Bol. Soc. Paran. Mat. (3s.) **v. 37** 3 (2019): 67–74.
©SPM –ISSN-2175-1188 on line ISSN-00378712 in press
SPM: www.spm.uem.br/bspm doi:10.5269/bspm.v37i3.31983

On the Existence Results for a Class of Singular Elliptic System Involving Indefinite Weight Functions and Asymptotically Linear Growth Forcing Terms

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ABSTRACT: In this work, we study the existence of positive solutions to the singular system

$$\begin{cases} -\Delta_p u = \lambda a(x)f(v) - u^{-\alpha} & \text{in } \Omega, \\ -\Delta_p v = \lambda b(x)g(u) - v^{-\alpha} & \text{in } \Omega, \\ u = v = 0 & \text{on } \partial\Omega, \end{cases}$$

where λ is positive parameter, $\Delta_p u = \text{div}(|\nabla u|^{p-2}\nabla u)$, $p > 1$, $\Omega \subset R^n$ some for $n > 1$, is a bounded domain with smooth boundary $\partial\Omega$, $0 < \alpha < 1$, and $f, g : [0, \infty] \to R$ are continuous, nondecreasing functions which are asymptotically p-linear at ∞. We prove the existence of a positive solution for a certain range of λ using the method of sub-supersolutions.

Key Words: Infinite semipositone problems; Indefinite weight; Asymptotically linear growth forcing terms; Sub-supersolution method.

Contents

1. Introduction

In this article, we mainly consider the existence of a positive solution of the following singular elliptic system

$$\begin{cases} -\Delta_p u = \lambda a(x)f(v) - u^{-\alpha} & \text{in } \Omega, \\ -\Delta_p v = \lambda b(x)g(u) - v^{-\alpha} & \text{in } \Omega, \\ u = v = 0 & \text{on } \partial\Omega, \end{cases} \tag{1.1}$$

where λ is a positive parameter, $\Delta_p u = \text{div}(|\nabla u|^{p-2}\nabla u)$, $p > 1$, $\Omega \subset R^n$some for $n > 1$, is a bounded domain with smooth boundary $\partial\Omega$, $0 < \alpha < 1$, and $f, g : [0, \infty] \to R$ are continuous, nondecreasing functions which are asymptotically p-linear at ∞. We prove the existence of a positive solution for a certain range of λ.

We consider problem (1.1) under the following assumptions.

2010 *Mathematics Subject Classification:* 35J60; 35B30; 35B40.
Submitted May 18, 2016. Published April 08, 2017

(H_1) There exist $\sigma_1 > 0, k_1 > 0$ and $s_1 > 1$ such that

$$f(s) \geq \sigma_1 s^{p-1} - k_1$$

for every $s \in [0, s_1]$

and that there exist $\sigma_2 > 0, k_2 > 0$ and $s_2 > 1$ such that

$$g(s) \geq \sigma_2 s^{p-1} - k_2$$

for every $s \in [0, s_2]$,

(H_2) For all $M > 0$, $\lim_{s\to+\infty} \frac{f(M[g(s)]^{\frac{1}{p-1}})}{s^{p-1}} = \sigma$ for some $\sigma > 0$.

(H_3) $a, b : \overline{\Omega} \to (0, \infty)$ are continuous functions such that $a_1 = \min_{x\in\overline{\Omega}} a(x)$, $b_1 = \min_{x\in\overline{\Omega}} b(x)$, $a_2 = \max_{x\in\overline{\Omega}} a(x)$ and $b_2 = \max_{x\in\overline{\Omega}} b(x)$.

(H_4) There exists $\tau \in \mathbb{R}$ such that for each $M > 0, f(Ms) \leq M^\tau f(s)$ for $s \gg 1$.

