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# A Covering Property with respect to Generalized Preopen Sets

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abstract: In this paper, we introduce and study the notion of $\mu$-precompact spaces on the observation that each $\mu$-preopen set of a generalized topological space is contained in a $\mu$-open set. The $\mu$-precompactness is weaker than $\mu$-compactness but stronger than weakly $\mu$-compactness of generalized topological spaces.

Key Words: $\mu$-preopen, $\mu$-compact, weakly $\mu$-compact, $\mu$-precompact.

## Contents



## 1. Introduction

Let $(X, \mathscr{P})$ be a topological space. We find that certain subsets like semi-open sets (Levine [10], also called $\beta$-sets by Njåstad [13]), pre-open sets (Mashhour et al. [11]), semi-pre-open sets (Andrijević [1], also called $\beta$-open sets by El-Monsef et al. [9]), $\alpha$-sets (Njåstad [13]) of a topological space $X$ possess properties more or less similar to those of open sets of $X$. Also topological properties generated by sets like semi-open, pre-open etc. had impacts in developing the study of classical objects, see e.g. [7,8,18]. On this observation, Császár [6] introduced and studied $\gamma$-open sets in $X$. Again following the properties of $\gamma$-open sets of a topological space, Császár [4] introduced and studied the concept of generalized topology.

Let $X$ be a nonempty set and $\mu$ be a subcollection of the power set $\exp(X)$ of $X$. $\mu$ is called a generalized topology on $X$ if $\emptyset \in \mu$ and the union of arbitrary number of elements of $\mu$ is again a member of $\mu$. A nonempty set $X$ endowed with a generalized topology $\mu$ is called a generalized topological space and it is denoted by $(X, \mu)$. We write GT (resp. GTS) to denote the generalized topology $\mu$ (resp. generalized topological space $(X, \mu)$). An element of $\mu$ is called a $\mu$-open set of $(X, \mu)$. The complement of a $\mu$-open set is called a $\mu$-closed set of $(X, \mu)$. A generalized topological space $(X, \mu)$ is called strong [3] (also called $\mu$-space by Noiri [14]) if $X \in \mu$. For brevity, we retain the term $\mu$-space due to Noiri [14] to mean the strongly generalized topological space $(X, \mu)$ as well.

Henceforth, we write $X$ to denote a GTS or $\mu$-space to be understood from the context. For a subset $A$ of a GTS $X$, the generalized closure [2] of $A$ is denoted by $c_\mu(A)$ which is the intersection of all $\mu$-closed sets containing $A$ and the generalized interior [2] of $A$ is denoted by $i_\mu(A)$ which is the union of all $\mu$-open sets contained

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in $A$. It can be proved that a subset $A$ of $X$ is $\mu$-open (resp. $\mu$-closed) if and only if $A = i_\mu(A)$ (resp. $A = c_\mu(A)$). Also for any subset $A$ of $X$, we have $c_\mu(A) = X - i_\mu(X - A)$.

Throughout the paper, $N$ denotes the set of natural numbers and $R$, the set of real numbers.

## 2. $\mu$-precompact spaces

We begin by recalling some known definitions and results to use in the sequel.

**Definition 2.1** (Császár [2]). *A subset $A$ of $X$ is called $\mu$-preopen if $A \subset i_\mu(c_\mu(A))$ and $\mu$-semiopen if $A \subset c_\mu(i_\mu(A))$.*

**Definition 2.2** (Sarsak [17]). *A subset $A$ of a GTS $X$ is called $\mu$-regularly closed if $A = c_\mu(i_\mu(A))$. The complement of a $\mu$-regularly closed set is called a $\mu$-regularly open set. So a subset $A$ of a GTS is $\mu$-regularly open if $A = i_\mu(c_\mu(A))$.*

Note that if $G$ is a $\mu$-open set in $X$, then $i_\mu(c_\mu(G))$ is $\mu$-regularly open in $X$.

