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Soft Ideal Topological Space and Mixed Fuzzy Soft Ideal Topological Space *

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ABSTRACT: In this paper we introduce fuzzy soft ideal and mixed fuzzy soft ideal topological spaces and some properties of this space. Also we introduce fuzzy soft I-open set, fuzzy soft α-I-open set, fuzzy soft pre-I-open set, fuzzy soft semi-I-open set and fuzzy soft β-I-open set and discuss some of their properties.

Key Words: Fuzzy soft sets, Fuzzy soft ideal, Mixed fuzzy soft topology, Soft open set, Soft semi-open set.

Contents

1. Introduction

The notion of ideal in topological spaces has been introduced by Kuratowski [8] and follwed by Vaidyanathaswamy [25]. Thereafter many topologists have contributed more to this topic. An ideal on a set X is a non-empty collection of subsets of X with heredity property which is also closed under finite unions.

Mixed topology lies in the theory of strict topology of the spaces of continuous functions on locally compact spaces. The concept of mixed topology is very old. Mixed topology is a technique of mixing two topologies on a set to get a third topology on that set. Cooper [5] Das and Baishya [6], Tripathy and Ray [21], [22], [23], [24], Wiweger [26], Mukherjee [15] and many others worked on mixed topology. The technique of mixing topologies has a number of applications in various branches of analysis, notably summability theory, measure theory, locally compact spaces, and interpolation theorems for analytic functions.

There are many theories like theory of probability, theory of fuzzy sets, theory of intuitionistic fuzzy sets, theory of rough sets etc. which can be considered as mathematical tools for dealing with uncertain data, obtained in various fields of

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engineering, physics, computer science, economics, social science, medical science, and of many other diverse fields. But all these theories have their own difficulties. The most appropriate theory for dealing with uncertainties is the theory of fuzzy sets, introduced by L.A. Zadeh [27] in 1965. This theory brought a paradigmatic change in mathematics. But there exists difficulty, how to set the membership function in each particular case. The theory of intuitionistic fuzzy sets is a more generalized concept than the theory of fuzzy sets, but this theory has the same difficulties. All the above mentioned theories are successful to some extent in dealing with problems arising due to vagueness present in the real world. But there are also cases where these theories failed to give satisfactory results, possibly due to inadequacy of the parameterization tool in them. As a necessary supplement to the existing mathematical tools for handling uncertainty, in 1999, Molodtsov [13] initiated the theory of soft sets as a new mathematical tool to deal with uncertainties while modelling the problems in engineering, physics, computer science, economics, social sciences, and medical sciences. In [14], Molodtsov et al. successfully applied soft sets in directions such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability, and theory of measurement. Maji et al. [10] gave the first practical application of soft sets in decision-making problems. In 2003, Maji et al. [11] defined and studied several basic notions of the soft set theory. Also Çağman et al. [1] studied several basic notions of the soft set theory. In 2005, Pei and Miao [17] and Chen [4] improved the work of Maji et al. [10,11]. Some properties of soft topology studied by Hussai and Ahmed [7] and Tanay et al. [20]. In 1968, C. L. Chang [3] introduced fuzzy topological space and in 2011, subsequently Çağman et al. [2] and Shabir et al. [19] introduced fuzzy soft topological spaces and they defined basic notions of soft topological spaces such as soft open and closed sets, soft subspace, soft closure, soft neighbourhood of a point, soft T_i spaces, $i = 1, 2, 3, 4$, soft regular spaces, and soft normal spaces and established their several properties. In 2012, Mahanta et al. [9], Neog et al. [16] and Ray et al. [18] introduced fuzzy soft topological spaces in different direction.

In this paper first we define and study fuzzy soft ideal and fuzzy soft ideal topological space. We establish some interesting properties of this notion. Secondly we define and study on mixed fuzzy soft ideal topological space and establish several interesting properties.

2. Preliminary Results

In this section we recall some basic concepts and definitions regarding fuzzy soft sets and fuzzy soft topological space.

Definition 2.1. *[12] Let U be an initial universe and F be a set of parameters. Let $\tilde{P}(U)$ denote the power set of U and A be a non-empty subset of F. A pair (F, A) is called a fuzzy soft set over U where $F : A \to \tilde{P}(U)$is a mapping from A into $\tilde{P}(U)$.*

Definition 2.2. *[13] A pair (F, E) is called a soft set over U if and only if F is a mapping of E into the set of all subsets of the set U.*

In other words, the soft set is a parameterized family of subsets of the set U. Every set $F(\epsilon)$, $\epsilon\tilde{\in}E$, from this family may be considered as the set of ϵ-element of the soft set (F,E), or the set of ϵ-approximate elements of the soft set.

Definition 2.3. *[18] A fuzzy soft topology τ on (U,E) is a family of fuzzy soft sets over (U,E) satisfying the following properties*

(i) $\tilde{\phi}, \tilde{E}\tilde{\in}\tau$

(ii) *if* $F_A, G_B\tilde{\in}\tau$, *then* $F_A\tilde{\cap}G_B\tilde{\in}\tau$.

(iii) *if* $F_{A_\alpha}\tilde{\in}\tau$ *for all* $\alpha\tilde{\in}\Delta$ *an index set, then* $\bigcup_{\alpha\in\Delta} F_{A_\alpha}\tilde{\in}\tau$.

