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On Evolute Curves In Terms Of Inextensible Flows Of In $\mathbb{E}^3$

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ABSTRACT: In this paper, we study inextensible flows of evolute curve of curves in $\mathbb{E}^3$. We research inextensible flows evolute curves of in the $\mathbb{E}^3$.

Key Words: Inextensible flows, evolute curve.

Contents

1. Introduction

The flow of a curve or surface is said to be inextensible if, in the former case, the arclength is preserved, and in the latter case, if the intrinsic curvature is preserved. Physically, inextensible curve and surface flows are characterized by the absence of any strain energy induced from themotion. Kwon investigated inextensible flows of curves and developable surfaces in $\mathbb{R}^3$. Necessary and sufficient conditions for an inextensible curve flow first expressed as a partial differential equation involving the curvature and torsion. Then, they derived the corresponding equations for the inextensible flow of a developable surface, and showed that it suffices to describe its evolution in terms of two inextensible curve flows, [8,9].

In the past two decades, for the need to explain certain physical phenomena and to solve practical problems, geometers and geometric analysis have begun to deal with curves and surfaces which are subject to various forces and which flow or evolve with time in response to those forces so that the metrics are changing. Now, various geometric flows have become one of the central topics in geometric analysis. Many authors have studied geometric flow problems, [10].

In this paper, we study evolute curve of inextensible flows of curves in $\mathbb{E}^3$. We research evolute curve by means of inextensible flows curves of in the $\mathbb{E}^3$.

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2. Preliminaries

The variable s is employed to denote arc length along a space curve. Note that the arc-length parameterization $\mathbf{r} : s \to \mathbf{r}(s)$ of a curve satisfies $\|\mathbf{r}'(s)\| = 1$ and $\mathbf{r}'(s) \perp \mathbf{r}''(s)$ for all s. However, in this paper, a general parameterization $\mathbf{r} : t \to \mathbf{r}(t)$ is often used in the surface construction problem. The parameters of functions may sometimes be omitted when no confusion can arise.

With each point $\mathbf{r}(s)$ of a curve satisfying $\mathbf{r}''(s) \neq 0$, we associate the *Serret–Frenet frame* $(\mathbf{T}(s), \mathbf{N}(s), \mathbf{B}(s))$ where $\mathbf{T}(s) = \mathbf{r}'(s)$, $\mathbf{N}(s) = \mathbf{r}''(s)/\left\|\mathbf{r}''(s)\right\|$, and $\mathbf{B}(s) = \mathbf{T}(s) \times \mathbf{N}(s)$ are, respectively, the unit *tangent, principal normal*, and *binormal* vectors of the curve at the point $\mathbf{r}(s)$. The arc-length derivative of the Serret–Frenet frame is governed by the relations

$$\frac{d}{ds}\begin{bmatrix} \mathbf{T}(s) \\ \mathbf{N}(s) \\ \mathbf{B}(s) \end{bmatrix} = \begin{bmatrix} 0 & \kappa(s) & 0 \\ -\kappa(s) & 0 & \tau(s) \\ 0 & -\tau(s) & 0 \end{bmatrix} \begin{bmatrix} \mathbf{T}(s) \\ \mathbf{N}(s) \\ \mathbf{B}(s) \end{bmatrix}, \tag{2.1}$$

where the *curvature* $\kappa(s)$ *and torsion* $\tau(s)$ of the curve $\mathbf{r}(s)$ are defined by

$$\kappa(s) = \|\mathbf{r}''(s)\| \quad \text{and} \quad \tau(s) = \frac{\det(\mathbf{r}'(s), \mathbf{r}''(s), \mathbf{r}'''(s))}{\left\|\mathbf{r}''(s)\right\|^2}. \tag{2.2}$$

Definition 2.1. Let unit speed curve $\mathbf{r}$ and the curve β with the same interval be given. For $\forall s \in I,$ the curve β is called the evolute of the curve $\mathbf{r}$, if the tangent at the point $\beta(s)$ to the curve β passes through the tangent at the point $\mathbf{r}(s)$ to the curve β and

$$\langle \mathbf{T}^*(s), \mathbf{T}(s) \rangle = 0.$$

Let the Frenet-Serret frames of the curves $\mathbf{r}$ and β be $\{\mathbf{T}, \mathbf{N}, \mathbf{B}\}$ and $\{\mathbf{T}^*, \mathbf{N}^*, \mathbf{B}^*\}$, respectively.

