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Some Fixed and Common Fixed Point Results in G-Metric Spaces Which Can't be Obtained From Metric Spaces *

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ABSTRACT: In this paper, we use the concepts of (A, B)-weakly increasing mappings and altering distance functions to establish new contractive conditions for the pair of mappings in the setting of G–metric spaces. Many fixed and common fixed point results in the setting of G–metric spaces are formulated. Note that our new contractive conditions can't be reduces to contractive conditions in standard metric spaces.

Key Words: Nonlinear contractions, Common fixed point, G-metric spaces, Cyclic maps.

Contents

1. Introduction

The notion of G-metric spaces was initiated by Mustafa and Sims [1] in 2006. After that many scientists established many theorems in complete G-metric spaces for example see [2]- [11].

Recently, Jleli and Samet [12] and Samet *et.al* [13] pointed out some fixed and common fixed point theorem can be obtained from known results in standard metric spaces.

Saadati et al. [14] initiated the notion of Ω-distance. They employed the notion of Ω-distance to created and proved some fixed point results in G-metric spaces. After that, Some authors obtained many fixed and common fixed point theorems in the setting of G-metric space by using the notion of Ω-distance [15]- [19]. Moreover, the techniques of Jleli and Samet [12] and Samet *et.al* [13] are not working in the notion of Ω-distance.

The notion of cyclic mappings was introduced by Kirk et al. [20]. Moreover, Kirk et al. [20] initiated the study of fixed point for cyclic mappings. For more fixed point theorems for cyclic mappings see [21]- [28]. The notion of (A, B)-weakly increasing mappings initiated by Shatanawi and Postolache [29] where many fixed

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and common fixed point results for mappings of cyclic form are generalized by them.

Now, we introduce the necessary background for G–metric spaces that will be used in our work.

Definition 1.1. *[1]. Let X be a nonempty set, and let $G : X \times X \times X \to \mathbb{R}^+$ be a function satisfying:*

(G1) $G(x, y, z) = 0 \, if \, x = y = z$,

(G2) $G(x, x, y) > 0 \, for \, all \, x, y \in X \, , with \, x \neq y$,

(G3) $G(x, y, y) \leq G(x, y, z) \, for \, all \, x, y, z \in X \, , with \, y \neq z$,

(G4) $G(x, y, z) = G(x, z, y) = G(y, x, z) = ... = G(p\{x, y, z\})$, *where* $p\{x, y, z\}$ *is the all possible permutation of x,y,z (symmetry in all three variables),*

(G5) $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$ *for all* $x, y, z, a \in X$ *(rectangle inequality).*

Then the function G is called a generalized metric space, or more specifically G-metric on X, and the pair (X,G) is called a G-metric space.

Definition 1.2. *[1] Let (X, G) be a G-metric space, and let (x_n) be a sequence of points of X, we say that (x_n) is G-convergent to x if $lim_{n,m\to\infty} G(x, x_n, x_m) = 0$; that is for any $\epsilon > 0$, there exists $k \in \mathbb{N}$ such that $G(x, x_n, x_m) < \epsilon$, for all $n, m \geq k$.*

Proposition 1.3. *[1] Let (X, G) be a G-metric space then the following are equivalent.*

(1) (x_n) *is G-convergent to x.*

(2) $G(x_n, x_n, x) \to 0$, *as* $n \to \infty$.

(3) $G(x_n, x, x) \to 0$, *as* $n \to \infty$.

Definition 1.4. *[1] Let (X, G) be G-metric space, a sequence $(x_n) \subseteq X$ is said to be G-Cauchy if for every $\epsilon > 0$, there exists $k \in N$ such that $G(x_n, x_m, x_l) < \epsilon$ for all n,m,l $\geq k$.*

Proposition 1.5. *[1] In a G-metric space, the following are equivalent.*
(1) The sequence (x_n) is G-Cauchy.
(2)For every $\epsilon > 0$, there exists $k \in N$ such that $G(x_n, x_m, x_m) < \epsilon$ for all n,m,l $\geq k$.

