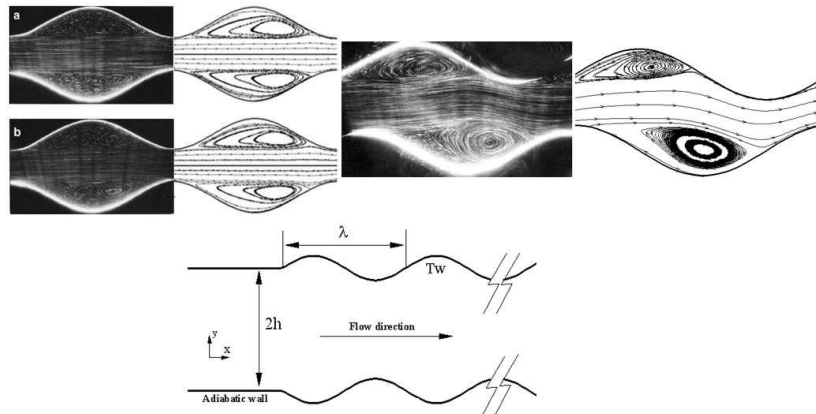




## Flow and Heat Transfer Investigation of Forced Convection of Nanofluid in a Wavy Channel at Different Wavelengths and Phase Difference

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ABSTRACT:  
Graphical Abstract:



Changing the fluid properties and flow geometry are two common ways of making an improvement in heat transfer rate. Recent investigations on nanofluid, as such suspensions are often called, indicate that the suspended nanoparticles remarkably change the transport properties and heat transfer characteristics of the suspension. Bending walls can also improve heat transfer by increasing the total heat transfer surface and changing the behavior of the flow. In this work a two dimensional incompressible laminar nanofluid flow in a wavy channel with sinusoidal curved walls is numerically investigated. The finite volume method, and Rhie and Chow interpolation with a collocated mesh are used for solving the governing equations. The effects of the volume fraction of nanoparticles, Reynolds number, the wavelength, Phase lag and amplitude on the heat transfer rate are studied. The present work showed good agreement with existing experimental and Numerical results. Increasing the amplitude of the wave and nanoparticles volume fraction, and decreasing wavelength of the wave, has great effects on enhancement of heat transfer rate.

Key Words: Laminar, Nanofluid, Nusselt number, Wavy channel, Nanoparticles.

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#### 1 Introduction

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**Nomenclature**

$c_p$	specific heat (kJ/kg.K)
$d$	channel diameter (m)
$h$	average height of the channel (m)
$k$	thermal conductivity (W/m.K)
$k_b$	Boltzmann number (J/K)
$L$	length of the channel (m)
$\dot{m}$	mass flow rate (kg/s)
$\hat{n}$	vector normal to the surface
$Nu$	Nusselt number
$P$	pressure (Pa)
$Pr$	Prandtl number
$\dot{Q}$	heat energy rate (J/s)
$Re$	Reynolds number
$T$	temperature (K)
$u$	x component of the velocity (m/s)
$v$	y component of the velocity (m/s)
$V_b$	Brownian velocity of nanoparticles (m/s)

**Greek symbols**

$\alpha_f$	thermal diffusivity (m <sup>2</sup> /s)
$\alpha$	amplitude of the waves (m)
$\phi$	volume fraction of nanoparticles
$\lambda$	wavelength (m)
$\mu$	viscosity (Pa.s)
$\nu$	kinematic viscosity (m <sup>2</sup> /s)
$\rho$	density (kg/m <sup>3</sup> )
$\bar{\sigma}$	length of curve (m)
$\delta$	center to center distance of nanoparticles (m)

**Subscripts**

$f$	base fluid
$nf$	nanofluid
$p$	particle
$w$	wall

**1. Introduction**

Changing the fluid properties and flow geometry are two common ways of making an improvement in heat transfer rate. Using porous media, micro scale channels, increasing the surface and placing a shackle in the way of the fluid are ways of increasing heat transfer by changing the geometry.

Adding nanoparticles in the base fluid is an active method for increasing the heat transfer rate. Many researchers have focused on this subject. Wang et al. [1] analyzed the rates of heat transfer for flow through a sinusoidally curved converging-diverging channel using a simple coordinate transformation method and the spline alternating-direction implicit method. They studied the effects of wavy geometry,

the Reynolds number and the Prandtl number on the skin-friction and the Nusselt number. They observed that as the wavy amplitude-wavelength and Reynolds number increase, the total heat transfer and local Nusselt number increase in the converging-diverging part of the channel. They concluded that a corrugated channel is an effective heat transfer enhancement device.

