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Tauberian Conditions Under Which Statistical Convergence Follows From Statistical Summability $(EC)^1_n$

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ABSTRACT: Let (x_k), for $k \in \mathbb{N} \cup \{0\}$ be a sequence of real or complex numbers and set $(EC)^1_n = \frac{1}{2^n}\sum_{j=0}^{n}\binom{n}{j}\frac{1}{j+1}\sum_{v=0}^{j} x_v$, $n \in \mathbb{N}\cup\{0\}$. We present necessary and sufficient conditions, under which $st - \lim x_k = L$ follows from $st - \lim (EC)^1_n = L$, where L is a finite number. If (x_k) is a sequence of real numbers, then these are one-sided Tauberian conditions. If (x_k) is a sequence of complex numbers, then these are two-sided Tauberian conditions.

Key Words: Statistical Convergence; $(EC)^1_n-$ Summability; $(EC)^1_n-$ Statistically Convergent; One-sided and two-sided Tauberian Conditions.

Contents

1. Introduction and preliminaries

We shall denote by $\mathbb{N}$ the set of all natural numbers. Let $K \in \mathbb{N}$ and $K_n = \{k \leq n : k \in K\}$. Then the natural density of K is defined by $d(K) = \lim_{n\to\infty} \frac{|K_n|}{n}$ if the limit exists, where the vertical bars indicate the number of elements in the enclosed set. The sequence $x = (x_k)$ is said to be statistically convergent to L if for every $\epsilon > 0$, the set $K_\epsilon = \{k \in \mathbb{N} : |x_k - L| \geq \epsilon\}$ has natural density zero ([5], [8])i.e. for each $\epsilon > 0$,

$$\lim_{n\to\infty} \frac{1}{n} |\{k \leq n : |x_k - L| \geq \epsilon\}| = 0.$$

In this case, we write $L = st - \lim x_n$. Note that every convergent sequence is statistically convergent but not conversely.

Let us define the $(EC)^1_n-$ summability method as follows:

$$(EC)^1_n = \frac{1}{2^n}\sum_{k=0}^{n}\binom{n}{k} C^1_k,$$

where C^1_k denotes the Cesaro summability method. The summability method $(EC)^1_n$ is a regular.

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We say that the series $\sum_{n=1}^{\infty} x_n$ is $(EC)_n^1-$ summable to L if

$$\lim_n \frac{1}{2^n}\sum_{k=0}^{n}\binom{n}{k}\frac{1}{k+1}\sum_{v=0}^{k} x_v = L.$$

Definition 1.1. *A sequence (x_n) is weighted $(EC)_n^1-$statistically convergent to L if for every $\epsilon > 0$,*

$$\lim_{n\to\infty}\frac{1}{2^n}\left|\left\{k \le 2^n : \left|\frac{1}{2^n}\sum_{k=0}^{n}\binom{n}{k}\frac{1}{k+1}\sum_{v=0}^{k} x_v - L\right| \ge \epsilon\right\}\right| = 0.$$

Theorem 1.2. *If sequence $x = (x_n)$ is $(EC)_n^1$ summable to L, then sequence $x = (x_n)$ is $(EC)_n^1-$ statistically convergent to L. But not conversely.*

Proof: The first part of the proof is obvious. To prove the second part we will show this example:
Example We will define

$$x_k = \begin{cases} \sqrt{2^k} & , \quad \text{for} \quad k = 2^n \\ 0 & , \quad \text{otherwise} \end{cases}$$

Under this conditions we get:

$$\frac{1}{2^n}\left|\left\{k \le 2^n : \left|\frac{1}{2^n}\sum_{k=0}^{n}\binom{n}{k}\frac{1}{k+1}\sum_{v=0}^{k} x_v - 0\right| \ge \epsilon\right\}\right| \le \frac{\sqrt{2^n}}{2^n} \to 0.$$

On the other hand, if we assume that $k = 2^n$, then we obtain:

$$\frac{1}{2^n}\sum_{k=0}^{n}\binom{n}{k}\frac{1}{k+1}\sum_{v=0}^{k} x_v \to \infty \quad \text{as} \quad n \to \infty.$$

