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A Variation on Absolute Weighted Mean Summability Factors of Fourier Series and its Conjugate Series

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ABSTRACT: In this paper, theorems concerning $\left|\bar{N}, p_n, \theta_n\right|_k$ factors of Fourier series and its conjugate series are studied. The presented results are generalizations two theorems of Mazhar on conjugate series and Fourier series.

Key Words: Absolute summability, Fourier series, Conjugate series.

Contents

1. Introduction

Let $\sum a_n$ be a given series with the sequence of partial sums (s_n). Let (p_n) be a sequence of positive real numbers such that

$$P_n = \sum_{v=0}^{n} p_v \to \infty \text{ as } n \to \infty, \ \left(P_{-i} = p_{-i} = 0, \ i \geq 1\right).$$

Let (θ_n) be any sequence of positive constants. The series $\sum a_n$ is said to be summable $\left|\bar{N}, p_n, \theta_n\right|_k$, $k \geq 1$, if (see [12])

$$\sum_{n=1}^{\infty} \theta_n^{k-1} |T_n - T_{n-1}|^k < \infty, \tag{1.1}$$

where

$$T_n = \frac{1}{P_n} \sum_{v=0}^{n} p_v s_v.$$

Let f be a periodic function with period 2π, and integrable (L) over $[-\pi, \pi]$. Let the Fourier series of f be

$$\frac{a_0}{2} + \sum_{1}^{\infty} (a_n \cos nt + b_n \sin nt) = \sum_{0}^{\infty} C_n(t).$$

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Then the conjugate series of the Fourier series is

$$\sum_{1}^{\infty}(b_n \cos nt - a_n \sin nt) = \sum_{1}^{\infty} B_n(t).$$

We denote

$$\phi(t) = \frac{1}{2} f(x+t) + f(x-t)$$
$$\psi(t) = \frac{1}{2} f(x+t) - f(x-t)$$
$$\Lambda(t) = \frac{1}{t}\int_{0}^{t} u d\phi(u).$$

We can write $|R, \lambda_n, 1|$ for $|\bar{N}, p_n|$, where $P_n = \lambda_n$, and $0 < \lambda_1 < ... < \lambda_n \to \infty$, $n \to \infty$. Many studies have been done for absolute summability factors of infinite series and Fourier series by using different summability methods (see [1]- [2], [6], [8]- [11], [13]- [17]). Furthermore, in this paper, we obtained new generalizations of absolute summability factors of Fourier series and its conjugate series.

2. Absolute summability of conjugate series

By using the result in [7], Mazhar [5] has proved the following theorems concerning absolute summability of conjugate series $\sum_{1}^{\infty} B_n(x)$.

Theorem 2.1. *If $\psi(+0) = 0$ and*

$$\int_{0}^{\pi} \log \frac{C}{t} |d\psi(t)|^k < \infty, \quad k \geq 1, \tag{2.1}$$

then $\sum_{1}^{\infty} B_n(x)$ is summable $|\bar{N}, p_n|_k$, where

$$\left\{\frac{P_n}{n}\right\} \uparrow \tag{2.2}$$

$$n^{1-\alpha} p_n = O(P_n), \quad 0 < \alpha < 1. \tag{2.3}$$

For $k = 1$, $P_n = e^{n^\alpha}$, $0 < \alpha < 1$, Theorem 2.1 reduces to Mohanty's theorem concerning $|R, e^{n^\alpha}, 1|$ summability (see [7]). Firstly, we have generalized Theorem 2.1 to $|\bar{N}, p_n, \theta_n|_k$ summability as in the following theorem.

Theorem 2.2. *Let $\left(\frac{\theta_n p_n}{P_n}\right)$ be a non-increasing sequence and the conditions* (2.1)-(2.1) *of Theorem 2.1 are satisfied. If*

$$\left(\frac{\theta_n p_n}{P_n}\right)^{k-1} \frac{p_n}{P_n} = O\left(\frac{1}{n^{1-\alpha}}\right), \quad 0 < \alpha < 1, \tag{2.4}$$

then the series $\sum_{n=1}^{\infty} B_n(x)$ is summable $|\bar{N}, p_n, \theta_n|_k$.

If we take $\theta_n = \frac{P_n}{p_n}$ in above theorem, we can obtain Theorem 2.1.

