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Fuzzy m-Structures, m-Open Multifunctions and Bitopological Spaces

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abstract: In this paper we study different weak forms of open multifunctions from a fuzzy topological space into a fuzzy m-space. Further we study the same from a fuzzy topological space into a fuzzy bitopological space.

Key Words: Fuzzy m-structure; Fuzzy $K-m$-Open Multifunction; (i,j)-Fuzzy $K-m$-Open Multifunction; Fuzzy Bitopological Space.

Contents

1. Introduction and preliminaries

The notion of fuzzy set was introduced by L.A. Zadeh in 1965. Since than the importance of the introduced notion was realised by researchers in various fields of science and has successfully been for investigations. The notion has been applied for introducing different types of fuzzy topological spaces and investigate their properties by Alimohammady and Roohi [1], Tripathy and Debnath [21-22], Tripathy and Ray [23-24] and many others.

The notion of fuzzy bitopological spaces was introduced by Kelley [9]. Two topologies τ_1 and τ_2 are defined on the same topological space X and combining these two topologies a toplogical sapce (X, τ_1, τ_2) is obtained. Recently different properties of bitopological spaces has been investigated by Tripathy and Acharjee [20], Tripathy and Sarma [25-28] and many others.

Noiri and Popa [16] introduced the notions of minimal structures, m-spaces and m-continuity. Alimohammady and Roohi [1] introduced the concept of fuzzy minimal structure, fuzzy m-continuity and fuzzy minimal vector spaces. Using these concepts, several authors introduced and studied various types of modifications of open functions and open multifunction in m-spaces. In this paper we define fuzzy K-open function based on K-open function in the sense of Kuratowski [10] and

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introduce the notion of fuzzy $K-m$-open multifunction from a fuzzy topological space to fuzzy m-space, also between fuzzy bitopological spaces.

Let (X,τ) be a fuzzy topological space and A be a fuzzy subset of X. The closure of A and the interior of A are denoted by $Cl(A)$ and $Int(A)$, respectively.

Definition 1.1. A subset A is said to be fuzzy semi-open (respectively preopen, α-open, β-open, b-open) if $A \leq Cl(Int(A)), (A \leq Int(Cl(A)), A \leq Int(Cl(Int(A))), A \leq Cl(Int(Cl(A))), A \leq Int(Cl(A)) \bigcup Cl(Int(A)))$.

For the above definition, one may refer to Azad [2], Bin Shahna [3] and Mashhour, Ghanim and Fathalla [12].

The family of all fuzzy semi-open (respectively preopen, $\alpha-open$, $\beta-open$, b-open) sets in X is denoted by $FSO(X)$,(respectively $FPO(X)$, $F\alpha O(X)$, $F\beta O(X)$, $FBO(X)$). The complement of fuzzy semi-open (respectively preopen, $\alpha-open$, $\beta-open$, b-open) set is called fuzzy semi-closed (respectively preclosed, $\alpha-closed$, $\beta-closed$, b-closed). The intersection of all fuzzy semi-closed (respectively preclosed, $\alpha-closed$, $\beta-closed$, b-closed) sets of X containing A is called the fuzzy semi-closure (respectively preclosure, $\alpha-closure$, $\beta-closure$, b-closure) of A and is denoted by $sCl(A)$ (respectively $pCl(A)$, $\alpha Cl(A)$, $\beta Cl(A)$, $bCl(A)$). The union of all fuzzy semi-open (respectively preopen, $\alpha-open$, $\beta-open$, b-open) sets of X contained in A is called the fuzzy semi-interior (respectively pre-interior, $\alpha-interior$, $\beta-interior$, b-interior) of A and is denoted by $sInt(A)$ (respectively $pInt(A)$, $\alpha Int(A)$, $\beta Int(A)$, $bInt(A)$).

Definition 1.2. A fuzzy point x_α is said to be quasi-coincident with A, denoted by $x_\alpha qA$, if and only if $\alpha + A(x) > 1$ or $\alpha > A^c(x)$.

