-$ metric space is a family of parameterized $D-$metric space.*

Definition 3.5. *Let $\left(\widetilde{X}, D, E\right)$ be a soft $D-$ metric space.*

(a) A soft sequence $\{x_{a_n}^n\}$ in $\left(\widetilde{X}, D, E\right)$ converges to a soft point $x_b \in SP(\widetilde{X})$ if for each $\widetilde{\varepsilon} > \widetilde{0}$, there exists $n_0 \in \mathbb{N}$ such that, for all $n, m \geq n_0$, $D\left(x_{a_n}^n, x_{a_m}^m, x_b\right) < \widetilde{\varepsilon}$.

(b) A soft sequence $\{x_{a_n}^n\}$ in $\left(\widetilde{X}, D, E\right)$ is called a Cauchy sequence if for $\widetilde{\varepsilon} > \widetilde{0}$, there exists $n_0 \in \mathbb{N}$ such that, for all $m > n$, $p \geq n_0$, $D\left(x_{a_n}^n, x_{a_m}^m, x_{a_p}^p\right) < \widetilde{\varepsilon}$.

(c) The soft $D-$ metric space $\left(\widetilde{X}, D, E\right)$ is said to be complete if every Cauchy sequence is convergent.

Definition 3.6. *Let $\widetilde{X}$ be a soft $D-$ metric space with soft metric D. Then a mapping $\Delta : SP(\widetilde{X}) \times SP(\widetilde{X}) \times SP(\widetilde{X}) \to \mathbb{R}(E)^*$ is called a soft $\Delta-$ distance on the soft set $\widetilde{X}$ if the following conditions are satisfied:*

(1) $\Delta(x_a, y_b, z_c) \leq \Delta(x_a, y_b, u_d) + \Delta(x_a, u_d, z_c) + \Delta(u_d, y_b, z_c)$ for all soft points $x_a, y_b, z_c, u_d \in SP(\widetilde{X})$,

(2) for any $x_a, y_b \in SP(\widetilde{X}), \Delta(x_a, y_b, .) : SP(\widetilde{X}) \to \mathbb{R}(E)^*$ is soft continuous,

(3) for any $\widetilde{\varepsilon} > \widetilde{0}$, there exists $\widetilde{\delta} > \widetilde{0}$ such that $\Delta(u_d, x_a, y_b) \leq \widetilde{\delta}, \Delta(u_d, x_a, z_c) \leq \widetilde{\delta}$ and $\Delta(u_d, y_b, z_c) \leq \widetilde{\delta}$ imply that $\Delta(x_a, y_b, z_c) \leq \widetilde{\varepsilon}$.

Example 3.7. *Let X be a non-emty set and $E \subset \mathbb{R}$ be a non-emty set of parameters. Let (X, d^*) be an ordinary metric on X. Therefore $d_s^*(x_a, y_b) = |a-b| + d^*(x, y)$ is a soft metric. It is clear that*

$$D(x_a, y_b, z_c) = \max\{d_s^*(x_a, y_b), d_s^*(y_b, z_c), d_s^*(x_a, z_c)\}$$

is a soft $D-$ metric for all soft points $x_a, y_b, z_c \in SP(\widetilde{X})$. Then $\Delta = D$ is a soft $\Delta-$ distance on the soft set $\widetilde{X}$.

For the soft $D-$ metric space conditions (1) and (2) are clear. We want to show only the condition (3). Let $\widetilde{\varepsilon} > \widetilde{0}$ be given and put $\widetilde{\delta} = \widetilde{\varepsilon}$. If $\Delta(u_d, x_a, y_b) \leq \widetilde{\delta}, \Delta(u_d, x_a, z_c) \leq \widetilde{\delta}$ and $\Delta(u_d, y_b, z_c) \leq \widetilde{\delta}$, we have $d_s^*(x_a, y_b) \leq \widetilde{\delta}, d_s^*(y_b, z_c)$ and $d_s^*(x_a, z_c) \leq \widetilde{\delta}$, which implies that $D(x_a, y_b, z_c) \leq \widetilde{\delta} = \widetilde{\varepsilon}$.