From last relation follows that $x = (x_n)$ is not $(EC)_n^1$ summable to 0. □

Theorem 1.3. *Let us suppose that sequence (x_n)-statistically convergent to L, and $|x_n - L| \le M$ for every $n \in \mathbb{N}$. Then it converges $(EC)_n^1$-statistically to L. Converse is not true.*

Proof: From fact that (x_n) converges statistically to L, we get

$$\lim_{n\to\infty}\frac{|\{k \le n : |x_k - L| \ge \epsilon\}|}{n} = 0.$$

Let us denote by $B_\epsilon = \{k \le n : |x_k - L| \ge \epsilon\}$ and $\overline{B_\epsilon} = \{k \le n : |x_k - L| \le \epsilon\}$. Then

$$\left|\frac{1}{2^n}\sum_{k=0}^{n}\binom{n}{k}\frac{1}{k+1}\sum_{v=0}^{k} x_v - L\right| = \left|\frac{1}{2^n}\sum_{k=0}^{n}\binom{n}{k}\frac{1}{k+1}\sum_{v=0}^{k}(x_v - L)\right| \le$$

$$\frac{1}{2^n}\sum_{\substack{k=0\\k\in B_\epsilon}}^{n}\binom{n}{k}\frac{1}{k+1}\sum_{v=0}^{k}|x_v-L|+\frac{1}{2^n}\sum_{\substack{k=0\\k\in \overline{B}_\epsilon}}^{n}\binom{n}{k}\frac{1}{k+1}\sum_{v=0}^{k}|x_v-L|\leq$$

$$\leq M|B_\epsilon|\frac{1}{2^n}\sum_{k=0}^{n}\binom{n}{k}\frac{1}{k+1}+\epsilon|\overline{B_\epsilon}|\frac{1}{2^n}\sum_{k=0}^{n}\binom{n}{k}\frac{1}{k+1}\leq$$

$$M|B_\epsilon|\frac{2^{n+1}-1}{2^n(n+1)}+\epsilon|\overline{B_\epsilon}|\frac{2^{n+1}-1}{2^n(n+1)}\to 0+\epsilon,\quad \text{as}\quad n\to\infty.$$

To show that converse is not true we will use into consideration this

Example Let us consider the following sequence $x=(x_n)$, which is defined as follows:

$$x_k=\begin{cases}1 & , \quad \text{for}\quad k=m^2-m,\cdots,m^2-1\\ -\frac{1}{m} & , \quad \text{for}\quad k=m^2, m=2,\cdots\\ 0 & , \quad \text{otherwise}\end{cases}$$

Under this conditions, after some calculations we get:

$$\frac{1}{2^n}\sum_{k=0}^{n}\binom{n}{k}\frac{1}{k+1}\sum_{v=0}^{k}x_v\leq\frac{1}{2^n}\sum_{k=0}^{n}\binom{n}{k}\frac{k}{k+1}-\frac{1}{2^n}\sum_{k=0}^{n}\binom{n}{k}\frac{(1+\frac{1}{2}+\cdots+\frac{1}{k})}{k+1}=$$

$$\frac{1}{2^n}\sum_{k=0}^{n}\binom{n}{k}\frac{k}{k+1}-\frac{1}{2^n}\sum_{k=0}^{n}\binom{n}{k}\frac{\ln k+C}{k+1},$$

where $C-$ is Euler constant,

$$\leq\frac{1}{2^n}\sum_{k=0}^{n}\binom{n}{k}\frac{k}{k+1}-\frac{1}{2^n}\sum_{k=0}^{n}\binom{n}{k}\frac{1}{k+1}\cdot\frac{k-1}{k}-\frac{C(2^{n+1}-1)}{2^n(n+1)}\leq$$

$$\frac{1}{2^n}\sum_{k=0}^{n}\binom{n}{k}\frac{1}{k+1}-\frac{C(2^{n+1}-1)}{2^n(n+1)}\to 0,\quad \text{as}\quad n\to\infty,$$

for every k. From last relation follows that $x=(x_n)$ is $(EC)^1_n$ summable to 0. Hence from Theorem 1.3, (x_n) is $(EC)^1_n-$ statistically convergent. On the other hand, the sequence $(m^2; m=2,3\cdots,)$ has natural density zero and it is clear that $st-\liminf_n x_n=0$ and $st-\limsup_n x_n=1$. Thus, (x_k) is not statistically convergent.