Definition 2.4. *[20] A fuzzy soft set F_A in a fuzzy soft topological space (U,E,τ) is a neighborhood of a fuzzy soft set G_B if and only if there exists an open fuzzy soft set H_C i.e. $H_C\tilde{\in}\tau$ such that $G_B\tilde{\subseteq}H_C\tilde{\subseteq}F_A$.*

Definition 2.5. *[20] Let (U,E,τ_1) and (U,E,τ_2) be two fuzzy soft topological spaces. If each $F_A\tilde{\in}\tau_1$ is in τ_2, then τ_2 is called fuzzy soft finer than τ_1, or τ_1 is fuzzy soft coarser than τ_2.*

Definition 2.6. *[16] The fuzzy soft set F_A over (U,E) is called a fuzzy soft point in (U,E) denoted by $e(F_A)$, if for the element $e\tilde{\in}A, F(e)\neq\bar{0}$ and $F(e^{'})=\bar{0}$ for all $e^{'}\tilde{\in}A-\{e\}$.*

Definition 2.7. *[16] Let (U,E,τ) be a fuzzy soft topological space. Let F_A be a fuzzy soft set over (U,E). The fuzzy soft closure of F_A is defined as the intersection of all fuzzy soft closed sets which contained F_A and is denoted by $cl(F_A)$ or $\bar{F}_A$. we write*

$$cl(F_A)=\tilde{\bigcap}\{G_B : G_B \textit{ is fuzzy soft closed and } F_A\tilde{\subseteq}G_B\}.$$

Definition 2.8. *[16] Let (U,E,τ) be a fuzzy soft topological space. Let F_A be a fuzzy soft set over (U,E). The fuzzy soft interior of F_A is defined as the union of all fuzzy soft open sets which contained F_A and is denoted by $int(F_A)$ or F_A°. we write*

$$int(F_A)=\tilde{\cup}\{G_B : G_B \textit{ is fuzzy soft closed and } G_B\tilde{\subseteq}F_A\}.$$

Definition 2.9. *[16] A fuzzy soft set F_A in a fuzzy soft topological space (U,E,τ) is a fuzzy soft neighborhood of the fuzzy soft point $e(G_B)\tilde{\in}(U,E)$ if there is an open fuzzy soft set H_C such that $e(G_B)\tilde{\in}H_C\tilde{\subseteq}F_A$.*

3. Fuzzy soft ideal topological spaces

Definition 3.1. *A non-empty collection of fuzzy soft sets $\tilde{I}$ of a soft set U is called a fuzzy soft ideal if the following properties are satisfies.*

(i) *if* $F_A\tilde{\in}\tilde{I}$ *and* $G_A\tilde{\in}\tilde{I}$ *then* $F_A\tilde{\cup}G_B\tilde{\in}\tilde{I}$.

(ii) *if* $F_A\tilde{\in}\tilde{I}$ *and* $G_A\tilde{\subseteq}F_A$ *then* $G_A\tilde{\in}\tilde{I}$.

Problem 1. *Let* $\tilde{I} = \{\tilde{\phi}, F_A, G_A\}$, *where* $F_A = \{F(e_1) = \{(a, .7), (b, .9)\}, F(e_2) = \{(a, .3), (b, .6)\}\}$ *and* $G_A = \{G(e_1) = \{(a, .6), (b, .7)\}, G(e_2) = \{(a, .2), (b, .4)\}\}$. *Therefore* $\tilde{I}$ *is a fuzzy soft ideal on* (U, E), *because* $F_A \tilde{\cup} G_A = \{(e_1) = \{(a, .7), (b, .9)\}, (e_2) = \{(a, .3), (b, .6)\}\} = F_A \tilde{\in} \tilde{I}$, $G_A \tilde{\subseteq} F_A$ *and* $G_A \tilde{\in} \tilde{I}$.

Definition 3.2. *Let* (U, E, τ) *be fuzzy soft topological space and* $\tilde{I}$ *be a fuzzy soft ideal on* (U, E). *Then the triplet* $(U, \tau, \tilde{I})$ *is called fuzzy soft ideal topological space.*

Problem 2. *Let* $F_A = \{F(e_1) = \{(a, .8), (b, .8)\}, F(e_2) = \{(a, .6), (b, .6)\}\}$ *and* $G_A = \{G(e_1) = \{(a, .3), (b, .3)\}, G(e_2) = \{(a, .5), (b, .4)\}\}$, $\tau = \{\tilde{\phi}, \tilde{E}, \tilde{F_A}, \tilde{G_A}\}$ *and* $\tilde{I} = \{\tilde{\phi}, \tilde{F_A}, \tilde{G_A}\}$. *Therefore* $(U, \tau, \tilde{I})$ *is a fuzzy soft ideal topological space.*

Definition 3.3. *Let* (U, E, τ) *be fuzzy soft topological space and* $\tilde{I}$ *be a fuzzy soft ideal on* (U, E). *The fuzzy soft local function* F_A *with respect to* τ *and* $\tilde{I}$ *is denoted by* ${F_A}^*(\tau, \tilde{I})$ *or* ${F_A}^*$ *and defined as*

$$ {F_A}^* = \tilde{\cup}\{e(X_A) \tilde{\in} U : F_A \tilde{\cap} Q_A \tilde{\notin} \tilde{I}, \ \textit{for every } Q_A \tilde{\in} \tau\}. $$

Problem 3. *Let* $F_A = \{F(e_1) = \{(a, .6), (b, .2)\}, F(e_2) = \{(a, .8), (b, .7)\}\}$, $\tau = \{\tilde{\phi}, \tilde{E}, \tilde{F_A}\}$ *and* $\tilde{I} = \{\tilde{\phi}\}$. *Let* $G_A = \{G(e_1) = \{(a, .4), (b, .8)\}, G(e_2) = \{(a, .2), (b, .9)\}\}$ *and* $e(H_A) = \{H(e_1) = \{(a, .3), (b, .1)\}, H(e_2) = \{(a, 0), (b, 0)\}\}$. *The open set in* τ *containing* $e(H_A)$ *are* F_A *and* $\tilde{E}$. *Therefore* $F_A \tilde{\cap} G_A = \{(e_1) = \{(a, .4), (b, .2)\}, (e_2) = \{(a, .2), (b, .7)\}\} \tilde{\notin} \tilde{I}$ $\tilde{E} \tilde{\cap} G_A = \{(e_1) = \{(a, .4), (b, .8)\}, (e_2) = \{(a, .2), (b, .9)\}\} \tilde{\notin} \tilde{I}$. *Therefore* $e(H_A) \tilde{\in} {G_A}^*$.