Example 3.8. *Let us consider Example 2. Then the mapping $\Delta : SP(\widetilde{X}) \times SP(\widetilde{X}) \times SP(\widetilde{X}) \to \mathbb{R}(E)^*$ defined by $\Delta(x_a, y_b, z_c) = \widetilde{r}$, for all $x_a, y_b, z_c \in SP(\widetilde{X})$, is a soft $\Delta-$ distance on the soft set $\widetilde{X}$, where $\widetilde{r}$ is a non-negative soft real number.*

For the soft $D-$ metric space conditions (1) and (2) are clear. To show the condition (3), for arbitrary $\widetilde{\varepsilon} > \widetilde{0}$, take $\widetilde{\delta} = \frac{\widetilde{\varepsilon}}{3}$. Then $\Delta(u_d, x_a, y_b) \leq \widetilde{\delta}, \Delta(u_d, x_a, z_c) \leq \widetilde{\delta}$ and $\Delta(u_d, y_b, z_c) \leq \widetilde{\delta}$ imply that $D(x_a, y_b, z_c) \leq \widetilde{\varepsilon}$.

Lemma 3.9. *Let $\left(\widetilde{X}, D, E\right)$ be a soft $D-$ metric space and $\Delta-$ be a soft distance on the soft set $\widetilde{X}$. Let $\left\{x_{a_n}^n\right\}$and $\left\{y_{b_n}^n\right\}$ be two soft sequences in $\widetilde{X}$ and $\{\widetilde{\alpha}_n\}, \left\{\widetilde{\beta}_n\right\}$ and $\{\widetilde{\gamma}_n\}$ be sequences in $\mathbb{R}(E)^*$ converging to $\widetilde{0}$ and assume that soft points $x_a, y_b, z_c, u_d \in SP(\widetilde{X})$. Then we have the following statements:*

(a) If $\Delta\left(x_{a_n}^n, \widetilde{\alpha}_n, y_{b_n}^n\right) \leq \widetilde{\alpha}_n$, $\Delta\left(x_{a_n}^n, \widetilde{\alpha}_n, z_c\right) \leq \widetilde{\beta}_n$ and $\Delta\left(x_{a_n}^n, y_{b_n}^n, z_c\right) \leq \widetilde{\gamma}_n$, for any $n \in \mathbb{N}$, then $D\left(\widetilde{\alpha}_n, y_{b_n}^n, z_c\right) \to \widetilde{0}$.

(b) If $\Delta\left(x_{a_n}^n, x_{a_m}^m, x_{a_p}^p\right) \leq \widetilde{\alpha}_n$, for any $p, n, m \in \mathbb{N}$ with $m < n < p$, then $\left\{x_{a_n}^n\right\}$ is a Cauchy sequence in $\left(\widetilde{X}, D, E\right)$.

Proof. (a) Let arbitrary $\widetilde{\varepsilon} > \widetilde{0}$ be given. From definition of $\Delta-$distance, there exists $\widetilde{\delta} > \widetilde{0}$ such that $\Delta(u_d, x_a, y_b) \leq \widetilde{\delta}, \Delta(u_d, x_a, z_c) \leq \widetilde{\delta}$ and $\Delta(u_d, y_b, z_c) \leq \widetilde{\delta}$ imply that $D(x_a, y_b, z_c) \leq \widetilde{\varepsilon}$.Choose $n_0 \in \mathbb{N}$ such that $\widetilde{\alpha}_n \leq \widetilde{\delta}$, $\widetilde{\beta}_n \leq \widetilde{\delta}$ and $\widetilde{\gamma}_n \leq \widetilde{\delta}$ for every $n \geq n_0$. Then for any $n \geq n_0$ we have $\Delta\left(x_{a_n}^n, \widetilde{\alpha}_n, y_{b_n}^n\right) \leq \widetilde{\alpha}_n \leq \widetilde{\delta}$, $\Delta\left(x_{a_n}^n, \widetilde{\alpha}_n, z_c\right) \leq \widetilde{\beta}_n \leq \widetilde{\delta}, \Delta\left(x_{a_n}^n, y_{b_n}^n, z_c\right) \leq \widetilde{\gamma}_n \leq \widetilde{\delta}$, and hence $D\left(\widetilde{\alpha}_n, y_{b_n}^n, z_c\right) \leq \widetilde{\varepsilon}$.If we replace $\{\widetilde{\alpha}_n\}$ with $\left\{y_{b_n}^n\right\}$, then $\left\{y_{b_n}^n\right\}$ converges to z_c.