3. Evolute Curves By Using Inextensible Flows In $\mathbb{E}^3$

We recall the following result from [9].

Lemma 3.1. *Let* $\dfrac{\partial \mathbf{r}}{\partial t} = f\mathbf{T} + g\mathbf{N} + h\mathbf{B}$ *be a smooth flow of the curve* $\mathbf{r}$ *in* $\mathbb{E}^3$. *Then,*

$$\begin{aligned} \frac{\partial \mathbf{T}}{\partial t} &= (f\kappa - h\tau + \frac{\partial g}{\partial s})\mathbf{N} + (g\tau + \frac{\partial h}{\partial s})\mathbf{B}, \\ \frac{\partial \mathbf{N}}{\partial t} &= -(f\kappa - h\tau + \frac{\partial g}{\partial s})\mathbf{T} + \psi\mathbf{B}, \\ \frac{\partial \mathbf{B}}{\partial t} &= -(g\tau + \frac{\partial h}{\partial s})\mathbf{T} - \psi\mathbf{N}, \end{aligned} \tag{3.1}$$

where $\psi = < \dfrac{\partial \mathbf{N}}{\partial t}, \mathbf{B} >$.

Theorem 3.2. *Let* β *be evolute of a space curve* $\mathbf{r}$ and $\dfrac{\partial \beta}{\partial t}$ *its inextensible flow in* $\mathbb{E}^3$. *If* $\dfrac{\partial \mathbf{r}}{\partial t}$ *is inextensible, then*

$$\begin{aligned}
\frac{\partial \mathbf{T}^*}{\partial t} &= (-(f\kappa - h\tau + \frac{\partial g}{\partial s})\cos(\varphi + c) + (g\tau + \frac{\partial h}{\partial s})\sin(\varphi + c))\mathbf{T} \\
&\quad +(\frac{\partial}{\partial t}(\cos(\varphi + c)) + \psi \sin(\varphi + c))\mathbf{N} + (-\frac{\partial}{\partial t}(\sin(\varphi + c)) + \psi \cos(\varphi + c))\mathbf{B}, \\
\frac{\partial \mathbf{N}^*}{\partial t} &= -(f\kappa - h\tau + \frac{\partial g}{\partial s})\mathbf{N} - (g\tau + \frac{\partial h}{\partial s})\mathbf{B}, \qquad (3.2) \\
\frac{\partial \mathbf{B}^*}{\partial t} &= (-(f\kappa - h\tau + \frac{\partial g}{\partial s})\sin(\varphi + c) - (g\tau + \frac{\partial h}{\partial s})\cos(\varphi + c))\mathbf{T} \\
&\quad +(\frac{\partial}{\partial t}(\sin(\varphi + c)) - \psi \cos(\varphi + c))\mathbf{N} + (\frac{\partial}{\partial t}\cos(\varphi + c)\psi \sin(\varphi + c))\mathbf{B}.
\end{aligned}$$

where

$$c \in \mathbb{R}\ ,\varphi(s) = \int \tau(s)ds \ \text{and}\ \psi = < \frac{\partial \mathbf{N}}{\partial t}, \mathbf{B} > . \qquad (3.3)$$

