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Some Common Fixed Point Theorems in Fuzzy Metric Spaces and Their Applications

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ABSTRACT: The main aim of this paper is to prove fixed point theorems via notion of pairwise semi-compatible mappings and occasionally weakly compatible mappings(owc) in fuzzy metric spaces satisfying contractive type condition.

Key Words: Fuzzy metric space(FM-space), fixed point, property E.A., compatible mapping.

Contents

1. Introduction

The concept of Fuzzy sets was introduced initially by Zadeh [6]. Since then, to use this concept in topology and analysis many authors have expansively developed the theory of Fuzzy sets and applications. Especially, Erceg [8], Kramosil and Michalek [12], Kaleva and Seikkala [13], Deng [29] have introduced the concept of Fuzzy metric space in different ways.

The study of common fixed points of mapping, satisfying some contractive type condition has been at the center of vigorous research activities; and a number of interesting results have been obtained by various authors. Most of these results deal either with commuting mappings or assume the notion of weak commutativity of mappings introduced by Seesa [19]. In 1986, Jungck [5] introduced the notion of compatible maps. This concept was frequently used to prove existence theorems in common fixed point theory. In 2002, Aamir and Moutawakil [9] studied a property for pair of maps namely the property E.A, which is the generalization of the concept of non compatible maps. Further, Pant and Pant [28] studied the common fixed points of a pair of non compatible maps and the property E.A in FM-space.

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The result obtained by us gives generalization of many important fixed point theorems and open up a wider scope for the study of common fixed points under contractive type conditions.After these no fixed point theorems have been investigated to find the fixed point in fuzzy metric spaces. (See [7] , [15,16,17,18], [22, 23,24,25,26,27])

In this paper, our objective is to prove some common fixed point theorems by removing the assumption of continuity and replacing the completeness of the space with a set of three conditions for self mappings in Fuzzy metric space. Our result generalizes the result of Tanmony Som [21].

2. Preliminaries

Definition 2.1. *[14] A binary operation* $* : [0,1] \times [0,1] \to [0,1]$ *is a continuous t-norm if* $*$ *satisfies the following conditions:*

1. $*$ *is commutative and associative,*
2. $*$ *is continuous,*
3. $a * 1 = a$ *for all* $a \in [0,1]$,
4. $a * b \le c * d$ *whenever* $a \le c$ *and* $b \le d$ *and* $a, b, c, d \in [0,1]$.

Example of t-norm are $a * b = ab$ and $a * b = min\{a, b\}$.

Definition 2.2. *[20] The 3-tuple* $(X, M, *)$ *is said to be Fuzzy metric space if* X *is an arbitrary set,* $*$ *is a continuous t-norm and* M *is a Fuzzy set in* $X^2 \times [0, \infty)$ *satisfying the following conditions:*

1. $M(x, y, 0) = 0$
2. $M(x, y, t) = 1$ *for all* $t > 0$ *if and only if* $x = y$
3. $M(x, y, t) = M(y, x, t)$
4. $M(x, y, t) * M(y, z, s) \le M(x, z, t + s)$
5. $M(x, y, .) : [0, \infty) \to [0, 1]$ *is left continuous*
6. $lim_{t\to\infty} M(x, y, t) = 1$

for all $x, y, z \in X$ *and* $s, t > 0$.

Definition 2.3. *[20] A sequence* $\{x_n\}$ *in a Fuzzy metric space* $(X, M, *)$ *is said to be Cauchy sequence if and only if for each* $\epsilon > 0$, $t > 0$, *there exist* $n_0 \in N$ *such that* $M(x_n, x_m, t) > 1 - \epsilon$ *for all* $n, m \ge n_0$.

Definition 2.4. *[20] A sequence* $\{x_n\}$ *in a Fuzzy metric space* $(X, M, *)$ *is said to be convergent sequence to a point* x *in* X *if and only if for each* $\epsilon > 0$, $t > 0$, *there exist* $n_0 \in N$ *such that* $M(x_n, x, t) > 1 - \epsilon$ *for all* $n \ge n_0$.