□

The theory of Tauberian theorems was investigated intensively from many authors(see [1,2,3], [6,7], [9], [4]). In this paper our aim is to find conditions (so-called

Tauberian) under which the converse implication holds, in Theorem 1.3, for defined convergence. Exactly, we will prove under which conditions statistical convergence $st-\lim x_n$, follows from $(EC)^1_n$-statistically convergence. This method generalized method given in [6].

2. Main results

Theorem 2.1. *If*

$$st-\liminf_n \frac{2^{t_n}}{2^n} > 1, \quad t > 1 \tag{2.1}$$

where t_n, denotes the integral parts of the $[tn]$ for every $n \in \mathbb{N}$, and let (x_k) be a sequence of real numbers which converges to L, $(EC)^1_n-$ statistically. Then (x_k) is $st-$ convergent to the same number L if and only if the following two conditions holds:

$$\inf_{t>1} \limsup_n \frac{1}{2^n} \left| \left\{ k \le 2^n : \frac{1}{2^{t_k}-2^k} \sum_{j=k+1}^{t_k} \binom{t_k}{j} \frac{1}{j+1} \sum_{v=0}^{j} (x_v - x_k) \le -\epsilon \right\} \right| = 0 \tag{2.2}$$

and

$$\inf_{0<t<1} \limsup_n \frac{1}{2^n} \left| \left\{ k \le 2^n : \frac{1}{2^k-2^{t_k}} \sum_{j=t_k+1}^{k} \binom{k}{j} \frac{1}{j+1} \sum_{v=0}^{j} (x_k - x_v) \le -\epsilon \right\} \right| = 0. \tag{2.3}$$

Remark 2.2. *Let us suppose that $st-\lim_k x_k = L$; (x_n) is $(EC)^1_n-$ statistically convergent and relation (2.1) satisfies, then for every $t > 1$, is valid the following relation:*

$$st-\lim_k \frac{1}{2^{t_k}-2^k} \sum_{j=k+1}^{t_k} \binom{t_k}{j} \frac{1}{j+1} \sum_{v=0}^{j} (x_v - x_k) = 0 \tag{2.4}$$

and in case where $0 < t < 1$,

$$st-\lim_k \frac{1}{2^k-2^{t_k}} \sum_{j=t_k+1}^{k} \binom{k}{j} \frac{1}{j+1} \sum_{v=0}^{j} (x_k - x_v) = 0. \tag{2.5}$$

In what follows we will show some auxiliary lemmas which are needful in the sequel.

Lemma 2.3. *Condition given by relation (2.1) is equivalent to this one:*

$$st-\liminf_n \frac{2^n}{2^{t_n}} > 1, \quad 0 < t < 1. \tag{2.6}$$

Proof: Let us suppose that relation (2.1) is valid, $0 < t < 1$ and $m = t_n = [t \cdot n]$, $n \in \mathbb{N}$. Then it follows that

$$\frac{1}{t} > 1 \Rightarrow \frac{m}{t} = \frac{[t \cdot n]}{t} \le n,$$

from above relation we obtain:

$$\frac{2^n}{2^{t_n}} \geq \frac{2^{\left[\frac{m}{t}\right]}}{2^{t_n}} \Rightarrow st - \liminf_n \frac{2^n}{2^{t_n}} \geq st - \liminf_n \frac{2^{\left[\frac{m}{t}\right]}}{2^{t_n}} > 1.$$

Conversely, let use suppose that relation (2.6) is valid. Let $t > 1$ be given number and let t_1 be chosen such that $1 < t_1 < t$. Set $m = t_n = [t \cdot n]$. From $0 < \frac{1}{t} < \frac{1}{t_1} < 1$, it follows that:

$$n \leq \frac{tn - 1}{t_1} < \frac{[tn]}{t_1} = \frac{m}{t_1},$$

provided $t_1 \leq t - \frac{1}{n}$, which is a case where if n is large enough. Under this conditions we have:

$$\frac{2^{t_n}}{2^n} \geq \frac{2^{t_n}}{2^{\left[\frac{m}{t_1}\right]}} \Rightarrow st_\lambda - \liminf_n \frac{2^{t_n}}{2^n} \geq st_\lambda - \liminf_n \frac{2^{t_n}}{2^{\left[\frac{m}{t_1}\right]}} > 1.$$

