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Notes on the Higher Derivations of Prime Rings

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ABSTRACT: The main purpose of these notes investigate some certain properties and relation between the higher derivation (HD, for short) and Lie ideal of semiprime ring and prime rings, we gave some results about that.

Key Words: Higher Derivation, Jordan Higher Derivation, Lie Ideal, Prime Rings, Semiprime Ring, Commutative Ring.

Contents

1. Introduction

For almost 50 years, the study of Lie isomorphisms and Lie derivations was carried on mainly by W.S.Martindal III and his students. In (1964) Martindale generalized an unpublished results of Kaplansky(obtained in the case of matrix ring over field), described Lie derivations of primitive ring of characteristic not 2 with nontrivial idempotents [1]. In subsequent papers of several authors, the analogues problem was considered either in the context of prime rings with involution [2] or in the context of von Neumann algebras under a similar assumption. In the year (1961) Herstein [3] in his AMS hour talk about Lie and Jordan Structure in Simple, Associative Rings, posted a number of problems on Lie (Jordan) isomorphisms and derivations.
Beidar and Chebotar [4] consider the Lie derivations of prime rings. Banning and Mathieu [5] extended to semiprime rings the description of Lie derivations obtained by Bresar in prime case.Villena [6]considered D a Lie derivation on an unital complex Banach algebra then for every primitive ideal P of A, except for a finite of them which have finite codimension greater than one ,there exists a derivation d from A/P to itself and a linear functional τ on A such that $Q_qD(a) = d(a+p)+\tau(a)$ for all $a \in A$(where Q_q denotes the quotient map from A onto A/P). The relation between usual derivations and Lie ideal of prime rings has been extensively studied in the last 30 years. In particular ,when this relationship involves the action of the derivations on Lie ideals. Many of these results extend other ones proven previously just for the action of the derivations on the whole ring. In (1984), Awtar [7]

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extended to Lie ideal a well-know result proved by Herstein concerning derivations in prime rings.
In fact, Awtar proved that if U is a Lie ideal of prime ring R of characteristic different of 2 such that $u^2 \in U$ for every $u \in U$ and $d : R \longrightarrow R$ is an additive mapping such that $d\backslash U$ is a Jordan derivation of U into R, then $d\backslash U$ is a derivation of U into R, where the additive mapping $d : R \longrightarrow R$ is called Jordan derivation when d satisfying the condition that $d(x^2) = d(x)x + xd(x)$ for all $x \in R$. Lanski and Montgomery [8] proved, let U_1, U_2 be Lie ideals of R then $[U_1, U_2] \subset Z(R)$, then either $U_1 \subset Z(R)$ or $U_2 \subset Z(R)$. Basudeb Dhara [9] proved, let R be a prime ring of characteristic different from 2, L a noncentral Lie ideal of R, H and G two nonzero generalized derivations of R. Suppose $us(H(u)u - uG(u))ut = 0$ for all $u \in L$, where $s, t \geq 0$ are fixed integers. Then either (i) there exists $p \in U$ such that $H(x) = xp$ for all $x \in R$ and $G(x) = px$ for all $x \in R$ unless R satisfies S_4, the standard identity in four variables; or (ii) R satisfies S_4 and there exist $p, q \in U$ such that $H(x) = px + xq$ for all $x \in R$ and $G(x) = qx + xp$ for all $x \in R$. Öznur and Emine [10] proved, let R be a prime ring with characteristic different from two,U a nonzero Lie ideal of R and f be a generalized derivation associated with d.
They prove the following results: (i) If $[u, f(u)] \in Z(R)$, for all $u \in U$, then $U \subset Z(R)$. (ii) (f, d) and (g, h) be two generalized derivations of R such that $f(u)v = ug(v)$, for all $u, v \in U$, then $U \subset Z(R)$. (iii) $f([u, v]) = \pm[u, v]$, for all $u, v \in U$, then $U \subset Z(R)$. Vincenzo De Filippis, Nadeem UR Rehman, and Abu Zaid Ansari [11] proved, let be a 2-torsion free ring and let L be a noncentral Lie ideal of R and let $F : R \longrightarrow R$ and $G : R \longrightarrow R$ be two generalized derivations of R, they analyze the structure of R in the cases:R is prime and R is semiprime ring after satisfy some conditions. In this notes we gave some results about the higher derivation (HD, for short) and Lie ideal of semiprime ring and prime rings.