Definition 1.6. *[7] A G-metric space (X, G) is said to be G-complete or complete G-metric space if every G-Cauchy sequence in (X, G) is G-convergent in (X, G).*

Definition 1.7. *[7] Let (X, G) and (X', G') be two G-metric spaces and let $f : X \to X'$ be a function, then f is said to be G-continuous at a point $a \in X$ if given $\epsilon > 0$, there exists $\delta > 0$ such that x,y $\in X$; $G(a, x, y) < \delta$ implies $G'(fa, fx, fy) < \epsilon$. A function f is G-continuous on X if and only if it is G-continuous at every point $a \in X$.*

Proposition 1.8. *[7] Let (X, G) and (X', G') be two G-metric spaces and let $f : X \to X'$ be a function, then f is said to be G-continuous at a point $x \in X$ if and only if it is G-sequentially continuous at x; that is, whenever (x_n) is G-convergent to x, $(f(x_n))$ is G'-convergent to $f(x)$.*

Proposition 1.9. *[1] Let (X, G) be G-metric space. Then the function G is jointly continuous in all three of it's variables.*

The notion of (A, B)-weakly increasing mapping is given as follows:

Definition 1.10. *[29] Let $(X, \preceq)$ be a partially ordered set and A, B be two nonempty subsets of X with $X = A \cup B$. Let $f, g : X \to X$ be two mappings. Then the pair (f, g) is said to be (A, B)-weakly increasing if $fx \preceq gfx$ for all $x \in A$ and $gx \preceq fgx$ for all $x \in B$*

The notion of altering distance functions plays an important role in our works. Here is the definition of the altering distance function.

Definition 1.11. *[30] The function $\phi : [0, \infty) \to [0, \infty)$ is called an altering distance function if the following properties are satisfied*
(1) ϕ is continuous and nondecreasing.
(2) $\phi(t) = 0$ if and only if $t = 0$.

For more results related to altering distance functions, see [31]-[33].

In this paper, we initiated new contractive conditions by utilizing the notion of (A, B)-weakly increasing mappings in the sense of Shatanawi, and Postolache [29] and the notion of altering distance function in the sense of Khan et al. [30]. Then after, we proved some fixed point results in the setting of G-metric spaces. The techniques of Samet [12] and Samet et al. [13] can't be used to reduce our works to some results known in standard metric spaces.

2. Main Result

Now, we are ready to present our main result:

Theorem 2.1. *Let $\preceq$ be an ordered relation in a set X. Let (X, G) be a complete G-metric space. Let $X = A \cup B$, where A and B are nonempty closed subsets of X. Let f, g be self mapping on X that satisfy the following conditions:*

1. *The pair (f, g) is (A, B)-weakly increasing.*
2. *$f(A) \subseteq B$ and $g(B) \subseteq A$.*
3. *There exist two altering distance functions ϕ and ψ such that*

$$\phi G(fx, gfx, gy) \leq \phi G(x, fx, y) - \psi G(x, fx, y)$$

holds for all comparative elements $x, y \in X$ with $x \in A$ and $y \in B$ and

$$\phi G(gx, fgx, fy) \leq \phi G(x, gx, y) - \psi G(x, gx, y)$$

holds for all comparative elements $x, y \in X$ with $x \in B$ and $y \in A$

4. f or g is continuous.