□

Lemma 2.4. *Let us suppose that relation (2.1) is satisfied and let $x = (x_k)$ be a sequence of complex numbers which is $(EC)_n^1-$statistically convergent to L. Then for every $t > 0$,*

$$st - \lim_n (EC)_{t_n}^1 = L.$$

Proof: (I) Let us consider that $t > 1$. Then

$$\lim_{n\to\infty} \frac{1}{2^n} \sum_{k=0}^{n} \binom{n}{k} \frac{1}{k+1} \sum_{v=0}^{k} x_v = \lim_{n\to\infty} \frac{1}{2^{t_n}} \sum_{k=0}^{t_n} \binom{t_n}{k} \frac{1}{k+1} \sum_{v=0}^{k} x_v, \tag{2.7}$$

and for every $\epsilon > 0$ we have:

$$\{k \leq 2^{t_n} : |(EC)_{t_n}^1 - L| \geq \epsilon\} \subset \{k \leq 2^n : |(EC)_n^1 - L| \geq \epsilon\} \cup$$

$$\left\{ k \leq 2^n : \frac{1}{2^n} \sum_{k=0}^{n} \binom{n}{k} \frac{1}{k+1} \sum_{v=0}^{k} x_v \neq \frac{1}{2^{t_n}} \sum_{k=0}^{t_n} \binom{t_n}{k} \frac{1}{k+1} \sum_{v=0}^{k} x_v \right\}.$$

Now proof of the lemma in this case follows from relation (2.2) and $st - \lim_n (EC)_n^1 = L$.

(II) In this case we have that $0 < t < 1$. For $t_n = [t \cdot n]$, for any natural number n, we can conclude that $(EC)_{t_n}^1$ does not appears more than $[1 + t^{-1}]$ times in the sequence $(EC)_n^1$. In fact if there exist integers k, l such that

$$n \leq t \cdot k < t(k+1) < \cdots < t(k+l-1) < n+1 \leq t(k+l),$$

then

$$n + t(l-1) \leq t(k+l-1) < n+1 \Rightarrow l < 1 + \frac{1}{t}.$$

And we have this estimation

$$\frac{1}{2^n}\left|\left\{k \leq 2^n : |(EC)^1_{t_n} - L| \geq \epsilon\right\}\right| \leq \left(1+\frac{1}{t}\right)\frac{1}{2^n}\left|\left\{k \leq 2^{t_n} : |(EC^1_n - L| \geq \epsilon\right\}\right| \leq$$

$$2(1+t)\frac{1}{2^{t_n}}\left|\left\{k \leq 2^{t_n} : |(EC)^1_n - L| \geq \epsilon\right\}\right|,$$

provided $\frac{1}{2^n}(\frac{t+1}{t}) \leq 2(t+1)\frac{1}{2^{t_n}}$, which is the case where n is large enough. From last relation it follows: $st - \lim_n (EC)^1_{t_n} = L$. □

Lemma 2.5. *Let us suppose that relation (2.1) is satisfied and let $x = (x_k)$ be a sequence of complex numbers which is $(EC)^1_n-$statistically convergent to L. Then for every $t > 1$,*

$$st - \lim_k \frac{1}{2^{t_k} - 2^k}\sum_{j=k+1}^{t_k}\binom{t_k}{j}\frac{1}{j+1}\sum_{v=0}^{j} x_v = L; \tag{2.8}$$

and for every $0 < t < 1$,

$$st - \lim_k \frac{1}{2^k - 2^{t_k}}\sum_{j=t_k+1}^{k}\binom{k}{j}\frac{1}{j+1}\sum_{v=0}^{j} x_v = L. \tag{2.9}$$