Proof of Theorem 2.2 Let $T_n(x)$ denote the $(\bar{N}, p_n)$ mean of the series $\sum_1^\infty B_n(x)$ and let C_1 denote a positive constant not necessarily the same at each occurance. Then,

$$\begin{aligned} T_n(x) - T_{n-1}(x) &= \frac{p_n}{P_n P_{n-1}} \sum_{v=1}^{n} P_{v-1} B_v(x) \\ &= \frac{p_n}{P_n P_{n-1}} \sum_{v=1}^{n} P_{v-1} \frac{2}{\pi} \int_0^\pi d\psi(t) \int_t^\pi \sin vu du \end{aligned}$$

since $\psi(+0) = 0$, so that

$$\begin{aligned} &\sum_1^\infty \theta_n^{k-1} |T_n(x) - T_{n-1}(x)|^k = \sum_1^\infty \theta_n^{k-1} \left| \frac{p_n}{P_n P_{n-1}} \sum_{v=1}^{n} P_{v-1} \frac{2}{\pi} \int_0^\pi d\psi(t) \int_t^\pi \sin vu du \right|^k \\ &\leq C_1 \sum_1^\infty \theta_n^{k-1} \left(\frac{p_n}{P_n P_{n-1}} \right)^k \left| \sum_{v=1}^{n} P_{v-1} \int_0^\pi d\psi(t) \int_t^\pi \sin vu du \right|^k \\ &= C_1 \sum_1^\infty \theta_n^{k-1} \left(\frac{p_n}{P_n P_{n-1}} \right)^k \left| \sum_{v=1}^{n} \frac{P_{v-1}}{v} \int_0^\pi d\psi(t) (\cos vt - \cos v\pi) \right|^k \\ &\leq C_1 \sum_1^\infty \theta_n^{k-1} \left(\frac{p_n}{P_n P_{n-1}} \right)^k \left(\int_0^\pi |d\psi(t)| \left| \sum_{v=1}^{n} \frac{P_{v-1}}{v} (\cos vt - \cos v\pi) \right| \right)^k \\ &\leq C_1 \sum_1^\infty \theta_n^{k-1} \left(\frac{p_n}{P_n P_{n-1}} \right)^k \left(\int_0^\pi |d\psi(t)|^k \left| \sum_{v=1}^{n} \frac{P_{v-1}}{v} (\cos vt - \cos v\pi) \right| \right) \\ &\times \left(\int_0^\pi |d\psi(t)| \left| \sum_{v=1}^{n} \frac{P_{v-1}}{v} (\cos vt - \cos v\pi) \right| \right)^{k-1} \\ &\leq C_1 \sum_1^\infty \left(\frac{\theta_n p_n}{P_n} \right)^{k-1} \left(\frac{p_n}{P_n P_{n-1}} \right) \int_0^\pi |d\psi(t)|^k \left| \sum_{v=1}^{n} \frac{P_{v-1}}{v} (\cos vt - \cos v\pi) \right| \\ &= C_1 \int_0^\pi |d\psi(t)|^k L_n(t), \end{aligned}$$

where

$$\begin{aligned} L_n(t) &= \sum_1^\infty \left(\frac{\theta_n p_n}{P_n} \right)^{k-1} \left(\frac{p_n}{P_n P_{n-1}} \right) \left| \sum_{v=1}^{n} \frac{P_{v-1}}{v} (\cos vt - \cos v\pi) \right| \\ &= \sum_1^T + \sum_{T+1}^\infty = L_1 + L_2, \end{aligned}$$

and T is the integer part of $\left(\frac{1}{t}\right)^{\frac{1}{1-\alpha}}$, $T=\left(\frac{1}{t}\right)^{\frac{1}{1-\alpha}}$, $0<\alpha<1$.

$$\begin{aligned}
L_1 &\le C_1 \sum_{1}^{T}\left(\frac{\theta_n p_n}{P_n}\right)^{k-1}\left(\frac{p_n}{P_n P_{n-1}}\right)\sum_{v=1}^{n}\frac{P_{v-1}}{v}\\
&\le C_1 \sum_{1}^{T}\frac{P_{v-1}}{v}\sum_{n=v}^{\infty}\left(\frac{\theta_n p_n}{P_n}\right)^{k-1}\left(\frac{p_n}{P_n P_{n-1}}\right)\\
&\le C_1 \sum_{v=1}^{T}\left(\frac{\theta_v p_v}{P_v}\right)^{k-1}\frac{P_{v-1}}{v}\sum_{n=v}^{\infty}\left(\frac{p_n}{P_n P_{n-1}}\right)\\
&\le C_1 \sum_{v=1}^{T}\left(\frac{\theta_v p_v}{P_v}\right)^{k-1}\frac{1}{v}=C_1\left(\frac{\theta_1 p_1}{P_1}\right)\sum_{v=1}^{T}\frac{1}{v}=O(\log T)=O(\log\frac{C}{t})
\end{aligned}$$