We see that a subset $A$ of $X$ is $\mu$-preopen if and only if there exists a $\mu$-open set $G$ such that $A \subset G \subset c_\mu(A)$. Also a subset $A$ of $X$ is $\mu$-semiopen if and only there exists a $\mu$-open set $G$ such that $G \subset A \subset c_\mu(G)$.

We write '$\mu$-open collection' and '$\mu$-preopen collection' to mean a collection consisting $\mu$-open sets and $\mu$-preopen sets respectively of a $\mu$-space. A cover of a $\mu$-space $X$ is a collection $\mathscr{A}$ of subsets of $X$ such that $\bigcup_{A \in \mathscr{A}} A = X$. $\mathscr{A}$ is called a $\mu$-open cover (resp. $\mu$-preopen cover) of $X$ if $\mathscr{A}$ is a $\mu$-open collection (resp. $\mu$-preopen collection) of $X$ and covers $X$. The terms 'regularly $\mu$-open collection', 'regularly $\mu$-open cover' '$\mu$-semiopen collection', '$\mu$-semiopen cover' are apparent.

**Definition 2.3** (Sarsak [16]). *A $\mu$-space is called $\mu$-compact if each $\mu$-open cover of $X$ has a finite subcover.*

**Definition 2.4** (Sarsak [17]). *A $\mu$-space is called weakly $\mu$-compact (briefly, w$\mu$-compact) if each $\mu$-open cover $\mathscr{G}$ of $X$ has a finite subcollection $\mathscr{G}_n$ such that $\bigcup_{G \in \mathscr{G}_n} c_\mu(G) = X$.*

**Definition 2.5** (Sarsak [15]). *A $\mu$-space is called $\mu$-S-closed if each $\mu$-semiopen cover $\mathscr{G}$ of $X$ has a finite subcollection $\mathscr{G}_n$ such that $\bigcup_{G \in \mathscr{G}_n} c_\mu(G) = X$.*

We now introduce the following.

**Definition 2.6.** *Let $\mathscr{S}$ be a $\mu$-preopen collection of $X$. For each $A \in \mathscr{S}$, there exists a $\mu$-open set $U$ such that $A \subset U \subset c_\mu(A)$. We define $\mathscr{U} = \{U \mid A \in \mathscr{S}, A \subset U \subset c_\mu(A)\}$. Then $\mathscr{U}$ is said to be a '$\mu$-open super collection' of $\mathscr{S}$.*

It follows that there always exists a $\mu$-open super collection of a $\mu$-preopen collection of a $\mu$-space $X$. We also see that $\mathscr{U}$ is a cover of $X$ if $\mathscr{S}$ is a cover of $X$. In this case, $\mathscr{U}$ is said to be a $\mu$-open super cover of the $\mu$-preopen cover $\mathscr{S}$.

**Definition 2.7.** *A $\mu$-space $X$ is said to be $\mu$-precompact if each $\mu$-preopen cover of $X$ has a finite $\mu$-open super cover.*

If $\mathscr{U}$ is a finite $\mu$-open super cover of a $\mu$-preopen cover $\mathscr{S}$ of a $\mu$-precompact space $X$, then for each $U \in \mathscr{U}$, there exists a $\mu$-preopen set $A \in \mathscr{S}$ such that $A \subset U \subset c_\mu(A)$. Thus we have a finite subcollection $\{A \mid U \in \mathscr{U}, A \subset U \subset Cl(A)\}$ of $\mathscr{S}$ corresponding to $\mathscr{U}$.

It is easy to see that a $\mu$-compact space is a $\mu$-precompact space and a $\mu$-precompact space is a weakly $\mu$-compact space but reverse implication relations are not true.