Proof: (I) Let us suppose that $t > 1$. After some calculations we obtain

$$(2^{t_n} - 2^n)^{-1}\sum_{j=n+1}^{t_n}\binom{t_n}{j}\frac{1}{j+1}\sum_{v=0}^{j} x_v = \frac{1}{2^n}\sum_{j=1}^{n}\binom{n}{j}\frac{1}{j+1}\sum_{v=0}^{j} x_v +$$

$$\frac{2^{t_n}}{2^{t_n} - 2^n}\left(\frac{1}{2^{t_n}}\sum_{j=1}^{t_n}\binom{t_n}{j}\frac{1}{j+1}\sum_{v=0}^{j} x_v - \frac{1}{2^n}\sum_{j=1}^{n}\binom{n}{j}\frac{1}{j+1}\sum_{v=0}^{j} x_v\right). \tag{2.10}$$

From definition of the sequence (t_n), we get

$$st - \limsup_n \frac{2^{t_n}}{2^{t_n} - 2^n} < \infty. \tag{2.11}$$

Now relation (2.8) follows from relations (2.10), (2.11) and Lemma 2.4.

(II) Case where $0 < t < 1$. In this case we have

$$(2^n - 2^{t_n})^{-1}\sum_{j=t_n+1}^{n}\binom{n}{j}\frac{1}{j+1}\sum_{v=0}^{j} x_v = \frac{1}{2^n}\sum_{j=1}^{n}\binom{n}{j}\frac{1}{j+1}\sum_{v=0}^{j} x_v +$$

$$\frac{2^{t_n}}{2^n - 2^{t_n}}\left(\frac{1}{2^n}\sum_{j=1}^{n}\binom{n}{j}\frac{1}{j+1}\sum_{v=0}^{j} x_v - \frac{1}{2^{t_n}}\sum_{j=1}^{t_n}\binom{t_n}{j}\frac{1}{j+1}\sum_{v=0}^{j} x_v\right). \tag{2.12}$$

Following Lemma 2.4, relation (2.12) and the conclusions like as in the previous case we get that relation (2.8) is valid. □

In what follows we will prove the Theorem 2.1.

Proof of Theorem 2.1

Proof: Necessity. Let us suppose that $st-\lim_k x_k = L$, and $st-\lim_k (EC)^1_k = L$. For every $t>1$ following Lemma 2.4 we get relation (2.2) and in case where $0<t<1$, again applying Lemma 2.4 we obtain relation (2.3).

Sufficient: Assume that $st-\lim_n (EC)^1_n = L$, and conditions (2.1), (2.2) and (2.3) are satisfied. In what follows we will prove that $st-\lim_n x_n = L$. Or equivalently, $st-\lim_n ((EC)^1_n - x_n) = 0$.
First we consider the case where $t>1$. We will start from this estimation

$$x_n - (EC)^1_n = \frac{2^{t_n}}{2^{t_n}-2^n}\left[\frac{1}{2^{t_n}}\sum_{j=0}^{t_n}\binom{t_n}{j}\frac{1}{j+1}\sum_{v=0}^{j}x_v - \frac{1}{2^n}\sum_{j=0}^{n}\binom{n}{j}\frac{1}{j+1}\sum_{v=0}^{j}x_v\right] -$$

$$\frac{1}{2^{t_n}-2^n}\sum_{j=n+1}^{t_n}\binom{t_n}{j}\frac{1}{j+1}\sum_{v=0}^{j}(x_v-x_n).$$

For any $\epsilon>0$, we obtain:

$$\{k\le 2^n : x_k-(EC)^1_k\ge\epsilon\}\subset\left\{k\le 2^n : \frac{2^{t_k}}{2^{t_k}-2^k}((EC)^1_{t_k}-(EC)^1_k)\ge\frac{\epsilon}{2}\right\}\cup$$

$$\left\{k\le 2^n : \frac{1}{2^{t_k}-2^k}\sum_{j=k+1}^{t_k}\binom{t_k}{j}\frac{1}{j+1}\sum_{v=0}^{j}(x_v-x_k)\le-\frac{\epsilon}{2}\right\}.$$

From relation (2.2), it follows that for every $\gamma>0$, exists a $t>1$ such that

$$\limsup_n \frac{1}{2^n}\left|\left\{k\le 2^n : \frac{1}{2^{t_k}-2^k}\sum_{j=k+1}^{t_k}\binom{t_k}{j}\frac{1}{j+1}\sum_{v=0}^{j}(x_v-x_k)\le-\epsilon\right\}\right|\le\gamma.$$