Then f and g have a common fixed point in $A \cap B$

Proof. Since A is not empty, we start with $x_0 \in A$. By using condition (2), we construct a sequence (x_n) in X such that $fx_{2n} = x_{2n+1}$, $x_{2n} \in A$ and $gx_{2n+1} = x_{2n+2}$, $x_{2n+1} \in B$ $n \in \mathbb{N}$. By using condition (1), we have $x_n \preceq x_{n+1}$ for all $n \in \mathbb{N}$. If $x_{2n+1} = x_{2n+2}$ for some $n \in \mathbb{N}$, then x_{2n+1} is a fixed point for g in $A \cap B$. Since $x_{2n+1} \preceq x_{2n+2}$, by condition (3) we have

$$\begin{aligned} \phi G(x_{2n+2}, x_{2n+3}, x_{2n+3}) &= \phi G(gx_{2n+1}, fgx_{2n+1}, fx_{2n+2}) \\ &\leq \phi G(x_{2n+1}, x_{2n+2}, x_{2n+2}) \\ &\quad -\psi G(x_{2n+1}, x_{2n+2}, x_{2n+2}). \end{aligned}$$

Since $x_{2n+1} = x_{2n+2}$, we have $G(x_{2n+2}, x_{2n+3}, x_{2n+3}) = 0$. Hence $x_{2n+3} = x_{2n+2}$, and so $x_{2n+3} = x_{2n+2} = x_{2n+1}$. Therefore, x_{2n+1} is also a fixed point for f. Hence x_{2n+1} is a common fixed point for f and g in $A \cap B$.
Now assume that $x_{n+1} \neq x_n$, for all $n \in \mathbb{N}$. Let $n \in \mathbb{N}$. Since $x_{2n+1} \preceq x_{2n+2}$, then by condition (3) we have

$$\begin{aligned} \phi G(x_{2n+2}, x_{2n+3}, x_{2n+3}) &= \phi G(gx_{2n+1}, fgx_{2n+1}, fx_{2n+2}) \\ &\leq \phi G(x_{2n+1}, x_{2n+2}, x_{2n+2}) \\ &\quad -\psi G(x_{2n+1}, x_{2n+2}, x_{2n+2}) \\ &\leq \phi G(x_{2n+1}, x_{2n+2}, x_{2n+2}). \end{aligned} \tag{2.1}$$

Again since $x_{2n} \preceq x_{2n+1}$, then by condition (3) we have

$$\begin{aligned} \phi G(x_{2n+1}, x_{2n+2}, x_{2n+2}) &= \phi G(fx_{2n}, gfx_{2n}, gx_{2n+1}) \\ &\leq \phi G(x_{2n}, x_{2n+1}, x_{2n+1}) \\ &\quad -\psi G(x_{2n}, x_{2n+1}, x_{2n+1}) \\ &\leq \phi G(x_{2n}, x_{2n+1}, x_{2n+1}). \end{aligned} \tag{2.2}$$

From (2.1) and (2.2), we conclude that for all $n \in \mathbb{N}$

$$\phi G(x_{n+1}, x_{n+2}, x_{n+2}) \leq \phi G(x_n, x_{n+1}, x_{n+1}) - \psi \phi G(x_n, x_{n+1}, x_{n+1}). \tag{2.3}$$

From (2.3), we have

$$\phi G(x_{n+1}, x_{n+2}, x_{n+2}) \leq \phi G(x_n, x_{n+1}, x_{n+1}). \tag{2.4}$$

which means that $\{G(x_n, x_{n+1}, x_{n+1}) : \ n \in \mathbb{N}\}$ is a nonnegative decreasing sequence. Therefore, there exists$r \geq 0$ such that $\lim_{n\to\infty} G(x_n, x_{n+1}, x_{n+1}) = r$. By taking the limit as $n \to \infty$ in (2.3) and using the fact that ϕ and ψ are continuous we get
$\phi r \leq \phi r - \psi r$. Therefore $\psi r = 0$ and so $r = 0$.
Hence

$$\lim_{n\to\infty} G(x_n, x_{n+1}, x_{n+1}) = 0. \tag{2.5}$$

By using the definition of G-metric spaces, we have

$$\lim_{n\to\infty} G(x_n, x_n, x_{n+1}) = 0. \tag{2.6}$$

Now, we will show that (x_n) is G-Cauchy. It is sufficient to show that (x_{2n}) is a G-Cauchy sequence.
Suppose to the contrary that (x_{2n}) is not G-Cauchy sequence. Then there exist $\epsilon > 0$ and two subsequences (x_{2n_k}),(x_{2m_k}) of (x_{2n}) such that m_k is chosen as the smallest index for which