Let $F(u) := \lambda a(x)f(u) - u^{-\alpha}$. The case when $F(0) < 0$ (and finite) is referred to in the literature as a semipositone problem. Finding a positive solution for a semipositone problem is well known to be challenging (see [2,5]) . Here we consider the more challenging case when $\lim_{u\to0^+} F(u) = -\infty$, which has received attention very recently and is referred to as an infinite semipositone problem. However, most of these studies have concentrated on the case when the nonlinear function satisfies a sublinear condition at ∞ (see [6,7,9]). We refer to [15,16,17,18,19] for additional results on elliptic problems. The only paper to our knowledge dealing with an infinite semipositone problem with an asymptotically linear nonlinearity is [8], where the author is restricted to the case $p = 2$. Also here the existence of a positive solution is focused near $\frac{\lambda_1}{\sigma}$, where λ_1 is the first eigenvalue of $-\Delta$. See also [1,11], where asymptotically linear nonlinearities have been discussed in he case of a nonsingular semipostione problem and an infinite positone problem. Motivated by the above papers, in this note, we are interested in the existence of positive solution for problem (1.1), where a, b are continuous functions in $\overline{\Omega}$ and λ is a positive parameter. Our main goal is to improve the result introduced in [12], in which the authors study the existence of positive solutions for an infinite semipositone problem with the nonlinearity f being not dependent of x. We shall establish our an existence result via the method of sub and supersolutions.

Definition 1.1. *We say that* (ψ_1, ψ_2) *(resp.* (z_1, z_2)*) in* $(W^{1,p}(\Omega)\cap C(\overline{\Omega}), W^{1,p}(\Omega)\cap C(\overline{\Omega}))$ *are called a subsolution (resp. a supersolution) of (1.1), if* ψ_i $(i = 1, 2)$ *satisfy*

$$\begin{cases} \int_\Omega |\nabla\psi_1(x)|^{p-2}\nabla\psi_1 \cdot \nabla w_1 dx \leq \int_\Omega \Big(\lambda a(x)f(\psi_2) - \psi_1^{-\alpha}\Big) w_1(x)dx \\ \int_\Omega |\nabla\psi_2(x)|^{p-2}\nabla\psi_2(x) \cdot \nabla w_2 dx \leq \int_\Omega \Big(\lambda b(x)g(\psi_1) - \psi_2^{-\alpha}\Big) w_2(x)dx \\ \psi_1, \psi_2 > 0 & \text{in } \Omega, \\ \psi_1 = \psi_2 = 0 & \text{on } \partial\Omega, \end{cases} \tag{1.2}$$

$\Big($*resp.* z_i $(i = 1, 2)$ *satisfy:*

$$\begin{cases} \int_\Omega |\nabla z_1|^{p-2}\nabla z_1 \cdot \nabla w_1(x)dx \geq \int_\Omega \Big(\lambda a(x) f(z_2) - z_1^{-\alpha}\Big) w_1(x)dx \\ \int_\Omega |\nabla z_2|^{p-2}\nabla z_2 \cdot \nabla w_2(x)dx \geq \int_\Omega \Big(\lambda b(x) g(z_1) - z_2^{-\alpha}\Big) w_2(x)dx \\ z_1, z_2 > 0 & \text{in } \Omega, \\ z_1 = z_2 = 0 & \text{on } \partial\Omega \end{cases}\Bigg) \tag{1.3}$$

for all non-negative test functions $w_i(i = 1, 2) \in W$, *where* $W = \{\xi \in C_0^\infty(\Omega) : \xi \geq 0 \text{ in } \Omega\}$.

The following lemma was established by Miyagaki in [14]:

Lemma 1.1 (See [14]). *If there exist sub-supersolutions* (ψ_1, ψ_2) *and* (z_1, z_2), *respectively, such that* $0 \leq \psi_i(x) \leq z_i(x)$ $(i = 1, 2)$ *for all* $x \in \Omega$, *then (1.1) has a positive solution* (u, v) *such that* $\psi_1(x) \leq u(x) \leq z_1(x)$ *and* $\psi_2(x) \leq v(x) \leq z_2(x)$ *for all* $x \in \Omega$.

2. Main result

With the hypotheses introduced in previous section, the main result of this paper is given by the following theorem.

Theorem 2.1. *Assume the conditions* $(H_1) - (H_4)$ *are satisfed. Then there exist positive constants* $s_0^*(\sigma, \Omega)$, $J^*(\Omega)$, λ_*, *and* $\lambda_{**}(> \lambda_*)$ *such that if* $min\,\{s_1, s_2\} \geq s_0^*$ *and* $\frac{min\{a_1,b_1\}min\{\sigma_1,\sigma_2\}}{(\sigma)^{\frac{p-1}{p-1+\tau}}} \geq J^*$, *problem (1.1) has a positive solution for* $\lambda \in [\lambda_*, \lambda_{**}]$.