Ko et al. [2] numerically investigated developing laminar forced convection and entropy generation in a wavy channel. The effects of aspect ratio and Reynolds number on entropy generation were their major concerns. Their result demonstrated the enhancement in heat transfer by increasing aspect ratio and Reynolds number.

Flow through in a channel with sinusoidal plates was experimentally investigated by Tolentino et al. [3]. The experiments were operated in a water tunnel; and laser illuminated particle tracking was used as the technique of flow visualization. They reported that, for a set of flow and geometric parameters, eight waves were enough to develop the flow in the channel used for their study.

Chang et al. [4] experimentally studied enhancement of heat transfer in a rotating furrowed channel with two opposite walls. Their results include Nusselt number scans which were generated by means of infra-red thermographs over two opposite leading and trailing wavy walls of a radially rotating furrowed channel. For the transverse and skewed wavy channels, the Nusselt number ratio ranges fall, between 5–8.8 and 5.4–11.3, respectively, with  $1000 < Re < 2000$ .

Choi et al. [5] applied large eddy simulation (LES) to the thermal fields of a turbulent flow in a channel having one wavy wall with various wall wave amplitudes. They showed that both the friction coefficient and Nusselt number have maximum values in the upper part of the wavy wall. Wall bending is found to effectively enhance the wall heat transfer.

A numerical investigation of heat transfer and flow field in a wavy channel with a nanofluid was performed by Heidary et al. [6]. They reported that adding nanoparticles to the base fluid and the construction of wavy walls can significantly enhance the heat transfer. They also observed that the skin friction coefficient is almost insensitive to the volume fraction of nanoparticles. However, they only studied the effect of amplitude on the heat transfer. It seems that studying the effects of amplitude alone is not enough to fully understand the effect of wavy channels on Nusselt number.

So in this paper the effect of other parameters, such as wavelength and phase lag have been investigated.

Falahaty et al. [7] numerically studied forced convection of laminar nanofluid flow in a diverging sinusoidal channel. The results showed that by increasing aspect ratio, wall wave amplitude and Reynolds number and by decreasing wall wavelength, the length and area of recirculation zones are increased. Also, it was demonstrated that increase of the nanoparticle volume fraction and decrease of nanoparticles size, increase the fluid heat conductivity. Both recirculation growth and conductivity enhancement have an increasing impact on the local Nusselt number along the walls and a hence on the averaged Nusselt number of the duct.

Esmaili et al. [8] synthesized the porous superparamagnetic Fe<sub>3</sub>O<sub>4</sub> nanoparticles

with an average particle size of 75 nm through a potential solvothermal method in order to investigate the influence of a 3-D low voltage magnetic field on the convective heat transfer measurements. According to their results, use of the magnetic field had a significant effect on the heat transfer values induced by both Brownian motions and viscosity gradients; however, this enhancement was strongly sensitive to the applied field strength. Conclusively, their findings showed promising insights into the enhancement of the forced-convective heat transfer properties of nanofluids by applying an appropriate magnetic field.

In this work laminar flow of water containing  $\text{Al}_2\text{O}_3$  nanoparticles is numerically simulated in a channel with sinusoidally curved walls. For this purpose the governing equations were discretized and solved using finite volume and a SIMPLE based method. Effects of different Reynolds numbers, nanoparticle volume fraction, amplitude of sinusoidal curves, and wavelength of sinusoidal curves on the Nusselt number have been studied.

#### Physics of the problem and governing equations

The geometry of the problem is a channel with two direct sections at the entrance and exit. Wavy walls are located at the middle section as shown in Figure 1. The fluid enters the channel from the left and exits from the right side. The wavy walls in the top and bottom are considered isothermal and the direct sections of the channel are considered as adiabatic walls. The equation for describing the wavy wall is a function of variable amplitude and wavelength. The length of the direct section varies from  $0.6\text{m} < L < 2.0\text{m}$  and the maximum length for all cases is 20 m. In this study six complete sinusoidal waves have been considered for all cases.

