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Weakly b-Open Functions in Bitopological Spaces

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abstract: The aim of this paper is to introduce the notion of weakly b-open functions in a bitopological spaces. Some properties of this function are established and the relationships with some other types of spaces are also investigated.

Key Words: Bitopological spaces; (i,j)-b-open; (i,j)-b-closed; (i,j)-θ-closed; (i,j)-regular open.

Contents

1. Introduction

The notion of bitopological spaces (X, τ_1, τ_2), where X is a non-empty set and τ_1, τ_2 are different topologies on X was introduced by Kelly [9]. In 1996, Andrijevic [2] introduced the concept of b-open sets in topological spaces. After that Al-Hawary and Al-Omari [1] defined the notion of b-open sets in bitopological spaces and established several fundamental properties. Noiri et al. [11] defined the notion of weakly b-open functions in topological spaces and established several properties of this notion.

The purpose of this paper is to present the concept of weakly b-open functions in bitopological spaces and to obtain several characterizations and properties of this concept.

2. Preliminaries

Throughout this paper, X and Y represents bitopological spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) on which no separation axioms are assumed and (i,j) means the topologies τ_i, τ_j where $i, j \in \{1,2\}, i \neq j$. For a subset A of (X, τ_1, τ_2), i-$int(A)$ (respectively, i-$cl(A)$) denotes the interior(respectively, closure) of A with respect to the topology τ_i, where $i \in \{1,2\}$.

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Now, we list some definitions and results those will be used throughout this article.

Definition 2.1. *A subset A of a bitopological space (X, τ_1, τ_2) is said to be*

(a) (i, j)-*b-open* ([1]) *if* $A \subset i\text{-}int(j\text{-}cl(A)) \cup j\text{-}cl(i\text{-}int(A))$.

(b) (i, j)-*regular open* ([3]) *if* $A = i\text{-}int(j\text{-}cl(A))$.

(c) (i, j)-*regular closed* ([4]) *if* $A = i\text{-}cl(j\text{-}int(A))$.

(d) (i, j)-*preopen* ([6]) *if* $A \subset i\text{-}int(j\text{-}cl(A))$

(e) (i, j)-α-*open* ([7]) *if* $A \subset i\text{-}int(j\text{-}cl(i\text{-}int(A)))$

The complement of (i, j)-b-open set is (i, j)-b-closed.

Definition 2.2. [2] *Let A be a subset of a bitopological space (X, τ_1, τ_2). Then,*

(a) *the (i, j)-b-closure of A denoted by (i, j)-bcl(A), is defined by the intersection of all (i, j)-b-closed sets containing A.*

(b) *the (i, j)-b-interior of A denoted by (i, j)-bint(A), is defined by the union of all (i, j)-b-open sets contained in A.*

Lemma 2.1. [1] *Let (X, τ_1, τ_2) be a bitopological space and A be a subset of X. Then,*

(a) *(i, j)-bint(A) is (i, j)-b-open.*

(b) *(i, j)-bcl(A) is (i, j)-b-closed.*

(c) *A is (i, j)-b-open if and only if $A = (i, j)$-bint(A).*

(d) *A is (i, j)-b-closed if and only if $A = (i, j)$-bcl(A).*

Lemma 2.2. [14] *Let A be a subset of a bitopological space (X, τ_1, τ_2). Then,*

(a) $X \setminus (i, j)\text{-}bcl(A) = (i, j)\text{-}bint(X \setminus A)$

(b) $X \setminus (i, j)\text{-}bint(A) = (i, j)\text{-}bcl(X \setminus A)$

Lemma 2.3. [1] *Let (X, τ_1, τ_2) be a bitopological space and A be a subset of X. Then $x \in (i, j)$-bcl(A) if and only if for every (i, j)-b-open set U containing x, $U \cap A \neq \emptyset$.*

Definition 2.3. [8] *Let (X, τ_1, τ_2) be a bitopological space and A be a subset of X. A point x of X is said to be in (i,j)-θ-closure of A, denoted by (i,j)-$cl_\theta(A)$, if $A \cap j$-$cl(U) \neq \emptyset$ for every τ_i-open set U containing x, where $i, j \in \{1, 2\}$ and $i \neq j$.*

A subset A of X is said to be (i,j)-θ-closed if $A = (i,j)$-$cl_\theta(A)$. A subset A of X is said to be (i,j)-θ-open if $X \setminus A$ is (i,j)-θ-closed. The (i,j)-θ-interior of A, denoted by (i,j)-$int_\theta(A)$ is defined as the union of all (i,j)-θ-open sets contained in A. Therefore $x \in (i,j)$-$int_\theta(A)$ if and only if there exists a τ_i-open set U containing x such that $x \in U \subset j$-$cl(U) \subset A$.