Definition 2.5. *[20] A Fuzzy metric space $(X, M, *)$ is said to be complete if every Cauchy sequence in it converges to a point in it.*

Definition 2.6. *[2] Two self mappings A and S of a Fuzzy metric space $(X, M, *)$ are said to be compatible if and only if $M(ASx_n, SAx_n, t) \to 1$ for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that Sx_n, $Ax_n \to p$ for some p in X, as $n \to \infty$.*

Definition 2.7. *[3] Two self mappings A and S of a Fuzzy metric space $(X, M, *)$ are said to be semi-compatible if and only if $M(ASx_n, Sp, t) \to 1$ for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that Sx_n, $Ax_n \to p$ for some pinX, as $n \to \infty$.*

Definition 2.8. *[4] Two self mappings A and S of a Fuzzy metric space $(X, M, *)$ are said to be weakly-compatible if they commute at their coincidence points, i.e $Ax = Sx$ implies ASx=SAx.*

Definition 2.9. *[10] Two self mappings A and S of a Fuzzy metric space $(X, M, *)$ are said to be occasionally-weakly compatible if and only if there is a point x in X which is coincidence point of A and S at which A and S commute.*

Definition 2.10. *[11] Two self mappings A and S of a Fuzzy metric space $(X, M, *)$ are said to satisfy the property E.A if there exist sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$ for some $z \in X$.*

3. Main results

Theorem 3.1. *Let A, B, S and T be self mappings on a Fuzzy metric space $(X, M, *)$ satisfying the following condition:*

$$M(Ax, By, t) \geq r\Big[\min\Big\{M(Sx, Ty, t), M(Sx, Ax, t), M(Sx, By, t), M(Ty, Ax, t)\Big\}\Big] \tag{3.1}$$

for all $x, y \in X$, where $r : [0, 1] \to [0, 1]$ is a continuous function such that

$$r(t) > t \quad for\,each \quad t < 1 \quad and \quad r(t) = 1 \quad for \quad t = 1. \tag{3.2}$$

Also, suppose the pair (A, S) and (B, T) share the common property (E.A), and $S(X)$ and $T(X)$ are closed subsets of X, then the pair (A, S) as well as (B, T) have a coincidence point.
Further A, B, S,T have a unique common fixed point provided the pair (A, S) is semi-compatible and (B, T) is occasionally weakly compatible.

Proof: Since the pair (A, S) and (B, T) share the common property (E.A) then there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that
$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z$,

for some $z \in X$.
Also, $S(X)$ is closed subset of X, therefore $\lim_{n\to\infty} Sx_n = z \in S(X)$ and there is a point u in X such that $Su = z$.

Now, we claim that $Au = z$. If not, then by using (3.1), we have

$$M(Au, By_n, t) \geq r\Big[\min\Big\{M(Su, Ty_n, t), M(Su, Au, t), M(Su, By_n, t), M(Ty_n, Au, t)\Big\}\Big].$$

Taking $n \to \infty$, we get

$$\begin{aligned} M(Au, z, t) &\geq r[\min\{M(z, z, t), M(z, Au, t), M(z, z, t), M(z, Au, t)\}] \\ &= r[M(Au, z, t)] \\ &> M(Au, z, t), \end{aligned}$$

this is a contradiction. Hence $Au = z$.
Thus we have $Au = Su$ or u is a coincidence point of the pair (A, S).
Since $T(X)$ is closed subset of X, therefore $\lim_{n\to\infty} Ty_n = z \in T(X)$ and there exists $w \in X$ such that $Tw = z$.
Again using (3.1), we obtain

$$M(Ax_n, Bw, t) \geq r\Big[\min\Big\{M(Sx_n, Tw, t), M(Sx_n, Ax_n, t), M(Sx_n, Bw, t), M(Tw, Ax_n, t)\Big\}\Big].$$

On taking $n \to \infty$, we get

$$\begin{aligned} M(z, Bw, t) &\geq r[\min\{M(z, z, t), M(z, z, t), M(z, Bw, t), M(z, z, t)\}] \\ &= r[M(z, Bw, t)] \\ &> M(z, Bw, t). \end{aligned}$$