$$\begin{aligned}
L_2 \le &\sum_{n=T+1}^{\infty}\left(\frac{\theta_n p_n}{P_n}\right)^{k-1}\left(\frac{p_n}{P_n P_{n-1}}\right)\left|\sum_{v=1}^{n}\frac{P_{v-1}}{v}\cos vt\right|\\
&+\sum_{n=T+1}^{\infty}\left(\frac{\theta_n p_n}{P_n}\right)^{k-1}\left(\frac{p_n}{P_n P_{n-1}}\right)\left|\sum_{v=1}^{n}\frac{P_{v-1}}{v}\cos v\pi\right|
\end{aligned}$$

by using condition (2.3) of Theorem 2.1 and Abel's partial sums for $\left|\sum_{v=1}^{n}\frac{P_{v-1}}{v}\cos vt\right|$, $0<t\le\pi$, we have

$$\begin{aligned}
&\le C_1\sum_{n=T+1}^{\infty}\left(\frac{\theta_n p_n}{P_n}\right)^{k-1}\left(\frac{p_n}{P_n P_{n-1}}\right)\frac{P_{n-1}}{n}t^{-1}\\
&\quad + C_1\sum_{n=T+1}^{\infty}\left(\frac{\theta_n p_n}{P_n}\right)^{k-1}\left(\frac{p_n}{P_n P_{n-1}}\right)\frac{P_{n-1}}{n}\\
&\le C_1\sum_{n=T+1}^{\infty}\left(\frac{\theta_n p_n}{P_n}\right)^{k-1}\left(\frac{p_n}{nP_n}\right)t^{-1}\\
&\quad + C_1\sum_{n=T+1}^{\infty}\left(\frac{\theta_n p_n}{P_n}\right)^{k-1}\left(\frac{p_n}{nP_n}\right)\\
&\le C_1\sum_{n=T+1}^{\infty}\frac{t^{-1}}{n^{2-\alpha}}\le C_1.
\end{aligned}$$

Therefore

$$L_n(t)=O(\log\frac{C}{t}).$$

Thus

$$\sum_{1}^{\infty}\theta_n^{k-1}|T_n(x)-T_{n-1}(x)|^k\le C_1\int_0^{\pi}\log\frac{C}{t}|d\psi(t)|^k<\infty.$$

This completes the proof of Theorem 2.2.

3. Absolute summability of Fourier series

Concerning about $\sum_{n=0}^{\infty} C_n(x)$, Prem Chandra has proved main theorems in [3] and [4]. Later on, Mazhar [5] obtained the following result for summability $|\bar{N}, p_n|_k$ for $k \geq 1$.

Theorem 3.1. *Let*

$$\left\{\frac{P_n}{n}\right\} \uparrow \tag{3.1}$$

and

$$\frac{p_n}{P_n} = O\left(\frac{1}{n^{1-\alpha}}\right), \quad 0 < \alpha < 1. \tag{3.2}$$

Then, $\phi(t) \in BV[0, \pi]$ *and* $\Lambda(t)g(\frac{C}{t}) \in BV[0, \pi]$*, under the following conditions*

$$\log \frac{\pi}{t} = O(g(C/t)), \quad t \to 0 \tag{3.3}$$

$$\frac{x}{g(x)} \uparrow \tag{3.4}$$

$$x \frac{d}{dx} \frac{1}{g(C/x)} \uparrow \quad \textit{with} \quad x \tag{3.5}$$

$$\frac{d}{dx}\left(\frac{1}{g(\frac{C}{x})}\right) \downarrow \quad \textit{with} \quad x \tag{3.6}$$

$$\left[\frac{d}{dt}\left(\frac{1}{g(C/t)}\right)\right]_{t=1/n} = O\left(\frac{n}{g(n)}\right) \tag{3.7}$$

$$\sum_{1}^{\infty} \frac{1}{ng(n)} < \infty, \tag{3.8}$$

the series $\sum_{n=0}^{\infty} C_n(x)$ *is summable* $|\bar{N}, p_n|_k$*, where* C *is a positive constant such that* $g(C/t) > 0$ *for* $t > 0$.