**Example 2.8.** *On $R$, we define $\mu = \{\emptyset, R\} \cup \{(-\infty, n) \mid n \in N\} \cup \{[1, \infty)\}$. The $\mu$-space $(R, \mu)$ is $\mu$-precompact but not a $\mu$-compact space.*

**Lemma 2.9.** *If $A$ is $\mu$-preopen in $X$, then $i_\mu(c_\mu(A))$ is $\mu$-regularly open in $X$.*

**Proof:** Since $A$ is a $\mu$-preopen set in $X$, there exists a $\mu$-open set $G$ such that $A \subset G \subset c_\mu(A)$ which implies that $c_\mu(A) = c_\mu(G)$. Thus we have $i_\mu(c_\mu(A)) = i_\mu(c_\mu(G))$. Since $i_\mu(c_\mu(G))$ is $\mu$-regularly open, $i_\mu(c_\mu(A))$ is $\mu$-regularly open in $X$. □

**Example 2.10** (cf. Example 1 [12]). *We define $\mu = \{\emptyset, (-\infty, b), (-\infty, b]\}$ where $b \in R$. So $(X, \mu)$ is a GTS. We put $A = (-\infty, a)$, $a \in R$ and $a > b$. We see that $i_\mu(c_\mu(A)) = (-\infty, b]$ and $i_\mu(c_\mu((-\infty, b])) = (-\infty, b]$. It means that $i_\mu(c_\mu(A))$ is $\mu$-regularly open in $(X, \mu)$. As $A \not\subset i_\mu(c_\mu(A))$, $A$ is not $\mu$-preopen in $X$.*

*So we conclude that the converse of Lemma 2.9 need not be true in general.*

**Theorem 2.11.** *A $\mu$-space $X$ is $\mu$-precompact if and only if each $\mu$-preopen cover $\mathscr{S}$ of $X$ has a finite $\mu$-regularly open super cover $\{i_\mu(c_\mu(A)) \mid A \in \mathscr{T}\}$ where $\mathscr{T}$ is a finite subcollection of $\mathscr{S}$.*

**Proof:** By $\mu$-precompactness of $X$, we obtain a finite $\mu$-open super cover $\mathscr{G}$ of $\mathscr{S}$. For each $G \in \mathscr{G}$, there exists $A \in \mathscr{S}$ such that $A \subset G \subset c_\mu(A)$ which implies that $A \subset G \subset i_\mu(c_\mu(A)) \subset c_\mu(A)$. We put $\mathscr{T} = \{A \in \mathscr{S} \mid G \in \mathscr{G}, A \subset G \subset c_\mu(A)\}$. It means that $\mathscr{T}$ is a finite subcollection of $\mathscr{S}$. $\mathscr{G}$ being a cover of $X$, $\{i_\mu(c_\mu(A)) \mid A \in \mathscr{T}\}$ is also a cover of $X$. By Lemma 2.9, $i_\mu(c_\mu(B))$ is regularly open for each $B \in \mathscr{T}$. So $\mathscr{T}$ is a finite subcollection of $\mathscr{S}$ such that $\{i_\mu(c_\mu(B)) \mid B \in \mathscr{T}\}$ is a $\mu$-regularly open super cover of the $\mu$-preopen cover $\mathscr{S}$ of $X$.

Conversely, since $i_\mu(c_\mu(A))$ is $\mu$-open and $A \subset i_\mu(c_\mu(A)) \subset c_\mu(A)$ for each $A \in \mathscr{T}$, $\{i_\mu(c_\mu(A)) \mid A \in \mathscr{T}\}$ is a finite $\mu$-open super cover of $\mathscr{S}$. So $X$ is $\mu$-precompact. □

**Theorem 2.12.** *In a $\mu$-space $X$, the following statements are equivalent.*

1. *$X$ is $\mu$-precompact.*
2. *Each $\mu$-preopen cover $\mathscr{A}$ of $X$ has a finite subcollection $\mathscr{B}$ such that $\{i_\mu(c_\mu(B)) \mid B \in \mathscr{B}\}$ covers $X$.*
3. *If $\mathscr{E}$ is a collection of $\mu$-preclosed sets of $X$ such that $\bigcap_{E \in \mathscr{E}} E = \emptyset$, then there exists a finite subcollection $\mathscr{F}$ of $\mathscr{E}$ such that $\bigcap_{F \in \mathscr{F}} i_\mu(c_\mu(F)) = \emptyset$.*