Problem 4. *In problem 3 we have considered* $\tilde{I} = \{\tilde{\phi}, K_A\}$, *where* $K_A = \{K(e_1) = \{(a, .7), (b, .8)\}, K(e_2) = \{(a, .2), (b, .9)\}\}$. *The open set in* τ *containing* $e(H_A)$ *are* F_A *and* $\tilde{E}$. *Therefore* $F_A \tilde{\cap} G_A = \{(e_1) = \{(a, .4), (b, .2)\}, (e_2) = \{(a, .2), (b, .7)\}\} \tilde{\in} \tilde{I}$ $\tilde{E} \tilde{\cap} G_A = \{(e_1) = \{(a, .4), (b, .8)\}, (e_2) = \{(a, .2), (b, .9)\}\} \tilde{\in} \tilde{I}$. *Hence* $e(H_A) \tilde{\notin} {G_A}^*$.

Proposition 3.4. *Let* (U, E, τ) *be a fuzzy soft topological space and* $\tilde{I}$ *be a fuzzy soft ideal on* (U, E). *Let* F_A *and* G_A *are two fuzzy soft set. Then the followings are true.*

(i) $\tilde{\phi}^* = \tilde{\phi}$

(ii) $F_A \tilde{\subseteq} G_A \Rightarrow F_A^* \tilde{\subseteq} G_A^*$

(iii) $(F_A \tilde{\cup} G_A)^* = F_A^* \tilde{\cup} G_A^*$

(iv) $(F_A \tilde{\cap} G_A)^* \tilde{\subseteq} F_A^* \tilde{\cap} G_A^*$.

Proof: (i) Follows directly from the definitions.
(ii) Let $e(X_A) \tilde{\in} U$. Suppose $F_A \tilde{\subseteq} G_A$. Therefore we have $F_A \tilde{\subseteq} G_A$ which implies that $F_A \tilde{\cap} Q_A \tilde{\subseteq} G_A \tilde{\cap} Q_A, Q_A \tilde{\in} \tau$. Now
${F_A}^* = \{e(X_A) \tilde{\in} U : F_A \tilde{\cap} Q_A \tilde{\notin} \tilde{I}, \ \text{for every } Q_A \tilde{\in} \tau\}$
$\tilde{\subseteq} \{e(X_A) \tilde{\in} U : G_A \tilde{\cap} Q_A \tilde{\notin} \tilde{I}, \ \text{for every } Q_A \tilde{\in} \tau\} = G_A^*$.
Therefore $F_A^* \tilde{\subseteq} G_A^*$.

(iii) Let $e(X_A)\tilde{\in}(F_A\tilde{\cup}G_A)^*$. Now
$(F_A\tilde{\cup}G_A)^* = \{e(X_A)\tilde{\in}U : (F_A\tilde{\cup}G_A)\tilde{\cap}Q_A\tilde{\notin}\tilde{I},$ for every $Q_A\tilde{\in}\tau\}$
$\Rightarrow \{e(X_A)\tilde{\in}U : (F_A\tilde{\cap}Q_A)\tilde{\cup}(G_A\tilde{\cap}Q_A)\tilde{\notin}\tilde{I},$ for every $Q_A\tilde{\in}\tau\}$
$\Rightarrow \{e(X_A)\tilde{\in}U : (F_A\tilde{\cap}Q_A)\tilde{\notin}\tilde{I}or(G_A\tilde{\cap}Q_A)\tilde{\notin}\tilde{I},$ for every $Q_A\tilde{\in}\tau\}$
$\Rightarrow \{e(X_A)\tilde{\in}U : F_A\tilde{\cap}Q_A\tilde{\notin}\tilde{I},$ for every $Q_A\tilde{\in}\tau\}$
or$\{e(X_A)\tilde{\in}U : G_A\tilde{\cap}Q_A\tilde{\notin}\tilde{I},$ for every $Q_A\tilde{\in}\tau\}$
$\Rightarrow e(X_A)\tilde{\in}F_A^*\tilde{\cup}G_A^*$.
Therefore $(F_A\tilde{\cup}G_A)^*\tilde{\subseteq}F_A^*\tilde{\cup}G_A^*$. Again we have $F_A\tilde{\subseteq}F_A\tilde{\cup}G_A$ and $G_A\tilde{\subseteq}F_A\tilde{\cup}G_A$. From part (ii) we have $F_A^*\tilde{\subseteq}(F_A\tilde{\cup}G_A)^*$ and $G_A^*\tilde{\subseteq}(F_A\tilde{\cup}G_A)^*$. Therefore $F_A^*\tilde{\cup}G_A^*\tilde{\subseteq}(F_A\tilde{\cup}G_A)^*$. Hence $(F_A\tilde{\cup}G_A)^* = F_A^*\tilde{\cup}G_A^*$.
(iv) We have $F_A\tilde{\cap}G_A\tilde{\subseteq}F_A$ and $F_A\tilde{\cap}G_A\tilde{\subseteq}G_A$. From part (ii) we have $(F_A\tilde{\cap}G_A)^*\tilde{\subseteq}F_A^*$ and $(F_A\tilde{\cap}G_A)^*\tilde{\subseteq}G_A^*$. Therefore $(F_A\tilde{\cap}G_A)^*\tilde{\subseteq}F_A^*\tilde{\cap}G_A^*$. □