Proof: Let μ_1 is the principal eigenvalue of operator $-\Delta_p$ with Dirichlet boundary condition. By anti-maximum principle (see [10]), there exists $\xi = \xi(\Omega) > 0$ such that the solution z_μ of

$$\begin{cases} -\Delta_p z - \mu|z|^{p-2}z = -1 & \text{in } \Omega, \\ z = 0 & \text{on } \partial\Omega \end{cases}$$

for $\mu \in (\mu_1, \mu_1 + \xi)$ is positive in Ω and is such that $\frac{\partial z_\mu}{\partial \nu} < 0$ on $\partial\Omega$, where ν is outward normal vector at $\partial\Omega$.

Since $z_\mu > 0$ in Ω and $\frac{\partial z_\mu}{\partial \nu} < 0$ there exist $m > 0$, $A > 0$, and $\delta > 0$ be such that $|\nabla z_\mu| \geq m$ in $\overline{\Omega}_\delta$ and $z_\mu \geq A$ in $\Omega \setminus \overline{\Omega}_\delta$, where $\Omega_\delta = \{x \in \Omega : d(x, \partial\Omega) \leq \delta\}$.

We first construct a supersolution for (1.1). Let

$$(z_1, z_2) = \Big(M_\lambda e_p, [\lambda b_2 g(M_\lambda \|e_p\|_\infty)]^{\frac{1}{p-1}} e_p\Big),$$

where $M_\lambda \gg 1$ is a large positive constant and e_p is the unique positive solution of

$$\begin{cases} -\Delta_p e_p = 1 & \text{in } \Omega, \\ e_p = 0 & \text{on } \partial\Omega. \end{cases}$$

By the hypothesis (H_2), we can choose $M_\lambda \gg 1$ such that

$$\frac{2\sigma}{a_2} \geq \frac{f\Big([b_2 g(M_\lambda \|e_p\|_\infty)]^{\frac{1}{p-1}}\Big)}{\Big(M_\lambda \|e_p\|_\infty\Big)^{p-1}}.$$

Then

$$-\Delta_p z_1 = M_\lambda^{p-1} \geq \frac{a_2 f\Big([b_2 g(M_\lambda \|e_p\|_\infty)]^{\frac{1}{p-1}}\Big)}{2\sigma \|e_p\|_\infty^{p-1}}.$$

Now since $\lambda \leq \frac{1}{\|e_p\|_\infty^{p-1}(2\sigma)^{\frac{p-1}{p-1+\tau}}} = \lambda_{**}$ we have

$$\begin{aligned} -\Delta_p z_1 \geq & \frac{\lambda^{\frac{p-1+\tau}{p-1}} a_2 \|e_p\|_\infty^{p-1+\tau} f\Big([b_2 g(M_\lambda \|e_p\|_\infty)]^{\frac{1}{p-1}}\Big)}{\|e_p\|_\infty^{p-1}} \\ \geq & \lambda a_2 \lambda^{\frac{\tau}{p-1}} \|e_p\|_\infty^{\tau} f\Big([b_2 g(M_\lambda \|e_p\|_\infty)]^{\frac{1}{p-1}}\Big). \end{aligned}$$

Note that (H_2) implies $g(s) \to \infty$ as $s \to \infty$. Hence from (H_3) for $M_\lambda \gg 1$ we get

$$\begin{aligned} -\Delta_p z_1 &\geq \lambda a_2 f\Big(\lambda^{\frac{1}{p-1}} \|e_p\|_\infty [b_2 g(M_\lambda \|e_p\|_\infty)]^{\frac{1}{p-1}}\Big) \\ &= \lambda a_2 f\Big(\|e_p\|_\infty [\lambda b_2 g(M_\lambda \|e_p\|_\infty)]^{\frac{1}{p-1}}\Big) \\ &\geq \lambda a_2 f(z_2) - \frac{1}{z_1^\alpha} \\ &\geq \lambda a(x) f(z_2) - \frac{1}{z_1^\alpha}. \end{aligned} \tag{2.1}$$

Also

$$-\Delta_p z_2 = \lambda b_2 g(M_\lambda \|e_p\|_\infty) \geq \lambda b_2 g(M_\lambda e_p) \geq \lambda b_2 g(z_1) - \frac{1}{z_2^\alpha} \geq \lambda b(x) g(z_1) - \frac{1}{z_2^\alpha}. \tag{2.2}$$

Hence, from relations (2.1) and (2.2) we see that (z_1, z_2) is a supersolution of problem (1.1) when $\lambda \leq \frac{1}{\|e_p\|_\infty^{p-1}(2\sigma)^{\frac{p-1}{p-1+\tau}}}$.