**Proof:** $(a) \Rightarrow (b)$: Follows from Theorem 2.11.

$(b) \Rightarrow (c)$: Let $\mathscr{E} = \{E_\alpha \mid \alpha \in \Delta\}$ be a collection of $\mu$-preclosed sets such that $\bigcap_{\alpha\in\Delta} E_\alpha = \emptyset$. It means that $\{X - E_\alpha \mid \alpha \in \Delta\}$ is a $\mu$-preopen cover of $X$. By $(b)$, we find a finite subcollection $\{X - E_{\alpha_k} \mid \alpha_k \in \Delta, k \in \{1, 2, \dots, n\}\}$ of $\{X - E_\alpha \mid \alpha \in \Delta\}$ such that $\{i_\mu(c_\mu(X - E_{\alpha_k})) \mid k \in \{1, 2, \dots, n\}\}$ covers $X$. It means that $X - \bigcup_{k=1}^n i_\mu(c_\mu(X - E_{\alpha_k})) = \emptyset$ and hence $\bigcap_{k=1}^n c_\mu(i_\mu(E_{\alpha_k})) = \emptyset$.

$(c) \Rightarrow (a)$: Let $X$ be a $\mu$-space satisfying (c). Suppose $\mathscr{W} = \{W_\alpha \mid \alpha \in A\}$ is a $\mu$-preopen cover of $X$. So we find that $\mathscr{E} = \{X - W_\alpha \mid \alpha \in A\}$ is a collection of $\mu$-preclosed sets such that $\bigcap\{X - W_\alpha \mid \alpha \in A\} = \emptyset$. By $(c)$, we obtain a finite subcollection $\{X - W_{\alpha_k} \mid \alpha_k \in A, k \in \{1, 2, \dots, n\}\}$ such that $\bigcap_{k=1}^n c_\mu(i_\mu(X - W_{\alpha_k})) = \emptyset$ which in turn implies that $\bigcup_{k=1}^n i_\mu(c_\mu(W_{\alpha_k})) = X$. So $\{W_{\alpha_k} \mid \alpha_k \in A, k \in \{1, 2, \dots, n\}\}$ is a finite subcollection $\mathscr{W}$ such that $\{i_\mu(c_\mu(W_{\alpha_k})) \mid \alpha_k \in A, k \in \{1, 2, \dots, n\}\}$ covers $X$. Then by Theorem 2.11, $X$ is $\mu$-precompact. □

**Definition 2.13.** *A collection $\mathscr{A}$ of subsets of $X$ is called a $\mu$-proximate cover of $X$ if $c_\mu\left(\bigcup_{A\in\mathscr{A}} A\right) = X$.*

**Theorem 2.14.** *Each $\mu$-preopen cover of a $\mu$-precompact space $X$ has a finite $\mu$-proximate $\mu$-preopen cover.*

**Proof:** Let $\mathscr{S} = \{A_\alpha \mid \alpha \in \Delta\}$ be a $\mu$-preopen cover of a $\mu$-precompact space $X$. By $\mu$-precompactness of $X$, we obtain a finite $\mu$-open super cover $\{G_1, G_2, \dots, G_n\}$ of $\mathscr{S}$. For each $k \in \{1, 2, \dots, n\}$, there exist an $\alpha_k \in \Delta$ such that $A_{\alpha_k} \subset G_k \subset c_\mu(A_{\alpha_k})$. Since $\{G_1, G_2, \dots, G_n\}$ is a cover of $X$, we have $X = \bigcup_{k=1}^n c_\mu(A_{\alpha_k}) = c_\mu\left(\bigcup_{k=1}^n A_{\alpha_k}\right)$. So $\{A_{\alpha_1}, A_{\alpha_2}, \dots, A_{\alpha_n}\}$ is a finite $\mu$-proximate $\mu$-preopen cover of $X$. □

**Definition 2.15** (Császár [3]). *A $\mu$-space $X$ is called $\mu$-extremally disconnected if $c_\mu(G)$ is $\mu$-open for each $\mu$-open set $G$ of $X$.*