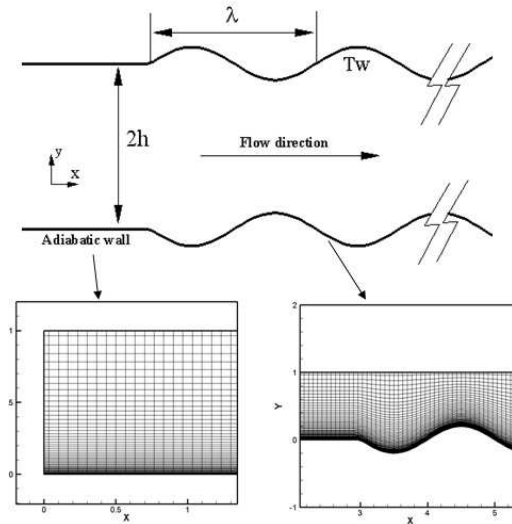


Figure 1: Physical model and grids used in present computing. (total 6 pitches for present work)

The continuity, momentum and energy equations for the present 2D incompressible flow can be expressed as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1.1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial x} + \nu_{nf} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \quad (1.2)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial y} + \nu_{nf} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \quad (1.3)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \quad (1.4)$$

The boundary conditions for this kind of flow are described as:

$$\frac{\partial T}{\partial x} = 0, \quad \frac{\partial u}{\partial x} = 0 \text{ for outlet boundary condition} \quad (1.5)$$

$$T = T_w, \quad u = v = 0 \text{ for isothermal walls} \quad (1.6)$$

$$\frac{\partial T}{\partial \hat{n}} = 0, \quad u = v = 0 \text{ for adiabatic boundary conditions} \quad (1.7)$$

The Reynolds number is defined as:

$$Re = \frac{\rho_f U_{in} h}{\mu_f}. \quad (1.8)$$

The characteristics length for calculating Re is the average height of the wavy walls (h).

Local Nusselt number can be defined as:

$$Nu_x = -\frac{k_{nf} h (\partial T / \partial \hat{n})}{k_f (T_{in} - T_y)} \Big|_{y=s(x)}, \quad (1.9)$$

where  $\frac{\partial T}{\partial \hat{n}}$  is the temperature gradient normal to the walls.

$$\frac{\partial T}{\partial \hat{n}} = \vec{\nabla} T \cdot \hat{n} \quad (1.10)$$

The averaged Nusselt number  $\overline{Nu}$  is:

$$\overline{Nu} = \frac{\dot{Q} L}{Ak_f \Delta T \bar{x}}, \quad (1.11)$$

where  $\dot{Q}$  is total heat transferred.

$$\dot{Q} = \dot{m} c_f (T_{bulk} - T_{in}) \Big|_{x=L} \quad (1.12)$$

And  $\bar{\sigma}$  is the length of the curve and can be calculated as:

$$\bar{\sigma} = \int_{x_s}^x \left( 1 + \left( \frac{\partial y}{\partial x} \right)^2 \right)^{1/2} \quad (1.13)$$

here  $T_{bulk}$  is the mean temperature and can be calculated as defined below:

$$T_{bulk} = \frac{\int_{A_c} \rho u c_f T dA_c}{\dot{m} c_f} \quad (1.14)$$

The thermal conductivity of the nanofluid is calculated from Chon et al. [9], which is expressed in the following form:

$$\frac{k_{nf}}{k_f} = 1 + 64.7 \phi^{0.746} (d_f/d_p)^{0.369} \times (k_p/k_f)^{0.7476} \text{Pr}_f^{0.9955} \text{Re}_p^{1.2321} \quad (1.15)$$

where  $\text{Pr}_f$ ,  $\text{Re}_p$  and  $d_p$  are the Prandtl number of the fluid, Reynolds number and diameter of the nanoparticles, respectively.

$$\text{Pr}_f = \mu_f / \rho_f \alpha_f \quad (1.16)$$

$$\text{Re}_p = \rho_f k_b T / 3 \pi \mu_f^2 l_f \quad (1.17)$$

$$d_p = 36 \times 10^{-9} (m) \quad (1.18)$$

$k_b = 1.3807 \times 10^{-23}$  is the Boltzmann constant and  $l_f$  is the mean free path of water molecules that according to the suggestion of Chon and et al. [9] is taken as 17 nm. Some researchers approved the accuracy of this model [10].