Proof. From definition of evolute curve, the curve $\beta(s)$ may be given as

$$\begin{aligned}
\frac{\partial \mathbf{T}^*}{\partial t} &= (-(f\kappa - h\tau + \frac{\partial g}{\partial s})\cos(\varphi + c) + (g\tau + \frac{\partial h}{\partial s})\sin(\varphi + c))\mathbf{T} \\
&\quad +(\frac{\partial}{\partial t}\cos(\varphi + c) + \psi \sin(\varphi + c))\mathbf{N} \qquad (3.4) \\
&\quad +(-\frac{\partial}{\partial t}\sin(\varphi + c) + \psi \cos(\varphi + c))\mathbf{B}.
\end{aligned}$$

On the other hand, we obtain

$$\frac{\partial \mathbf{N}^*}{\partial t} = -(f\kappa - h\tau + \frac{\partial g}{\partial s})\mathbf{N} - (g\tau + \frac{\partial h}{\partial s})\mathbf{B}.$$

Thus,

$$\begin{aligned}
\frac{\partial \mathbf{B}^*}{\partial t} &= (-(f\kappa - h\tau + \frac{\partial g}{\partial s})\sin(\varphi + c) - (g\tau + \frac{\partial h}{\partial s})\cos(\varphi + c))\mathbf{T} \\
&\quad +(\frac{\partial}{\partial t}(\sin(\varphi + c)) - \psi \cos(\varphi + c))\mathbf{N} \qquad (3.5) \\
&\quad +(\frac{\partial}{\partial t}(\cos(\varphi + c)) + \psi \sin(\varphi + c))\mathbf{B},
\end{aligned}$$

where

$$c \in \mathbb{R}\ ,\ \varphi(s) = \int \tau(s)ds \ \text{and}\ \psi = < \frac{\partial \mathbf{N}}{\partial t}, \mathbf{B} > .$$

Combining (3.4) and (3.5), we have theorem. This concludes the proof of theorem.

Theorem 3.3. *Let* β *be evolute of a space curve* $\mathbf{r}$ and $\frac{\partial \beta}{\partial t}$ *its inextensible flow in* $\mathbb{E}^3$. *If* $\frac{\partial \mathbf{r}}{\partial t}$ *is inextensible, then*

$$\begin{aligned}
& \tau \frac{\partial}{\partial t}(\cos(\varphi + c)) + \tau\psi \sin(\varphi + c) + \frac{\partial}{\partial s}(-\frac{\partial}{\partial t}(\sin(\varphi + c)) + \psi \cos(\varphi + c)) \\
= & \frac{\partial}{\partial t}(\tau \cos(\varphi + c)) - \frac{\partial^2}{\partial s \partial t}(\varphi \cos(\varphi + c)) - \psi \frac{\partial}{\partial s}(\varphi \sin(\varphi + c)) + \psi\tau \sin(\varphi + c) \\
& -(\kappa \cos(\varphi + c))(g\tau + \frac{\partial h}{\partial s}),
\end{aligned} \tag{3.6}$$

where

$$c \in \mathbb{R}\ , \varphi(s) = \int \tau(s) ds \ \text{and} \ \psi = < \frac{\partial \mathbf{N}}{\partial t}, \mathbf{B} > .$$

Proof. Using (3.2), we easily have

$$\begin{aligned}
\frac{\partial}{\partial t}\frac{\partial \mathbf{T}^*}{\partial s} = & [\frac{\partial}{\partial t}(\kappa \cos(\varphi + c)) + (-\frac{\partial}{\partial s}(\varphi \sin(\varphi + c)) + \tau \sin(\varphi + c))(-f\kappa + h\tau - \frac{\partial g}{\partial s}) \\
& -(\tau \cos(\varphi + c) - \frac{\partial}{\partial s}\varphi \cos(\varphi + c))(g\tau + \frac{\partial h}{\partial s})]\mathbf{T} + [-\frac{\partial^2}{\partial s \partial t}(\varphi \sin(\varphi + c)) \\
& + \frac{\partial}{\partial t}(\tau \sin(\varphi + c)) - (\kappa \cos(\varphi + c))(f\kappa - h\tau + \frac{\partial g}{\partial s})) - (\tau\psi \cos(\varphi + c) \\
& -\psi \frac{\partial}{\partial s}(\varphi \cos(\varphi + c))]\mathbf{N} + [\frac{\partial}{\partial t}(\tau \cos(\varphi + c)) - \frac{\partial^2}{\partial s \partial t}(\varphi \cos(\varphi + c)) \\
& -\psi \frac{\partial}{\partial s}(\varphi \sin(\varphi + c)) + \psi\tau \sin(\varphi + c) - (\kappa \cos(\varphi + c))(g\tau + \frac{\partial h}{\partial s})]\mathbf{B}.
\end{aligned}$$