2. Preliminaries

Throughout, let R be an associative ring with the center $Z(R)$. For any $a, b \in R$, the symbol $[a, b]$ stands for the commutater $ab - ba$. Given two subset A, B of R,$[A, B]$ will denote the additive subgroups of R generated by all elements of the form $[a, b]$,where $a \in A, b \in B$. During these notes we suppose that R is a prime ring i.e. if $aRb = 0$ this implies to either $a = 0$ or $b = 0$ and semiprime if $xRx = 0$ implies $x = 0$. In fact, a prime ring is semiprime but the converse is not true in general. A ring R is n-torsion free, where $n > 1$ is an integer in case $nx = 0$ implies that $x = 0$ for any $x \in R$. An additive subgroup U of R is said to be a Lie ideal of R if the commutator $[u, r] = ur - ru \in U$ for every $u \in U, r \in R$. It is clear to see that every bilateral ideal A of R is Lie ideal of R. An additive mapping $d : R \longrightarrow R$ is called a derivation if $d(xy) = d(x)y + xd(y)$, for all $x, y \in R$, and is Jordan derivation in case $d(x^2) = d(x)x + xd(x)$, is fulfilled for all $x \in R$.
Every derivation is Jordan derivation. The converse is in general not true. A derivation d is inner in case there exists $a \in R$, such that $d(x) = [a, x]$ holds for all $x \in R$.

Moreover, if U be a Lie ideal of R, D is said to be higher a derivation(HD, for short) of U into R if for every $n \in N$, we have $d_n(xy) = \sum_{i+j=n} d_i(x)d_j(y)$ for all $x, y \in U$, Jordan higher derivation (JHD, for short) of U into R if for every $n \in N$, we have $d_n(x^2) = \sum_{i+j=n} d_i(x)d_j(x)$ for all $x \in U$, where N denotes the set of natural numbers including 0, and $D = (d_i \neq 0)$ for all $i \in N$ is the family of additive mappings of R such that $d_0 = id_R$. We denote by $\tau_n(a, b, c)$ the element of R is defined by $\tau_n(a, b, c) = d_n(a, b, c) - \sum_{i+j+k=n} d_i(a)d_j(b)d_k(c)$. Then τ_n is an additive in each argument.
To achieve our purpose, we mention the following lemmas.

Lemma 2.1. *[12] If U is a Lie ideal of a semiprime ring R and $[U, U] \subseteq Z(R)$,then $U \subseteq Z(R)$.*

Lemma 2.2. *[13, Lemma3] If a prime ring R contains a non-zero commutative right ideal, then R is commutative.*

Lemma 2.3. *[14] Let R be a 2-torsion free semiprime ring (resp. prime ring and U an admissible Lie ideal of R). Then $\tau_n(a, b, c) = 0$ for every $a, b, c \in R$(resp.$a, b, c \in U$),$n \in N, N$ denotes the set of natural numbers.*

3. The main results

Theorem 3.1. *Let R be a prime ring,U be a Lie ideal of R and $D = (d_i \neq 0)i \in N$ a HD of U into R such that $[a, d_i(u)] \in Z(R)$ and $d_i(Z(R)) \neq 0$, for all $i \in N, u \in U, a \in R$, then $U \subseteq Z(R)$.*

Proof. Chose $\pi \in Z(R)$, with $d_i(\pi) \neq 0$.Then it is easy seen that $d_i(\pi) \in Z(R), i \in N$.
For $u \in U, x \in R$, we have $\pi([u, x]) = [u, \pi(x)] \in U$. So according to the hypothesis, we have $[a, d_i(\pi([u, x]))] \in Z(R)$. Then

$$\begin{aligned}
[a, d_n(\pi([u, x]))] &= [a, \sum_{i+j} d_i(\pi)d_j([u, x]) \\
&= [a, d_n(\pi)([u, x])] + [a, d_{n-1}(\pi)d_1([u, x]))] \\
&\quad +[a, d_{n-2}(\pi)d_2([u, x])] + \cdots + [a, d_1(\pi)d_{n-1}([u, x])] \\
&\quad +[a, \pi(d_n([u, x]))] \\
&= d_n(\pi)[a, [u, x]] + d_{n-1}(\pi)[a, d_1([u, x])] + \cdots \\
&\quad +d_1(\pi)[a, d_{n-1}([u, x])] + \pi[a, d_n([u, x])]
\end{aligned}$$

Since $d_j(\pi) \in Z(R)$,$\pi \in Z(R)$ and $[a, d_i([u, x])] \in Z(R), i \in N$, we obtain $d_n(\pi)([a, [u, x]]) \in Z(R)$. But we have $d_n(\pi) \in Z(R)$ and $d_n(\pi) \neq 0$. Then it follows that $[a, [u, x]] \in Z(R)$ for all $u \in U, a \in R$. This relation leads to $[a, [U, R]] \subseteq Z(R)$ this relation implies $[a, [U, U]] \subseteq Z(R)$. Thus depend on Lemma 1, we complete the proof. □

Depend on Theorem 3.1 and Lemma 2, we can easy prove the following corollary.