Lemma 2.4. [8] *For a subset A of a bitopological space (X, τ_1, τ_2), the following properties hold :*

(a) $X \setminus (i,j)$-$cl_\theta(A) = (i,j)$-$int_\theta(X \setminus A)$

(b) $X \setminus (i,j)$-$int_\theta(A) = (i,j)$-$cl_\theta(X \setminus A)$

Lemma 2.5. [8] *Let (X, τ_1, τ_2) be a bitopological space. If U is a τ_j-open set of X, then (i,j)-$cl_\theta(U) = i$-$cl(U)$.*

3. (i,j)-Weakly b-Open Functions

Definition 3.1. *A function $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ is said to be (i,j)-weakly b-open if $f(U) \subset (i,j)$-$bint(f(j$-$cl(U)))$, for every τ_i-open set U of X.*

Theorem 3.1. *The following statements are equivalent for a function $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$.*

(a) *f is (i,j)-weakly b-open.*

(b) *$f((i,j)$-$int_\theta(A)) \subset (i,j)$-$bint(f(A))$, for every subset A of X.*

(c) *(i,j)-$int_\theta(f^{-1}(B)) \subset f^{-1}((i,j)$-$bint(B))$, for every subset B of Y.*

(d) *$f^{-1}((i,j)$-$bcl(B)) \subset (i,j)$-$cl_\theta(f^{-1}(B))$, for every subset B of Y.*

(e) *For every $x \in X$ and every τ_i-open set U of X containing x, there exists an (i,j)-b-open set V containing $f(x)$ such that $V \subset f(j$-$cl(U))$.*

Proof: (a)⇒(b) Let $A \subset X$ and $x \in (i,j)$-$int_\theta(A)$. Then there exists a τ_i-open set U of X such that $x \in U \subset j$-$cl(U) \subset A$. Thus $f(x) \in f(U) \subset f(j$-$cl(U)) \subset f(A)$. Since f is (i,j)-weakly b-open function, therefore $f(U) \subset (i,j)$-$bint(f(j$-$cl(U))) \subset (i,j)$-$bint(f(A))$. Thus $f(x) \in (i,j)$-$bint(f(A))$. This implies that $x \in f^{-1}((i,j)$-$bint(f(A)))$. So, (i,j)-$int_\theta(A) \subset f^{-1}((i,j)$-$bint(f(A)))$. Hence $f((i,j)$-$int_\theta(A)) \subset (i,j)$-$bint(f(A))$.

(b)⇒(c) Let $B \subset Y$. Then $f^{-1}(B)$ is a subset of X. Next by (b), $f((i,j)\text{-}int_\theta(f^{-1}(B))) \subset (i,j)\text{-}bint(f(f^{-1}(B))) \subset (i,j)\text{-}bint(B)$. This implies $(i,j)\text{-}int_\theta(f^{-1}(B)) \subset f^{-1}((i,j)\text{-}bint(B))$.

(c)⇒(d) Let $B \subset Y$ and $x \notin (i,j)\text{-}cl_\theta(f^{-1}(B))$. Then $x \in X \setminus (i,j)\text{-}cl_\theta(f^{-1}(B)) = (i,j)\text{-}int_\theta(X \setminus f^{-1}(B)) = (i,j)\text{-}int_\theta(f^{-1}(Y \setminus B)) \subset f^{-1}((i,j)\text{-}bint(Y \setminus B)) = f^{-1}(Y \setminus (i,j)\text{-}bcl(B)) = X \setminus f^{-1}((i,j)\text{-}bcl(B))$. So, $x \notin f^{-1}((i,j)\text{-}bcl(B))$. Hence $f^{-1}((i,j)\text{-}bcl(B)) \subset (i,j)\text{-}cl_\theta(f^{-1}(B))$.