Definition 1.3. A fuzzy set A is said to be quasi-coincident with B and is denoted by AqB, if and only if there exists a $x \in X$ such that $A(x) + B(x) > 1$.

It is clear that A and B are quasi-coincident at x both $A(x)$ and $B(x)$ are not zero at x and hence A and B intersect at x.

Definition 1.4. A fuzzy set A in a fts (X,τ) is called a quasi-neighborhood of x_λ if and only if A_1 such that $A_1 \subseteq A$ and $x_\lambda qA_1$. The family of all Q-neighborhoods of x_λ is called the system of Q-neighborhood of x_λ. Intersection of two quasi-neighborhoods of x_λ is a quasi-neighborhood.

In a fuzzy topological space (X,τ) a fuzzy point x_p is called a fuzzy θ-cluster point of a fuzzy set A if $cl(V)qA$ holds for every open Q-neighbourhood V of x_p (one may refer to Mukharjee and Sinha [13]). The union of all fuzzy θ-cluster points of A is called a fuzzy θ-closure of A, written as $Cl_\theta(A)$ and A is called fuzzy θ-closed if $A = Cl_\theta(A)$. The complements of fuzzy θ-closed sets are called fuzzy

θ-open (one may refer to Mukharjee and Sinha [13]). The fuzzy θ-interior of a fuzzy set A in X, written as $Int_\theta(A)$, is defined to be the fuzzy set $(Cl_\theta(A^c))^c$ (see for instance Ghosh [5]).

In a fuzzy topological spaces (X,τ) a fuzzy point x_p is called a fuzzy δ-cluster point of a fuzzy set A if every fuzzy regular open Q-neighbourhood of x_p is quasi-coincident with A (one may refer to Sinha [18]). The union of all fuzzy δ-cluster points of A is called a fuzzy δ-closure of A, written as $Cl_\delta(A)$; and A is called fuzzy δ-closed if $A = Cl_\delta(A)$. The complements of fuzzy δ-closed sets are called fuzzy δ-open (please refer to Sinha [17]). The fuzzy δ-interior of a fuzzy set A in X, written as $Int_\delta(A)$, is defined to be the fuzzy set $(Cl_\delta(A^c))^c$ (one may refer to Ghosh [5]).

Throughout the paper, (X,τ) and (Y,σ) (briefly X and Y) denote fuzzy topological spaces and $F : X \to Y$ (respectively $f : X \to Y$) presents a multivalued (respectively single valued) function. For a fuzzy set $A \leq X$, $F^+(A)$ and $F^-(A)$ are defined by $F^+(A) = \{x \in X : F(x) \leq A\}$ and $F^-(A) = \{x \in X : F(x)qA\}$, respectively. It is obvious that $F^-(1-A) = 1-F^+(A)$ (see for instance Mukharjee and Malakar [14]).

Definition 1.5. A function $f : (X,\tau) \to (Y,\sigma)$ is said to be fuzzy semi-open (respectively pre-open, α-open, β-open) if $f(U)$ is semi open (respectively pre-open, α-open, β-open) for each fuzzy open set U of X.

Definition 1.6. A function $F : (X,\tau) \to (Y,\sigma)$ is said to be fuzzy open (respectively semi-open, pre-open, α-open, β-open) if $f(U)$ is open (respectively semi-open, pre-open, α-open, β-open) for each fuzzy open set U of X.

2. Fuzzy m-structures and $K - m$-open multifunctions

Definition 2.1. A subfamily m_X of I^X (where I^X is the collections of all fuzzy sets from X into $I = [0,1]$) of a non empty set X is called fuzzy minimal structure (or briefly, fuzzy m-structure on X if $\alpha 1_X \in m_X$ for any $\alpha \in I = [0,1]$ (one may refer to Alimohammady and Roohi [1])

By (X, m_X) (or briefly (X,m)), we denote a non-empty set X with a fuzzy minimal structure m_X on X and call it a fuzzy m-space. Each member of m_X is said to be fuzzy m_X-open (or briefly m-open) and the complement of an m_X-open set is said to be m_X-closed (or briefly m-closed).