(b)Let arbitrary $\widetilde{\varepsilon} > \widetilde{0}$ be given. As in the proof of (a), choose $\widetilde{\delta} \geq \widetilde{0}$ and then $n_0 \in \mathbb{N}$. Then, for any $p > n > m \geq n_0 + 1$,

$$\begin{aligned} \Delta\left(x_{a_{n_0}}^{n_0}, x_{a_n}^n, x_{a_m}^m\right) &\leq \widetilde{\alpha}_{n_0} \leq \widetilde{\delta}, \\ \Delta\left(x_{a_{n_0}}^{n_0}, x_{a_n}^n, x_{a_p}^p\right) &\leq \widetilde{\beta}_{n_0} \leq \widetilde{\delta}, \\ \Delta\left(x_{a_{n_0}}^{n_0}, x_{a_m}^m, x_{a_p}^p\right) &\leq \widetilde{\gamma}_{n_0} \leq \widetilde{\delta} \end{aligned}$$

and hence

$$D\left(x_{a_n}^n, x_{a_m}^m, x_{a_p}^p\right) < \widetilde{\varepsilon}.$$

This implies that $\left\{x_{a_n}^n\right\}$ is a Cauchy sequence in $\left(\widetilde{X}, D, E\right)$. □

Definition 3.10. *Let $\widetilde{X}$ be an absolute soft set. $\widetilde{X}$ is said to be $\Delta-$bounded if there is a constant $\widetilde{M} \geq \widetilde{0}$ such that $\Delta\left(x_a, y_b, z_c\right) \leq \widetilde{M}$ for all $x_a, y_b, z_c \in SP(\widetilde{X})$.*

Theorem 3.11. *Let $\left(\widetilde{X}, D, E\right)$ be a complete $D-$metric space and $\Delta-$be a distance on $\widetilde{X}$, $(f,\varphi) : \left(\widetilde{X}, D, E\right) \to \left(\widetilde{X}, D, E\right)$ be a soft mapping. Let $\widetilde{X}$ be a $\Delta-$bounded. Suppose that there exists a soft real number $\widetilde{r} \in \mathbb{R}(E)$,$\widetilde{0} \leq \widetilde{r} < \widetilde{1}$ ($\mathbb{R}(E)$ denotes the soft real numbers set) such that*

$$\Delta\left(\left(f,\varphi\right)\left(x_a\right), \left(f,\varphi\right)^2\left(x_a\right), \left(f,\varphi\right)\left(y_b\right)\right) \leq \widetilde{r}\Delta\left(x_a, \left(f,\varphi\right)\left(x_a\right), y_b\right)$$

for all $x_a, y_b \in SP(\widetilde{X})$.Then there exists $z_c \in SP(\widetilde{X})$ such that $z_c = (f,\varphi)(z_c)$ .In addition to,if $v_s = (f,\varphi)(v_s)$, then $\Delta(v_s, v_s, v_s) = \widetilde{0}$.

Proof. We claim that

$$\inf\left\{\begin{array}{c}\Delta\left(x_a, \left(f,\varphi\right)\left(x_a\right), y_b\right) + \Delta\left(x_a, \left(f,\varphi\right)\left(x_a\right), \left(f,\varphi\right)^2\left(x_a\right)\right) + \\ \Delta\left(\left(f,\varphi\right)\left(x_a\right), \left(f,\varphi\right)^2\left(x_a\right), \left(f,\varphi\right)\left(y_b\right)\right) : x_a \in SP(\widetilde{X})\end{array}\right\} > \widetilde{0},$$

for all $y_b \in SP(\widetilde{X})$ with $y_b \neq (f,\varphi)(y_b)$. Suppose that the claim is true. Let $u_d \in SP(\widetilde{X})$ and define a soft sequence $\left\{u^n_{d_n}\right\}$ in $\widetilde{X}$ by $u^n_{d_n} = (f,\varphi)^n(u_d)$, for all $n \in \mathbb{N}$.Then, for all $n, t \in \mathbb{N}$, we have

$$\Delta\left(u^n_{d_n}, u^{n+1}_{d_{n+1}}, u^{n+t}_{d_{n+t}}\right) \leq \widetilde{r}\Delta\left(u^{n-1}_{d_{n-1}}, u^n_{d_n}, u^{n+t-1}_{d_{n+t-1}}\right) \leq ... \leq \widetilde{r}^n\Delta\left(u_d, u^1_{d_1}, u^t_{d_t}\right).$$