(d)⇒(e) Let $x \in X$ and U be a τ_i-open set of X containing x. Let $B = Y \setminus f(j\text{-}cl(U))$. By (d), then we have

$$\begin{aligned} f^{-1}((i,j)\text{-}bcl(Y \setminus f(j\text{-}cl(U)))) &\subset (i,j)\text{-}cl_\theta(f^{-1}(Y \setminus f(j\text{-}cl(U)))) \\ &= (i,j)\text{-}cl_\theta(X \setminus f^{-1}(f(j\text{-}cl(U)))) \\ &\subset (i,j)\text{-}cl_\theta(X \setminus j\text{-}cl(U)) \\ &= i\text{-}cl(X \setminus j\text{-}cl(U)), \text{ by Lemma 2.4.} \\ &= X \setminus i\text{-}int(j\text{-}cl(U)) \\ &\subset X \setminus i\text{-}int(U) = X \setminus U. \end{aligned}$$

Thus $f^{-1}((i,j)\text{-}bcl(Y \setminus f(j\text{-}cl(U)))) \subset X \setminus U$.

$\Rightarrow f^{-1}(Y \setminus (i,j)\text{-}bint(f(j\text{-}cl(U)))) \subset X \setminus U$.

$\Rightarrow X \setminus f^{-1}((i,j)\text{-}bint(f(j\text{-}cl(U)))) \subset X \setminus U$.

Hence $U \subset f^{-1}((i,j)\text{-}bint(f(j\text{-}cl(U))))$. This implies that $f(x) \in f(U) \subset (i,j)\text{-}bint(f(j\text{-}cl(U))) \subset f(j\text{-}cl(U))$. Let $V = (i,j)\text{-}bint(f(j\text{-}cl(U)))$. Then V is (i,j)-b-open and $f(x) \in V \subset f(j\text{-}cl(U))$.

(e)⇒(a) Let U be a τ_i-open set of X containing x. By (e), there exists an (i,j)-b-open set V of Y containing $f(x)$ such that $V \subset f(j\text{-}cl(U))$. Thus $f(x) \in V \subset (i,j)\text{-}bint(f(j\text{-}cl(U)))$. Hence $f(U) \subset (i,j)\text{-}bint(f(j\text{-}cl(U)))$ and so f is (i,j)-weakly b-open. □

Theorem 3.2. *The following statements are equivalent for a function $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$.*

(a) f is (i,j)-weakly b-open.

(b) $(i,j)\text{-}bcl(f(j\text{-}int(i\text{-}cl(U)))) \subset f(i\text{-}cl(U))$, for every subset U of X.

(c) $(i,j)\text{-}bcl(f(j\text{-}int(F))) \subset f(F)$, for every (i,j)-regular closed set F of X.

(d) $(i,j)\text{-}bcl(f(U)) \subset f(i\text{-}cl(U))$, for every τ_j-open set U of X.

Proof: (a)⇒(b) Let $x \in X$ and $U \subset X$ such that $x \in U$. Let $f(x) \in Y \setminus f(i\text{-}cl(U))$. Then $x \in X \setminus i\text{-}cl(U)$. This implies that, there exists a τ_i-open set V containing x such that $V \cap U = \emptyset$. Thus $j\text{-}cl(V) \cap j\text{-}int(i\text{-}cl(U)) = \emptyset$. Since f is (i,j)-weakly b-open, therefore by theorem 3.1, there exists an (i,j)-b-open set W containing $f(x)$ such that $W \subset f(j\text{-}cl(V))$. So, $W \cap f(j\text{-}int(i\text{-}cl(U)) = \emptyset$ and therefore

$f(x) \in X \setminus (i,j)\text{-}bcl(f(j\text{-}int(i\text{-}cl(U))))$. Hence $(i,j)\text{-}bcl(f(j\text{-}int(i\text{-}cl(U)))) \subset f(i\text{-}cl(U))$.

(b)⇒(c) Let F be (i,j)-regular closed set in X. Therefore $F = i\text{-}cl(j\text{-}int(F))$. Now $(i,j)\text{-}bcl(f(j\text{-}int(F))) = (i,j)\text{-}bcl(f(j\text{-}int(i\text{-}cl(j\text{-}int(F))))) \subset f(i\text{-}cl(j\text{-}int(F))) = f(F)$. Hence $(i,j)\text{-}bcl(f(j\text{-}int(F))) \subset f(F)$.