The viscosity of the nanofluid is approximated by the suggested correlation of Masoumi et al. [11]:

$$\frac{\mu_{nf}}{\mu_f} = 1 + \frac{\rho_p V_b d_p^2}{72 N \delta} \quad (1.19)$$

where  $\delta$  is the center to center distance of nanoparticles and  $V_b$  is the Brownian velocity of the nanoparticles.

$$\delta = \sqrt[3]{\pi/6 \phi \times d_p} \quad (1.20)$$

$$V_b = (1/d_p) \sqrt{18 k_b T / \pi \rho_p d_p} \quad (1.21)$$

$N = (c_1 \phi + c_2) d_p + (c_3 \phi + c_4)$  is a parameter for adapting the results with experimental data where  $c_1 = -1.133 \times 10^{-6}$ ,  $c_2 = -2.771 \times 10^{-6}$ ,  $c_3 = 9.0 \times 10^{-8}$  and  $c_4 = -3.93 \times 10^{-7}$ .

The density and specific heat of the nanofluid are calculated using the correlation provided by Pak and Cho [12], which are defined as follows:

$$\rho_{nf} = \phi \rho_p + (1 - \phi) \rho_f \quad (1.22)$$

$$c_p)_{nf} = \frac{(1-\phi) \times \rho_f \times (c_p)_f + \phi \times (\rho \times c_p)_p}{\rho_{nf}} \quad (1.23)$$

$$\text{Pr}_{nf} = \mu_{nf} \times c_p)_{nf} / k_{nf} \quad (1.24)$$

### Results and Discussion

As shown in Figure 2, the results of several grids ( $100 \times 40$ ,  $150 \times 40$ ,  $200 \times 50$ ,  $300 \times 50$ ,  $350 \times 60$  and  $400 \times 80$ ) have been compared with the results provided by Wang et al. [1]. It is observed that  $300 \times 50$  grid guaranties the mesh independency. Present calculations were performed for  $Re = 100$  to  $500$ ,  $Pr = 6.5$ ,  $\alpha = 0.0$  to  $0.03$ ,  $\phi = 0.0$  to  $0.05$  and  $\lambda = 1$  to  $12m$ . The fluid considered here is a suspension of  $Al_2O_3$  nanoparticles in water. For all cases wavelength remains a constant value of 2 meters, except for the cases that wave length mentioned to have a specific value.

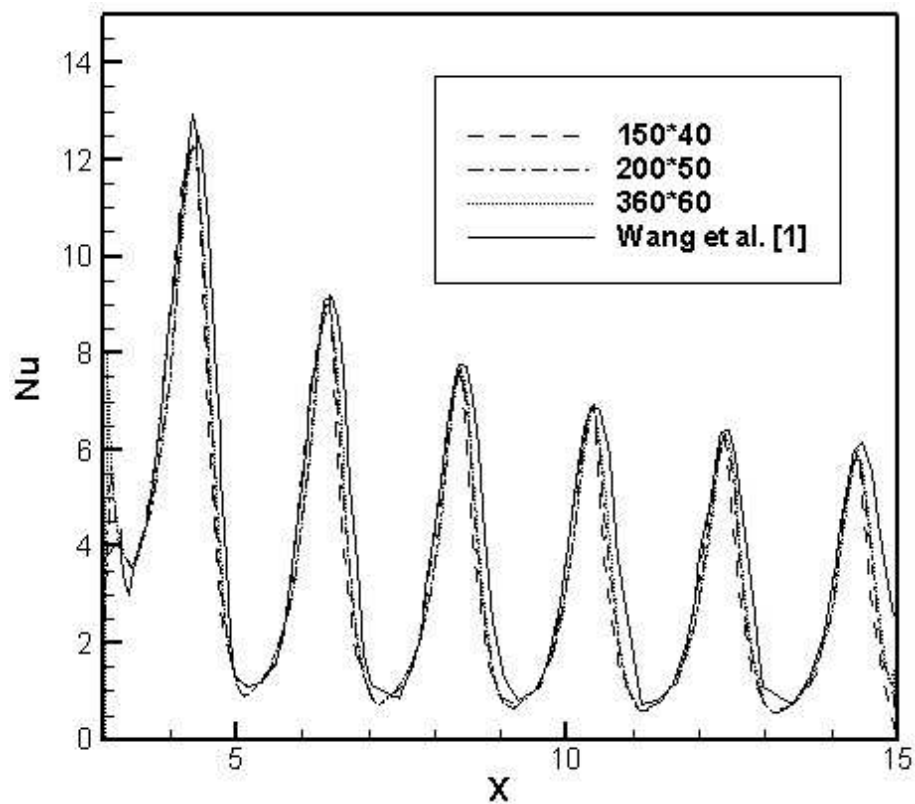


Figure 2: Local Nusselt number for different grids.  $\alpha = 0.2$  and  $Re = 100$ .