Thus, for any $p > m > n$ for which $m = n + k$ and $p = m + t$ $(k, t \in \mathbb{N})$, we have

$$\begin{array}{rcl}\Delta\left(u^n_{d_n}, u^m_{d_m}, u^p_{d_p}\right) & \leq & \Delta\left(u^n_{d_n}, u^{n+1}_{d_{n+1}}, u^{n+2}_{d_{n+2}}\right) + ... + \Delta\left(u^{p-2}_{d_{p-2}}, u^{p-1}_{d_{p-1}}, u^p_{d_p}\right) \\ \leq \displaystyle\sum_{i=n}^{p} 2\widetilde{M}\widetilde{r}^j & \leq & \dfrac{\widetilde{r}^n}{\widetilde{1} - \widetilde{r}}2\widetilde{M}.\end{array}$$

By part (b) of Lemma 3.9, the soft sequence $\left\{u^n_{d_n}\right\}$ is a Cauchy sequence in $\left(\widetilde{X}, D, E\right)$. Since $\left(\widetilde{X}, D, E\right)$ is a complete, the soft sequence $\left\{u^n_{d_n}\right\}$ converges to a soft point $z_c \in SP(\widetilde{X})$. Let $n \in \mathbb{N}$ be fixed. Then by soft continuous of Δ, we have

$$\Delta\left(u^n_{d_n}, u^m_{d_m}, z_c\right) \leq \lim_{p\to\infty} \inf \Delta\left(u^n_{d_n}, u^m_{d_m}, u^p_{d_p}\right) \leq \frac{\widetilde{r}^n}{\widetilde{1} - \widetilde{r}}2\widetilde{M}.$$

Assume that $z_c \neq (f,\varphi)(z_c)$. Then, by hypothesis, we have

$$\begin{aligned}
\widetilde{0} &< \left[\inf\left\{\begin{array}{c}\Delta\left(x_a,(f,\varphi)(x_a),z_c\right)+\Delta\Big(x_a,(f,\varphi)(x_a),(f,\varphi)^2(x_a)\Big)\\ +\Delta\Big(x_a,(f,\varphi)^2(x_a),z_c\Big)\end{array}\right\}\right]\\
&\leq \left[\inf\left\{\begin{array}{c}\Delta\Big(u^n_{d_n},u^{n+1}_{d_{n+1}},z_c\Big)+\Delta\Big(u^n_{d_n},u^{n+1}_{d_{n+1}},u^{n+2}_{d_{n+2}}\Big)\\ +\Delta\Big(u^n_{d_n},u^{n+2}_{d_{n+2}},z_c\Big):n\in\mathbb{N}\end{array}\right\}\right]\\
&\leq \inf\left\{\frac{\widetilde{r}^n}{\widetilde{1}-\widetilde{r}}2\widetilde{M}+\widetilde{r}^n\widetilde{M}+\frac{\widetilde{r}^{n+1}}{\widetilde{1}-\widetilde{r}}2\widetilde{M}:n\in\mathbb{N}\right\}=\widetilde{0}.
\end{aligned}$$

This is a contradiction. Therefore, we have $z_c = (f,\varphi)(z_c)$ .Now, if $v_s = (f,\varphi)(v_s)$, we have

$$\begin{aligned}
\Delta(v_s,v_s,v_s) &= \Delta\Big((f,\varphi)(v_s),(f,\varphi)^2(v_s),(f,\varphi)^3(v_s)\Big)\\
&\leq \widetilde{r}\Delta\Big(v_s,(f,\varphi)(v_s),(f,\varphi)^2(v_s)\Big)\\
&= \widetilde{r}\Delta(v_s,v_s,v_s),
\end{aligned}$$

and so $\Delta(v_s,v_s,v_s)=\widetilde{0}$.

Now, we prove the claim. Assume that there exists $y_b \in SP(\widetilde{X})$ with $y_b \neq (f,\varphi)(y_b)$ and

$$\left[\inf\left\{\begin{array}{c}\Delta\left(x_a,(f,\varphi)(x_a),y_b\right)+\Delta\Big(x_a,(f,\varphi)(x_a),(f,\varphi)^2(x_a)\Big)\\ +\Delta\Big(x_a,(f,\varphi)^2(x_a),y_b\Big)\end{array}\right\}=\widetilde{0}.\right]$$