This implies $Bw = z$. Hence, we get $Tw = Bw = z$. Thus w is a coincidence point of the pair (B, T).
Also, (A, S) is semi-compatible pair, so $\lim_{n\to\infty} ASx_n = Sz$ and $\lim_{n\to\infty} ASx_n = Az$.
Since the limit in Fuzzy metric space is unique so $Sz = Az$.
Now, we claim that z is a common fixed point of the pair (A, S).
Again, from (3.1), we obtain

$$M(Az, Bw, t) \geq r\Big[\min\Big\{M(Sz, Tw, t), M(Sz, Az, t), M(Sz, Bw, t), M(Tw, Az, t)\Big\}\Big].$$

Taking $n \to \infty$, we get

$$\begin{aligned} M(Az, z, t) &\geq r[\min\{M(Az, z, t), M(Az, Az, t), M(Az, z, t), M(z, Az, t)\}] \\ &= r[M(Az, z, t)] \\ &> M(Az, z, t), \end{aligned}$$

implies $Az = z$. Thus $Az = z = Sz$.
Since w is a coincidence point of B and T and the pair (B, T) is occasionally weakly compatible, so we have, $BTw = TBw \Rightarrow Bz = Tz = z$.
Hence, z is the common fixed point of A, B, S and T.

For Uniqueness,
Let v be another common fixed point of A, B, S and T.
Take $x = z$ and $y = v$ in (3.1), we get

$$M(Az, Bv, t) \geq r\left[\min\left\{M(Sz, Tv, t), M(Sz, Az, t), M(Sz, Bv, t), M(Tv, Az, t)\right\}\right].$$

Taking $n \to \infty$, we get

$$\begin{aligned} M(z, v, t) &\geq r\left[\min\left\{M(z, v, t), M(z, z, t), M(z, v, t), M(v, z, t)\right\}\right] \\ &= r\left[M(Az, z, t)\right] \\ &> M(Az, z, t), \end{aligned}$$

this implies $z = v$. Thus z is the unique common fixed point of the mappings A, B, S and T.

□

Example 3.1. *Let $X = [2, 20]$ and d be the usual metric on X. For each $t \in [0, \infty)$, define $M(x, y, t) = \frac{t}{t + |x - y|}$.*
Clearly $(X, M, *)$ is a Fuzzy metric space, where $*$ is defined as $a * b = ab$. Define the mapping A, B, S, T as follows:

$$A(x) = \begin{cases} 5, & \text{if } x=2 \\ 6, & \text{if } 2 < x \leq 5 \\ 7, & \text{if } x > 5 \end{cases} \qquad S(x) = \begin{cases} 5, & \text{if } x=2 \\ 7, & \text{if } 2 < x \leq 5 \\ \frac{4x+10}{15}, & \text{if } x > 5 \end{cases}$$

$$B(x) = \begin{cases} 5, & \text{if } x=2 \\ 7, & \text{if } 2 < x \leq 20 \end{cases} \qquad T(x) = \begin{cases} 5, & \text{if } x=2 \\ 3, & \text{if } 2 < x \leq 5 \\ x-3, & \text{if } x > 5 \end{cases}$$

Consider $x_n = 5 + \frac{1}{n}$, then one can say the pairs (A, S) and (B, T) share the common property (E.A) and the pair (A, S) is semi-compatible and (B, T) is occasionally weakly compatible.
Also, for different values of x, equation (3.1) is satisfied. We will discuss it in the following three cases:

Case I: when $x = y = 2$
$M(Ax, By, t) = M(A2, B2, t) = M(2, 2, t) = 1$ *and*
$r\Big[\min\left\{M(Sx, Ty, t), M(Sx, Ax, t), M(Sx, By, t), M(Ty, Ax, t)\right\}\Big] = 1$ *therefore, equation (3.1) is satisfied.*
Case II: when $2 < x \leq 5$
$M(Ax, By, t) = M(6, 7, t) = \frac{t}{t+1}$ *and*
$r\left[min\left\{M(Sx, Ty, t), M(Sx, Ax, t) f, M(Sx, By, t), M(Ty, Ax, t)\right\}\right]$