Define

$$(\psi_1, \psi_2) := \Big(k_0 z_\mu^{\frac{p}{p-1+\alpha}}, k_0 z_\mu^{\frac{p}{p-1+\alpha}}\Big)$$

where $k_0 > 0$ is such that

$$\frac{1}{k_0^{p-1+\alpha}} \Big(1 + \frac{k k_0^\alpha z_\mu^{\frac{\alpha p}{p-1+\alpha}}}{\|e_p\|_\infty^{p-1}(2\sigma)^{\frac{p-1}{p-1+\tau}}}\Big) \leq \min\{(x_1, x_2)\} \tag{2.3}$$

with $k=\max\{k_1,k_2\}$ and $(x_1,x_2)=\left(\left(\frac{m^p(1-\alpha)(p-1)p^{p-1}}{(p-1+\alpha)^p}\right),\left(\frac{p}{p-1+\alpha}\right)^{p-1}A\right)$. Then

$$\nabla\psi_1=k_0\Big(\frac{p}{p-1+\alpha}\Big)z_\mu^{\frac{1-\alpha}{p-1+\alpha}}\nabla z_\mu$$

and

$$\begin{aligned}
&-\Delta_p\psi_1\\
&=-\mathrm{div}(|\nabla\psi_1|^{p-2}\nabla\psi_1)\\
&=-k_0^{p-1}\Big(\frac{p}{p-1+\alpha}\Big)^{p-1}\mathrm{div}\Big(z_\mu^{\frac{(1-\alpha)(p-1)}{p-1+\alpha}}|\nabla z_\mu|^{p-2}\nabla z_\mu\Big)\\
&=-k_0^{p-1}\Big(\frac{p}{p-1+\alpha}\Big)^{p-1}\Big\{\nabla[z_\mu^{\frac{(1-\alpha)(p-1)}{p-1+\alpha}}].|\nabla z_\mu|^{p-2}\nabla z_\mu+z_\mu^{\frac{(1-\alpha)(p-1)}{p-1+\alpha}}\Delta_p z_\mu\Big\}\\
&=-k_0^{p-1}\Big(\frac{p}{p-1+\alpha}\Big)^{p-1}\Big\{\frac{(1-\alpha)(p-1)}{p-1+\alpha}z_\mu^{\frac{-\alpha p}{p-1+\alpha}}.|\nabla z_\mu|^p+z_\mu^{\frac{(1-\alpha)(p-1)}{p-1+\alpha}}(1-\mu z_\mu^{p-1})\Big\}\\
&=k_0^{p-1}\Big(\frac{p}{p-1+\alpha}\Big)^{p-1}\mu z_\mu^{\frac{p(p-1)}{p-1+\alpha}}-k_0^{p-1}\Big(\frac{p}{p-1+\alpha}\Big)^{p-1}z_\mu^{\frac{(1-\alpha)(p-1)}{p-1+\alpha}}-\\
&\qquad\frac{k_0^{p-1}p^{p-1}(1-\alpha)(p-1)|\nabla z_\mu|^p}{(p-1+\alpha)^p z_\mu^{\frac{\alpha p}{p-1+\alpha}}}.
\end{aligned}\tag{2.4}$$

Now we let $s_0^*(\sigma,\Omega)=k_0\|z_\mu^{\frac{p}{p-1+\alpha}}\|_\infty$. If we can prove

$$-\Delta_p\psi_1\le\lambda a_1\sigma_1k_0^{p-1}z_\mu^{\frac{p(p-1)}{p-1+\alpha}}-\lambda k-\frac{1}{k_0^\alpha z_\mu^{\frac{\alpha p}{p-1+\alpha}}},\tag{2.5}$$

then it implies from (H_1) that

$$-\Delta_p\psi_1\le\lambda a_1f(\psi_2)-\frac{1}{\psi_1^\alpha}\le\lambda a(x)f(\psi_2)-\frac{1}{\psi_1^\alpha}.$$