Definition 3.5. *Let (U, E, τ) be a fuzzy soft topological space and $\tilde{I}$ be a fuzzy soft ideal on (U, E). The fuzzy soft closure of a fuzzy soft set F_A in $(U, \tau, \tilde{I})$ is defined as $cl^*(F_A) = F_A\tilde{\cup}F_A^*$.*

Proposition 3.6. *If $F_A\tilde{\subseteq}G_A$, then $cl^*(F_A)\tilde{\subseteq}cl^*(G_A)$.*

Proof: Since $cl^*(F_A)(x) = (F_A\tilde{\cup}F_A^*)(x) = \max\{F_A(x), F_A^*(x)\}\tilde{\subseteq} \max\{G_A(x), G_A^*(x)\} = Cl^*(G_A)(x)$. Hence $cl^*(F_A)\tilde{\subseteq}cl^*(G_A)$. □

Definition 3.7. *Let $(U, \tau, \tilde{I})$ be a fuzzy soft ideal topological space. A fuzzy soft set F_A in (U, E) is said to be fuzzy soft I-open set if $F_A\tilde{\subseteq}int(F_A^*)$, where F_A^* is the fuzzy soft local function of F_A.*

Problem 5. *Let $F_A = \{F(e_1) = \{(a, .4), (b, .3)\}, F(e_2) = \{(a, .7), (b, .5)\}\}$, $\tau = \{\tilde{\phi}, \tilde{E}, \tilde{F_A}\}$ and $\tilde{I} = \{\tilde{\phi}\}$. Therefore $(U, \tau, \tilde{I})$ is a fuzzy soft topological space. Let $G_A = \{G(e_1) = \{(a, .6), (b, .4)\}, G(e_2) = \{(a, .2), (b, .3)\}\}$ be a fuzzy soft set in U. Then ${G_A}^* = \tilde{\cup}\{e(X_A)\tilde{\in}U : G_A\tilde{\cap}Q_A\tilde{\notin}\tilde{I},$ for every $Q_A\tilde{\in}\tau\}$. Let $e_1(G_A) = \{e_1 = \{(a, .3), (b, .1)\}, e_2 = \{(a, 0), (b, 0)\}\}\tilde{\in}U$. Therefore $e_1(G_A)$ containing F_A and $\tilde{E}$ in τ. Then $F_A\tilde{\cap}G_A\tilde{\notin}\tilde{I}$, $\tilde{E}\tilde{\cap}G_A\tilde{\notin}\tilde{I}$. Therefore $e_1(G_A)\tilde{\in}G_A^*$. Hence $G_A\tilde{\subseteq}int(G_A^*)$, i.e. G_A is a fuzzy soft I-open set in U.*

Definition 3.8. *Let $(U, \tau, \tilde{I})$ is a fuzzy soft ideal topological space. A fuzzy soft set F_A in (U, E) is said to be*

(a) fuzzy soft α-I-open set if $F_A\tilde{\subseteq}int(cl^*(int(F_A)))$.

(b) fuzzy soft pre-I-open set if $F_A\tilde{\subseteq}int(cl^*(F_A)$.

(c) fuzzy soft semi-I-open set if $F_A\tilde{\subseteq}cl^*(int(F_A)$.

(d) fuzzy soft β-I-open set if $F_A\tilde{\subseteq}cl(int(cl^*(F_A))$.

Problem 6. *Let* $F_A = \{F(e_1) = \{(a, .4), (b, .3)\}, F(e_2) = \{(a, .7), (b, .5)\}\}$, $\tau = \{\tilde{\phi}, \tilde{E}, \tilde{F_A}\}$ *and* $\tilde{I} = \{\tilde{\phi}\}$. *Therefore* $(U, \tau, \tilde{I})$ *is a fuzzy soft topological space. Let* $L_A = \{L(e_1) = \{(a, .6), (b, .7)\}, L(e_2) = \{(a, .8), (b, .9)\}\}$ *be a fuzzy soft set in* U. *Then,* $int(L_A) = \{(e_1) = \{(a, .4), (b, .3)\}, (e_2) = \{(a, .7), (b, .5)\}\} = F_A$. *Then* $cl^*(int(L_A)) = cl^*(F_A)$. *Since* $F_A^* \tilde{\notin} \tilde{\phi}$ *and* $F_A^* \tilde{\notin} \tilde{E}$. *So,* F_A^* *is fuzzy soft closed in* τ. *Hence* $F_A^* = \{e_1 = \{(a, .6), (b, .7)\}, e_2 = \{(a, .3), (b, .5)\}\}$. *Therefore* $F_A \tilde{\cup} F_A^* = \{e_1 = \{(a, .6), (b, .7)\}, e_2 = \{(a, .7), (b, .5)\}\}$. *i.e.* $cl^*(F_A) = \{e_1 = \{(a, .6), (b, .7)\}, e_2 = \{(a, .7), (b, .5)\}\}$. *Therefore* $cl^*(int(L_A)) = cl^*(F_A) = \{e_1 = \{(a, .6), (b, .7)\}, e_2 = \{(a, .7), (b, .5)\}\}$. *Hence* $int(cl^*(int(L_A))) = \{e_1 = \{(a, 1), (b, 1)\}, e_2 = \{(a, 1), (b, 1)\}\}$. *Therefore* $L_A \tilde{\subseteq} int(cl^*(int(L_A)))$. *i.e.* L_A *be a fuzzy soft* α-I-*open set.*