$= r\left[min\left\{\frac{t}{t+4},\frac{t}{t+1},\frac{t}{t+0},\frac{t}{t+3}\right]\right\} = r\left(\frac{t}{t+4}\right) > \frac{t}{t+4}.$

Thus for all values of $t > 0$, $\frac{t}{t+1} \geq \frac{t}{t+4}$ *and hence equation (3.1) is satisfied.*

Case III: when $x > 5$

$M(Ax, By, t) = M\left(2, \frac{4x+10}{15}, t\right) = \frac{t}{t+\left|\frac{20-4x}{15}\right|}$ *and*

$r\left[min\left\{M(Sx,Ty,t), M(Sx,Ax,t)f, M(Sx,By,t), M(Ty,Ax,t)\right\}\right]$

$= r\left[min\left\{\frac{t}{t+\left|\frac{55-11x}{15}\right|},\frac{t}{t+\left|\frac{4x-20}{15}\right|},\frac{t}{t+\left|\frac{4x-95}{15}\right|},\frac{t}{t+|x-5|}\right]\right\}$

$= r\left(\frac{t}{t+\left|\frac{4x-95}{15}\right|}\right) > \frac{t}{t+\left|\frac{4x-95}{15}\right|}$

Thus for all values of $t > 0$ *and* $x > 5$, $\frac{t}{t+\left|\frac{20-4x}{15}\right|} \geq \frac{t}{t+\left|\frac{4x-95}{15}\right|}$ *and hence equation (3.1) is satisfied.*

Theorem 3.2. *Let* A, B, S *and* T *be four self mappings on a Fuzzy metric space* $(X, M, *)$ *satisfying the following conditions:*

1. *The pairs* (A, S) *and* (B, T) *share the common property (E.A)*
2. $S(X)$ *and* $T(X)$ *are closed subsets of* X
3. $qM(Ax, By, t) \geq aM(Ty, Sx, t) + bM(Sx, By, t) + cM(Ax, By, t)$
   $+max\{M(Ax, Sx, t), M(By, Ty, t)\}$

for all $x, y \in X$, $a, b, c \geq 0$, $q > 0$ *and* $q < a + b + c$,

then each pair (A, S) *and* (B, T) *have a point of coincidence .*
Further, if the pair (A, S) *is semi-compatible and* (B, T) *is occasionally weakly compatible, then* A, B, S *and* T *have a unique common fixed point.*

Proof: As the pair (A, S) and (B, T) share the common property (E.A), then there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that
$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z$,
for some $z \in X$.
Since $S(X)$ is a closed subset of X; therefore, there exists a point $u \in X$ such that $Su = z$. Using the above condition (iii), we have

$$qM(Au, By_n, t) \geq aM(Ty_n, Su, t) + bM(Su, By_n, t) + cM(Au, By_n, t) + max\{M(Au, Su, t), M(By_n, Ty_n, t)\}.$$

Taking $n \to \infty$, we obtain

$$\begin{aligned} qM(Au, z, t) &\geq aM(z, z, t) + bM(z, z, t) + cM(Au, z, t) \\ &+ max\{M(Au, z, t), M(z, z, t)\}, \end{aligned}$$

this gives,

$$(q-c)M(Au,z,t) \geq (a+b)M(z,z,t)+1 > (a+b)M(z,z,t),$$

this implies,

$$\begin{aligned} M(Au,z,t) &> \frac{a+b}{q-c} \\ &> 1. \end{aligned}$$

for all $t>0$, this implies, $Au=z$. Hence $Au=Su$, which shows that u is the coincidence point of (A,S).
Again, $T(X)$ is closed subset of X, therefore there is a point w in X such that $Tw=z$.
Now take $x=x_n$ and $y=w$ in condition (iii), we get

$$qM(Ax_n,Bw,t) \geq aM(Tw,Sx_n,t)+bM(Sx_n,Bw,t)+cM(Ax_n,Bw,t) \\ +max\{M(Ax_n,Sx_n,t),M(Bw,Tw,t)\}.$$

Taking $n\to\infty$, we obtain

$$\begin{aligned} qM(z,Bw,t) &\geq aM(z,z,t)+bM(z,Bw,t)+cM(z,Bw,t) \\ &+ max\{M(z,z,t),M(Bw,z,t)\}, \\ (q-b-c)M(z,Bw,t) &\geq aM(z,z,t)+1 \\ &> aM(z,z,t), \end{aligned}$$

this gives

$$M(z,Bw,t) > \frac{a}{q-b-c} > 1 \;\; for\, all\, t>0.$$

This implies $Bw=z$. Hence $Tw=Bw=z$ and thus w is the coincidence point of (B,T).