(c)⇒(d) Let U be a τ_j-open subset of X. Then $i\text{-}cl(U)$ is (i,j)-regular closed in X. Now $(i,j)\text{-}bcl(f(U)) \subset (i,j)\text{-}bcl(f(j\text{-}int(i\text{-}cl(U)))) \subset f(i\text{-}cl(U))$.

(d)⇒(a) Let U be a τ_i-open subset of X. Then $j\text{-}cl(U)$ is τ_j-closed. Now $Y \setminus (i,j)\text{-}bint(f(j\text{-}cl(U))) = (i,j)\text{-}bcl(Y \setminus f(j\text{-}cl(U))) = (i,j)\text{-}bcl(f(X \setminus j\text{-}cl(U))) \subset f(i\text{-}cl(X \setminus j\text{-}cl(U))) = f(X \setminus i\text{-}int(j\text{-}cl(U))) \subset f(X \setminus i\text{-}int(U)) = f(X \setminus U) = Y \setminus f(U)$. Thus $f(U) \subset (i,j)\text{-}bint(f(j\text{-}cl(U)))$ and hence f is (i,j)-weakly b-open. □

Theorem 3.3. *The following statements are equivalent for a function* $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$.

(*a*) f *is* (i,j)*-weakly* b*-open.*

(*b*) $f(i\text{-}int(F)) \subset (i,j)\text{-}bint(f(F))$*, for every* τ_j*-closed set* F *of* X.

(*c*) $f(U) \subset (i,j)\text{-}bint(f(j\text{-}cl(U)))$*, for every* (i,j)*-preopen set* U *of* X.

(*d*) $f(U) \subset (i,j)\text{-}bint(f(j\text{-}cl(U)))$*, for every* (i,j)*-*α*-open set* U *of* X.

Proof: (a)⇒(b) Let F be a τ_j-closed subset of X. Then we have $i\text{-}int(F)$ is τ_i-open. Since f is (i,j)-weakly b-open, therefore $f(i\text{-}int(F)) \subset (i,j)\text{-}bint(f(j\text{-}cl(i\text{-}int(F)))) \subset (i,j)\text{-}bint(f(F))$.

(b)⇒(c) Let U be a (i,j)-preopen set in X. Then by (b), we have $f(U) \subset f(i\text{-}int(j\text{-}cl(U))) \subset (i,j)\text{-}bint(f(j\text{-}cl(U)))$.

(c)⇒(d) Since every (i,j)-α-open set is (i,j)-preopen, so the result follows immediately.

(d)⇒(a) Let U be a τ_i-open set in X. Then U is (i,j)-α-open in X. Therefore by (d), we have $f(U) \subset (i,j)\text{-}bint(f(j\text{-}cl(U)))$. Hence f is (i,j)-weakly b-open. □

Theorem 3.4. *The following statements are equivalent for a function* $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$.

(*a*) f *is* (i,j)*-weakly* b*-open.*

(b) $f^{-1}((i,j)\text{-}bcl(B)) \subset (i,j)\text{-}cl_\theta(f^{-1}(B))$, *for every subset* B *of* Y.

(c) $(i,j)\text{-}bcl(f(A)) \subset f((i,j)\text{-}cl_\theta(A))$, *for every subset* A *of* X.

(d) $(i,j)\text{-}bcl(f(i\text{-}int((i,j)\text{-}cl_\theta(A)))) \subset f((i,j)\text{-}cl_\theta(A))$, *for every subset* A *of* X.

Proof: (a)⇒(b) Assume that f is (i,j)-weakly b-open. Let B be any subset of Y and $x \in f^{-1}((i,j)\text{-}bcl(B))$. Then $f(x) \in (i,j)\text{-}bcl(B)$. Let V be a τ_i-open set of X containing x. Since f is (i,j)-weakly b-open, therefore by theorem 3.1, there exists an (i,j)-b-open set U containing $f(x)$ such that $U \subset f(j\text{-}cl(V))$. Also, $f(x) \in (i,j)\text{-}bcl(B)$, therefore we get $U \cap A \neq \emptyset$ and hence $\emptyset \neq f^{-1}(U) \cap f^{-1}(B) \subset j\text{-}cl(V) \cap f^{-1}(B)$. Therefore we get $x \in (i,j)\text{-}cl_\theta(f^{-1}(B))$. Thus $f^{-1}((i,j)\text{-}bcl(B)) \subset (i,j)\text{-}cl_\theta(f^{-1}(B))$.