Problem 7. *Let* $F_A = \{F(e_1) = \{(a, .8), (b, .8)\}, F(e_2) = \{(a, .6), (b, .6)\}\}$, $G_A = \{G(e_1) = \{(a, .2), (b, .2)\}, G(e_2) = \{(a, .4), (b, .4)\}\}$, $\tilde{I} = \{\tilde{\phi}\}$ *and* $\tau = \{\tilde{\phi}, \tilde{E}, \tilde{F_A}, \tilde{G_A}\}$. *Then* $(U, \tau, \tilde{I})$ *be a fuzzy soft ideal topological space. Again consider the fuzzy soft set* $H_A = \{H(e_1) = \{(a, .6), (b, .3)\}, H(e_2) = \{(a, .9), (b, .2)\}\}$ *in* U. *Since* $H_A^* \tilde{\notin} \tilde{\phi}$ *and* $H_A^* \tilde{\notin} \tilde{E}$. *So,* H_A^* *is fuzzy soft closed in* τ. *Hence* $H_A^* = \{e_1 = \{(a, .4), (b, .7)\}, e_2 = \{(a, .1), (b, .8)\}\}$. *Therefore* $H_A \tilde{\cup} H_A^* = \{e_1 = \{(a, .6), (b, .7)\}, e_2 = \{(a, .9), (b, .5)\}\}$. *i.e.* $cl^*(H_A) = \{e_1 = \{(a, .6), (b, .7)\}, e_2 = \{(a, .9), (b, .5)\}\}$. *Therefore* $H_A \tilde{\subseteq} int(Cl^*(H_A))$. *Hence* H_A *is a fuzzy soft pre-I-open set.*
Also $H_A \tilde{\subseteq} cl(int(cl^*(H_A)))$. *Hence* H_A *is a fuzzy soft* β-I-*open set.*

Problem 8. *Let* $F_A = \{F(e_1) = \{(a, .4), (b, .3)\}, F(e_2) = \{(a, .7), (b, .5)\}\}$, $\tau = \{\tilde{\phi}, \tilde{E}, \tilde{F_A}\}$ *and* $\tilde{I} = \{\tilde{\phi}\}$. *Therefore* $(U, \tau, \tilde{I})$ *is a fuzzy soft topological space. Let* $P_A = \{P(e_1) = \{(a, .6), (b, .7)\}, P(e_2) = \{(a, .3), (b, .5)\}\}$ *be a fuzzy soft set in* U. *Then* $int(P_A) = \{(e_1) = \{(a, .4), (b, .3)\}, (e_2) = \{(a, .7), (b, .5)\}\} = F_A$. *Then* $cl^*(int(L_A)) = cl^*(F_A)$. *Since* $F_A^* \tilde{\notin} \tilde{\phi}$ *and* $F_A^* \tilde{\notin} \tilde{E}$. *So* F_A^* *is fuzzy soft closed in* τ. *Hence* $F_A^* = \{e_1 = \{(a, .6), (b, .7)\}, e_2 = \{(a, .3), (b, .5)\}\}$. *Therefore* $F_A \tilde{\cup} F_A^* = \{e_1 = \{(a, .6), (b, .7)\}, e_2 = \{(a, .7), (b, .5)\}\}$. *i.e.* $cl^*(F_A) = \{e_1 = \{(a, .6), (b, .7)\}, e_2 = \{(a, .7), (b, .5)\}\}$. *Therefore* $cl^*(int(P_A)) = cl^*(F_A) = \{e_1 = \{(a, .6), (b, .7)\}, e_2 = \{(a, .7), (b, .5)\}\}$. *Hence* $P_A \tilde{\subseteq} cl^*(int(P_A))$ *i.e.* P_A *be a fuzzy soft semi-I-open set.*

Definition 3.9. *A fuzzy soft set* F_A *in a fuzzy soft ideal topological space* $(U, \tau, \tilde{I})$ *is said to be fuzzy soft regularly I-open set if* $F_A = int(cl^*(F_A))$.

Proposition 3.10. *Let* F_A *be a fuzzy soft set in a fuzzy soft ideal topological space* $(U, \tau, \tilde{I})$. *Then*

(i) if $\tilde{I} = \{\tilde{\phi}\}$, *then it is fuzzy soft semi-I-(respectively pre-I-,* α-I- *and* β-I)*-open set if and only if it is fuzzy soft semi-(respectively pre-,*α*-,*β)*-open set.*

(ii) if $\tilde{I} = P(U)$, *then it is fuzzy soft semi-I-(respectively pre-I-,* α-I- *and* β-I)*-open set if and only if it is fuzzy soft* τ*-open set.*

Proof: Proof of the result is easy, so omitted. □

Proposition 3.11. *Every fuzzy soft pre-I-open set is fuzzy soft pre-open.*

Proof: Let $(U, \tau, \tilde{I})$ be a fuzzy soft ideal topological space and F_A be a fuzzy soft pre-I-open set. Therefore we have $F_A \tilde{\subseteq} \text{int}(cl^*(F_A)) = \text{int}(F_A \tilde{\cup} F_A^*) \tilde{\subseteq} \text{int}(F_A \tilde{\cup} cl(F_A))$ $= \text{int}(cl(F_A))$. Hence F_A be a fuzzy soft pre-open set. □

Proposition 3.12. *Every fuzzy soft I-open set is fuzzy soft pre-I-open.*

Proof: Let $(U, \tau, \tilde{I})$ be a fuzzy soft ideal topological space and F_A be a fuzzy soft-I-open set. Therefore, we have $F_A \tilde{\subseteq} \text{int}(F_A^*) \tilde{\subseteq} \text{int}(F_A \tilde{\cup} F_A^*) = \text{int}(cl^*(F_A))$. Hence F_A be a fuzzy soft pre-I-open set. □