Corollary 3.2. *Let R be a prime ring, U be a Lie ideal of R and $D = (d_i \neq 0)_{i\in N}$ a HD of U into R such that $[a, d_i(u)] \in Z(R)$ and $d_i(Z(R)) \neq 0$, for all $i \in N, u \in U, a \in R$, then R is commutative.*

Theorem 3.3. *Let R be a prime ring, U be a Lie ideal of R and $D = (d_i \neq 0)_{i\in N}$ a HD of U into R such that $[d_i(U), d_i(U)] \subseteq Z(R)$ and $d_i(Z(R)) \neq 0, i \in N$ then R is commutative.*

Proof. According to Theorem 3.1, we obtain $d_i(U) \subseteq Z(R)$, so for all $x, y \in U$, we get $d_i(xy) \subseteq Z(R)$,that is $d_n(xy) = \sum_{i+j=n} d_i x) d_j(y)$. Thus commuting above relation with x, we obtain $[x, d_n(x)d_0(y)] + [x, d_{n-1}(x)d_1(y)] + [x, d_{n-2}(x)d_2(y)] + \cdots + [x, d_0 x)d_n(y)] = [x, d_n(x)y] = d_n(x)[x, y] + [x, d_n(x)]y$. Since $d_i(U) \subseteq Z(R), i \in N$, the above relation reduces to $d_n(x)[x, y] = 0$ for all $x, y \in U$. Left multiplying by r, we obtain $rd_n(x)[x, y] = 0$ for all $x, y \in U, r \in R$. Since $d_n(x) \in Z(R)$, therefore, we get $d_n(x)r[x, y] = 0$ for all $x, y \in U, r \in R$. Then from above equation, we achieve $d_n(x)R[x, y] = 0$. Since R is prime ring, then either $d_n(x) = 0$ or $[x, y] = 0$. The additive group of U is the union of two subgroups, then $Kerd_n$ (kernel of d_n) and $Z(U)$ is the center of U, so one of them must be whole group. According to our hypothesis $d_i \neq 0$ for all $i \in N, Kerd_n \neq U$. Thus $Z(U) = U$, therefore, U is commutative. According to Lemma 2, we complete the proof. □

Corollary 3.4. *Let R be a prime ring, U be a Lie ideal of R and $D = (d_i \neq 0)_{i\in N}$ a HD of U into R such that $[d_i(U), d_i(U)] \subseteq Z(R)$ and $d_i(Z(R)) \neq 0, i \in N$ then U is central ideal.*

Proof. By use the same method in Theorem 3.2 with depend on the fact, every central ideal is commutative, we complete the proof.

□

Theorem 3.5. *Let R be a 2-torsion free prime ring, U be a Lie ideal of R and $D = (d_i \neq 0)_{i\in N}$ a HD of U into R such that $d_i^2(U) \subseteq Z(R), d_i(Z(R)) \neq 0$ and $d_i d_j(U) \subseteq Z(R), i, j \in N$ then R is commutative.*

Proof. At first, we expanding $d_n^2([U, U]) \subseteq Z(R)$ with using $d_i^2(U) \subseteq Z(R)$. Then

$$\begin{aligned} d_n([U, U]) &= \sum_{i+j=n} [d_i(U), d_j(U)] \\ &= [d_n(U), U] + [d_{n-1}(U), d_1(U)] \\ &\quad + [d_{n-2}(U), d_2(U)] + \cdots + [U, d_n(U)], \end{aligned}$$

such that,

$$\begin{aligned} d_n^2([U, U]) &= d_n(d_n([U, U]) \\ &= \sum_{i+j=n} [d_i(d_n(U)), d_j(U)] + \cdots + \sum_{i+j=n} [d_i(U), d_j d_n(U)] \\ &= d_n^2([U, U]) + \cdots + [d_n(U), d_n(U)]. \end{aligned}$$

According to our assumption that $d_i d_j(U) \subseteq Z(R), i, j \in N$, and $d_2(U) \subseteq Z(R)$ with R be a 2-torsion free prime ring, we obtain $[d_n(U), d_n(U)] \subseteq Z(R)$. Thus, we complete the by same technique of Theorem 3.2. □