(b)⇒(c) Let A be any subset of X. Then we have $f^{-1}((i,j)\text{-}bcl(f(A))) \subset (i,j)\text{-}cl_\theta(f^{-1}(f(A))) \subset (i,j)\text{-}cl_\theta(A)$. Hence $(i,j)\text{-}bcl(f(A)) \subset f((i,j)\text{-}cl_\theta(A))$.

(c)⇒(d) Let A be any subset of X. Since $(i,j)\text{-}cl_\theta(A)$ is τ_i-closed in X, therefore by (b) and Lemma 2.4, we have $(i,j)\text{-}bcl(f(j\text{-}int((i,j)\text{-}cl_\theta(A)))) \subset f((i,j)\text{-}cl_\theta(j\text{-}int((i,j)\text{-}cl_\theta(A)))) = f(i\text{-}cl(j\text{-}int((i,j)\text{-}cl_\theta(A)))) \subset f(i\text{-}cl((i,j)\text{-}cl_\theta(A))) = f((i,j)\text{-}cl_\theta(A))$.

(d)⇒(a) Let V be any τ_j-open sub-set of X. Then by Lemma 2.4, $V \subset j\text{-}int(i\text{-}cl(V)) = j\text{-}int((i,j)\text{-}cl_\theta(V))$. Now, by (d) and Lemma 2.4, $(i,j)\text{-}bcl(f(V)) \subset (i,j)\text{-}bcl(f(j\text{-}int((i,j)\text{-}cl_\theta(V)))) \subset f((i,j)\text{-}cl_\theta(V)) = f(i\text{-}cl(V))$. Thus we obtain $(i,j)\text{-}bcl(f(V)) \subset f(i\text{-}cl(V))$ and hence by Theorem 3.2, we have f is (i,j)-weakly b-open. □

Definition 3.2. [9] *A bitopological space* (X, τ_1, τ_2) *is said to be* (i,j)*-regular if for each* $x \in X$ *and each* τ_i*-open set* U *containing* x*, there exists a* τ_i*-open set* V *such that* $x \in V \subset j\text{-}cl(V) \subset U$.

Theorem 3.5. *If* X *is* (i,j)*-regular, then for a bijective function* $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$*, the following statements are equivalent :*

(a) f *is* (i,j)*-weakly* b*-open.*

(b) $f(A)$ *is* (i,j)*-*b*-closed in* Y *for every* (i,j)*-*θ*-closed set* A *of* X.

(c) $f(B)$ *is* (i,j)*-*b*-open in* Y *for every* (i,j)*-*θ*-open set* B *of* X.

(d) *For every subset* C *of* Y *and for every* (i,j)*-*θ*-closed sub-set* A *of* X *such that* $f^{-1}(C) \subset A$*, there exists an* (i,j)*-*b*-closed sub-set* F *in* Y *containing* C *such that* $f^{-1}(F) \subset A$.

Proof: (a)⇒(b) Let A be any (i,j)-θ-closed set of X. Since f is (i,j)-weakly b-open, therefore by theorem 3.4, we have (i,j)-$bcl(f(A)) \subset f((i,j)$-$cl_\theta(A)) = f(A)$. Hence $f(A)$ is (i,j)-b-closed subset in Y.

(b)⇒(c) Let B be any (i,j)-θ-open sub-set of X. Then $X \setminus B$ is (i,j)-θ-closed sub-set in X. By (b), $f(X \setminus B) = Y \setminus f(B)$ is (i,j)-b-closed in Y. Hence $f(B)$ is (i,j)-b-open in Y.

(c)⇒(d) Let C be any subset of Y and A be an (i,j)-θ-closed set in X such that $f^{-1}(C) \subset A$. Since $X \setminus A$ is (i,j)-θ-open in X, therefore by (c), $f(X \setminus A)$ is (i,j)-b-open in Y. Let $F = Y \setminus f(X \setminus A)$. Then F is (i,j)-b-closed and $C \subset F$. Now, $f^{-1}(F) = f^{-1}(Y \setminus f(X \setminus A)) = f^{-1}(f(A)) \subset A$. Thus there exists an (i,j)-b-closed set F containing C such that $f^{-1}(F) \subset A$.