4. Mixed fuzzy soft ideal topological spaces

Definition 4.1. *Let $\tilde{I}$ be a fuzzy soft ideal on (U, E). Consider a mixed fuzzy soft topological space $(U, E, \tau_1(\tau_2))$. Then the triplet $(U, \tau_1(\tau_2), \tilde{I})$ is said to be mixed fuzzy soft ideal topological space.*

Problem 9. *Suppose $F_A = \{F(e_1) = \{(a, .6), (b, .2)\}, F(e_2) = \{(a, .8), (b, .7)\}\}$ and $G_A = \{G(e_1) = \{(a, .4), (b, .8)\}, G(e_2) = \{(a, .2), (b, .3)\}\}$. Then the collection of fuzzy soft sets $\tau_1 = \{\tilde{\phi}, \tilde{E}, \tilde{F_A}\}$ and $\tau_2 = \{\tilde{\phi}, \tilde{E}, \tilde{G_A}\}$ will form a fuzzy soft topology on (U, E). Therefore $\tau_1(\tau_2) = \{\tilde{\phi}, \tilde{E}, \tilde{G_A}\}$ is a mixed fuzzy soft topology on (U, E). Let $\tilde{I} = \{\tilde{\phi}, \tilde{E}\}$ be a fuzzy soft ideal on (U, E). Therefore $(U, \tau_1(\tau_2), \tilde{I})$ is a mixed fuzzy soft ideal topological space.*

Proposition 4.2. *In a mixed fuzzy soft ideal topological space $(U, \tau_1(\tau_2), \tilde{I})$, if $\tilde{I} = P(U)$ then $F_A^* = \tilde{\phi}$, where F_A^* is the fuzzy soft local function of F_A.*

Proof: Let F_A be a fuzzy soft set in U. Then ${F_A}^*(\tau_1(\tau_2), \tilde{I}) = \{e(X_A) \tilde{\in} U : F_A \tilde{\cap} Q_A \tilde{\notin} \tilde{I},$ for every $Q_A \tilde{\in} \tau_1(\tau_2)\}$. Since $\tilde{I} = P(U)$. Therefore $F_A \tilde{\cap} Q_A \tilde{\in} \tilde{I}$. Hence any fuzzy soft point does not contain in F_A^*. i.e. $F_A^* = \tilde{\phi}$. □

Definition 4.3. *Let $(U, \tau_1(\tau_2), \tilde{I})$ is a mixed fuzzy soft ideal topological space. A fuzzy soft set F_A in (U, E) is said to be fuzzy soft I-open set if $F_A \tilde{\subseteq} int(F_A^*)$, where F_A^* is the fuzzy soft local function of F_A.*

Problem 10. *In problem 3, we can show that for any fuzzy soft point $e_1(X_A)$, where grade of membership $a < .6, b < .2$. and $e_2(X_A)$, where grade of membership $a < .8, b < .7$. in (U, E) is a member of G_A^*. Therefore $G_A \tilde{\subseteq} int(G_A^*)$. Hence G_A is a fuzzy soft I-open set in $(U, \tau_1(\tau_2), \tilde{I})$.*

Definition 4.4. *Let $(U, \tau_1(\tau_2), \tilde{I})$ is a mixed fuzzy soft ideal topological space. A fuzzy soft set F_A in (U, E) is said to be*

(a) *fuzzy soft α-I-open set if $F_A \tilde{\subseteq} int(cl^*(int(F_A)))$.*

(b) *fuzzy soft pre-I-open set if $F_A \tilde{\subseteq} int(cl^*(F_A))$.*

(c) *fuzzy soft semi-I-open set if $F_A \tilde{\subseteq} cl^*(int(F_A))$.*

(d) fuzzy soft β-I-open set if $F_A \tilde{\subseteq} cl(int(cl^*(F_A)))$.

Definition 4.5. *A fuzzy soft set F_A in a mixed fuzzy soft ideal topological space $(U, \tau_1(\tau_2), \tilde{I})$ is said to be fuzzy soft regularly I-open set if $F_A = int(cl^*(F_A))$.*

Problem 11. *Let $F_A = \{F(e_1) = \{(a, .8), (b, .8)\}, F(e_2) = \{(a, .6), (b, .6)\}\}$ and $G_A = \{G(e_1) = \{(a, .2), (b, .2)\}, G(e_2) = \{(a, .4), (b, .4)\}\}$. Also let $\tau_1 = \{\tilde{\phi}, \tilde{E}\}$, $\tau_2 = \{\tilde{\phi}, \tilde{E}, \tilde{F_A}, \tilde{G_A}\}$ and $\tau_1(\tau_2) = \{\tilde{\phi}, \tilde{E}, \tilde{F_A}, \tilde{G_A}\}$. Then $(U, \tau_1(\tau_2), \tilde{I})$ be a mixed fuzzy soft ideal topological space. Consider the fuzzy soft set $Q_A = \{Q(e_1) = \{(a, .1), (b, .9)\}, Q(e_2) = \{(a, .8), (b, .2)\}\}$, then $V_A = Q_A^C = \{Q^C(e_1) = \{(a, .9), (b, .1)\}, Q^C(e_2) = \{(a, .2), (b, .8)\}\}$. Consider the fuzzy soft point $(e_2) = \{(a, .2), (b, .8)\}$ in V_A and let $H_A = \{H(e_1) = \{(a, .8), (b, 1)\}, H(e_2) = \{(a, .9), (b, .4)\}\}$ be a fuzzy soft set in U. Then H_A is fuzzy soft I-regularly open set, because $H_A \tilde{\notin} \tilde{\phi}$ and $H_A \notin \tilde{E}$. So we must have $H_A^* = \{H^*(e_1) = \{(a, .2), (b, 0)\}, H^*(e_2) = \{(a, .1), (b, .3)\}\}$. Hence $H_A \tilde{\cup} H_A^* = \{(e_1) = \{(a, .8), (b, 1)\}, (e_2) = \{(a, .9), (b, .4)\}\} \Rightarrow cl^*(H_A) = \{(e_1) = \{(a, .8), (b, 1)\}, (e_2) = \{(a, .9), (b, .4)\}\}$. Therefore $int(cl^*(H_A)) = H_A$.*