Theorem 3.6. *Let R be a prime ring, U be a Lie ideal of R and $D = (d_i \neq 0)_{i \in N}$ a HD of U into R such that $ad_i(U) \subseteq Z(R)$ and $d_i(Z(R)) \neq 0$ then either $U \subseteq Z(R)$ or $a[U, R] \subseteq Z(R)$, where $a \in R$.*

Proof. Suppose that $a \in Z(R)$, then we have either $a = 0$ or $d(U) \subseteq Z(R)$, which implies that $U \subseteq Z(R)$. We, now suppose that a not in $Z(R)$,with proceed to show that $U \subseteq Z(R)$. According to the hypothesis $d_i(Z(R)) \neq 0$,let $\vartheta \in Z(R)$ with $d_i(\vartheta) \neq 0 \in Z(R)$. For $a \in U, y \in R$, we gain $\vartheta([x, y]) = [x, \vartheta(y)] \in U$. That is $ad_n(\vartheta([x, y]) \in Z(R)$, which leads to the relation

$$\begin{aligned} ad_n(\vartheta([x,y]) &= a \sum d_i(\vartheta) d_j([x,y]) \\ &= ad_n(\vartheta)[x,y] + ad_{n-1}d_1([x,y]) + \cdots \\ &\quad + \vartheta(a) d_n([x,y]) \in Z(R). \end{aligned}$$

Again by using that $ad_i(U) \subseteq Z(R)$ and $\vartheta \in Z(R)$, with $d_i(\vartheta) \neq 0 \in Z(R)$,we arrive $d_n(\vartheta)a([x, y]) \in Z(R)$, then we have $a[x, y] \in Z(R)$ for all $x \in U, y \in R$, which implies that $a[U, R] \subseteq Z(R)$. Depend on the style of the above the proof of following corollary is evident. □

Corollary 3.7. *Let R be a prime ring, U be a Lie ideal of R and $D = (d_i \neq 0)_{i \in N}$ a HD of U into R such that $ad_i(U) \subseteq Z(R)$ and $d_i(Z(R)) \neq 0$ then either $a = 0$ or $a[U, R] \subseteq Z(R)$, where $a \in R$.*

Lemma 3.8. *Let R be a 2-torsion free ring and U is a Lie ideal of R, $D = (d_i)_{i \in N}$ a JHD of R into R and $n \in N$, then $d_n(abc + cda) = \sum_{h+j+k=n} d_h(a)d_j(b)d_k(c) + d_h(c)d_j(b)d_k(a)$ for all $a, b, c \in R$.*

Proof. First of all, we denote by $\xi_n(a, b, c)$ to the element of R is defined by $\xi_n(a, b, c) = d_n(abc) - \sum_{h+j+k=n} d_h(a)d_j(b)d_k(c)$. Then ξ_n is additive in each argument and $\xi_n(a, b, c) = 0$ for all $a, b, c \in R$. So, we have

$$\begin{aligned} \xi_n(a+c, b, a+c) &= d_n(a+c)b(a+c) - \sum_{h+j+k=n} d_h(a+c)d_j(b)d_k(a+c) \\ &= d_n(aba + cbc + abc + cba) \\ &= \sum_{i+j+k=n} d_i(a)d_j(b)d_k(a) + d_i(c)d_j(b)d_k(c) \\ &\quad + \sum_{i+j+k=n} d_h(a)d_j(b)d_k(c) + d_i(c)d_j(b)d_k(a) \\ &= 0. \end{aligned}$$

Since ξ_n is additive mapping, from above relation, we get $d_n(abc + cda) = \sum_{h+j+k=n} d_h(a)d_j(b)d_k(c) + d_h(c)d_j(b)d_k(a)$ for all $a, b, c \in R$. After above lemma, we can easy give the proof of the following corollary. □

Corollary 3.9. *Let R be a 2-torsion free ring and U is a Lie ideal of R, $D = (d_i)_{i\in N}$ a JHD of R into R and $n \in N$, then $\xi_n(c, b, a) = -\xi_n(a, b, c)$ for all $a, b, c \in R$.*

Theorem 3.10. *Let R be a 2-torsion free ring and U is a Lie ideal of R, $D = (d_i)_{i\in N}$ a JHD of R into R and $n \in N$. If $\tau_m(a, b, c) = 0$ for every $m < n, a, b, c \in U$, then $\tau_n(a, b, c)[a, b, c] + [a, b, c]r\tau_n(a, b, c) = 0$ for every $a, b, c, r \in U$.*

Proof. According to our hypothesis, we have D is JHD of R.