(d)⇒(a) Let C be any subset of Y. Let $A = (i,j)$-$cl_\theta(f^{-1}(C))$. Since X is (i,j)-regular, then A is (i,j)-θ-closed set in X and $f^{-1}(C) \subset A$. By (d), there exists an (i,j)-b-closed set F in Y containing C such that $f^{-1}(F) \subset A$. Since F is (i,j)-b-closed, we have $f^{-1}((i,j)$-$bcl(C)) \subset f^{-1}(F) \subset A = (i,j)$-$cl_\theta(f^{-1}(C))$. Hence by theorem 3.4, f is (i,j)-weakly b-open. □

Definition 3.3. [10] *A function $f : (X, \tau) \longrightarrow (Y, \sigma)$ is said to be strongly continuous if $f(cl(A)) \subset f(A)$, for every subset A of X.*

Theorem 3.6. *If a function $f : (X_1, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ is (i,j)-weakly b-open and strongly j-continuous, then $f(A)$ is (i,j)-b-open in Y, for every τ_i-open sub-set A in X.*

Proof: Let A be a τ_i-open set in X. Since f is (i,j)-weakly b-open and strongly j-continuous, therefore $f(A) \subset (i,j)$-$bint(f(j$-$cl(A))) \subset (i,j)$-$bint(f(A))$. Thus $f(A) = (i,j)$-$bint(f(A))$ and so $f(A)$ is (i,j)-b-open in Y. □

Theorem 3.7. *Let $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ be a bijective function. If $f((i,j)$-$cl_\theta(A))$ is (i,j)-b-closed in Y for every subset A of X, then f is (i,j)-weakly b-open.*

Proof: Let A be any subset of X. Since $f((i,j)$-$cl_\theta(A))$ is (i,j)-b-closed, therefore (i,j)-$bcl(f(A)) \subset (i,j)$-$bcl(f((i,j)$-$cl_\theta(A))) = f((i,j)$-$cl_\theta(A))$. Hence by theorem 3.4, f is (i,j)-weakly b-open. □

Definition 3.4. *A function $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ is said to be (i,j)-contra b-open (respectively, (i,j)-contra b-closed) if $f(U)$ is (i,j)-b-closed (respectively, (i,j)-b-open) in Y for every τ_j-open (respectively,τ_j-closed) subset U of X.*

Theorem 3.8. *If a function $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ is (i,j)-contra b-closed, then f is (i,j)-weakly b-open.*

Proof: Let A be any τ_i-open subset of X. Then $j\text{-}cl(A)$ is τ_j-closed in X. Therefore $f(A) \subset f(j\text{-}cl(A)) = (i,j)\text{-}bint(f(j\text{-}cl(A)))$. Hence f is (i,j)-weakly b-open. □

Theorem 3.9. *If a bijective function $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ is (i,j)-contra b-open, then f is (i,j)-weakly b-open.*

Proof: Let A be any τ_j-open subset of X. Then $i\text{-}cl(A)$ is τ_i-closed in X. Since f is (i,j)-contra b-open, therefore $f(A)$ is (i,j)-b-closed. Now $(i,j)\text{-}bcl(f(A)) = f(A) \subset f(i\text{-}cl(A))$. By Theorem 3. 2, f is (i,j)-weakly b-open. □

Definition 3.5. [13] *A bitopological space (X, τ_1, τ_2) is said to be pairwise connected if it cannot be expressed as the union of two non-empty disjoint sub-sets A and B such that A is τ_i-open and B is τ_j-open.*

Definition 3.6. *A bitopological space (X, τ_1, τ_2) is said to be pairwise b-connected if it cannot be expressed as the union of two non-empty disjoint sets A and B such that A is (i,j)-b-open and B is (j,i)-b-open.*

Theorem 3.10. *If $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ is a bijective (i,j)-weakly b-open function of a space (X, τ_1, τ_2) onto a pairwise b-connected space (Y, σ_1, σ_2), then (X, τ_1, τ_2) is pairwise connected.*