**Theorem 2.16.** *A $w\mu$-compact and $\mu$-extremally disconnected space is a $\mu$-precompact space.*

**Proof:** Let $\mathscr{E} = \{E_\alpha \mid \alpha \in A\}$ be a $\mu$-preopen cover of a $w\mu$-compact $\mu$-extremally disconnected $\mu$-space $X$. For each $\alpha \in A$, there exists a $\mu$-open set $G_\alpha$ such that $E_\alpha \subset G_\alpha \subset c_\mu(E_\alpha) = c_\mu(G_\alpha)$. We see that $\mathscr{G} = \{G_\alpha \mid \alpha \in A\}$ is a $\mu$-open cover of $X$. Since $X$ is $w\mu$-compact, we obtain a finite subcollection $\{G_{\alpha_k} \mid \alpha_k \in A, k \in \{1, 2, \dots, n\}\}$ such that $\{c_\mu(G_{\alpha_k}) \mid \alpha_k \in A, k \in \{1, 2, \dots, n\}\}$ covers $X$. By $\mu$-extremal disconnectedness of $X$, we see that $\{c_\mu(G_{\alpha_k}) \mid \alpha_k \in A, k \in \{1, 2, \dots, n\}\}$ is a finite $\mu$-open super cover of $\mathscr{E}$. □

**Definition 2.17.** *A $\mu$-semiopen set $A$ in $X$ is said to be covered if $G \subset A \subset c_\mu(G)$ for some $\mu$-open set $G$, then there exists a $\mu$-open set $H$ such that $G \subset A \subset H \subset c_\mu(G)$.*

**Lemma 2.18.** *A covered $\mu$-semiopen set in $X$ is $\mu$-preopen in $X$.*

**Proof:** Let $A$ be a covered $\mu$-semiopen set and $G \subset A \subset c_\mu(G)$ for some $\mu$-open set. Then $c_\mu(A) = c_\mu(G)$. Also we have another $\mu$-open set $H$ such that $G \subset A \subset H \subset c_\mu(G)$ which implies that $A \subset i_\mu(c_\mu(G)) = i_\mu(c_\mu(A))$. Hence $A$ is $\mu$-preopen. □

In Example 2.8, $[1, \infty)$ is $\mu$-open and hence it is both $\mu$-semiopen and $\mu$-preopen. But there exist no $\mu$-open set $G$ such that $[1, \infty) \subset G$. So $[1, \infty)$ is not covered $\mu$-semiopen. So we conclude that the converse of Lemma 2.18 may not be true.

**Theorem 2.19.** *If each $\mu$-semiopen set of a $\mu$-precompact space $X$ is covered, then $X$ is $\mu$-S-closed also.*

**Proof:** Let $\mathscr{S}$ be a $\mu$-semiopen cover of $X$. By Lemma 2.18, $\mathscr{S}$ is a $\mu$-preopen cover of $X$. By Theorem 2.11, $\mathscr{S}$ has a finite subcollection $\mathscr{T}$ such that $\{i_\mu(c_\mu(A)) \mid A \in \mathscr{T}\}$ covers $X$. For each $A \in \mathscr{T}$, we have $A \subset i_\mu(c_\mu(A)) \subset c_\mu(A)$. So $\mathscr{T}$ is a finite subcollection of $\mathscr{S}$ such that $\{(c_\mu(A) \mid A \in \mathscr{T}\}$ covers $X$ and so $X$ is $\mu$-S-closed. □

A subset $A$ of a $\mu$-space is said to $\mu$-precompact with respect to $X$ if each $\mu$-preopen cover with respect to $X$ of $A$ has a finite $\mu$-open super cover. In view of Theorem 2.11, it can be showed that a subset $A$ of $X$ is $\mu$-precompact with respect to $X$ if each $\mu$-preopen cover $\mathscr{S}$ with respect to $X$ of $A$ has a finite subcollection $\mathscr{T}$ such that $\{i_\mu(c_\mu(G)) \mid G \in \mathscr{T}\}$ covers $A$.

