Bol. Soc. Paran. Mat. (3s.) **v. 38** 1 (2020): 221–226.
©SPM –ISSN-2175-1188 ON LINE ISSN-00378712 IN PRESS
SPM: www.spm.uem.br/bspm doi:10.5269/bspm.v38i1.33189

Construction For Fluid Flows Of Tangent Spherical Indicatrix By Flows

Talat Körpinar and Vedat Asil

ABSTRACT: In this article, we obtain the characterizations of tangent spherical indicatrix by inextensible flows in space. Moreover, we obtain some performances for curvatures of a tangent spherical indicatrix.

Key Words: Tangent spherical image, Inextensible flows, Curvatures.

Contents

1. Introduction

Recently, fluid flow theorist has been researching fluid flows in many advances, and now fluid flow is still an important field. This fields in which fluid flow plays a party are multitudinous. Gaseous flows are figured for progression of aircraft, cars and spacecrafts, and likewise for the construction of machines such as turbines and disturbance engines. Pulpy flow investigation is necessary for seafaring operation, such as ship construction, and is extensively used in civil engineering construction such as harbour construction and coastal conservation. In chemistry, learning of fluid flow in reactor tanks is significant; in medicine, the flow in blood vessels is obtained. Various other illustrations could be submited, [1-7].

In applied differential geometry, theory of curves in space is one of the significant study areas. In the theory of curves, helices, slant helices, and rectifying curves are the most fascinating curves. Flows of curves of a given curve are also widely studied, [7-9].

In our article, we obtain the characterizations of tangent spherical indicatrix by inextensible flows in space. Moreover, we obtain some performances for curvatures of a tangent spherical indicatrix.

2010 *Mathematics Subject Classification:* 31B30, 58E20.
Submitted August 22, 2016. Published July 16, 2017
[1] Corresponding author.

2. Basic Knowledge of Curves

Euclidean space $\mathbb{E}^3$ given by following standard flat metric

$$\langle , \rangle = dx_1^2 + dx_2^2 + dx_3^2,$$

where (x_1, x_2, x_3) is a rectangular coordinate system.

The sphere of radius $r > 0$ and with center in the origin in the space $\mathbb{E}^3$ is defined by

$$\mathbb{S}^2 = \left\{ p = (p_1, p_2, p_3) \in \mathbb{E}^3 : \langle p, p \rangle = r^2 \right\}.$$

$\{\mathbf{T}, \mathbf{N}, \mathbf{B}\}$ is denoted by the moving Frenet-Serret frame along the curve α in the space $\mathbb{E}^3$. For an arbitrary curve α with first and second curvature, κ and τ in the space $\mathbb{E}^3$, the following Frenet-Serret formulae are given in [2] written under matrix form

$$\begin{bmatrix} \mathbf{T}' \\ \mathbf{N}' \\ \mathbf{B}' \end{bmatrix} = \begin{bmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{bmatrix} \begin{bmatrix} \mathbf{T} \\ \mathbf{N} \\ \mathbf{B} \end{bmatrix}.$$

3. Inextensible Flows of Tangent Spherical Images in $\mathbb{E}^3$

Any flow of α can be represented by using time parameter

$$\frac{\partial \alpha}{\partial t} = \varphi_1 \mathbf{T} + \varphi_2 \mathbf{N} + \varphi_3 \mathbf{B},$$

where $\varphi_1, \varphi_2, \varphi_3$ are differentiable functions.

Theorem 3.1.

$$\begin{aligned}
\nabla_t \mathbf{T}^\beta &= -(\varphi_1 \kappa - \varphi_3 \tau + \frac{\partial \varphi_2}{\partial s})\mathbf{T} + \varpi \mathbf{B}, \\
\nabla_t \mathbf{N}^\beta &= -[\frac{\partial}{\partial t}[\frac{\kappa}{\pi}] + [\frac{\tau}{\pi}](\varphi_2 \tau + \frac{\partial \varphi_3}{\partial s})]\mathbf{T} \\
&\quad -[[\frac{\tau}{\pi}]\varpi + [\frac{\kappa}{\pi}](\varphi_1 \kappa - \varphi_3 \tau + \frac{\partial \varphi_2}{\partial s})]\mathbf{N} \\
&\quad +[\frac{\partial}{\partial t}[\frac{\tau}{\pi}] - [\frac{\kappa}{\pi}](\varphi_2 \tau + \frac{\partial \varphi_3}{\partial s})]\mathbf{B}, \qquad (3.1) \\
\nabla_t \mathbf{B}^\beta &= [\frac{\partial}{\partial t}[\frac{\tau}{\pi}] - [\frac{\kappa}{\pi}](\varphi_2 \tau + \frac{\partial \varphi_3}{\partial s})]\mathbf{T} \\
&\quad +[[\frac{\tau}{\pi}](\varphi_1 \kappa - \varphi_3 \tau + \frac{\partial \varphi_2}{\partial s}) - \varpi[\frac{\kappa}{\pi}]]\mathbf{N} \\
&\quad +[\frac{\partial}{\partial t}[\frac{\kappa}{\pi}] + [\frac{\tau}{\pi}](\varphi_2 \tau + \frac{\partial \varphi_3}{\partial s})]\mathbf{B},
\end{aligned}$$

where

$$\varpi = \langle \nabla_t \mathbf{N}, \mathbf{B} \rangle, \pi = \sqrt{\kappa^2 + \tau^2}.$$