Now we generalize Theorem 3.1 to $|\bar{N}, p_n, \theta_n|_k$ summability as in the following manner.

Theorem 3.2. *Let* $\left(\frac{\theta_n p_n}{P_n}\right)$ *be a non-increasing sequence. If* $\phi(t) \in BV[0, \pi]$ *and* $\Lambda(t)g(\frac{C}{t}) \in BV[0, \pi]$*, under the conditions* (3.1)-(3.7) *of Theorem 3.1, and*

$$\sum_{1}^{\infty} \left(\frac{\theta_n p_n}{P_n}\right)^{k-1} \frac{1}{ng(n)} < \infty, \tag{3.9}$$

then, the series $\sum_{n=0}^{\infty} C_n(x)$ *is summable* $|\bar{N}, p_n, \theta_n|_k$.

Proof of Theorem 3.2 Let $T_n(x)$ denote the $(\bar{N}, p_n)$ mean of $\sum_{n=0}^{\infty} C_n(x)$. Then

$$t_n(x) - t_{n-1}(x) = \frac{p_n}{P_n P_{n-1}} \sum_{v=1}^{n} P_{v-1} C_v(x)$$

where,

$$C_v(x) = \frac{2}{\pi} \int_0^{\pi} \phi(t) \cos vt dt$$

so that

$$\sum_{1}^{\infty} \theta_n^{k-1} |t_n(x) - t_{n-1}(x)|^k \leq \sum_{1}^{\infty} \theta_n^{k-1} \left(\frac{p_n}{P_n P_{n-1}} \right)^k$$
$$\times \left| \sum_{v=1}^{n} P_{v-1} \frac{2}{\pi} \int_0^{\pi} \Lambda(t) g(\frac{C}{t}). \frac{t}{g(C/t)}. \frac{d}{dt} \left(\frac{\sin vt}{vt} \right) dt \right|^k .$$

Since $\Lambda(t) g(\frac{C}{t}) \in BV[0, \pi]$ it is enough to prove that

$$\sum_{1}^{\infty} \theta_n^{k-1} \left(\frac{p_n}{P_n P_{n-1}} \right)^k \left| \sum_{v=1}^{n} \frac{P_{v-1}}{v} \int_0^{\pi} \frac{u}{g(C/u)}. \frac{d}{du} \left(\frac{\sin vu}{v} \right) du \right|^k < \infty$$

uniformly in $0 < t \leq \pi$. Now

$$\begin{aligned} \int_0^t \frac{u}{g(\frac{C}{u})} \frac{d}{du} \left(\frac{\sin vu}{u} \right) du =& \frac{\sin vt}{g(C/t)} - \int_0^t \frac{\sin vu}{u g(C/u)} du \\ & - \int_0^t \sin vu \frac{d}{du} \left(\frac{1}{g(C/u)} \right) du \\ =& M_1(t) + M_2(t) + M_3(t). \end{aligned}$$

Using the estimates [4] uniformly in $0 < t \leq \pi$.

$$\int_0^t \frac{\sin vu}{u g(C/u)} du = O\left(\frac{1}{g(v)} \right), \tag{3.10}$$

$$\int_0^t \sin vu \frac{d}{du} \left(\frac{1}{g(C/u)} \right) du = O\left(\frac{1}{g(v)} \right). \tag{3.11}$$

We have to show that

$$\sum_{1}^{\infty} \theta_n^{k-1} \left(\frac{p_n}{P_n P_{n-1}} \right)^k \left| \sum_{v=1}^{n} \frac{P_{v-1}}{v} M_1(t) \right|^k < \infty, \tag{3.12}$$

$$\sum_{1}^{\infty} \theta_n^{k-1} \left(\frac{p_n}{P_n P_{n-1}} \right)^k \left| \sum_{v=1}^{n} \frac{P_{v-1}}{v} M_2(t) \right|^k < \infty, \tag{3.13}$$

$$\sum_{1}^{\infty} \theta_n^{k-1} \left(\frac{p_n}{P_n P_{n-1}} \right)^k \left| \sum_{v=1}^{n} \frac{P_{v-1}}{v} M_3(t) \right|^k < \infty. \tag{3.14}$$