We procure the following two definitions, some lemmas and Remark due to Alimohammady and Roohi [1], those will be use in this article.

Definition 2.2. Let X be a non empty set and m_X an fuzzy m-structure on X. For a fuzzy subset A of X, the m_X-closure of A and the m_X-interior of A are

defined as follows:
(1) $m_X Cl(A) = \inf \{F : A \leq F, F^{'} \in m_X\}$
(2) $m_X Int(A) = \sup \{U : U \leq A, U \in m_X\}$.

Lemma 2.1. *Let (X, m_X) be an fuzzy m-space. For fuzzy subsets A and B of X, the following properties hold*:
(1) $(m_X Cl(A))^c$=$m_X Int(A^c)$ and $(m_X Int(A))^c$= $m_X Cl(A^c)$.
(2) $m_X Cl(A) = A$ *if A is a fuzzy m_X closed set. Specially,* $m_X Cl(\alpha 1_X)$= $\alpha 1_X$, *for all* $\alpha \in I$.
(3) $m_X Int(A)$ =A *if A is a fuzzy m_X open set. Specially,* $m_X Int(\alpha 1_X)$= $\alpha 1_X$, *for all* $\alpha \in I$.
(4)if $A \leq B$, *then* $m_X Cl(A) \leq m_X Cl(B)$ *and* $m_X Int(A) \leq m_X Int(B)$.
(5)$A \leq m_X Cl(A)$ *and* $m_X Int(A) \leq A$.
(6)$m_X Cl(m_X Cl(A)) = m_X Cl(A)$ *and* $m_X Int(m_X Int(A)) = m_X Int(A)$.

Definition 2.3. A fuzzy minimal structure m_X on a non- empty set X is said to have property **B** if the union of any family of fuzzy subsets belonging to m_X belongs to m_X.

Lemma 2.2. *Let (X, m_X) be an fuzzy m-space and m_X satisfy the property* **B**. *Then for a fuzzy subset A of X, the following properties hold*:
(1) $A \in m_X$ *if and only if* $m_X Int(A) = A$.
(2) *A is m_X-closed if and only if* $m_X Cl(A) = A$.
(3) $m_X Int(A) \in m_X$ *and* $m_X Cl(A)$ *is* m_X*-closed.*

Remark 2.1. Let (X, τ) be a fuzzy topological space and $m_X = FSO(X)$ (respectively $FPO(X)$, $F\alpha O(X)$, $F\beta O(X)$, $FBO(X)$), then m_X satisfies property **B**.

Koratowski [9] defined K-open function in topological spaces. Based on this definition, we define K-open function in fuzzy topological spaces as follows.

Definition 2.4. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be fuzzy K-open if for each fuzzy point y_p of Y and for each fuzzy open set U of X such that $y_p \in f(U)$, there exists a fuzzy open set V of Y such that $y_p \in V \leq f(U)$.

Remark 2.2. A function $f : (X, \tau) \to (Y, \sigma)$ is fuzzy K-open if and only if $f(U)$ is fuzzy open in Y for each fuzzy open set U of X.

Definition 2.5. Let (Y, m_Y) be a fuzzy m-space. A multifunction $F : (X, \tau) \to (Y, m_Y)$ is said to be fuzzy $K - m$-open if for each fuzzy point y_p of Y and for each fuzzy open set U of X such that $y_p \in F(U)$, there exists $V \in m_Y$ such that $y_p \in V \leq F(U)$.

Theorem 2.1. *For a multifunction $F : (X, \tau) \to (Y, m_Y)$, the following properties are equivalent*:
(1) *F is fuzzy $K - m$-open.*

(2) $F(U) = m_Y Int(F(U))$, *for every fuzzy open set* U *of* X.
(3) $F(Int(A)) \leq m_Y Int(F(A))$, *for any fuzzy subset* A *of* X.
(4) $Int(F^+(B)) \leq F^+(m_Y Int(B))$ *for any fuzzy subset* B *of* Y.