Proof. Using definition of β, we have

$$\nabla_t \mathbf{T}^\beta = -(\varphi_1 \kappa - \varphi_3 \tau + \frac{\partial \varphi_2}{\partial s})\mathbf{T} + \varpi \mathbf{B}.$$

Hence above equation becomes

$$\begin{aligned} \nabla_t \mathbf{N}^\beta &= -[\frac{\partial}{\partial t}[\frac{\kappa}{\pi}] + [\frac{\tau}{\pi}](\varphi_2 \tau + \frac{\partial \varphi_3}{\partial s})]\mathbf{T} \\ &\quad -[[\frac{\tau}{\pi}]\varpi + [\frac{\kappa}{\pi}](\varphi_1 \kappa - \varphi_3 \tau + \frac{\partial \varphi_2}{\partial s})]\mathbf{N} \\ &\quad +[\frac{\partial}{\partial t}[\frac{\tau}{\pi}] - [\frac{\kappa}{\pi}](\varphi_2 \tau + \frac{\partial \varphi_3}{\partial s})]\mathbf{B}. \end{aligned}$$

Also

$$\begin{aligned} \nabla_t \mathbf{B}^\beta &= [\frac{\partial}{\partial t}[\frac{\tau}{\pi}] - [\frac{\kappa}{\pi}](\varphi_2 \tau + \frac{\partial \varphi_3}{\partial s})]\mathbf{T} \\ &\quad +[[\frac{\tau}{\pi}](\varphi_1 \kappa - \varphi_3 \tau + \frac{\partial \varphi_2}{\partial s}) - \varpi[\frac{\kappa}{\pi}]]\mathbf{N} \\ &\quad +[\frac{\partial}{\partial t}[\frac{\kappa}{\pi}] + [\frac{\tau}{\pi}](\varphi_2 \tau + \frac{\partial \varphi_3}{\partial s})]\mathbf{B}. \end{aligned}$$

Corollary 3.2.

$$\frac{\partial \beta}{\partial t} = (\varphi_1 \kappa - \varphi_3 \tau + \frac{\partial \varphi_2}{\partial s})\mathbf{N} + (\varphi_2 \tau + \frac{\partial \varphi_3}{\partial s})\mathbf{B}.$$

Lemma 3.3. *If* $\frac{\partial \beta}{\partial t}$ *is inextensible, then*

$$[\frac{\partial}{\partial s}(\varphi_1 \kappa - \varphi_3 \tau + \frac{\partial \varphi_2}{\partial s}) - \tau(\varphi_2 \tau + \frac{\partial \varphi_3}{\partial s})] = 0.$$

Proof.

$$\begin{aligned} \nabla_s \frac{\partial \beta}{\partial t} &= -\kappa(\varphi_1 \kappa - \varphi_3 \tau + \frac{\partial \varphi_2}{\partial s})\mathbf{T} \\ &\quad +[\frac{\partial}{\partial s}(\varphi_1 \kappa - \varphi_3 \tau + \frac{\partial \varphi_2}{\partial s}) - \tau(\varphi_2 \tau + \frac{\partial \varphi_3}{\partial s})]\mathbf{N} \\ &\quad +[\frac{\partial}{\partial s}(\varphi_2 \tau + \frac{\partial \varphi_3}{\partial s}) + \tau(\varphi_1 \kappa - \varphi_3 \tau + \frac{\partial \varphi_2}{\partial s})]\mathbf{B}. \end{aligned}$$

From above equation, we have lemma.