Let us prove (2.5) holds true. Let $\lambda_*=\frac{\mu(\frac{p}{p-1+\alpha})^{p-1}}{\min\{a_1,b_1\}\min(\sigma_1,\sigma_2)}$. For $\lambda\ge\lambda_*$, we get

$$k_0^{p-1}\Big(\frac{p}{p-1+\alpha}\Big)^{p-1}\mu z_\mu^{\frac{p(p-1)}{p-1+\alpha}}\le\lambda a_1\sigma_1k_0^{p-1}z_\mu^{\frac{p(p-1)}{p-1+\alpha}},\tag{2.6}$$

$$k_0^{p-1}\Big(\frac{p}{p-1+\alpha}\Big)^{p-1}\mu z_\mu^{\frac{p(p-1)}{p-1+\alpha}}\le\lambda b_1\sigma_2k_0^{p-1}z_\mu^{\frac{p(p-1)}{p-1+\alpha}}.\tag{2.7}$$

Also since $\lambda\le\lambda_{**}=\frac{1}{\|e_p\|_\infty^{p-1}(2\sigma)^{\frac{p-1}{p-1+\tau}}}$

$$\begin{aligned}
\lambda k+\frac{1}{k_0^\alpha z_\mu^{\frac{\alpha p}{p-1+\alpha}}}&\le\frac{1}{k_0^\alpha z_\mu^{\frac{\alpha p}{p-1+\alpha}}}+\frac{k}{\|e_p\|_\infty^{p-1}(2\sigma)^{\frac{p-1}{p-1+\tau}}}\\
&=\frac{k_0^{p-1}}{z_\mu^{\frac{\alpha p}{p-1+\alpha}}}\left[\frac{1}{k_0^{p-1+\alpha}}\Big(1+\frac{kk_0^\alpha z_\mu^{\frac{\alpha p}{p-1+\alpha}}}{\|e_p\|_\infty^{p-1}(2\sigma)^{\frac{p-1}{p-1+\tau}}}\Big)\right].
\end{aligned}\tag{2.8}$$

Now in Ω_δ, we have $|\nabla z_\mu| \geq m$, and by (2.3)

$$\frac{1}{k_0^{p-1+\alpha}}\Big(1+\frac{k k_0^{\alpha} z_\mu^{\frac{\alpha p}{p-1+\alpha}}}{\|e_p\|_\infty^{p-1}(2\sigma)^{\frac{p-1}{p-1+\tau}}}\Big) \leq \frac{m^p(1-\alpha)(p-1)p^{p-1}}{(p-1+\alpha)^p}.$$

Hence,

$$\lambda k + \frac{1}{k_0^{\alpha} z_\mu^{\frac{\alpha p}{p-1+\alpha}}} \leq \frac{k_0^{p-1}p^{p-1}(1-\alpha)(p-1)|\nabla z_\mu|^p}{(p-1+\alpha)^p z_\mu^{\frac{\alpha p}{p-1+\alpha}}} \quad \text{in } \Omega_\delta. \tag{2.9}$$

From (2.6) and (2.9) it can be seen that (2.5) holds in Ω_δ. A similar argument shows that

$$-\Delta_p \psi_2 \leq \lambda b_1 g(\psi_1) - \frac{1}{\psi_2^{\alpha}} \leq \lambda b(x) g(\psi_1) - \frac{1}{\psi_2^{\alpha}}.$$

We will now prove (2.5) holds also in $\Omega \setminus \Omega_\delta$. Since $z_\mu \geq A$ in $\Omega \setminus \Omega_\delta$ and by (2.3) and (2.8) we get

$$\begin{aligned}\lambda k + \frac{1}{k_0^{\alpha} z_\mu^{\frac{\alpha p}{p-1+\alpha}}} &\leq \frac{k_0^{p-1}}{z_\mu^{\frac{\alpha p}{p-1+\alpha}}}\Big(\frac{p}{p-1+\alpha}\Big)^{p-1} z_\mu \\ &\leq k_0^{p-1}\Big(\frac{p}{p-1+\alpha}\Big)^{p-1} z_\mu^{\frac{(1-\alpha)(p-1)}{p-1+\alpha}} \quad \text{in } \Omega\setminus\Omega_\delta.\end{aligned} \tag{2.10}$$

From (2.6) and (2.10), (2.5) holds also in $\Omega \setminus \overline{\Omega}_\delta$.

