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Implementation of Homotopy Analysis Method on circular permeable slider containing of incompressible Newtonian fluid

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abstract: The aim of this paper is to examine the classical problem of an incompressible Newtonian fluid through the porous of a circular slider which is moving laterally on a horizontal plan. Employing the similarity variables, the governing differential equations have been reduced to ordinary ones and solved via Homotopy Analysis Method (HAM). The analytical solution for the coupled Nonlinear Ordinary Differential Equations resulting from the momentum equation is obtained and Velocity fields have been computed and discussed for different values of the Reynolds number. Also the fourth-order Runge-Kutta numerical method (NUM) is used for the validity of these analytical methods and excellent agreement are observed between the solutions obtained from HAM and numerical results.

Key Words: Analytical solution; Homotopy Analysis Method; Circular porous slider.

Contents

1. Introduction

Sliding friction is greatly reduced if a fluid of constant density is forced through the porous bottom of a circular slider between two solid surfaces moving relative to each other. Porous sliders are important in fluid cushioned moving pads. An interesting subject has been carried out by different authores [1,2,3,4]. The fluid dynamical and heat transfer of the circular porous slider bearing is discussed by [5]. Most of problems and scientific phenomena such as heat transfer are inherently of nonlinearity. We know that except a limited number of these problems, most of them do not have exact solutions. Therefore, these nonlinear equations should be solved approximately either numerically or analytically. In the numerical method,

2000 *Mathematics Subject Classification:* 76NXX, 76D10, 65Z05 stability and convergence should be considered so as to avoid divergence or inappropriate results. Time consuming is another problem of numerical techniques. Analytical solutions often fit under classical perturbation methods.

Perturbation method [6] provides the most versatile tools available in nonlinear analysis of engineering problem, but its limitations restrict its application [7] [8]: Perturbation method [9] is based on assuming a small parameter. The majority of nonlinear problems, especially those having strong nonlinearity, have no small parameters at all. The approximate solutions obtained by the perturbation methods, in most cases, are valid only for small values of the small parameter. Generally, the perturbation solutions are uniformly valid as long as a scientific system parameter is small. However, we cannot rely fully on the approximations, because there is no criterion on which the small parameter should exist. Thus, it is essential to check the validity of the approximations numerically and/or experimentally. To overcome these difficulties, some new methods have been proposed such as VIM, HPM, ADM and so on.

Disappointingly, the majority of nonlinear problems have no small parameter at all. Recently, several new techniques have been presented to overcome the mentioned difficulties. Some of these techniques include Variational Iteration Method (VIM) [9] [10], decomposition method [11], Homotopy Perturbation Method (HPM) [12] [13] and Homotopy Analysis Method [14,15,16,17,18,19,20,21,22] etc. The Homotopy Analysis method (HAM) has been introduced by Liao in 1992. In the present work, the governing equation of circular porous slider is solved through HAM. The convergence of the series solution is also explicitly discussed. Obtaining the analytical solution of the models and comparing with numerical result reveal the capability, effectiveness and convenience of HAM. This method gives successive approximations of high accuracy solution.

2. Problem statement and mathematical formulation

We consider the flow field due to a circular porous slider as shown in Fig.1. A fluid of constant density is forced through the porous bottom of the slider and thus separated the slider from the ground. An incompressible fluid is forced through the porous wall of the slider with a velocity W. Figure b, shows the slider which is fixed at the plane, $z = d$ with a viscous fluid injected through it. The base is the plane at, $z = 0$ which is moving in the x-direction with velocity U. For detail, please see [23,24]. As the gap d is small, it can be assumed that both planes are extended to infinity.