Proposition 4.6. *In a mixed fuzzy soft ideal topological space $(U, \tau_1(\tau_2), \tilde{I})$, a fuzzy soft I-regularly open set is not fuzzy soft I-open.*

Proof: Let F_A be any fuzzy soft I-regularly open set in U. Therefore we have $F_A = \text{int}(cl^*(F_A)) = \text{int}(F_A \tilde{\cup} F_A^*) \tilde{\supseteq} \text{int}(F_A^*)$ i.e. $F_A \tilde{\supseteq} \text{int}(F_A^*)$. Hence F_A is not fuzzy soft I-open set. □

Proposition 4.7. *Let $(U, \tau_1(\tau_2), \tilde{I})$ be a mixed fuzzy soft ideal topological space. Let F_A and G_A be two fuzzy soft sets in (U, E).*

(i) If $F_A \tilde{\subseteq} G_A$ then $F_A^* \tilde{\subseteq} G_A^*$.

(ii) $F_A^* \tilde{\subseteq} cl(F_A)$, where cl is the soft closure w.r.t $\tau_1(\tau_2)$.

(iii) If $F_A \tilde{\subseteq} G_A$ then $cl^*(F_A) \tilde{\subseteq} cl^*(G_A)$.

(iv) $(F_A \tilde{\cup} G_A)^* = F_A^* \tilde{\cup} G_A^*$.

(v) $(F_A \tilde{\cap} G_A)^* \tilde{\subseteq} F_A^* \tilde{\cap} G_A^*$.

Proof: (i) Let $e(X_A) \tilde{\in} U$. Therefore we have
$F_A \tilde{\subseteq} G_A \Rightarrow F_A \tilde{\cap} Q_A = G_A \tilde{\cap} Q_A, Q_A \tilde{\in} \tau_1(\tau_2)$.
Now
${F_A}^* = \{e(X_A) \tilde{\in} U : F_A \tilde{\cap} Q_A \tilde{\notin} \tilde{I}, \text{ for every } Q_A \tilde{\in} \tau_1(\tau_2)\}$
$\tilde{\subseteq} \{e(X_A) \tilde{\in} U : G_A \tilde{\cap} Q_A \tilde{\notin} \tilde{I}, \text{ for every } Q_A \tilde{\in} \tau_1(\tau_2)\} = G_A^*$.
Therefore, $F_A^* \tilde{\subseteq} G_A^*$.
(ii) Let $e(X_A) \tilde{\in} U$. Now
${F_A}^* = \{e(X_A) \tilde{\in} U : F_A \tilde{\cap} Q_A \tilde{\notin} \tilde{I}, \text{ for every } Q_A \tilde{\in} \tau_1(\tau_2)\}$
$\tilde{\subseteq} \{e(X_A) \tilde{\in} U : F_A^C \tilde{\cap} Q_A^C \tilde{\notin} \tilde{I}; F_A^C, Q_A^C \text{ are fuzzy soft closed sets in } \tau_1(\tau_2)\} = cl(F_A)$.
Hence $F_A^* \tilde{\subseteq} cl(F_A)$.

(iii) Same as Proposition 3.6.
(iv) Same as Proposition 3.4(iii).
(v) Same as Proposition 3.4(iv). □

Proposition 4.8. *In a mixed fuzzy soft ideal topological space $(U, \tau_1(\tau_2), \tilde{I})$ intersection of two fuzzy soft I-open sets is I-open.*

Proof: Let F_A and G_A be any two fuzzy soft I-open sets in U. Therefore we have $F_A \tilde{\subseteq} \text{int}(F_A^*)$ and $G_A \tilde{\subseteq} \text{int}(G_A^*)$. Now $F_A \tilde{\cap} G_A \tilde{\subseteq} \text{int}(F_A^*) \tilde{\cap} \text{int}(G_A^*) \tilde{\subseteq} \text{int}(F_A^* \tilde{\cap} G_A^*) \tilde{\subseteq} \text{int}((F_A \tilde{\cap} G_A)^*)$. Hence $F_A \tilde{\cap} G_A$ is a fuzzy soft I-open. □

Proposition 4.9. *In a mixed fuzzy soft ideal topological space $(U, \tau_1(\tau_2), \tilde{I})$ union of two fuzzy soft I-open sets is I-open.*

Proof: Let F_A and G_A be any two fuzzy soft I-open sets in U. Therefore we have $F_A \tilde{\subseteq} \text{int}(F_A^*)$ and $G_A \tilde{\subseteq} \text{int}(G_A^*)$. Now $F_A \tilde{\cup} G_A \tilde{\subseteq} \text{int}(F_A^*) \tilde{\cup} \text{int}(G_A^*) \tilde{\subseteq} \text{int}(F_A^* \tilde{\cup} G_A^*) = \text{int}((F_A \tilde{\cap} G_A)^*)$. Hence $F_A \tilde{\cup} G_A$ is a fuzzy soft I-open. □