Then,

$$\begin{aligned}
\frac{\partial}{\partial t}\frac{\partial \mathbf{T}^*}{\partial s} = & [\frac{\partial}{\partial t}(\kappa \cos(\varphi + c)) + (-\frac{\partial}{\partial s}(\varphi \sin(\varphi + c)) + \tau \sin(\varphi + c))(-f\kappa + h\tau - \frac{\partial g}{\partial s}) \\
& -(\tau \cos(\varphi + c) - \frac{\partial}{\partial s}\varphi \cos(\varphi + c))(g\tau + \frac{\partial h}{\partial s})]\mathbf{T} + [-\frac{\partial^2}{\partial s \partial t}(\varphi \sin(\varphi + c)) \\
& + \frac{\partial}{\partial t}(\tau \sin(\varphi + c)) - (\kappa \cos(\varphi + c))(f\kappa - h\tau + \frac{\partial g}{\partial s})) - (\tau\psi \cos(\varphi + c) \\
& -\psi \frac{\partial}{\partial s}(\varphi \cos(\varphi + c))]\mathbf{N} + [\frac{\partial}{\partial t}(\tau \cos(\varphi + c)) - \frac{\partial^2}{\partial s \partial t}(\varphi \cos(\varphi + c)) \\
& -\psi \frac{\partial}{\partial s}(\varphi \sin(\varphi + c)) + \psi\tau \sin(\varphi + c) - (\kappa \cos(\varphi + c))(g\tau + \frac{\partial h}{\partial s})]\mathbf{B}.
\end{aligned}$$

Also, we obtain

$$\begin{aligned}
& \tau\frac{\partial}{\partial t}(\cos(\varphi+c))+\tau\psi\sin(\varphi+c)+\frac{\partial}{\partial s}(-\frac{\partial}{\partial t}(\sin(\varphi+c))+\psi\cos(\varphi+c)) \\
= & \frac{\partial}{\partial t}(\tau\cos(\varphi+c))-\frac{\partial^2}{\partial s\partial t}(\varphi\cos(\varphi+c))-\psi\frac{\partial}{\partial s}(\varphi\sin(\varphi+c))+\psi\tau\sin(\varphi+c) \\
& -(\kappa\cos(\varphi+c))(g\tau+\frac{\partial h}{\partial s}),
\end{aligned}$$

where

$$c\in\mathbb{R}\ ,\varphi(s)=\int\tau(s)ds\ \ and\ \ \psi=<\frac{\partial\mathbf{N}}{\partial t},\mathbf{B}>.$$

On the other hand, we obtain

$$\begin{aligned}
\frac{\partial}{\partial s}\frac{\partial\mathbf{N}^*}{\partial t} &= \frac{\partial}{\partial s}(-(f\kappa-h\tau+\frac{\partial g}{\partial s})\mathbf{N}-(g\tau+\frac{\partial h}{\partial s})\mathbf{B}) \\
& -\frac{\partial}{\partial s}(f\kappa-h\tau+\frac{\partial g}{\partial s})\mathbf{N}-(f\kappa-h\tau+\frac{\partial g}{\partial s})(-\kappa\mathbf{T}+\tau\mathbf{B}) \\
& -\frac{\partial}{\partial s}(g\tau+\frac{\partial h}{\partial s})\mathbf{B}-(g\tau+\frac{\partial h}{\partial s})(-\tau\mathbf{N}) \\
&= [f\kappa^2-h\kappa\tau+\kappa\frac{\partial g}{\partial s}]\mathbf{T}+[-\frac{\partial}{\partial s}(f\kappa-h\tau+\frac{\partial g}{\partial s}+g\tau^2+\tau\frac{\partial h}{\partial s}]\mathbf{N} \\
& +[-f\kappa\tau+h\tau^2-\tau\frac{\partial g}{\partial s}-\frac{\partial}{\partial s}(g\tau+\frac{\partial h}{\partial s})]\mathbf{B}.
\end{aligned}$$