Proof. (1)$\Rightarrow$(2) Let U be any fuzzy open set of X and $y_p \in F(U)$. Since F is fuzzy $K - m$-open, there exists $V \in mY$ such that $y_p \in V \leq F(U)$. Therefore $y_p \in m_Y Int(F(U))$. Hence $F(U) \leq m_Y Int(F(U))$, by Lemma 2.1, $F(U) = m_Y Int(F(U))$.

(2)$\Rightarrow$(3) Let A be any fuzzy subset of X. Then by (2), $F(Int(A)) = m_Y Int(F(Int(A))) \leq m_Y Int(F(A))$.

(3)$\Rightarrow$(4) Let B be any fuzzy subset of Y. By (3) we have $F(Int(F^+(B))) \leq m_Y Int(F(F^+(B))) \leq m_Y Int(B)$. Hence $Int(F^+(B)) \leq F^+(m_Y Int(B))$.

(4)$\Rightarrow$(1) Let $y_p \in Y$ and U be any fuzzy open set of X such that $y_p \in F(U)$. Since $U \leq F(F^+(U))$, by hypothesis, $U \leq Int(F(F^+(U))) \leq F^+(m_Y Int(F(U)))$. Therefore $F(U) \leq m_Y Int(F(U))$ and $y_p \in m_Y Int(F(U))$. Hence there exists $V \in m_Y$ such that $y_p \in V \leq F(U)$. This shows that F is fuzzy $K - m$-open.

Remark 2.3.(a) Let m_Y have the property **B**. Then, it follows from Lemma 2.2 and Theorem 2.1 that F is fuzzy $K - m$-open if and only if $F(U)$ is fuzzy m_Y-open for each open set U of X.

(b) If $F : (X, \tau) \to (Y, \sigma)$ is a multifunction and $m_Y = \sigma$ (respectively $FSO(Y)$, $FPO(Y)$, FαO(Y), $F\beta O(Y)$), then by (a) we get definition 1.3.

(c) If $f : (X, \tau) \to (Y, \sigma)$ is a function and $m_Y = \sigma$ (respectively $FSO(Y)$, $FPO(Y)$, $F\alpha O(Y)$, $F\beta O(Y)$), then by (a) we get definition 1.2.

3. Fuzzy m-Structures in bitopological spaces and Fuzzy $K - m$-open multifunctions in bitopological spaces

Throughout (X, τ_1, τ_2) and (Y, σ_1, σ_2) will denote fuzzy bitopological spaces. For a fuzzy subset A of X, the closure of A and the interior of A with respect to τ_i are denoted by $iCl(A)$ and $iInt(A)$ respectively, for $i = 1, 2$. First we shall recall some definitions of weak forms of open sets in a fuzzy bitopological spaces.

Definition 3.1. A fuzzy subset A of a fuzzy bitopological space (X, τ_1, τ_2) is said to be

(1) (i, j)-fuzzy semi-open if $A \leq jCl(iInt(A))$, where $i \neq j, i, j = 1, 2$.(one may refer to Kumar [6])

(2) (i,j)-fuzzy preopen if $A \leq iInt(jCl(A))$, where $i \neq j, i,j = 1,2$ (one may refer to Kumar [7]).

(3) (i,j)-fuzzy α-open if $A \leq iInt(jCl(iInt(A)))$, where $i \neq j, i,j = 1,2$ (one may refer to Kumar [7]).

(4) (i,j)-fuzzy semi-preopen if there exists an (i,j)-fuzzy preopen set U such that $U \leq A \leq jCl(U)$, where $i \neq j, i,j = 1,2$ (one may refer to Park [17]).

The family of (i,j)-fuzzy semi-open (respectively (i,j)-fuzzy preopen, (i,j)-fuzzy α-open, (i,j)-fuzzy semi-preopen) sets of (X,τ_1,τ_2) is denoted by $(i,j)FSO(X)$ (respectively $(i,j)FPO(X)$, $(i,j)F\alpha O(X)$, $(i,j)FSPO(X)$).