Then, we take $a, b, c, r \in U$, with putting $z = abcrcba + cbarabc$, we obtain $d_n(z) = d_n(a(bcrcb)a + c(barab)c)$. Then

$$\begin{aligned}
&= \sum_{i+j+k=n} (d_i(a)d_j(b(crc)b)d_k a + d_i(c)d_j(b(ara)b)d_k(c)\\
&= \sum_{i+j+k=n} d_i(a) \sum_{h+t+u=j} d_h(b)d_t(crc)d_u(b)d_k(a)\\
&\quad + \sum_{i+j+k=n} d_i(c) \sum_{h+t+u=j} d_h(b)d_t(ara)d_u(b)d_k(a)\\
&= \sum_{i+j+k=n} d_i(a) \sum_{h+t+u=j} d_h(b) \sum_{s+p+g=t} d_s(c)d_p(r)d_g(c)d_u(b)d_k(a)\\
&\quad + \sum_{I+j+k=n} d_I(c) + \sum_{h+t+u=j} d_h(b) \sum_{s+p+g=t} d_s(a)d_p(r)d_g(a)d_u(b)d_k(c)\\
&= \sum_{w=n} d_i(a)d_h(b)d_s(c)d_p(r)d_g(c)d_u(b)d_k(a)\\
&\quad + \sum_{w=n} d_i(c)d_h(b)d_s(a)d_p(r)d_g(a)d_u(b)d_k(c),
\end{aligned}$$

where $w = i + h + s + p + g + u + k$. Suppose that

$$\eta = \sum_{w=n} d_i(a)d_h(b)d_s(c)d_p(r)d_g(c)d_u(b)d_k(a)d_i(c)d_h(b)d_s(a)d_p(r)d_g(a)d_u(b)d_k(c).$$

From Lemma 3.4, we obtain

$$\begin{aligned}
d_n(z) &= d_n((abc)r(cba) + (cba)r(abc))\\
&= \sum_{\psi+\alpha+\beta} (d_\alpha(abc)d_\beta(r)d(cba) + d(cba)d_\beta(r)d_\psi(abc)).
\end{aligned}$$

According to our assumption the expression

$$\xi = \sum_{\alpha+\beta+\psi} d_\alpha(abc)d_\beta(r)d_\psi(cba) + d_f(cba)d_\beta(r)d_\psi(abc).$$

Then, we have $\xi - \eta = 0$. Now, let

$$\xi_1 = \sum_{f+\beta+\psi=n} d_f(abc)d_\beta(r)d_\psi(cba),$$

and

$$\xi_2 = \sum_{f+\beta+\psi=n} d_f(cba)d_\beta(r)d_\psi(abc).$$

Furthermore, we suppose that

$$\eta_1 = \sum_{w=n} d_i(a)d_h(b)d_s(c)d_p(r)d_q(c)d_u(b)d_k(a),$$

and

$$\eta_2 = \sum_{w=n} d_i(c)d_h(b)d_s(a)d_p(r)d_q(a)d_u(b)d_k(c).$$

Substituting $d_\alpha(abc)$ by $\sum_{i+f+h=\alpha} d_i(a)d_j(b)d_h(c)$, where $\alpha < n$. Similarly for $d_\alpha(cba)$, where $\psi < n$. Then

$$\eta_1 - \xi_1 = \tau_n(a,b,c)rcba + abcr\tau_n(c,b,a),$$

and

$$\eta_1 - \xi_2 = \tau_n(c,b,a)rabc + cbar\tau_n(a,b,c).$$

Thus, according to our hypothesis which is $\eta - \xi = 0$, we get

$$\tau_n(a,b,c)rcba + abcr\tau_n(c,b,a) + \tau_n(c,b,a)rabc + cbar\tau_n(a,b,c) = 0.$$

Apply the Corollary 3.5, we obtain $\tau_n(a,b,c)r[a,b,c] + [a,b,c]r\tau_n(a,b,c) = 0$. Thus we complete the proof of our theorem. $\square$

Depend on same technique in above theorem with use Lemma 3, we can prove the following results.

Theorem 3.11. *Let R be a 2-torsion free semiprime ring and U is a Lie ideal of R, $D = (d_i)_{i\in N}$ a JHD of R into R and $n \in N$, then $\tau_n(a,b,c)[a,b,c] + [a,b,c]r\tau_n(a,b,c) = 0$ for every $a,b,c,r \in R$, where there exists $m < n, m \in N$.*

Corollary 3.12. *Let R be a 2-torsion free prime ring and U is a dmissible Lie ideal of R, $D = (d_i)_{i\in N}$ a JHD of R into R and $n \in N$, then $\tau_n(a,b,c)[a,b,c] + [a,b,c]r\tau_n(a,b,c) = 0$ for every $a,b,c,r \in U$, where there exists $m < n, m \in N$.*

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