Also,

$$\frac{\partial}{\partial t}\frac{\partial\mathbf{N}^*}{\partial s}=(-f\kappa^2+h\kappa\tau-\kappa\frac{\partial g}{\partial s})\mathbf{T}+\frac{\partial\kappa}{\partial t}\mathbf{N}+\boldsymbol{\kappa}\psi\mathbf{B}.$$

Similarly, we have

$$\begin{aligned}
\frac{\partial}{\partial s}\frac{\partial\mathbf{B}^*}{\partial t} &= =[(\frac{\partial}{\partial s}[(-(f\kappa-h\tau+\frac{\partial g}{\partial s})\sin(\varphi+c)-(g\tau+\frac{\partial h}{\partial s})\cos(\varphi+c)] \\
& +(-\kappa\frac{\partial}{\partial t}\sin(\varphi+c)+\kappa\psi\cos(\varphi+c)]\mathbf{T}+[\frac{\partial}{\partial s}[\sin(\varphi+c) \\
& -\psi\cos(\varphi+c)]-\tau\frac{\partial}{\partial t}\cos(\varphi+c)-\tau\psi\sin(\varphi+c)+(-(f\kappa-h\tau \\
& +\frac{\partial g}{\partial s})\kappa\sin(\varphi+c)-(g\tau+\frac{\partial h}{\partial s})\kappa\cos(\varphi+c)]\mathbf{N}+[\tau\frac{\partial}{\partial t}\sin(\varphi+c) \\
& -\tau\psi\cos(\varphi+c)+\frac{\partial}{\partial s}(\frac{\partial}{\partial t}\cos(\varphi+c)+\psi\sin(\varphi+c))]\mathbf{B}.
\end{aligned}$$

On the other hand, we have

$$\begin{aligned}
\frac{\partial}{\partial t}\frac{\partial \mathbf{B}^*}{\partial s} &= =[\frac{\partial}{\partial s}\sin(\varphi+c)(-(f\kappa-h\tau+\frac{\partial g}{\partial s})+\frac{\partial}{\partial t}(-\kappa\sin(\varphi+c))\\
&+\tau\sin(\varphi+c)(-g\tau-\frac{\partial h}{\partial s})+\frac{\partial}{\partial s}(\cos(\varphi+c))(-g\tau-\frac{\partial h}{\partial s})\\
&+\tau\cos(\varphi+c)(f\kappa-h\tau+\frac{\partial g}{\partial s})]\mathbf{T}+[\frac{\partial^2}{\partial t\partial s}(\sin(\varphi+c))\\
&-\kappa\sin(\varphi+c)(f\kappa-h\tau+\frac{\partial g}{\partial s})-\psi\tau\sin(\varphi+c)-\psi\frac{\partial}{\partial s}(\cos(\varphi+c))\\
&-\frac{\partial}{\partial t}(\cos(\varphi+c))]\mathbf{N}+[\psi\frac{\partial}{\partial s}(\sin(\varphi+c))-\kappa\sin(\varphi+c)(g\tau+\frac{\partial h}{\partial s})\\
&+\frac{\partial}{\partial t}(\tau\sin(\varphi+c))+\frac{\partial^2}{\partial t\partial s}(\cos(\varphi+c))-\psi\tau\cos(\varphi+c)]\mathbf{B}.
\end{aligned}$$

Thus, using above equations the proof is finished.