Figure 3 compares the present computational results with the experimental results provided by Tolentino et al. [3]. In the last waves a fluctuation in the fluid behavior at the experimental results of Tolentino et al. [3] is seen which cannot be detected by the numerical solution. It should be noted that there are many other numerical researchers that were unable to capture unsteady fluctuations. They considered this type of problems as a symmetric problem. For example some of these researches are: Wang et al. [1], Heidary et al. [5], Ko et al. [3].

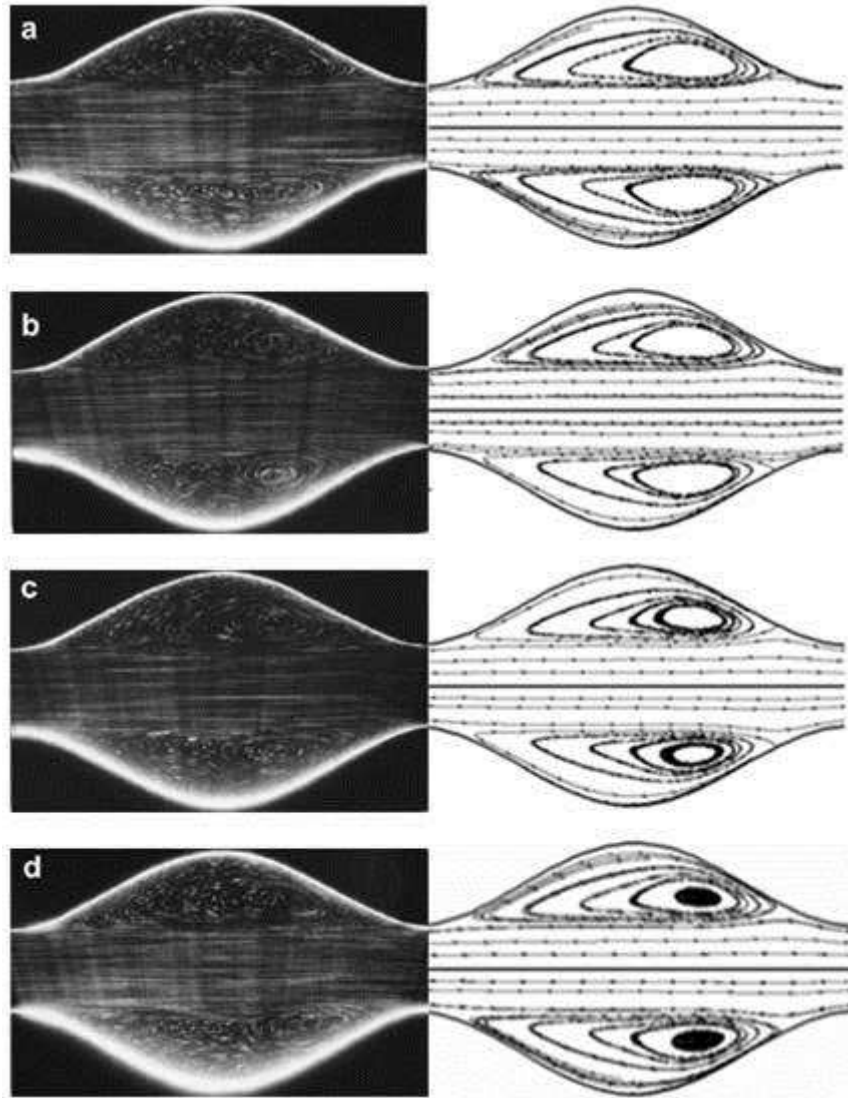


Figure 3: Comparison of stream lines for  $Re = 600$  from the first wave (a) to fourth wave (d).  $\lambda = 2.77$ ,  $\alpha = 0.25$ ,  $\phi = 0.0$ .

Figures 3 and 4 compare stream lines of the present work and experimentally generated photos that provided by Tolentino et al. [3]. Although the present code is capable to capturing these fluctuations and the stream lines for early waves are matched quite well with the experiments.



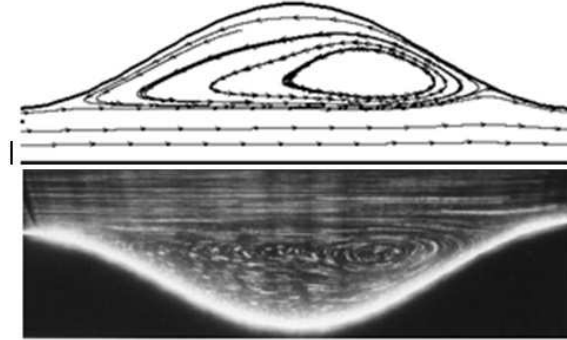


Figure 4: Comparison of stream lines in  $Re = 600$ .  $\lambda = 2.77$ ,  $\alpha = 0.25$ ,  $\phi = 0.0$ .