**Theorem 2.20.** *If each proper $\mu$-regularly closed set of a $\mu$-space $X$ is $\mu$-precompact with respect to $X$, then $X$ is $\mu$-precompact.*

**Proof:** Let $\mathscr{S} = \{A_\alpha \mid \alpha \in \Delta\}$ be a $\mu$-preopen cover of $X$. Since $\mathscr{S}$ is a cover of $X$, there exits an $A \in \mathscr{S}$ such that $A \neq \emptyset$. By Lemma 2.9, $i_\mu(c_\mu(A))$ is $\mu$-regularly open in $X$ and so $X - i_\mu(c_\mu(A))$ is $\mu$-regularly closed in $X$. By the assumption, we get a finite subcollection $\{A_{\alpha_k} \mid \alpha_k \in \Delta, k \in \{1, 2, \dots, n\}\}$ such that $X - i_\mu(c_\mu(A)) \subset \bigcup_{k=1}^{n} i_\mu(c_\mu(A_{\alpha_k}))$ and thus $X \subset \bigcup_{k=1}^{n} i_\mu(c_\mu(A_{\alpha_k})) \cup i_\mu(c_\mu(A))$. Therefore by Theorem 2.11, $X$ is $\mu$-precompact. □

Recall that a nonempty collection $\mathscr{C}$ of nonempty subsets of a set $S$ is called a filter base [19, p. 78] if $C_1, C_2 \in \mathscr{S}$, then $C_3 \subset C_1 \cap C_2$ for some $C_3 \in \mathscr{S}$. A filter base is called maximal [19, p. 80] if its not properly contained into another filter base. A filter base is always contains in a maximal filter base [19, p. 80].

**Definition 2.21.** *A filter base $\mathscr{F}$ on a $\mu$-space $X$ is called $p_\mu$-converges to a point $x \in X$ if for each $\mu$-preopen set $A$ of $X$ with $x \in A$, there exists $F \in \mathscr{F}$ such that $F \subset i_\mu(c_\mu(A))$.*

**Definition 2.22.** *A filter base $\mathscr{F}$ on a $\mu$-space $X$ is called $p_\mu$-accumulates to a point $x \in X$ if for each $\mu$-preopen set $A$ of $X$ with $x \in A$, $F \cap i_\mu(c_\mu(A)) \neq \emptyset$ for each $F \in \mathscr{F}$.*

**Lemma 2.23.** *If a filter base $\mathscr{F}$ in $X$ $p_\mu$-converges to a point $x \in X$, then the filter base is $p_\mu$-accumulates to $x$.*

**Proof:** By $p_\mu$-convergence of $\mathscr{F}$ to $x \in X$, there exists $F \in \mathscr{F}$ such that $F \subset i_\mu(c_\mu(A))$ for each $\mu$-preopen set $A$ with $x \in A$. Let $E \in \mathscr{F}$. Then there exists $D \in \mathscr{F}$ such that $D \subset E \cap F \subset F \subset i_\mu(c_\mu(A))$. So $D \cap i_\mu(c_\mu(A)) \neq \emptyset$. As $D \subset E$, we have $E \cap i_\mu(c_\mu(A)) \neq \emptyset$. So $\mathscr{F}$ $p_\mu$-accumulates to $x \in X$. □

**Lemma 2.24.** *Let $\mathscr{F}$ be a maximal filter base in $X$. Then $\mathscr{F}$ $p_\mu$-converges to $x \in X$ if and only if $\mathscr{F}$ is $p_\mu$-accumulates to $x \in X$.*

**Proof:** Since $\mathscr{F}$ is a filter base, $\mathscr{F}$ is $p_\mu$-accumulates to $x \in X$ by Lemma 2.23 if $\mathscr{F}$ is $p_\mu$-converges to $x \in X$.