$$G(x_{2n_k}, x_{2m_k}, x_{2m_k}) \geq \epsilon \qquad m_k > n_k. \tag{2.7}$$

This implies that that

$$G(x_{2n_k}, x_{2m_k-2}, x_{2m_k-2}) < \epsilon. \tag{2.8}$$

From (2.7), (2.8) and triangular inequality, we get

$$\begin{aligned}
\epsilon &\leq G(x_{2n_k}, x_{2m_k}, x_{2m_k}) \\
&\leq G(x_{2n_k}, x_{2n_k+1}, x_{2n_k+1}) + G(x_{2n_k+1}, x_{2m_k}, x_{2m_k}) \\
&\leq G(x_{2n_k}, x_{2n_k+1}, x_{2n_k+1}) + G(x_{2n_k+1}, x_{2m_k-1}, x_{2m_k-1}) \\
&\quad +G(x_{2m_k-1}, x_{2m_k}, x_{2m_k}) \\
&\leq G(x_{2n_k}, x_{2n_k+1}, x_{2n_k+1}) + G(x_{2n_k+1}, x_{2n_k}, x_{2n_k}) + G(x_{2n_k}, x_{2m_k-1}, x_{2m_k-1}) \\
&\quad +G(x_{2m_k-1}, x_{2m_k}, x_{2m_k}) \\
&\leq G(x_{2n_k}, x_{2n_k+1}, x_{2n_k+1}) + G(x_{2n_k+1}, x_{2n_k}, x_{2n_k}) + G(x_{2n_k}, x_{2m_k-2}, x_{2m_k-2}) \\
&\quad +G(x_{2m_k-2}, x_{2m_k-1}, x_{2m_k-1}) + G(x_{2m_k-1}, x_{2m_k}, x_{2m_k}) \\
&< G(x_{2n_k}, x_{2n_k+1}, x_{2n_k+1}) + G(x_{2n_k+1}, x_{2n_k}, x_{2n_k}) + \epsilon \\
&\quad +G(x_{2m_k-2}, x_{2m_k-1}, x_{2m_k-1}) + G(x_{2m_k-1}, x_{2m_k}, x_{2m_k}).
\end{aligned}$$

Letting $k \to \infty$ in above inequalities and using (2.5) and (2.6), we get

$$\begin{aligned}
\lim_{k\to\infty} G(x_{2n_k}, x_{2m_k}, x_{2m_k}) &= \lim_{k\to\infty} G(x_{2n_k+1}, x_{2m_k}, x_{2m_k}) \\
&= \lim_{k\to\infty} G(x_{2n_k+1}, x_{2m_k-1}, x_{2m_k-1}) \\
&= \lim_{k\to\infty} G(x_{2n_k}, x_{2m_k-1}, x_{2m_k-1}) = \epsilon.
\end{aligned} \tag{2.9}$$

Now, since $x_{2n_k} \preceq x_{2m_k-2}$, then by using condition (3) we get

$$\begin{aligned}
\phi G(x_{2n_k+1}, x_{2m_k}, x_{2m_k}) &= \phi G(f x_{2n_k}, g f x_{2m_k-2}, g x_{2m_k-1}) \\
&\leq \phi G(x_{2n_k}, x_{2m_k-1}, x_{2m_k-1}) \\
&\quad -\psi \max\{G(x_{2n_k}, x_{2m_k-1}, x_{2m_k-1}).
\end{aligned}$$