As it was expected the averaged Nusselt number increases as the volume fraction increases. However, in high volume fractions and high Reynolds numbers it has been seen that adding more nanoparticles to the base fluid will cause a slightly reduction in the averaged Nusselt number. This is a consequence of increasing the viscosity of the fluid. Figure 5 shows that up to a 25% enhancement of heat transfer by adding nanoparticles, is achieved.

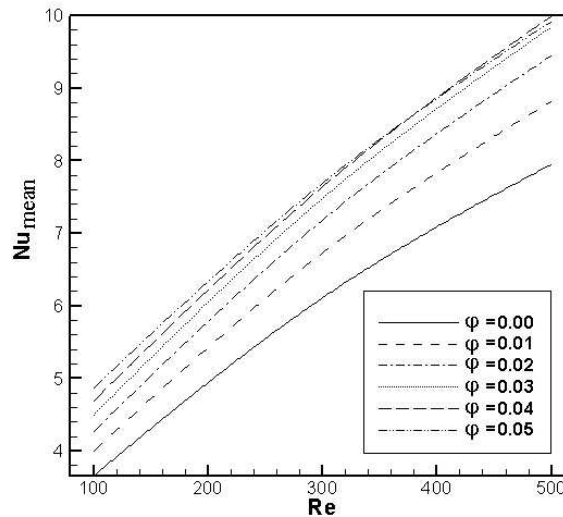


Figure 5: Heat transfer enhancement due to adding nanoparticles.  $\alpha = 0.2$ .

Figure 6 shows the distribution of local Nusselt number along the channel length. The local Nusselt number has a maximum value at the maximum position of the waves. The Local Nusselt in the lowest position of the waves was decreased because of the vortex formation. However, the averaged Nusselt number increased with the increasing amplitude of the waves. Effect of amplitude on heat transfer enhance-

ment, for a low Re is small, but for a high Re and high amplitudes it becomes larger.

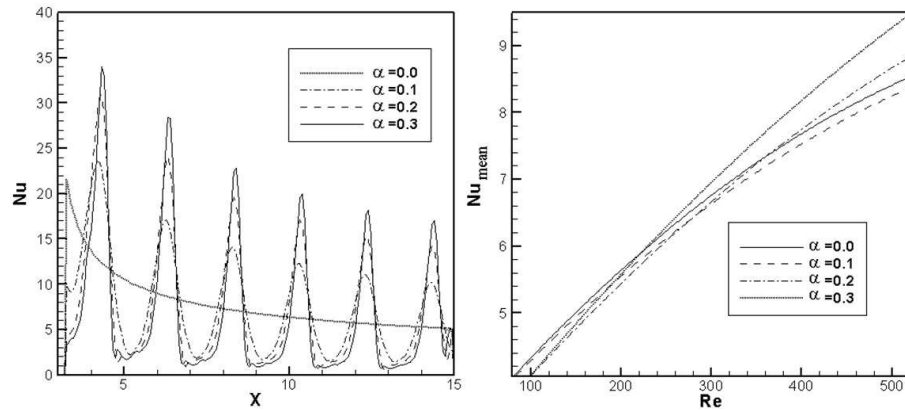


Figure 6: (a) Nusselt number for  $Re = 300$ ,  $\alpha = 0.2$  and (b) Averaged Nusselt number for different wave amplitude.  $\alpha = 0.2$ .

Figure 7 shows the averaged Nusselt number at different wavelengths. As the wavelength increases the averaged Nusselt number approaches the mean Nusselt number of a simple straight channel. There is an exceptional behavior near wavelength 3 and four. It is related to formation of recirculation zone. With decreasing wavelength vortices form near wall lead to circulating behavior flow. These circulating flow causes to decreasing of heat transfer. On the other hand decreasing of the wavelength will cause to increasing the heat transfer. Total enhancement or lessen of heat transfer is dependent on which term is stronger.