Remark 3.1. Let (X,τ_1,τ_2) be a fuzzy bitopological space and A be a fuzzy subsets of X. Then $(i,j)FSO(X)$, $(i,j)FPO(X)$, $(i,j)F\alpha O(X)$ and $(i,j)FSPO(X)$ are all m-structures on X. Hence, if $m_{ij} = (i,j)FSO(X)$ (respectively $(i,j)FPO(X)$, $(i,j)F\alpha O(X)$, $(i,j)FSPO(X)$), then we have

(1) $m_{ij} - Cl(A) = (i,j) - sCl(A)$ (respectively $(i,j) - pCl(A)$, $(i,j) - \alpha Cl(A)$, $(i,j) - spCl(A)$).

(2) $m_{ij} - Int(A) = (i,j) - sInt(A)$ (respectively $(i,j) - pInt(A)$, $(i,j) - \alpha Int(A), (i,j) - spInt(A)$).

Remark 3.3. Let (X,τ_1,τ_2) be a fuzzy bitopological space. Then $(i,j)FSO(X)$ (respectively $(i,j)FPO(X), (i,j)F\alpha O(X)$, $(i,j)FSPO(X)$) is an m-structures on X satisfying property **B**.

Definition 3.2. A multifunction $F : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is said to be (i,j)-fuzzy semi-open (respectively (i,j)-fuzzy preopen, (i,j)-fuzzy α-open, (i,j)-fuzzy semi-preopen) if for each τ_i open set U of X, $f(U)$ is (i,j)-fuzzy semi-open (respectively (i,j)-fuzzy preopen, (i,j)-fuzzy α-open, (i,j)-fuzzy semi-preopen) in Y.

Remark 3.3. By Remark 3.2, $(i,j)FSO(Y)$, $(i,j)FPO(Y)$, $(i,j)F\alpha O(Y)$ and $(i,j)FSPO(Y)$ are all m-structures on Y satisfying property **B**. Therefore, a multifunction $F : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is (i,j)-fuzzy semi-open (respectively (i,j)-fuzzy preopen, (i,j)-fuzzy α-open, (i,j)-fuzzy semi-preopen) if and only if $F : (X,\tau_i) \to (Y,m_{ij})$ is m-open, where $m_{ij} = (i,j)FSO(Y)$ (respectively $(i,j)FPO(Y)$, $(i,j)F\alpha O(Y)$, $(i,j)FSPO(Y)$).

Definition 3.3. Let (Y,σ_1,σ_2) be a fuzzy bitopological spaces and $m_{ij} = m(\sigma_1,\sigma_2)$, an m-structure on Y determined by σ_1 and σ_2. A multifunction $F : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is said to be (i,j)-fuzzy $K - m$-open if $F : (X,\tau_i) \to (Y,m_{ij})$ is fuzzy $K - m$-open.

Remark 3.4. (a) A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is (i, j)-fuzzy $K - m$-open if for each fuzzy point y_p of Y and for each fuzzy open set U of X such that $y_p \in F(U)$,there exists $V \in m_{ij}$ such that $y_p \in V \leq F(U)$.

(b) Let $m_{ij} = m(\sigma_1, \sigma_2)$ have property **B**. Then a multifunction $F : (X, \tau_1, \tau_2)$ $\to (Y, \sigma_1, \sigma_2)$ is (i, j)-fuzzy $K - m$-open if and only if $F(U)$ is m_{ij}-open for every τ_i-open set U of X.

(c)If $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is a multifunction, where m_{ij} has property **B**, then by definition 4.2 we get definition 4.1.

In view of Theorem 2.1 we state the following result.

Theorem 3.1. *Let (Y, σ_1, σ_2) be a fuzzy bitopological spaces and $m_{ij} = m(\sigma_1, \sigma_2)$, an m-structure on Y determined by σ_1 and σ_2. For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent :*

(1) *F is (i, j)-fuzzy $K - m$-open.*

(2) *$F(U) = m_{ij} - Int(F(U))$, for every τ_i-open set U of X.*

(3) *$F(Int(A)) \leq m_{ij} - Int(F(A))$, for any fuzzy subset A of X.*

(4) *$Int(F^+(B)) \leq F^+(m_{ij} - Int(B))$ for any fuzzy subset B of Y.*

Now we give some new forms of open multifunctions in fuzzy bitopological spaces.