Proposition 4.10. *Let $(U, \tau_1(\tau_2), \tilde{I})$ be a mixed fuzzy soft ideal topological space and $\tau_1 \tilde{\subseteq} \tau_2$. Let F_A be a fuzzy soft topology on (U, E). Then $F_A^*(\tau_1(\tau_2), \tilde{I}) \tilde{\subseteq} F_A^*(\tau_1, \tilde{I})$.*

Proof: Let $e(X_A) \tilde{\in} U$. Now
$F_A{}^*(\tau_1(\tau_2), \tilde{I}) = \{e(X_A) \tilde{\in} U : F_A \tilde{\cap} Q_A \tilde{\notin} \tilde{I}, \text{ for every } Q_A \tilde{\in} \tau_1(\tau_2)\}$
$\tilde{\subseteq} \{e(X_A) \tilde{\in} U : F_A \tilde{\cap} Q_A \tilde{\notin} \tilde{I}, \text{ for every } Q_A \tilde{\in} \tau_2\}$, since $\tau_1(\tau_2) \tilde{\subseteq} \tau_2$
$\tilde{\subseteq} \{e(X_A) \tilde{\in} U : F_A \tilde{\cap} Q_A \tilde{\notin} \tilde{I}, \text{ for every } Q_A \tilde{\in} \tau_1\}$, since $\tau_1 \tilde{\subseteq} \tau_2$
$= F_A^*(\tau_1, \tilde{I})$. □

5. Conclusion

Topology is an important and major area of mathematics and it can give many relationships between other scientific areas and mathematical models. In this paper we have introduced fuzzy soft ideal and mixed fuzzy soft ideal topological spaces which are defined over an initial universe with a fixed set of parameter. The notion of fuzzy soft I-open set, fuzzy soft α-I-open set, fuzzy soft pre-I-open set, fuzzy soft semi-I-open set and fuzzy soft β-I-open set are introduced and investigated some of their properties.

References

1. N. Çağman and N. S. Enginoğlu, Soft Set Theory and Uni-int Decision Making, Euro. J. Opera. Research 207(2010) 848-855.
2. N. Çağman, S. Karatas, N. S. Enginoğlu, Soft topology, Comput. Math. Appl. 62(2011) 351-358.
3. C. L. Chang, Fuzzy Topological Spaces, J. Math. Anal. Appl. 24(1)(1968) 182-190.
4. D. Chen, The parametrization reduction of soft sets and its applications, Comput. Math. Appl. 49(2005) 757-763.

5. J. B. Cooper, The Strict Topology and Spaces with mixed Topologies, Proc. Amer. Math. Soc. 30(3)(1971) 583-592.

6. N. R. Das and P. C. Baishya, Mixed Fuzzy Topological Spaces, J. Fuzzy Math. 3(4)(1995) 777-784.

7. S. Hussain, B. Ahmad, Some properties of soft topological spaces, Comput. Math. Appl. 62(2011) 4058-4067.

8. K. Kuratowski, Topology, Vol. I, Academic Press New York, 1966.

9. J. Mahanta and P. K. Das, Results on Fuzzy soft topological Spaces, arXiv:1203.0634v1.

10. P. K. Maji, R. Biswas, R. Roy, An application of soft sets in a decision making problem, Comput. Math. Appl. 44(2002) 1077-1083.

11. P. K. Maji, R. Biswas, R. Roy, Soft set theory, Comput. Math. Appl. 45(2003) 555-562.

12. P. K. Maji, R. Biswas and A. R. Roy, Fuzzy Soft Sets, J. Fuzzy Math. 9(3)(2001) 589-602.

13. D. A. Molodstov, Soft Set Theory-First Result, Comput. Math. Appl. 37(1999) 19-31.

14. D. A. Molodtsov, V. Y. Leonov, D.V. Kovkov, Soft sets technique and its application, Nechetkie Sistemy i Myagkie Vychisleniya 1(1)(2006) 8-39.

15. A. Mukherjee, Mixed δ-Semi Pre Fuzzy Topological Spaces, Bull. Cal. Math. Soc. 109(6)(2014) 441-452.

16. T. J. Neog, D. K. Sut and G. C. Hazarika, Fuzzy Soft Topological Spaces, Inter. J. Latest Trends Math. 2(1)(2012) 54-67.

17. D. Pie, D. Miao, From soft sets to information systems, Granular Comput. 2005 IEEE Inter. Conf 2(2005) 617-621.

18. S. Ray and T. K. Samanta, A note on Fuzzy Soft Topological Spaces, Annals Fuzzy Math. Inform. 3(2)(2012) 305-311.

19. M. Shabir and M. Naz, On soft topological spaces, Comput. Math. Appl. 61(2012) 412-418.

20. B. Tanay and M. Burç Kandemir, Topological Structure of fuzzy soft sets, Comput. Math. Appl. 61(2011) 2952-2957.

21. B. C. Tripathy and G. C. Ray, Mixed Fuzzy ideal topological spaces, Appl. Math. Comput. 220(2013) 602-607.

22. B. C. Tripathy and G. C. Ray, Weakly continuous functions on mixed fuzzy topological spaces, Acta Scientiarum Tech. 36(2)(2014) 331-335.

23. B. C. Tripathy and G. C. Ray, On Mixed fuzzy topological spaces and countability, Soft Computing 16(10)(2012) 1691-1695.

24. B. C. Tripathy and G. C. Ray, On δ-continuity in mixed fuzzy topological spaces, Bol. Soc. Parana. Mat. 32(2)(2014) 175-187.

25. R. Vaidyanathaswamy, Set Topology, 2nd ed., Chelsea Publishing, New York, 1960.

26. A. Wiweger, Linear spaces with mixed topology, Studia Math. 20(1)(1961) 47-68.

27. L. A. Zadeh, Fuzzy Sets, Inform. Control. 8(1965) 338-353.

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