Conversely, let a maximal filter base $\mathscr{F}$ $p_\mu$-accumulate to $x \in X$. If $\mathscr{F}$ does not $p_\mu$-converges to $x$, then for each $F \in \mathscr{F}$, there exists a $\mu$-preopen set $A$ containing $x$ such that $F \not\subset i_\mu(c_\mu(A))$ i.e. $F \cap c_\mu(i_\mu(X - A)) \neq \emptyset$. We put $\mathscr{E} = \mathscr{F} \cup \{F \cap c_\mu(i_\mu(X - A)) \mid F \in \mathscr{F}\}$. Then $\mathscr{E}$ is a filter base properly containing $\mathscr{F}$, a contradiction to the fact that $\mathscr{F}$ is a maximal filter base. □

**Theorem 2.25.** *The following statements are equivalent:*

1. *$X$ is $\mu$-precompact.*
2. *Each filter base $p_\mu$-accumulates to some $x_0 \in X$.*
3. *Each maximal filter base $p_\mu$-converges in $X$.*

**Proof:** $(a) \Rightarrow (b)$: Suppose that there exists a filter base $\mathscr{F} = \{F_\alpha \mid \alpha \in A\}$ in $X$ and $\mathscr{F}$ does not $p_\mu$-accumulates in $X$. It means that for each $x \in X$, there exists a $\mu$-preopen set $A_x$ containing $x$ and an $F_{\alpha(x)} \in \mathscr{F}$ such that $F_{\alpha(x)} \cap i_\mu(c_\mu(A_x)) = \emptyset$. So $\mathscr{S} = \{A_x \mid x \in X\}$ is a $\mu$-preopen cover of $X$. By Theorem 2.11, $\mathscr{S}$ has a finite subcollection $A_{x_1}, A_{x_2}, \ldots, A_{x_n}$ such that $\{i_\mu(c_\mu(A_{x_k})) \mid k \in \{1, 2, \ldots, n\}\}$ covers $X$. As $\mathscr{F}$ is a filter base, there exists an $F_0 \in \mathscr{F}$ such that $F_0 \subset \bigcap_{k=1}^{n} F_{\alpha(x_k)}$. It means that $F_0 \cap i_\mu(c_\mu(A_{x_k})) = \emptyset$ for each $k \in \{1, 2, \ldots, n\}\}$. Now $F_0 = F_0 \cap X = F_0 \cap (\bigcup_{k=1}^{n} i_\mu(c_\mu(A_{x_k}))) = \bigcup_{k=1}^{n} (F_0 \cap i_\mu(c_\mu(A_{x_k})) = \emptyset$, a contradiction to the fact that $F_0 \neq \emptyset$.

$(b) \Rightarrow (c)$: Let $\mathscr{F}$ be a maximal filter base in $X$. By (ii), $\mathscr{F}$ $p_\mu$-accumulates to some $x_0 \in X$. $\mathscr{F}$ being a maximal filter base in $X$, $\mathscr{F}$ $p_\mu$-converges to $x_0 \in X$ by Lemma 2.24.