Then there exists a soft sequence $\left\{x^n_{a_n}\right\}$ in $\widetilde{X}$ such that

$$\left[\lim_{n\to\infty}\left\{\begin{array}{c}\Delta\left(x^n_{a_n},(f,\varphi)\left(x^n_{a_n}\right),y_b\right)+\Delta\Big(x^n_{a_n},(f,\varphi)\left(x^n_{a_n}\right),(f,\varphi)^2\left(x^n_{a_n}\right)\Big)\\ +\Delta\Big(x^n_{a_n},(f,\varphi)^2\left(x^n_{a_n}\right),y_b\Big)\end{array}\right\}=\widetilde{0}.\right]$$

Thus we have

$$\begin{aligned}
\lim_{n\to\infty}\Delta\left(x^n_{a_n},(f,\varphi)\left(x^n_{a_n}\right),y_b\right) &= \widetilde{0},\\
\lim_{n\to\infty}\Delta\Big(x^n_{a_n},(f,\varphi)\left(x^n_{a_n}\right),(f,\varphi)^2\left(x^n_{a_n}\right)\Big) &= \widetilde{0},\\
\lim_{n\to\infty}\Delta\Big(x^n_{a_n},(f,\varphi)^2\left(x^n_{a_n}\right),y_b\Big) &= \widetilde{0},
\end{aligned}$$

and hence, by part (a) of Lemma 3.9, we have

$$\lim_{n\to\infty}D\Big((f,\varphi)\left(x^n_{a_n}\right),(f,\varphi)^2\left(x^n_{a_n}\right),y_b\Big)=\widetilde{0},$$

and by soft continuity of $D-$metric,

$$\lim_{n\to\infty}(f,\varphi)\left(x^n_{a_n}\right)=\lim_{n\to\infty}(f,\varphi)^2\left(x^n_{a_n}\right)=y_b.$$

We have

$$\left[\begin{array}{c} \Delta\left((f,\varphi)\left(x_{a_n}^n\right),(f,\varphi)^2\left(x_{a_n}^n\right),(f,\varphi)(y_b)\right) \\ \le\widetilde{r}\lim_{n\to\infty}\Delta\left(x_{a_n}^n,(f,\varphi)\left(x_{a_n}^n\right),y_b\right)=\widetilde{0}, \end{array}\right]$$

$$\left[\begin{array}{c} \lim_{n\to\infty}\Delta\left((f,\varphi)\left(x_{a_n}^n\right),y_b,(f,\varphi)(y_b)\right) \\ \le\lim_{n\to\infty}\inf\Delta\left((f,\varphi)\left(x_{a_n}^n\right),(f,\varphi)^2\left(x_{a_n}^n\right),(f,\varphi)(y_b)\right) \end{array}\right]$$

$$\le\widetilde{r}\lim_{n\to\infty}\inf\Delta\left(\left(x_{a_n}^n,(f,\varphi)\left(x_{a_n}^n\right),y_b\right)\right)=\widetilde{0},$$

and

$$\left[\begin{array}{c} \lim_{n\to\infty}\Delta\left((f,\varphi)\left(x_{a_n}^n\right),(f,\varphi)^2\left(x_{a_n}^n\right),y_b\right) \\ \le\lim_{n\to\infty}\inf\Delta\left((f,\varphi)\left(x_{a_n}^n\right),(f,\varphi)^2\left(x_{a_n}^n\right),(f,\varphi)^2\left(x_{a_n}^n\right)\right) \end{array}\right]$$

$$\le\widetilde{r}\lim_{n\to\infty}\inf\Delta\left(x_{a_n}^n,(f,\varphi)\left(x_{a_n}^n\right),(f,\varphi)\left(x_{a_n}^n\right)\right)$$

$$\le\widetilde{r}\lim_{n\to\infty}\inf\Delta\left(x_{a_n}^n,(f,\varphi)\left(x_{a_n}^n\right),(f,\varphi)^2\left(x_{a_n}^n\right)\right)=\widetilde{0}.$$

By part (a) of Lemma 3.9, we have $\lim_{n\to\infty}D\left((f,\varphi)^2\left(x_{a_n}^n\right),y_b,(f,\varphi)(y_b)\right)=\widetilde{0}$ and thus $y_b=(f,\varphi)(y_b)$. This is a contradiction. This completes the proof.

□

4. Conclusion

We have introduced soft $D-$ metric space which is based on soft point of soft sets and given some of its properties. In addition to this, we prove fixed point theorem of soft continuous mappings on soft $D-$ metric spaces.

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