Thus (ψ_1, ψ_2) is a positive subsolution of (1.1) if $\lambda \in [\lambda_*, \lambda_{**}]$. We can now choose $M_\lambda \gg 1$ such that $\psi_1 \leq z_1, \psi_2 \leq z_2$. Let

$$J^*(\Omega) = 2^{\frac{p-1}{p-1+\tau}} \|e_p\|_\infty^{p-1} \mu \Big(\frac{p}{p-1+\alpha}\Big)^{p-1}.$$

If $\frac{min\{a_1,b_1\}min(\sigma_1,\sigma_2)}{(2\sigma)^{\frac{p-1}{p-1+\tau}}} \geq J^*$ it is easy to see that $\lambda_* \leq \lambda_{**}$ and for $\lambda \in [\lambda_*, \lambda_{**}]$ we have a positive solution. This completes the proof of Theorem 2.1. □

Acknowledgments

The authors are extremely grateful to the referees for their helpful suggestions for the improvement of the paper.

References

1. A. Ambrosetti, D. Arcoya, and B. Buffoni, Positive solutions for some semi-positone problems via bifurcation theory, *Diff. Int. Eqs*, **7** (1994), 655-663.

2. H. Berestycki, L.A Caffarelli, and L.Nirenberg, Inequalities for second order elliptic equations with applications to unbounded domains, *Duke Math. J.*, A Celebration of John F. Nash Jr,**81**(1996), 467-494.

3. S. Cui, Existence and nonexistence of positive solutions for singular semilinear elliptic boundary value problems, *Nonlinear Anal.*, **41** (2000), 149-176.

4. T. Laetsch, The number of solutions of a nonlinear two point boundary value problem, *Indiana Univ. Math. J*, **20** (1970), 1-13.

5. P.L. LION, On the existence of positive solutions of semilinear elliptic equations, *SIAM Rev.*, **24** (9) (1982), 441-467.

6. M. RAMASWAMY, R. SHIVAJI AND J. YE, Classes of infinite semipositone systems, *Proc. Roy. Soc. Edin.*, **139**(A) (2009), 815-853.

7. E. LEE, R. SHIVAJI, AND J. YE, Positive solutions for elliptic equations involving nonlinearities with falling zeros, *Applied Mathematics Letters*, **22**(12) (2009), 846-851.

8. D.D. HAI, On an asymptotically linear singular boundary value problem, *Topo. Meth. in Nonlin. Anal*, **39** (2012), 83-92.

9. M. RAMASWAMY, R. SHIVAJI AND J. YE, Positive solutions for a class of infinite semipositone problems, *Differential Integral Equations*, **20**(12) (2007),1423-1433.

10. T. PETER, Degenerate elliptic equations in ordered Banach spaces and applications, *Nonlinear differential equations*, **404** II Chapman , Hall/CRC, Research Notes in Mathematic, 119-196.

11. Z. ZHANG, Critical points and positive solutions of singular elliptic boundary value problems,, *J. Math. Anal. Appl*, **302** (2005), 476-483.

12. D.D. HAI , L. SANKAR, R. SHIVAJI, Infinite semipositone problem with asymptotically linear growth forcing terms, *Differential Integral Equations*, **25**(12) (2012),1175-1188.

13. P. DRABEK, P. KERJCI, P. TAKAC, *Nonlinear differential equations*, Chapman and Hall/CRC, 1999.

14. O.H. MIYAGAKI, R.S. RODRIGUES, On positive solutions for a class of singular quasilinear elliptic systems, *J. Math. Anal. Appl*, **334** (2007), 818-33.

15. M.G. CRANDALL, P.H. RABINOWITZ, L. TARTAR, On a Dirichlet problem with a singular nonlinearity, *Comm. Partial Differential Equations*, **2** (1977), 193–222.

16. M. GHERGU, V. RADULESCU, Singular elliptic problems: bifurcation and asymptotic analysis, *Oxford Lecture Series in Mathematics and its Applications, 37. The Clarendon Press, Oxford University Press, Oxford*, (2008).

17. A. GHANMI, H. MÂAGLI, V. RADULESCU, N. ZEDDINI, Large and bounded solutions for a class of nonlinear Schrödinger stationary systems, *Anal. Appl. (Singap.)* , **7** (2009),no.4, 391-404.

18. V. RADULESCU, Singular phenomena in nonlinear elliptic problems: from blow-up boundary solutions to equations with singular nonlinearities. Handbook of differential equations: stationary partial differential equations, *Handb. Differ. Equ., Elsevier/North-Holland, Amsterdam*, **Vol. IV** (2007), 485–593.

19. Z. ZHANG, R. YUAN, Infinitely-many solutions for subquadratic fractional Hamiltonian systems with potential changing sign, *Adv. Nonlinear Anal.* , **1** (2015), 59-72.

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