Case I. By using $\mathbf{T}^{\beta}$ we obtain following equations.

i.

$$\begin{aligned} & -[\frac{\kappa}{\pi}\frac{\partial}{\partial t}(\kappa\kappa^{\beta})+\kappa\kappa^{\beta}[\frac{\partial}{\partial t}[\frac{\kappa}{\pi}]+[\frac{\tau}{\pi}](\varphi_2\tau+\frac{\partial\varphi_3}{\partial s})]] \\ = & -\frac{\partial}{\partial s}(\varphi_1\kappa-\varphi_3\tau+\frac{\partial\varphi_2}{\partial s})+\mathfrak{a}_1\kappa(\varphi_2\tau+\frac{\partial\varphi_3}{\partial s}). \end{aligned}$$

ii.

$$\begin{aligned} & -\kappa\kappa^{\beta}[[\frac{\tau}{\pi}]\varpi+[\frac{\kappa}{\pi}](\varphi_1\kappa-\varphi_3\tau+\frac{\partial\varphi_2}{\partial s})] \\ = & -[\kappa(\varphi_1\kappa-\varphi_3\tau+\frac{\partial\varphi_2}{\partial s})+\tau\varpi]+\mathfrak{a}_2\kappa(\varphi_2\tau+\frac{\partial\varphi_3}{\partial s}). \end{aligned}$$

iii.

$$\begin{aligned} & [\frac{\tau}{\pi}\frac{\partial}{\partial t}(\kappa\kappa^{\beta})+\kappa\kappa^{\beta}[\frac{\partial}{\partial t}[\frac{\tau}{\pi}] \\ & -[\frac{\kappa}{\pi}](\varphi_2\tau+\frac{\partial\varphi_3}{\partial s})]]=\frac{\partial\varpi}{\partial s}+\mathfrak{a}_3\kappa(\varphi_2\tau+\frac{\partial\varphi_3}{\partial s}). \end{aligned}$$

Case II. By using $\mathbf{N}^{\beta}$ we obtain following equations.
i.

$$\begin{aligned} & [\frac{\partial}{\partial t}(\kappa\tau^{\beta})\frac{\tau}{\pi}+(\kappa\kappa^{\beta})(\varphi_1\kappa-\varphi_3\tau+\frac{\partial\varphi_2}{\partial s}) \\ & +(\kappa\tau^{\beta})[\frac{\partial}{\partial t}[\frac{\tau}{\pi}]-[\frac{\kappa}{\pi}](\varphi_2\tau+\frac{\partial\varphi_3}{\partial s})]] \\ = & [-\frac{\partial}{\partial s}[\frac{\partial}{\partial t}[\frac{\kappa}{\pi}]+[\frac{\tau}{\pi}](\varphi_2\tau+\frac{\partial\varphi_3}{\partial s})] \\ & +\kappa[[\frac{\tau}{\pi}]\varpi+[\frac{\kappa}{\pi}](\varphi_1\kappa-\varphi_3\tau+\frac{\partial\varphi_2}{\partial s})]] \\ & +[-\mathfrak{q}_1\kappa(\varphi_2\tau+\frac{\partial\varphi_3}{\partial s})\frac{\kappa}{\pi}+\mathfrak{g}_1\kappa(\varphi_2\tau+\frac{\partial\varphi_3}{\partial s})\frac{\tau}{\pi}]. \end{aligned}$$

ii.

$$\begin{aligned} & [(\kappa\tau^{\beta})[[\frac{\tau}{\pi}](\varphi_1\kappa-\varphi_3\tau+\frac{\partial\varphi_2}{\partial s})-\varpi[\frac{\kappa}{\pi}]]-\frac{\partial}{\partial t}(\kappa\kappa^{\beta})] \\ = & -[\kappa[\frac{\partial}{\partial t}[\frac{\kappa}{\pi}]+[\frac{\tau}{\pi}](\varphi_2\tau+\frac{\partial\varphi_3}{\partial s})]+\frac{\partial}{\partial s}[[\frac{\tau}{\pi}]\varpi \\ & +[\frac{\kappa}{\pi}](\varphi_1\kappa-\varphi_3\tau+\frac{\partial\varphi_2}{\partial s})]+\tau[\frac{\partial}{\partial t}[\frac{\tau}{\pi}]-[\frac{\kappa}{\pi}](\varphi_2\tau+\frac{\partial\varphi_3}{\partial s})]] \\ & +[-\mathfrak{q}_2\kappa(\varphi_2\tau+\frac{\partial\varphi_3}{\partial s})\frac{\kappa}{\pi}+\mathfrak{g}_2\kappa(\varphi_2\tau+\frac{\partial\varphi_3}{\partial s})\frac{\tau}{\pi}]. \end{aligned}$$

iii.