Considering the u, v and w to be the velocity components in the direction x, y and z, respectively, the conservation mass and conservation momentum density Navier-Stokes Equations are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{2.1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + v\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) \tag{2.2}$$

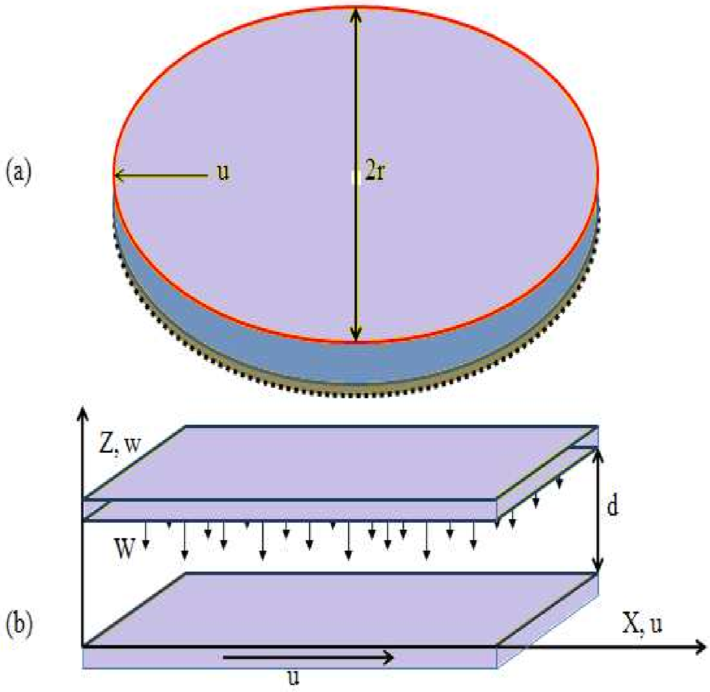

Figure 1: This cat is a eps file

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial y}+v\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right) \tag{2.3}$$

$$u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial z}+v\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right) \tag{2.4}$$

Where ρ is density of fluid,v is kinematic viscosity and P is pressure. The boundary conditions are as follows:

$$z=0, \quad u=U, \quad v=w=0 \tag{2.5}$$

$$z=d, \quad ,u=v=0, \quad w=-W \tag{2.6}$$

Where U is velocity of the slider in lateral and longitudinal direction and W is velocity of fluid injected through the porous bottom of the slider. For transforming (2)-(4), the following equations are defined:

$$\eta=\frac{z}{d}, \quad u=Uf(\eta)W\frac{x}{d}h(\eta), \quad v=\frac{W}{d}h(\eta), \quad w=-2W(\eta) \tag{2.7}$$

By substituting (7) into Navier-Stoks Equations (2)-(4), it can be obtained that

$$h''''(\eta) + 2Reh(\eta)h'''(\eta) = 0, \tag{2.8}$$

$$f''' + 2Reh(\eta)f'(\eta) - Reh'(\eta)f(\eta) = 0, \tag{2.9}$$

$$-\frac{P}{\rho} = \frac{WK}{2d^2}(x^2 + y^2) + \frac{1}{2}w^2 - v\frac{\partial w}{\partial z} + A \tag{2.10}$$

Where $Re = \frac{Wd}{v}$ is the cross-flow Reynolds number, A and K are constants which will have to be determined. The boundary conditions for the transformation are as follows:

$$h(0) = 0, \quad h'(0) = 0, \quad h'(1) = 0, \quad h(1) = \frac{1}{2} \tag{2.11}$$

$$f(0) = 0, \quad f(1) = 0 \tag{2.12}$$

3. Implementation of the Homotopy Analysis Method

For HAM solutions, we choose the initial guess and auxiliary linear operator in the following form:

$$h_0(\eta) = -\eta^3 + \frac{3}{2}\eta^2, \quad f_0(\eta) = 1 - \eta, \tag{3.1}$$

$$L_1(h) = h'''', \quad L_2(f) = f'', \tag{3.2}$$

$$L_1\left(\frac{1}{6}c_1\eta^3 + \frac{1}{2}c_2\eta^2 + c_3\eta + c_4\right) = 0, \quad L_2(c_5\eta + c_6) = 0, \tag{3.3}$$

where $c_i (i = 1 - 6)$ are constants. Let $P \in [0, 1]$ denotes the embedding parameter and h_1, h_2 indicates non-zero auxiliary parameters. We then construct the following equations:

Zeroth-order deformation equations

$$(1 - P)L_1[H(\eta; p) - h_0(\eta)] = ph_1H'(\eta)N[h(\eta; p)] \tag{3.4}$$

$$h(0; p) = 0, \quad h'(0; p) = 0, \quad h'(1; p) = 0, \quad h(1, p) = \frac{1}{2} \tag{3.5}$$

$$(1 - P)L_2[F(\eta; p) - f_0(\eta)] = ph_2H'(\eta)N[f(\eta; p)] \tag{3.6}$$

$$f(0; p) = 1; \quad f(1; p) = 0 \tag{3.7}$$

$$N[H(\eta; p)] = \frac{d^4H(\eta; p)}{d\eta^4} + 2ReH(\eta; p)\frac{d^4H(\eta; p)}{d\eta^4} = 0 \tag{3.8}$$

$$N[F(\eta; p)] = \frac{d^2F(\eta; p)}{d\eta^2} + 2ReH(\eta; p)\frac{dF(\eta; p)}{d\eta} - ReF(\eta; p)\frac{dH(\eta; p)}{d\eta} \tag{3.9}$$

For $p = 0$ and $p = 1$ we have

$$H(\eta; 0) = h_0(\eta), \qquad H(\eta; 1) = h(\eta) \tag{3.10}$$

$$F(\eta;0)=f_0(\eta),\qquad F(\eta;1)=f(\eta) \tag{3.11}$$

When p increases from 0 to 1 then $H(\eta;0)$ and $F(\eta;0)$ varies from $h_0(\eta)$ and $f_0(\eta)$ to $h(\eta)$ and $f(\eta)$ respectively. By Taylor's theorem and using Eq. (22) and Eq. (23), $H(\eta;0)$ and $F(\eta;0)$ can be expanded in a power series of p as follows:

$$H(\eta;p)=h_0(\eta)+\sum_{m-1}^{\infty}h_m(\eta)p^m,\quad h_m(\eta)=\frac{1}{m!}\frac{\partial^m(H(\eta;p))}{\partial p^m}\bigg|_{p=0} \tag{3.12}$$

$$F(\eta;p)=f_0(\eta)+\sum_{m-1}^{\infty}f_m(\eta)p^m,\quad f_m(\eta)=\frac{1}{m!}\frac{\partial^m(F(\eta;p))}{\partial p^m}\bigg|_{p=0} \tag{3.13}$$

In which h is chosen in such a way that this series is convergent at $p=1$, therefore we have through Eq. (24) and Eq. (25) that

$$h(\eta)=h_0(\eta)+\sum_{m-1}^{\infty}h_m(\eta),\quad f(\eta)=f_0(\eta)+\sum_{m-1}^{\infty}f_m(\eta), \tag{3.14}$$

mth-order deformation equations

$$L_1[h_m(\eta)-\chi_m h_{m-1}(\eta)]=h_1H'(\eta)R_m(\eta) \tag{3.15}$$

$$h_m(0;p)=0;\quad h'_m(0;p)=0,\quad h'_m(1;p)=0,\quad h_m(1;p)=0 \tag{3.16}$$

$$L_1[f_m(\eta)-\chi_m f_{m-1}(\eta)]=h_2H'(\eta)R_m(\eta) \tag{3.17}$$

$$f_m(0;p)=0;\quad f_m(1;p)=0 \tag{3.18}$$

$$R_m(\eta)=f''''_{m-1}+2\sum_{k=0}^{m-1}Re(h_{m-1-k}h'''_k)=0 \tag{3.19}$$

$$R_m(\eta)=h''_{m-1}+\sum_{k=0}^{m-1}[2Reh_{m-1-k}f'_k-Ref_{m-1-k}h']=0 \tag{3.20}$$

Now we determine the convergency of the result, the differential equation, and the auxiliary function according to the solution expression. So let us assume:

$$H'(\eta)=1 \tag{3.21}$$

We have found the answer by maple analytic solution device. For second deformation of the solution are presented below

$$h_1(\eta)=\frac{1}{70}h_1Re\eta^7-\frac{1}{20}h_1Re\eta^6+\frac{9}{70}h_1Re\eta^3-\frac{13}{140}h_1Re\eta^2 \tag{3.22}$$

$$f_1(\eta)=-\frac{1}{20}h_2Re\eta^5+\frac{1}{4}h_2Re\eta^4-\frac{1}{2}h_1Re\eta^3+\frac{10}{3}h_2Re\eta \tag{3.23}$$

The solutions $h(\eta)$ and $f(\eta)$ were too long to be mentioned here, therefore, they are shown graphically.