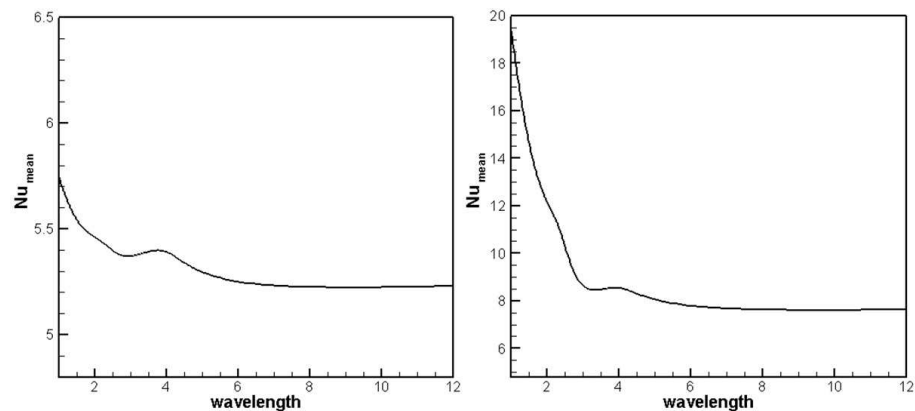


Figure 7: Averaged Nusselt changes with different wavelength for (a)  $Re = 200$  (b)  $Re = 400$ .  $\alpha = 0.2$   $h = 0.5$ .

The effect of wavelength can be seen in Figure 8 for  $Re = 200$  and  $Re = 400$ . This increasing is due to increasing the hot area around the fluid. By increasing the wavelength of the channel the averaged Nusselt tends to approach the averaged Nusselt in a simple channel. Around a wavelength of 3 to 4 there is an unusual behavior for the Nusselt number. If we consider an inviscid fluid, increasing the velocity and  $Re$  leads to an enhancement of heat transfer. On the other hand for viscous fluids the adverse pressure gradient separates the flow from the surface and vortex is formed.

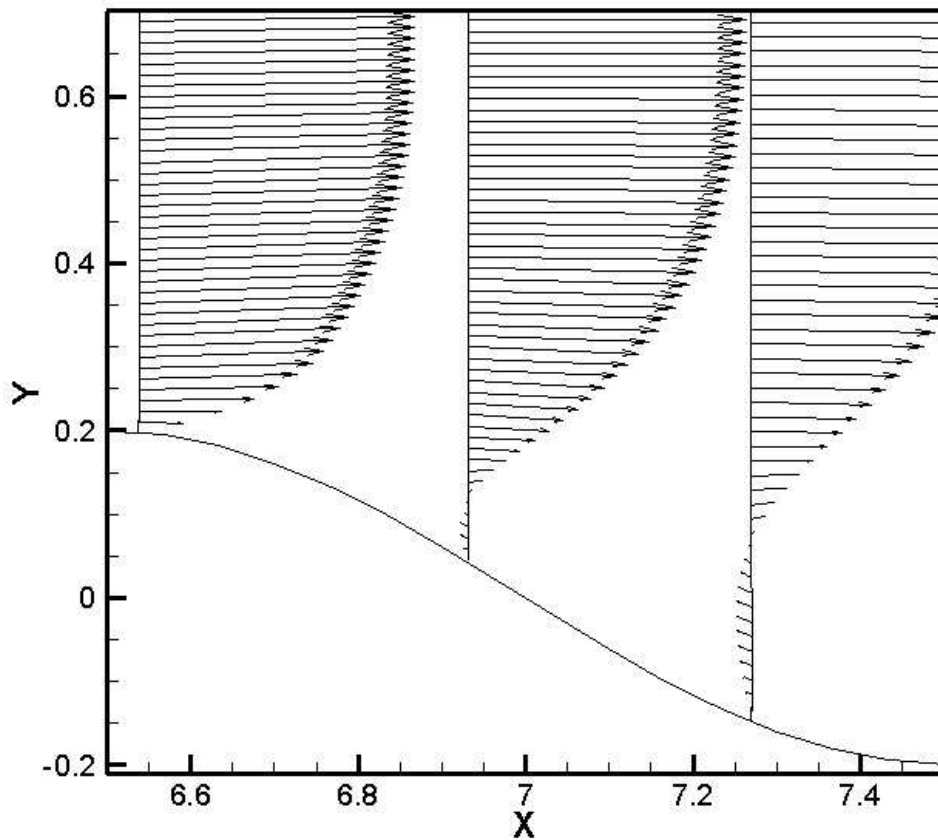


Figure 8: Separation behind the peaks and formation of vortex.  $\alpha = 0.2$ ,  $Re = 100$ .