Writing (3.12) as

$$\sum_{1}^{T} + \sum_{n=T+1}^{\infty} = N_1 + N_2, \quad T = \left(\frac{1}{t}\right)^{\frac{1}{1-\alpha}},$$

$$\begin{aligned}
N_1 &\leq C_1 \sum_{n=1}^{T} \theta_n^{k-1} \left(\frac{p_n}{P_n P_{n-1}}\right)^k \left|\sum_{v=1}^{n} \frac{P_{v-1}}{v} \frac{\sin vt}{g(C/t)}\right|^k \\
&\leq \frac{C_1}{(g(C/t))^k} \sum_{n=1}^{T} \theta_n^{k-1} \left(\frac{p_n}{P_n P_{n-1}}\right)^k \left|\sum_{v=1}^{n} \frac{P_{v-1}}{v} \sin vt\right| \times \left|\sum_{v=1}^{n} \frac{P_{v-1}}{v} \sin vt\right|^{k-1} \\
&\leq \frac{C_1}{(g(C/t))^k} \sum_{n=1}^{T} \left(\frac{\theta_n p_n}{P_n}\right)^{k-1} \left(\frac{p_n}{P_n P_{n-1}}\right) \sum_{v=1}^{n} \frac{P_{v-1}}{v} \\
&\leq \frac{C_1}{(g(C/t))^k} \sum_{v=1}^{T} \frac{P_{v-1}}{v} \sum_{n=v}^{T} \left(\frac{\theta_n p_n}{P_n}\right)^{k-1} \left(\frac{p_n}{P_n P_{n-1}}\right) \\
&\leq \frac{C_1}{(g(C/t))^k} \sum_{v=1}^{T} \left(\frac{\theta_v p_v}{P_v}\right)^{k-1} \frac{P_{v-1}}{v} \sum_{n=v}^{T} \left(\frac{p_n}{P_n P_{n-1}}\right) \\
&\leq \frac{C_1}{(g(C/t))^k} \sum_{v=1}^{T} \left(\frac{\theta_v p_v}{P_v}\right)^{k-1} \frac{1}{v} \leq \frac{C_1}{(g(C/t))^k} \left(\frac{\theta_1 p_1}{P_1}\right)^{k-1} \sum_{v=1}^{T} \frac{1}{v} \\
&= O(1) \frac{(logC/t)}{(g(C/t))^k} = O(1) \frac{1}{(g(C/t))^{k-1}} = O(1).
\end{aligned}$$

$$\begin{aligned}
N_2 &\leq \frac{1}{(g(C/t))^k} \sum_{n=T+1}^{\infty} \left(\frac{\theta_n p_n}{P_n}\right)^{k-1} \left(\frac{p_n}{P_n P_{n-1}^k}\right) \left|\sum_{v=1}^{n} \frac{P_{v-1}}{v} \sin vt\right| \left|\sum_{v=1}^{n} \frac{P_{v-1}}{v} \sin vt\right|^{k-1} \\
&= \frac{1}{(g(C/t))^k} \sum_{n=T+1}^{\infty} \left(\frac{\theta_n p_n}{P_n}\right)^{k-1} \left(\frac{p_n}{P_n P_{n-1}}\right) \left|\sum_{v=1}^{n} \frac{P_{v-1}}{v} \sin vt\right| \\
&\leq \frac{C_1 t^{-1}}{(g(C/t))^k} \sum_{n=T+1}^{\infty} \left(\frac{\theta_n p_n}{P_n}\right)^{k-1} \left(\frac{p_n}{P_n P_{n-1}}\right) \frac{P_{n-1}}{n} \\
&= C_1 \frac{t^{-1}}{(g(C/t))^k} \sum_{n=T+1}^{\infty} \left(\frac{\theta_n p_n}{P_n}\right)^{k-1} \left(\frac{p_n}{n P_n}\right) = O(1) \frac{t^{-1}}{(g(C/t))^k} \sum_{n=T+1}^{\infty} \frac{1}{n^{2-\alpha}} \\
&= O(1) \frac{1}{(g(C/t))^k} = O(1).
\end{aligned}$$