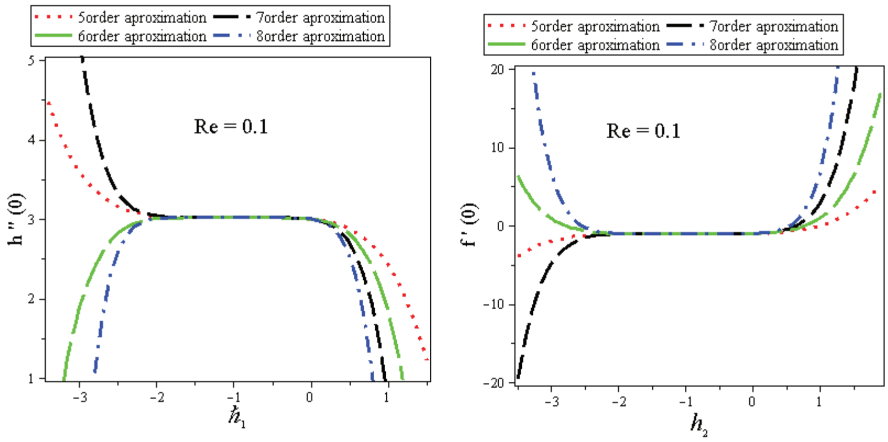

Figure 2: The h_1 and h_2-curve of $h''(0)$and $f'(0)$ given by the 5,6,7 and 8th−order approximation solution for $Re = 0.1$

4. Convergence of the HAM solution

As pointed out by [16], the convergence region and rate of solution series can be adjusted and controlled by means of the auxiliary parameter $\hbar$.

To influence of $\hbar$ on the convergence of solution, we plot the so-called $\hbar$-curve of $h''(0)$ and $f'(0)$, as shown in Fig. 2. The solutions converge for $\hbar$ values which are corresponding to the horizontal line segment in $\hbar$ curve. In our case study, it is easy to discover that $h_1 = -1$ and $h_2 = -0.5$ is suitable value which is used for values of $0.1 < Re < 6$

5. Results and discussion

The objective of the present study is to apply Homotopy Analysis method to obtain an explicit analytic solution of circular porous slider (Fig. 1). For showing the efficiency of analytical applied method a special case is considered and results are compared with numerical method as shown in Fig. 3. The numerical solution is performed using the algebra package Maple 16.0, to solve the present case. The package uses a fourth order Runge-Kutta procedure for solving nonlinear boundary value (B-V) problem [25]. Furthermore, Validity of HAM is shown in Table 1. In this tables, the % Error is defined as:

$$\%Error = |h(\eta)_{NUM} - h(\eta)_{Analytical}| \tag{5.1}$$

From the graphical representation, the results are proved to be precise and accurate in solving a wide range of mathematical and engineering problems especially Fluid mechanic cases. This accuracy gives high confidence to us about validity of this problem and reveals an excellent agreement of engineering accuracy.

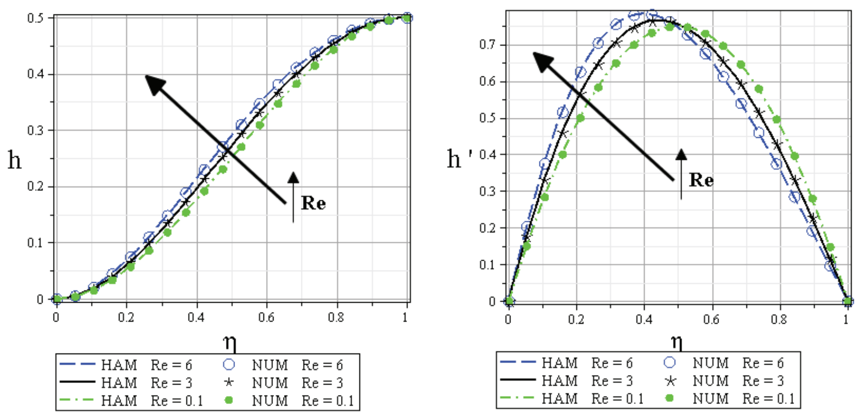

Figure 3: Effects of Reynolds numbers on 20th-order approximation of the velocity profile $h(\eta)$ and $h'(\eta)$

Moreover, Figs. 3 and 4 are prepared in order to see the effects of Reynolds number on the velocity profiles. Figs. 3 and 4 are depicted for showing the effect of Reynolds number on velocity profile $h(\eta)$and $h'(\eta)$, respectively. As seen in these figures by increasing Re number, velocity profiles increases. In addition, the lateral velocity $f(\eta)$ with Re varying is depicted in Figure 4. As the result shows, in the case of $Re = 0.01$, the lateral velocity is linear.