Taking the limit as $k \to \infty$ and using the fact that ϕ and ψ are continuous and using 2.9, we get

$$\phi\epsilon \leq \phi\epsilon - \psi\epsilon.$$

Therefore, $\psi\epsilon = 0$. Hence $\epsilon = 0$ a contradiction. Hence, (x_{2n}) is G-Cauchy. So (x_n) is a G-Cauchy sequence. Since (X, G) is a complete G-metric space, there exists $u \in X$ such that (x_n) is G-converges to u. Therefore the subsequences $(gx_{2n+1}), f(x_{2n}), (x_{2n+1})$, and (x_{2n}) are G-converge to u. Since $(x_{2n}) \subseteq A$ and A is closed then $u \in A$. Also, since $(x_{2n+1}) \subseteq B$ and B is closed, then $u \in B$. Without lose of generality, we may assume that f is continuous. So $\lim_{n\to\infty} fx_{2n} = fu$ and $\lim_{n\to\infty} fx_{2n} = \lim_{n\to\infty} x_{2n+1} = u$. By uniqueness of the limit we have $fu = u$. Since $u \preceq u$, by condition (3) we have

$$\begin{aligned} \phi G(u, gu, gu) &= \phi(fu, gfu, gu) \\ &\leq \phi G(u, fu, u) - \psi G(u, fu, u) \\ &= \phi G(u, u, u) - \psi G(u, u, u) = 0. \end{aligned}$$

Therefore, $\psi G(gu, gu, u) = 0$. Since ϕ is an altering distance function, we get $G(gu, gu, u) = 0$. Hence $gu = u$. Thus u is a common fixed point for f and g in $A \cap B$.

Corollary 2.2. *Let $\preceq$ be an ordered relation on a set X and suppose that there exists a G-metric on X such that (X, G) is a complete G-metric space. Let A,B be two nonempty closed subsets of X with $X = A \cup B$. Let $f : X \to X$ be a continuous function satisfying the following conditions:*

1. *$fx \preceq f^2x$ for all $x \in X$.*
2. *$f(A) \subseteq B$ and $f(B) \subseteq A$.*
3. *There exist two altering distance functions ϕ, ψ such that*

$$\phi G(fx, f^2x, fy) \leq \phi G(x, fx, y) - \psi G(x, fx, y)$$

holds for all comparative elements $x, y \in X$.

Then f has a fixed point in $A \cap B$.

Proof. It follows from Theorem 2.1 by taking $g = f$.

Corollary 2.3. *Let $\preceq$ be an ordered relation on a set X and suppose that there exists a G-metric on X such that (X, G) is complete G-metric space. Let A,B be two nonempty closed subsets of X with $X = A \cup B$ and f, g be two self mappings on X that satisfy the following conditions:*

1. *The pair (f, g) is (A, B)-weakly increasing.*
2. *$f(A) \subseteq B$ and $g(B) \subseteq A$.*

3. *There exists $r \in [0, 1)$ such that*

$$G(fx, gfx, gy) \leq rG(x, fx, y)$$

holds for all comparative elements $x, y \in X$ with $x \in A, y \in B$, and

$$G(gx, fgx, fy) \leq r\,G(x, gx, y)$$

holds for all comparative elements $x, y \in X$ with $x \in B, y \in A$.

4. *f or g is continuous.*

Then f and g have a common fixed point in $A \cap B$.

Proof. Define $\phi, \psi : [0, \infty) \to [0, \infty)$ by $\phi(t) = t$ and $\psi(t) = (1 - r)t$. Then the proof follows from Theorem 2.1.

Corollary 2.4. *Let $\preceq$ be an ordered relation on a set X and suppose that there exists a G-metric on X such that (X,G) is a complete G-metric space. Let A,B be two nonempty closed subsets of X with $X = A \cup B$. Let $f : X \to X$ be a continuous mapping satisfying the following conditions:*

1. *$fx \preceq f^2x$ for all $x \in X$.*
2. *$f(A) \subseteq B$ and $f(B) \subseteq A$.*
3. *Suppose that there exists $r \in [0, 1)$ such that*

$$G(fx, f^2x, fy) \leq r\,G(x, fx, y)$$

holds for all comparative $x, y \in X$.

Then f has a fixed point in $A \cap B$.

Proof. The proof follows from Corollary (2.3) by taking $g = f$.

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