Corollary 3.4.

$$\begin{aligned}
&-\frac{\partial^2}{\partial s\partial t}(\varphi\sin(\varphi+c))+\frac{\partial}{\partial t}(\tau\sin(\varphi+c))-(\kappa\cos(\varphi+c))\\
&(f\kappa-h\tau+\frac{\partial g}{\partial s}))-\tau\psi\cos(\varphi+c)-\psi\frac{\partial}{\partial s}(\varphi\cos(\varphi+c))\\
=\ &-(f\kappa^2-h\kappa\tau+\kappa\frac{\partial g}{\partial s})\cos(\varphi+c)+(g\kappa\tau+\kappa\frac{\partial h}{\partial s})\sin(\varphi+c)+\frac{\partial}{\partial s}(\frac{\partial}{\partial t}(\cos(\varphi+c))\\
&+\psi\sin(\varphi+c))+\tau\frac{\partial}{\partial t}(\sin(\varphi+c))-\tau\psi\cos(\varphi+c),
\end{aligned}$$

where

$$c\in\mathbb{R}\ ,\varphi(s)=\int\tau(s)ds\ \text{and}\ \psi=<\frac{\partial \mathbf{N}}{\partial t},\mathbf{B}>.$$

Corollary 3.5.

$$\frac{\partial\kappa}{\partial t}=-\frac{\partial}{\partial s}(f\kappa-h\tau+\frac{\partial g}{\partial s}+g\tau^2+\tau\frac{\partial h}{\partial s}.$$

Corollary 3.6.

$$-f\kappa\tau+h\tau^2-\tau\frac{\partial g}{\partial s}-\frac{\partial}{\partial s}(g\tau+\frac{\partial h}{\partial s})=\kappa\psi.$$

Corollary 3.7.

$$\frac{\partial^2}{\partial t\partial s}(\sin(\varphi+c)) - \kappa\sin(\varphi+c)(f\kappa-h\tau+\frac{\partial g}{\partial s}) - \psi\tau\sin(\varphi+c)$$
$$-\psi\frac{\partial}{\partial s}(\cos(\varphi+c)) - \frac{\partial}{\partial t}(\cos(\varphi+c)) = \frac{\partial}{\partial s}[\sin(\varphi+c)$$
$$-\psi\cos(\varphi+c)] - \tau\frac{\partial}{\partial t}\cos(\varphi+c) - \tau\psi\sin(\varphi+c) + (-(f\kappa-h\tau$$
$$+\frac{\partial g}{\partial s})\kappa\sin(\varphi+c) - (g\tau+\frac{\partial h}{\partial s})\kappa\cos(\varphi+c),$$

where

$$c\in\mathbb{R}\ ,\varphi(s)=\int\tau(s)ds\ \textit{and}\ \psi=<\frac{\partial\mathbf{N}}{\partial t},\mathbf{B}>.$$

Corollary 3.8.

$$\frac{\partial}{\partial s}\sin(\varphi+c)(-(f\kappa-h\tau+\frac{\partial g}{\partial s})+\frac{\partial}{\partial t}(-\kappa\sin(\varphi+c))$$
$$+\tau\sin(\varphi+c)(-g\tau-\frac{\partial h}{\partial s})+\frac{\partial}{\partial s}(\cos(\varphi+c))(-g\tau-\frac{\partial h}{\partial s})$$
$$+\tau\cos(\varphi+c)(f\kappa-h\tau+\frac{\partial g}{\partial s}) = \frac{\partial}{\partial s}[(-(f\kappa-h\tau+\frac{\partial g}{\partial s})$$
$$\sin(\varphi+c)-(g\tau+\frac{\partial h}{\partial s})\cos(\varphi+c)]+(-\kappa\frac{\partial}{\partial t}\sin(\varphi+c)$$
$$+\kappa\psi\cos(\varphi+c),$$

where

$$c\in\mathbb{R}\ ,\varphi(s)=\int\tau(s)ds\ \textit{and}\ \psi=<\frac{\partial\mathbf{N}}{\partial t},\mathbf{B}>.$$

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