This proves (3.12). Since $M_2(t) = O(1/g(v))$,

$$\sum_{1}^{\infty} \theta_n^{k-1} \left(\frac{p_n}{P_n P_{n-1}}\right)^k \left|\sum_{v=1}^{n} \frac{P_{v-1}}{v} M_2(t)\right|^k \leq C_1 \sum_{n=1}^{\infty} \theta_n^{k-1} \left(\frac{p_n}{P_n P_{n-1}}\right)^k \left(\sum_{v=1}^{n} \frac{P_{v-1}}{v} \frac{1}{g(v)}\right)^k$$

$$\le C_1 \sum_{n=1}^{\infty} \theta_n^{k-1} \left(\frac{p_n}{P_n P_{n-1}}\right)^k \sum_{v=1}^{n} \frac{P_{v-1}}{vg(v)} \left(\sum_{v=1}^{n} \frac{P_{v-1}}{vg(v)}\right)^{k-1}$$
$$\le C_1 \sum_{n=1}^{\infty} \left(\frac{\theta_n p_n}{P_n}\right)^{k-1} \left(\frac{p_n}{P_n P_{n-1}}\right) \sum_{v=1}^{n} \frac{P_{v-1}}{vg(v)}$$
$$= C_1 \sum_{v=1}^{\infty} \frac{P_{v-1}}{vg(v)} \sum_{n=v}^{\infty} \left(\frac{\theta_n p_n}{P_n}\right)^{k-1} \left(\frac{p_n}{P_n P_{n-1}}\right)$$
$$\le C_1 \sum_{v=1}^{\infty} \left(\frac{\theta_v p_v}{P_v}\right)^{k-1} \frac{P_{v-1}}{vg(v)} \sum_{n=v}^{\infty} \left(\frac{p_n}{P_n P_{n-1}}\right) \le C_1 \sum_{v=1}^{\infty} \left(\frac{\theta_v p_v}{P_v}\right)^{k-1} \frac{1}{vg(v)} < \infty.$$

Similarly, $M_3 = O(1/g(v))$ and as in the case of (3.13), (3.14) holds. This completes the proof of Theorem 3.2.

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References

1. H. Bor, *A note on $\mid \bar{N}, p_n \mid_k$ summability factors of infinite series*, Indian J. Pure Appl. Math., 18, 330-336, (1987).
2. H. Bor, *Absolute summability factors for infinite series*, Indian J. Pure Appl. Math., 19, 664-671, (1988).
3. P. Chandra, *Absolute Riesz summability and a new criterion for the absolute convergence of a Fourier series*, Rivista Mat. Univ. Parma, 12, 259-267, (1971).
4. P. Chandra, *Absolute summability by Riesz means*, Pacific J. Math., 34, 335-341, (1970).
5. S. M. Mazhar, *On $|\bar{N}, p_n|_k$ summability of Fourier series and its conjugate series*, Mathematica, 69, 89-96, (2004).
6. R. Mohanty, *A criterion for absolute convergence of a Fourier series*, Proc. London Math. Soc., 51, 186-196, (1949).
7. R. Mohanty, *On the absolute Riesz summability of Fourier series and allied series*, Proc. London Math. Soc., 52, 295-320, (1951).
8. H. S. Özarslan, S. Yıldız, *A new study on the absolute summability factors of Fourier series*, J. Math. Anal. 7, 31-36, (2016).
9. M. A. Sarıgöl, *On the local properties of factored Fourier series*, Appl. Math. Comp., 216, 3386-3390, (2010).
10. M. A. Sarıgöl, *Extension of Mazhar's theorem on summability factors*, Kuwait Journal of Science, 42, (2) 28-35, (2015).
11. M. A. Sarıgöl, *On two absolute Riesz summability factors of infinite series*, Proc. Amer. Math. Soc., 118, 485-488 (1993).
12. W.T. Sulaiman, *On some summability factors of infinite series*, Proc. Amer. Math. Soc., 115, 313-317, (1992).

13. S. Yıldız, *On Riesz summability factors of Fourier series*, Trans. A. Razmadze Math. Inst. 171, 328-331, (2017).

14. S. Yıldız, *A new generalization on absolute matrix summability factors of Fourier series*, J. Inequal. Spec. Funct. 8, (2), 65-73, (2017).

15. S. Yıldız, *On Absolute Matrix Summability Factors of infinite Series and Fourier Series*, Gazi Univ. J. Sci. 30, (1), 363-370, (2017).

16. S. Yıldız, *A new note on local property of factored Fourier series*, Bull. Math. Anal. Appl. 8, (4), 91-97, (2016).

17. S. Yıldız, *On Application of Matrix Summability to Fourier Series*, Math. Meth. Appl. Sci., 41, 664-670, DOI: 10.1002/mma.4635, (2018).

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