Further, we assume that (A,S) is a semi-compatible pair, so $\lim_{n\to\infty} ASx_n = Sz$ and $\lim_{n\to\infty} ASx_n = Az$.
Since the limit in fuzzy metric space is unique, so $Sz=Az$.
Now, we claim that z is a common fixed point of the pair A and S.
Using condition (iii), we get

$$qM(Az,Bw,t) \geq aM(Tw,Sz,t)+bM(Sz,Bw,t)+cM(Az,Bw,t) \\ +max\{M(Az,Sz,t),M(Bw,Tw,t)\}.$$

Taking $n \to \infty$, we obtain

$$\begin{aligned} qM(Az,z,t) &\geq aM(z,Az,t) + bM(Az,z,t) + cM(Az,z,t) \\ &+ max\{M(Az,Az,t), M(z,z,t)\} \\ (q-a-b-c)M(z,Bw,t) &\geq 1 \end{aligned}$$

This implies

$$\begin{aligned} M(Az,z,t) &\geq \frac{1}{q-a-b-c} \\ &> 1, \ for\, all\, t > 0. \end{aligned}$$

Thus $Az = z$. Hence $Az = z = Sz$.

Since w is a coincidence point B and T, and the pair (B,T) is occasionally weak compatible. So, $BTw = TBw$ this implies $Bz = Tz = z$.

Hence, z is the common fixed point of mappings A, S, B and T.
The uniqueness of fixed point follows from taking $x = z$ and $y = v$ in condition (iii).

□

Taking $A = B$ in the above theorem, we get the following corollary:

Corollary 3.3. *Let A, S and T be three self mappings of a Fuzzy metric space $(X, M, *)$, satisfying the following conditions:*

1. *The pairs (A,S) and (A,T) share the common property (E.A)*
2. *$S(X)$ and $T(X)$ are closed subsets of X*
3. $qM(Ax,Ay,t) \geq aM(Ty,Sx,t) + bM(Sx,Ay,t) + cM(Ax,Ay,t)$
   $+max\{M(Ax,Sx,t), M(Ay,Ty,t)\}$

for all $x, y \in X$, $a, b, c \geq 0$, $q > 0$ and $q < a+b+c$,

then the pairs (A,S) and (A,T) have a point of coincidence.
Further, if the pair (A,S) is semi-compatible and (A,T) is occasionally weakly compatible then A, S and T have a unique common fixed point.

4. Applications

Theorem 4.1. *Let A, B, S and T be self mappings on a Fuzzy metric space $(X, M, *)$ satisfying the condition*

$$\int_0^{M(Ax,By,t)} \phi(t)\,dt \geq \int_0^{r[m(x,y,t)]} \phi(t)\,dt \tag{4.1}$$

where $\phi : R^+ \to R^+$ is a Lebesgue-integrable mapping which is summable, non-negative such that $\int_0^{\epsilon} \phi(t)\,dt > 0$ for each $\epsilon > 0$,and

$m(x,y,t) = \min\Big\{M(Sx,Ty,t), M(Sx,Ax,t), M(Sx,By,t),$
$M(Ty,Ax,t)\Big\}$ *for all* $x,y \in X$, *and* $r : [0,1] \to [0,1]$ *is continuous function such that* $r(t) > t$ *for each* $t < 1$ *and* $r(t) = 1$ *for* $t = 1.$, *Also suppose that the pairs* (A,S) *and* (B,T) *share the common property (E.A), and* $S(X)$ *and* $T(X)$ *are closed subsets of* X.
Then the pair (A,S) *as well as* (B,T) *have a coincidence point.*
Further, if A, B, S *and* T *have a unique common fixed point provided the pair* (A,S) *is semi-compatible and* (B,T) *is occasionally weakly compatible.*

Proof: Since the pair (A,S) and (B,T) share the common property (E.A) then there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that
$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z$ for some $z \in X$.
Since $S(X)$ is closed subset of X, then $\lim_{n\to\infty} Sx_n = z \in S(X)$, therefore there is a point u in X such that $Su = z$.