$$
\begin{aligned}
& [\frac{\partial}{\partial t}(\kappa\tau^{\beta})\frac{\kappa}{\pi} - (\kappa\kappa^{\beta})\varpi + (\kappa\tau^{\beta})[\frac{\partial}{\partial t}[\frac{\kappa}{\pi}] + [\frac{\tau}{\pi}](\varphi_2\tau + \frac{\partial\varphi_3}{\partial s})]] \\
= \; & [\frac{\partial}{\partial s}[\frac{\partial}{\partial t}[\frac{\tau}{\pi}] - [\frac{\kappa}{\pi}](\varphi_2\tau + \frac{\partial\varphi_3}{\partial s})] - \tau[[\frac{\tau}{\pi}]\varpi + [\frac{\kappa}{\pi}](\varphi_1\kappa \\
& -\varphi_3\tau + \frac{\partial\varphi_2}{\partial s})]] + [-\mathfrak{q}_3\kappa(\varphi_2\tau + \frac{\partial\varphi_3}{\partial s})\frac{\kappa}{\pi} + \mathfrak{g}_3\kappa(\varphi_2\tau + \frac{\partial\varphi_3}{\partial s})\frac{\tau}{\pi}].
\end{aligned}
$$

Case III. By using $\mathbf{B}^{\beta}$ we obtain following equations.
i.

$$
\begin{aligned}
& -[\frac{\partial}{\partial t}(\kappa\tau^{\beta})\frac{\kappa}{\pi} + (\kappa\tau^{\beta})[\frac{\partial}{\partial t}[\frac{\tau}{\pi}] - [\frac{\kappa}{\pi}](\varphi_2\tau + \frac{\partial\varphi_3}{\partial s})]] \\
= \; & [\frac{\partial}{\partial s}[\frac{\partial}{\partial t}[\frac{\tau}{\pi}] - [\frac{\kappa}{\pi}](\varphi_2\tau + \frac{\partial\varphi_3}{\partial s})] - \kappa[[\frac{\tau}{\pi}](\varphi_1\kappa - \varphi_3\tau + \frac{\partial\varphi_2}{\partial s}) \\
& -\varpi[\frac{\kappa}{\pi}]]] + [\mathfrak{q}_1\kappa(\varphi_2\tau + \frac{\partial\varphi_3}{\partial s})\frac{\tau}{\pi} + \mathfrak{g}_1\kappa(\varphi_2\tau + \frac{\partial\varphi_3}{\partial s})\frac{\kappa}{\pi}].
\end{aligned}
$$

ii.

$$
\begin{aligned}
& -(\kappa\tau^{\beta})[[\frac{\tau}{\pi}](\varphi_1\kappa - \varphi_3\tau + \frac{\partial\varphi_2}{\partial s}) - \varpi[\frac{\kappa}{\pi}]] \\
& = [\kappa[\frac{\partial}{\partial t}[\frac{\tau}{\pi}] - [\frac{\kappa}{\pi}](\varphi_2\tau + \frac{\partial\varphi_3}{\partial s})] + \frac{\partial}{\partial s}[[\frac{\tau}{\pi}](\varphi_1\kappa - \varphi_3\tau + \frac{\partial\varphi_2}{\partial s}) \\
& - \varpi[\frac{\kappa}{\pi}]] - \tau[\frac{\partial}{\partial t}[\frac{\kappa}{\pi}] + [\frac{\tau}{\pi}](\varphi_2\tau + \frac{\partial\varphi_3}{\partial s})]] \\
& + [\mathfrak{q}_2\kappa(\varphi_2\tau + \frac{\partial\varphi_3}{\partial s})\frac{\tau}{\pi} + \mathfrak{g}_2\kappa(\varphi_2\tau + \frac{\partial\varphi_3}{\partial s})\frac{\kappa}{\pi}].
\end{aligned}
$$

iii.

$$
\begin{aligned}
& -[\frac{\partial}{\partial t}(\kappa\tau^{\beta})\frac{\tau}{\pi} + (\kappa\tau^{\beta})[\frac{\partial}{\partial t}[\frac{\kappa}{\pi}] + [\frac{\tau}{\pi}](\varphi_2\tau + \frac{\partial\varphi_3}{\partial s})]] \\
= \; & [\tau[[\frac{\tau}{\pi}](\varphi_1\kappa - \varphi_3\tau + \frac{\partial\varphi_2}{\partial s}) - \varpi[\frac{\kappa}{\pi}]] + \frac{\partial}{\partial s}[\frac{\partial}{\partial t}[\frac{\kappa}{\pi}] \\
& +[\frac{\tau}{\pi}](\varphi_2\tau + \frac{\partial\varphi_3}{\partial s})]] + [\mathfrak{q}_3\kappa(\varphi_2\tau + \frac{\partial\varphi_3}{\partial s})\frac{\tau}{\pi} + \mathfrak{g}_3\kappa(\varphi_2\tau + \frac{\partial\varphi_3}{\partial s})\frac{\kappa}{\pi}].
\end{aligned}
$$

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Talat Körpinar,
Muş Alparslan University,
Department of Mathematics,
49250, Muş, TURKEY.
E-mail address: talatkorpinar@gmail.com

and

Vedat Asil,
Fırat University,
Department of Mathematics,
23119, Elazığ, TURKEY.
E-mail address: vedatasil@gmail.comn