Definition 3.4. A fuzzy subset A of a fuzzy bitopological space (X, τ_1, τ_2) is said to be

(1) (i, j)-fuzzy δ-semi-open if $A \leq jCl(iInt_\delta(A))$, where $i \neq j, i, j = 1, 2$.

(2) (i, j)-fuzzy δ-preopen if $A \leq iInt(jCl_\delta(A))$, where $i \neq j, i, j = 1, 2$.

(3) (i, j)-fuzzy δ-semi-preopen if there exists an (i, j)-fuzzy δ- preopen set U such that $U \leq A \leq jCl(U)$, where $i \neq j, i, j = 1, 2$.

Definition 3.5. A fuzzy subset A of a fuzzy bitopological space (X, τ_1, τ_2) is said to be

(1) (i, j)-fuzzy θ-semi-open if $A \leq jCl(iInt_\theta(A))$, where $i \neq j, i, j = 1, 2$.

(2) (i,j)-fuzzy θ-preopen if $A \leq iInt(jCl_\theta(A))$, where $i \neq j, i, j = 1, 2$.

(3) (i,j)-fuzzy θ-semi-preopen if there exists an (i,j)-fuzzy θ-preopen set U such that $U \leq A \leq jCl(U)$, where $i \neq j, i, j = 1, 2$.

The family of (i,j)-fuzzy δ- semi-open (respectively (i,j)-fuzzy δ-preopen, (i,j)-fuzzy $\delta - sp$-open, (i,j)-fuzzy θ-semi-open, (i,j)-fuzzy θ-preopen, (i,j)-fuzzy $\theta - sp$-open) sets of (X, τ_1, τ_2) is denoted by $(i,j)F\delta SO(X)$ (respectively $(i,j)F\delta PO(X)$, $(i,j)F\delta SPO(X)$, $(i,j)F\theta SO(X)$, $(i,j)F\theta PO(X)$, $(i,j)F\theta SPO(X)$).

Remark 3.5. Let (X, τ_1, τ_2) be a fuzzy bitopological space. The family $(i,j)F\delta SO(X)$, $(i,j)F\delta PO(X)$, $(i,j)F\delta SPO(X)$, $(i,j)F\theta SO(X)$, $(i,j)F\theta PO(X)$ and $(i,j)F\theta SPO(X)$) are all m-structures with property **B**.

For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, one can define many new types of (i,j)-fuzzy $K - m$-open multifunctions. For example, in case $m_{ij} = (i,j)F\delta SO(Y)$ (respectively $(i,j)F\delta PO(Y)$, $(i,j)F\delta SPO(Y)$, $(i,j)F\theta SO(Y)$, $(i,j)F\theta PO(Y)$, $(i,j)F\theta SPO(Y)$), we can define new types of (i,j)-fuzzy $K - m$-open multifunctions as follows:

Definition 3.6. A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to (i,j)-fuzzy δ-semi-open (respectively (i,j)-fuzzy δ- preopen, (i,j)-fuzzy δ-semi-preopen) if $F : (X, \tau_i) \to (Y, m_{ij})$ is (i,j) fuzzy $K - m$-open and $m_{ij} = (i,j)\delta SO(Y)$ (respectively $(i,j)\delta PO(Y)$, $(i,j)\delta SPO(Y)$).

Definition 3.7. A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be (i,j)-fuzzy θ-semi-open (respectively (i,j)-fuzzy θ-preopen, (i,j)-fuzzy θ-semi-preopen) if $F : (X, \tau_i) \to (Y, m_{ij})$ is (i,j) fuzzy $K - m$-open and $m_{ij} = (i,j)\theta SO(Y)$ (respectively $(i,j)\theta PO(Y), (i,j)\theta SPO(Y)$).

Note 3.1. The characterization theorems for the above multifunctions can also be established as in the earlier cases.

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