We claim that $Au = z$. If not then by using (4.1), we have

$$\int_0^{M(Au,By_n,t)} \phi(t)\,dt \geq \int_0^{r[m(u,y_n,t)]} \phi(t)\,dt,$$

where
$r[m(x,y,t)]$
$= r\Big[min\Big\{M(Su,Ty_n,t), M(Su,Au,t), M(Su,By_n,t), M(Ty_n,Au,t)\Big\}\Big]$

$$so, \int_0^{M(Au,z,t)} \phi(t)\,dt \geq \int_0^{r[m(u,z,t)]} \phi(t)\,dt,$$

where

$$\begin{aligned} r[m(u,z,t)] &= r[min\{M(z,z,t), M(z,Au,t), M(z,z,t), M(z,Au,t)\}] \\ &= r[M(Au,z,t)] \\ &> M(Au,z,t) \end{aligned}$$

$$i.e \int_0^{M(Au,z,t)} \phi(t)\,dt \geq \int_0^{M(Au,z,t)} \phi(t)\,dt,$$

this is contradiction, this implies $Au = z$.
Hence $Au = Su$ or u is a coincidence point of the pair (A,S).
But $T(X)$ is closed subset of X, then $\lim_{n\to\infty} Ty_n = z \in T(X)$, therefore there exists $w \in X$ such that $Tw = z$.
Using (4.1), we obtain

$$\begin{aligned} \int_0^{M(Ax_n,Bw,t)} \phi(t)\,dt &\geq \int_0^{r[m(x_n,w,t)]} \phi(t)\,dt \\ or \int_0^{M(z,Bw,t)} \phi(t)\,dt &\geq \int_0^{r[m(z,w,t)]} \phi(t)\,dtV, \end{aligned}$$

$$
\begin{aligned}
&where\\
r\left[m\left(z,w,t\right)\right] =& r\left[min\left\{M\left(z,z,t\right),M\left(z,z,t\right),M\left(z,Bw,t\right),M\left(z,z,t\right)\right\}\right]\\
=& r\left[M\left(z,Bw,t\right)\right] > M\left(z,Bw,t\right),
\end{aligned}
$$

$$
i.e \int_0^{M(z,Bw,t)} \phi\left(t\right)dt \geq \int_0^{M(z,Bw,t)} \phi\left(t\right)dt.
$$

This gives, $Bw = z$. Hence $Tw = Bw = z$ or w is a coincidence point of the pair (B,T).

Also, (A,S) is a semi-compatible pair, so $\lim_{n\to\infty} ASx_n = Sz$ and $\lim_{n\to\infty} ASx_n = Az$.
Since the limit in Fuzzy metric space is unique, therefore $Sz = Az$.

We claim that z is a common fixed point of the pair (A,S).
from (4.1), we have

$$
\begin{aligned}
\int_0^{M(Az,Bw,t)} \phi\left(t\right)dt &\geq \int_0^{r[m(z,w,t)]} \phi\left(t\right)dt\\
or \int_0^{M(Az,z,t)} \phi\left(t\right)dt &\geq \int_0^{r[m(z,w,t)]} \phi\left(t\right)dt,
\end{aligned}
$$

$$
\begin{aligned}
&where\\
r\left[m\left(z,w,t\right)\right] =& r\left[min\left\{M\left(Az,z,t\right),M\left(Az,Az,t\right),M\left(Az,z,t\right),M\left(z,Az,t\right)\right\}\right]\\
=& r\left[M\left(Az,z,t\right)\right]\\
>& M\left(Az,z,t\right),
\end{aligned}
$$

$$
i.e \int_0^{M(Az,z,t)} \phi\left(t\right)dt \geq \int_0^{M(Az,z,t)} \phi\left(t\right)dt.
$$

this implies $Az = z$ and hence $Az = z = Sz$.