By Lemma 2.4 and relation (2.11) we get

$$\limsup_n \frac{1}{2^n}\left|\left\{k\le 2^n : |2^{t_k}(2^{t_k}-2^k)^{-1}((EC)^1_{t_k}-(EC)^1_k)|\ge\frac{\epsilon}{2}\right\}\right| = 0.$$

Combining last three relations we have:

$$\limsup_n \frac{1}{2^n}\left|\left\{k\le 2^n : x_k-(EC)^1_k\ge\epsilon\right\}\right|\le\gamma,$$

and γ is arbitrary, we conclude that for every $\epsilon>0$,

$$\limsup_n \frac{1}{2^n} \left|\left\{k \leq 2^n : x_k - (EC)^1_k \geq \epsilon\right\}\right| = 0. \tag{2.13}$$

Now we consider case where $0 < t < 1$. From above we get that:

$$x_n - (EC)^1_n = \frac{2^{t_n}}{2^n - 2^{t_n}} \left[\frac{1}{2^n} \sum_{j=0}^{n} \binom{n}{j} \frac{1}{j+1} \sum_{v=0}^{j} x_v - \frac{1}{2^{t_n}} \sum_{j=0}^{t_n} \binom{t_n}{j} \frac{1}{j+1} \sum_{v=0}^{j} x_v \right] +$$
$$\frac{1}{2^n - 2^{t_n}} \sum_{j=t_n+1}^{n} \binom{n}{j} \frac{1}{j+1} \sum_{v=0}^{j} (x_n - x_v).$$

For any $\epsilon > 0$,

$$\{k \leq 2^n : x_k - (EC)^1_k \geq \epsilon\} \subset \left\{k \leq 2^n : \frac{2^{t_k}}{2^k - 2^{t_k}} ((EC)^1_k - (EC)^1_{t_k}) \geq \frac{\epsilon}{2}\right\} \cup$$
$$\left\{k \leq 2^n : \frac{1}{2^k - 2^{t_k}} \sum_{j=t_k+1}^{k} \binom{k}{j} \frac{1}{j+1} \sum_{v=0}^{j} (x_k - x_v) \leq -\frac{\epsilon}{2}\right\}.$$

For same reasons as in the case where $t > 1$, by Lemma 2.4, we have that for every $\epsilon > 0$,

$$\limsup_n \frac{1}{2^n} \left|\left\{k \leq 2^n : x_k - (EC)^1_k \leq -\epsilon\right\}\right| = 0. \tag{2.14}$$

Finally from relations (2.13) and (2.14) we get:

$$\limsup_n \frac{1}{2^n} \left|\left\{k \leq 2^n : |x_n - (EC)^1_n| \geq \epsilon\right\}\right| = 0.$$

□

In the next result we will consider the case where $x = (x_n)$ is a sequence of complex numbers.

Theorem 2.6. *Let us suppose that relation (2.1) is satisfied. And (x_n) be a sequence of complex numbers, which is $(EC)^1_n-$ statistically convergent to L. Then (x_n) is $st-$ convergent to the same number L if and only if the following two conditions holds:*

$$\inf_{t>1} \limsup_n \frac{1}{2^n} \left|\left\{k \leq 2^n : \left|\frac{1}{2^{t_k} - 2^k} \sum_{j=k+1}^{t_k} \binom{t_k}{j} \frac{1}{j+1} \sum_{v=0}^{j} (x_v - x_k)\right| \geq \epsilon\right\}\right| = 0 \tag{2.15}$$

and

$$\inf_{0<t<1} \limsup_n \frac{1}{2^n} \left|\left\{k \leq 2^n : \left|\frac{1}{2^k - 2^{t_k}} \sum_{j=t_k+1}^{k} \binom{k}{j} \frac{1}{j+1} \sum_{v=0}^{j} (x_k - x_v)\right| \geq \epsilon\right\}\right| = 0. \tag{2.16}$$

Remark 2.7. *Let us suppose that* $st-\lim_k x_k = L$, $st-\lim_k (EC)^1_k = L$ *and relation (2.1) satisfies. Then for every* $t > 1$, *relation (2.4) holds, and in case where* $0 < t < 1$, *relation (2.5) is valid.*

Proof of Theorem 2.6. We omit it, because it is similar to the Theorem 2.1.

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