In Figure 9 the stream functions is compared with the experimental work established by Tolentiono et al. [3]. A good agreement is observed between them.

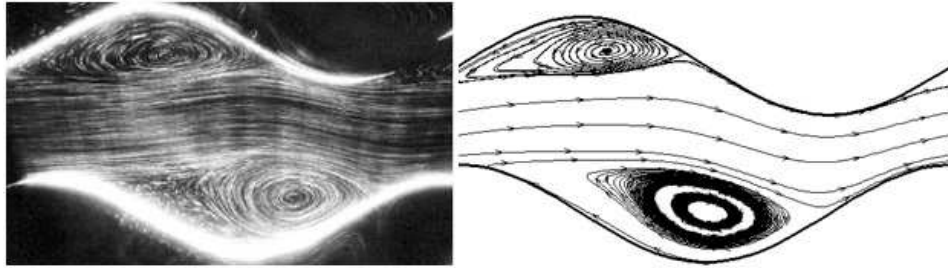


Figure 9: Behavior of stream lines in  $Re = 950$  and  $270$  degrees phase difference. (a) Experimental (b) Numerical.  $h = 0.6$ ,  $\alpha = 0.18$ ,  $\lambda = 2$ .

In Figure 10 the effect of phase lag on heat transfer is presented. Phase lag is the phase difference of the upper wall's wave from the lower wall's wave. A high heat transfer enhancement is observed for  $135$  and  $225$  phase lag degrees. These peaks are typical for any case, however the peak intensity is stronger when height of the channel is small.

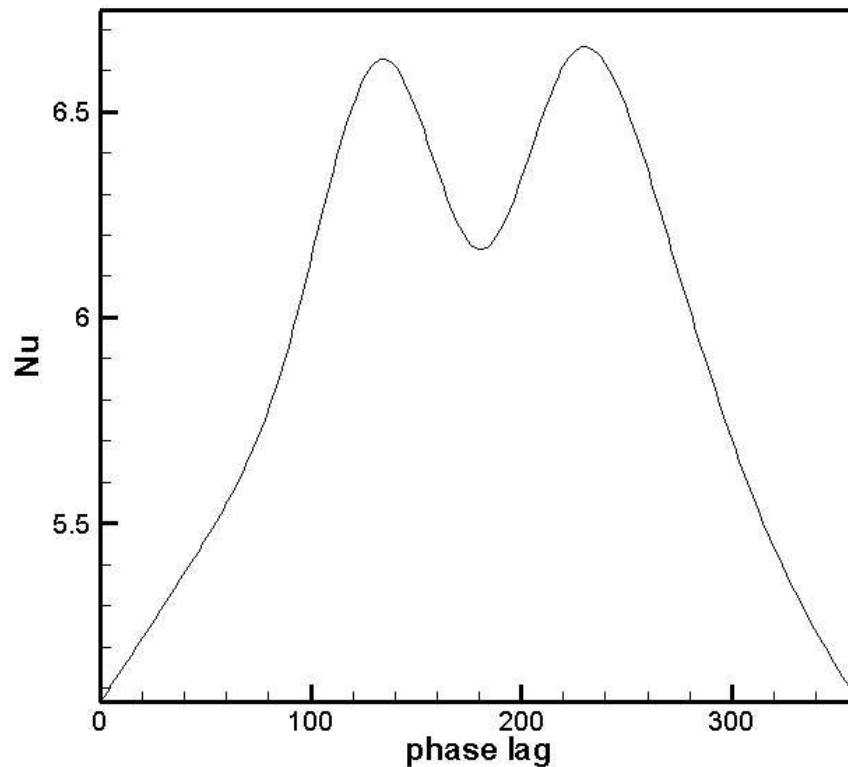


Figure 10: Averaged Nusselt change with angel  $Re = 300$ ,  $\alpha = 0.2$ ,  $\phi = 0.0$ .

## Conclusion

The nanofluid flow in a two dimensional channel with wavy walls has been investigated numerically. The effects of wavelengths, phase difference between waves of opposite walls, Reynolds number and volume fraction of nanoparticles on the local and total heat transfer has been studied. Adding nanoparticles to the base flow had a great effect on heat transfer. Also, it has been seen that at 135 or 225 degrees the angle difference will maximize the heat transfer. Another way to increase heat transfer was to decrease the wavelength; however it may not be a straight answer for industrial concerns due to pressure drop.

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