Proof: Suppose that (X, τ_1, τ_2) is not pairwise connected. Then there exists a non-empty τ_i-open set A and a non-empty τ_j-open set B such that $A \cap B = \emptyset$ and $A \cup B = X$. This implies that $f(A) \cap f(B) = \emptyset$ and $f(A) \cup f(B) = Y$. Also, $f(A) \neq \emptyset$ and $f(B) \neq \emptyset$. Since f is (i,j)-weakly b-open, therefore $f(A) \subset (i,j)\text{-}bint(f(j\text{-}cl(A)))$ and $f(B) \subset (j,i)\text{-}bint(f(i\text{-}cl(B)))$. Again since A and B are τ_j-closed and τ_i-closed respectively, therefore we have $f(A) \subset (i,j)\text{-}bint(f(A))$ and $f(B) \subset (j,i)\text{-}bint(f(B))$. Hence $f(A) = (i,j)\text{-}bint(f(A))$ and $f(B) = (j,i)\text{-}bint(f(B))$. Consequently, $f(A)$ and $f(B)$ are (i,j)-b-open and (j,i)-b-open respectively.Which is a contradiction to the hypothesis that Y is pairwise b-connected. Hence (X, τ_1, τ_2) is pairwise connected. □

Definition 3.7. [12] *A bitopological space (X, τ_1, τ_2) is said to be (i,j)- hyperconnected if $j\text{-}cl(A) = X$, for every τ_i-open sub-set A of X.*

Theorem 3.11. *Let (X, τ_1, τ_2) be an (i,j)-hyperconnected space. Then a function $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ is (i,j)-weakly b-open if and only if $f(X)$ is (i,j)-b-open in Y.*

Proof: Suppose that f is (i,j)-weakly b-open. Since X is τ_i-open, therefore we have $f(X) \subset (i,j)\text{-}bint(f(j\text{-}cl(X))) = (i,j)\text{-}bint(f(X))$. Hence $f(X)$ is (i,j)-b-open in Y.

Conversely, let $f(X)$ be (i,j)-b-open in Y and A be a τ_i-open set in X. Then $f(A) \subset f(X) = (i,j)\text{-}bint(f(X)) = (i,j)\text{-}bint(f(j\text{-}cl(A)))$. Hence f is (i,j)-weakly b-open. □

Definition 3.8. [3] *A subset A of a bitopological space (X, τ_1, τ_2) is said to be (i,j)-quasi H-closed relative to X if for each cover $\{B_\alpha : \alpha \in \wedge\}$ of A by τ_i-open sets of X, there exists a finite subset $\wedge_0$ of $\wedge$ such that $A \subset \bigcup\{j\text{-}cl(B_\alpha) : \alpha \in \wedge_0\}$.*

Definition 3.9. *A subset A of a bitopological space (X, τ_1, τ_2) is said to be (i,j)-b-compact relative to X if every cover of A by (i,j)-b-open sub-sets of X has a finite subcover.*

Theorem 3.12. *If a bijective function $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ is (i,j)-weakly b-open and A is (i,j)-b-compact relative to Y, then $f^{-1}(A)$ is (i,j)-quasi H-closed relative to X.*

Proof: Let A be (i,j)-b-compact relative to Y and $\{B_\alpha : \alpha \in \wedge\}$ be an open cover of $f^{-1}(A)$ by τ_i-open sets of X. Therefore $f^{-1}(A) \subset \bigcup\{B_\alpha : \alpha \in \wedge\}$ and so $A \subset \bigcup\{f(B_\alpha) : \alpha \in \wedge\}$. Since f is (i,j)-weakly b-open, therefore $f(B_\alpha) \subset (i,j)$-$bint(f(j\text{-}cl(B_\alpha)))$. Then $A \subset \bigcup\{(i,j)\text{-}bint(f(j\text{-}cl(B_\alpha))) : \alpha \in \wedge\}$. Also, A is (i,j)-b-compact relative to Y and (i,j)-$bint(f(j\text{-}cl(B_\alpha)))$ is (i,j)-b-open for each $\alpha \in \wedge$, therefore there exists a finite subset $\wedge_0$ of $\wedge$ such that $A \subset \bigcup\{(i,j)\text{-}bint(f(j\text{-}cl(B_\alpha))) : \alpha \in \wedge_0\}$. This implies that $f^{-1}(A) \subset \bigcup\{f^{-1}((i,j)\text{-}bint(f(j\text{-}cl(B_\alpha)))) : \alpha \in \wedge_0\} \subset \bigcup\{f^{-1}(f(j\text{-}cl(B_\alpha))) : \alpha \in \wedge_0\} \subset \bigcup\{j\text{-}cl(B_\alpha) : \alpha \in \wedge_0\}$. Hence $f^{-1}(A)$ is (i,j)-quasi H-closed relative to X. □

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