$(c) \Rightarrow (a)$: Let $\mathscr{S} = \{A_\alpha \mid \alpha \in \Delta\}$ be a $\mu$-preopen cover of $X$. If possible, let $X$ be not $\mu$-precompact. Then for each finite subcollection $\Delta_0$ of $\Delta$, we have $\bigcup_{\alpha \in \Delta_0} i_\mu(c_\mu(A_\alpha)) \neq X$ which implies that $\bigcap_{\alpha \in \Delta_0} c_\mu(i_\mu(X - A_\alpha)) \neq \emptyset$. We put $F_{\Delta_0} = \bigcap_{\alpha \in \Delta_0} c_\mu(i_\mu(X - A_\alpha))$. Let $\Lambda$ be the collection of all finite subcollection of $\Delta$. We write $\mathscr{F} = \{F_\lambda \mid \lambda \in \Lambda\}$ (each $F_\lambda$ bears the meaning as of $F_{\Delta_0}$). We see that $\mathscr{F}$ is a filterbase on $X$ and hence there exists a maximal filter base $\mathscr{M}$ containing $\mathscr{F}$. By $(c)$, $\mathscr{M}$ $p_\mu$-converges to some point $x_0 \in X$ and so $\mathscr{M}$ $p_\mu$-accumulates to some point $x_0 \in X$ by Lemma 2.24. As $\mathscr{S}$ is a cover of $X$, there exists $A_0 \in \mathscr{S}$ such that $x_0 \in A_0$. Then by construction, $c_\mu(i_\mu(X - A_0)) \in \mathscr{M}$. Since $\mathscr{M}$ $p_\mu$-accumulates to $x_0$ and $x_0 \in A_0$, we see that $M \cap i_\mu(c_\mu(A_0)) \neq \emptyset$ for each $M \in \mathscr{M}$, in particular, $c_\mu(i_\mu(X - A_0)) \cap i_\mu(c_\mu(A_0)) \neq \emptyset$, a contradiction to the fact that $c_\mu(i_\mu(X - A_0)) \cap i_\mu(c_\mu(A_0)) = \emptyset$. □

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## References

1. D. Andrijević, *Semi-preopen sets*, Mat. Vesnik **38** (1986), 24–32.
2. Á. Császár, *Generalized open sets in generalized topologies*, Acta Math. Hungar. **106** (1-2) (2005), 351–357.
3. Á. Császár, *Extremally disconnected generalized topologies*, Ann. Univ. Sci. Budapest. Eötvös Sect. Math. **47** (2004), 91–96.
4. Á. Császár, *Generalized topology, generalized continuity*, Acta Math. Hungar. **96** (4) (2002), 351–357.
5. Á. Császár, *$\gamma$-compact spaces*, Acta Math. Hungar. **87** (1-2) (2000), 99–107.
6. Á. Császár, *Generalized open sets*, Acta Math. Hungar. **75** (1–2) (1997), 65–87.
7. I. Dochviri and J. F. Peters, *Topological sorting of finitely near sets*, Math. Comput. Sci. **10** (2) (2016), 273–277.
8. I. Dochviri and T. Noiri, *Some remarks on mildly compact spaces in bitopologies*, Questions Answers Gen. Topology **32** (1) (2014), 69–72.
9. M. E. Abd El-Monsef, S. N. El-Deeb and R. A. Mahmoud, *$\beta$-open sets and $\beta$-continuous mapping*, Bull. Fac. Sci. Assiut Univ. **12** (1983), 77–90.
10. N. Levine, *Semi-open sets and semi-continuity in topological spaces*, Amer. Math. Monthly **70** (1963), 36–41.
11. A. S. Mashhour, M. E. Abd El-Monsef and S. N. El-Deeb, *On precontinuous and weak precontinuous mappings*, Proc. Math. Phy. Soc. Egypt **53** (1982), 47–53.
12. A. Mukharjee and R. M. Roy, Unification of maximality and minimality of open and $\mu$-open sets, Bull. Transilv. Univ. Braşov Ser. III **9(58)** (2016), 83–90.
13. O. Njåstad, *On some classes of nearly open sets*, Pacific J. Math. **15** (1965), 961–970.
14. T. Noiri, *Unified characterizations of modifications of $R_0$ and $R_1$ topological spaces*, Rend. Circ. Mat. Palermo (2) **55** (2006), 29–42.
15. M. S. Sarsak, *$\mu$-S-closed spaces*, Acta Math. Hungar. **146** (2) (2015), 285–299.
16. M. S. Sarsak, *On $\mu$-compact sets in $\mu$-spaces*, Questions Answers Gen. Topology **31** (2013), 49–57.
17. M. S. Sarsak, *Weakly $\mu$-compact spaces*, Demonstr. Math. **45** (4) (2012), 929–938.
18. T. Thomphson, *S-closed spaces*, Proc. Amer. Math. Soc. **60** (1976), 335–338.
19. S. Willard, *General Topology*, Addison-Wesley Publishing Company, Reading, 1970.

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