Since w is a coincidence point of B and T, and the pair (B,T) is occasionally weakly compatible, so we have $BTw = TBw \Rightarrow Bz = Tz = z$.
Hence z is the common fixed point of A,S,B and T.

For Uniqueness, let v be another common fixed point of A,B,S and T.
Take $x = z$ and $y = v$ in (4.1), we get

$$
\begin{aligned}
\int_0^{M(Az,Bv,t)} \phi\left(t\right)dt &\geq \int_0^{r[m(z,v,t)]} \phi\left(t\right)dt\\
or \int_0^{M(z,v,t)} \phi\left(t\right)dt &\geq \int_0^{r[m(z,v,t)]} \phi\left(t\right)dt,\\
where\, r\left[m\left(z,v,t\right)\right] &= r\left[min\left\{M\left(z,v,t\right),M\left(z,z,t\right),M\left(z,v,t\right),M\left(v,z,t\right)\right\}\right]\\
&= r\left[M\left(Az,z,t\right)\right]\\
&> M\left(Az,z,t\right),\\
i.e \int_0^{M(Az,z,t)} \phi\left(t\right)dt &\geq \int_0^{M(Az,z,t)} \phi\left(t\right)dt,
\end{aligned}
$$

this implies $z = v$ and thus z is the unique common fixed point of the mappings A, B, S and T.

□

Theorem 4.2. *Let A, B, S and T be four self mappings on a Fuzzy metric space $(X, M, *)$ satisfying the following conditions:*

1. *The pairs (A, S) and (B, T) share the common property (E.A);*
2. *$S(X)$ and $T(X)$ are closed subsets of X;*
3. $q \int_0^{M(Ax,By,t)} \phi(t)\,dt \geq a \int_0^{M(Ty,Sx,t)} \phi(t)\,dt + b \int_0^{M(Sx,By,t)} \phi(t)\,dt$
$+c \int_0^{M(Ax,By,t)} \phi(t)\,dt + \int_0^{max\{M(Ax,Sx,t),M(By,Ty,t)\}} \phi(t)\,dt$

for all $x, y \in X$, $a, b, c \geq 0$, $q > 0$ and $q < a + b + c$,

then the pairs (A, S) and (B, T) have a point of coincidence each.
Further, if the pair (A, S) is semi-compatible and (B, T) is occasionally weakly compatible, then A, B, S and T have a unique common fixed point.

Proof: The proof follows from Theorem 4.1. □

5. Conclusion

In this paper, Theorem 3.1, Theorem 3.2 are specially constructed for pairwise semi-compatible mappings and occasionally weakly compatible mappings(owc) in fuzzy metric spaces. An example and some applications are is given in support of our result.

References

1. A. Branciari, *A fixed point theorem for mappings satisfying a general contractive condition of integral type*, International Journal of Mathematics and Mathematical Sciences, 29, 531 – 536, (2002).
2. B. Singh, M.S. Chouhan, *Common fixed points of compatible maps in fuzzy metric spaces*, Fuzzy Sets and Systems, 115(3), 471–475, (2000).
3. B. Singh, S. Jain, *Semi-compatibility and fixed point theorems in fuzzy metric space using implicit relation*, Int. J. Math. Math. Sci., 2005(16), 2617–2629, (2005).
4. B.Singh, S. Jain, *Weak compatibility and fixed points in fuzzy metric spaces*, Ganita, 56(2), 167–176, (2005).
5. G. Jungck, *Compatible mappings and common fixed points*, Int. J. Math. Math. Sci., 9(4), 771–779, (1986).
6. L.A. Zadeh, *Fuzzy Sets*, Information and control, 8(3), 338–353, (1965).
7. I. Beg, V. Gupta, A. Kanwar, A., *Fixed Point on Intuitionistic Fuzzy Metric Spaces Using E.A. Property*, Journal of Nonlinear Functional Analysis, 2015, Article ID 20, (2015).
8. M.A. Erceg, *Metric Spaces in fuzzy set theory*, J. Math. Anal. Appl., 69(1), 205–230, (1979).

9. M. Aamir, M., D.El Moutawakil, *Some new common fixed point theorems under strict contractive conditions*, J. Math. Anal. Appl., 270(1), 181–188, (2002).

10. M.A. Al-Thagafi, N. Shahzad, *A note on occasionally weakly compatible maps*, Int. Journal of Math. Analysis, 3(2), 55–58, (2009).

11. M. Abbas, I. Altun, D. Gopal, *Common fixed point theorems for non-compatible mappings in fuzzy metric spaces*, Bull. Math. Anal. Appl., 1(2), 47–56, (2009).

12. O. Karmosil, G.N.V. Kishore, J. Michalek, *Fuzzy metric and statistical metric spaces*, Kybernetika, 11(5), 336–344, (1975).

13. O. Kaleva, S. Seikkala, *On fuzzy metric spaces*, Fuzzy Sets and Systems, 12(3), 215–229, (1984).

14. P.V. Subrahmanyam, *A common fixed point theorem in fuzzy metric spaces*, Information Sciences, 83(3-4), 109–112, (1995).

15. R.K. Saini, V. Gupta, S.B. Singh, *Fuzzy Version of Some Fixed Points Theorems On Expansion Type Maps in Fuzzy Metric Space*, Thai Journal of Mathematics, 5(2), 245–252, (2007).

16. R.K. Saini, V. Gupta, S.B. Singh, M. Kumar, *Common coincidence Points of R-Weakly Commuting Fuzzy Maps*, Thai Journal of Mathematics,6(1), 109–115, (2008).

17. S.M. Kang, V. Gupta, B. Singh, S. Kumar, *Some Common Fixed Point Theorems Using Implicit Relations In Fuzzy Metric Spaces*, International Journal of Pure and Applied Mathematics, 87(2), 333-347, (2013).

18. S.M. Kang, V. Gupta, B. Singh, S. Kumar, *Common fixed point theorems of R-weakly commuting mappings in fuzzy metric spaces*, International Journal of Mathematical Analysis, 9(2), 81-90, (2015).

19. S. Sessa,*On weak commutativity condition of mappings in fixed point considerations*, Publ. Inst. Math.,32(46), 149–153, (1982).

20. S.N. Mishra, S.L. Singh, *Common fixed points of maps in fuzzy metric space*, Int. J. Math. Math. Sci., 17(2), 253–258, (1994).

21. T. Som, *Some Results on Common fixed point in Fuzzy Metic Spaces*, Soochow J. Math., 33(4), 553–561, (2007).

22. V. Gupta, A. Kanwar, *Fixed Point Theorem in Fuzzy Metric Spaces Satisfying E.A Property*, Indian Journal of Science and Technology, 5(12),3767–3769, (2012).

23. V. Gupta, N. Mani, *Existence and Uniqueness of Fixed Point for Contractive Mapping of Integral Type*, International Journal of Computing Science and Mathematics, 4(1), 72–83, (2013).

24. V. Gupta, M. Verma, S. Devi, *Fixed Point Theorem in Fuzzy Metric Spaces Employing CLRg Property*, Advancements in the era of Multi-Disciplinary Systems, Elsevier, Chepter-76, 413-416, (2013).

25. V. Gupta, A. Kanwar, S. Devi, *A Common Fixed Point Theorem on Fuzzy Metric Spaces*, Proceeding of Information and Mathematical Sciences, Elsevier science and technology, 159-161, (2013).

26. V. Gupta, R.K. Saini, N. Mani, A.K Tripathi, *Fixed point theorems by using control function in fuzzy metric spaces*, Cogent Mathematics, 2(1), 7 pages, (2015).

27. V. Gupta, A. Kanwar, N. Gulati, *Common coupled fixed point result in fuzzy metric spaces using JCLR property*, Smart Innovation, Systems and Technologies, Springer, 43(1), 201-208, (2016).

28. V. Pant, R.V. Pant, *Fixed points in fuzzy metric space for non compatible maps*, Soochow J. Math., 33(4), 647–655, (2007).

29. Z.K. Deng, *Fuzzy pseudo metric spaces*, J. Math. Anal. Appl., 86(1), 74–95, (1982).

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