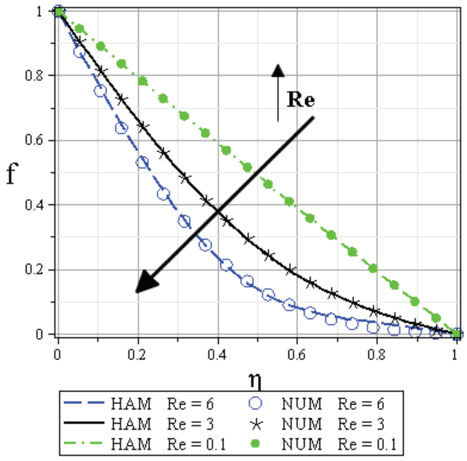

Figure 4: Effects of Reynolds numbers on 20th-order approximation of the lAll the illustrating results confirm the convenience, reliability and efficiency of the proposed method. HAM can be introduced to overcome the limitations and difficulties existing in other approximate methods. It is predicted that this method can be widely used in mathematical, physical and engineering problems, due to their simplicity and efficiency. ateral velocity $f(\eta)$

6. Conclusion

In this paper, an analytical method, called the Homotopy Analysis Method has been successfully applied to find explicit solutions of nonlinear problems, which occur in circular porous slider problem. The results obtained here were compared with the numerical solutions. The results show that these methods enable to convert a difficult problem into a simple problem which can easily be solved. Important objective of our research is the examination of the convergence of HAM. The comparisons of the results obtained here provide more realistic solutions, reinforcing the conclusions about the efficiency of these methods. Therefore the proposed method is powerful mathematical tools and can be applied to a large class of linear and nonlinear problems arising in heat transfer equations.

Table 1. The results of HAM and Numerical methods for $h(\eta)$ and $f(\eta)$ for Re=0.08.

η	$h(\eta)$			$f(\eta)$		
	HAM	NUM	Error	HAM	NUM	Error
0.0	0.0000000000	0.0000000000	0.0000E+00	1.0000000000	1.0000000000	0.0000E+00
0.1	0.0140640925	0.0140640923	2.2000E-10	0.8976382880	0.8976483290	1.00404E-05
0.2	0.0522153202	0.0522153200	1.8000E-10	0.7954904570	0.7955107670	2.03101E-05
0.3	0.1083937460	0.1083937460	1.0000E-10	0.6937318280	0.6937617210	2.98931E-05
0.4	0.1765447840	0.1765447840	1.0000E-10	0.5924992090	0.5925364520	3.72434E-05
0.5	0.2506245290	0.2506245290	0.0000E+00	0.4918948090	0.4919356570	4.08477E-05
0.6	0.3246062150	0.3246062150	0.0000E+00	0.3919903410	0.3920301100	3.97696E-05
0.7	0.3924871860	0.3924871860	1.0000E-10	0.2928314760	0.2928654510	3.39748E-05
0.8	0.4482957750	0.4482957760	1.0000E-10	0.1944427180	0.1944670910	2.43731E-05
0.9	0.4860975010	0.4860975000	3.0000E-10	0.0968327186	0.0968452434	1.25248E-05
1.0	0.5000000000	0.5000000000	4.0000E-10	0.0000000000	0.0000000000	0.0000E+00

References

1. Berman A. S.,(1953), *Laminar flow in channels with porous walls*, 24, 1232-1235

2. Elkouh A. F.,(1968), *Laminar flow between rotating porous disks*, Journal of the Engineering Mechanics Division, 94, 919-929.

3. Morgan V. T., Cameron A.,(1957), *Mechanism of lubrication in porous metal bearing*, in Proceeding Conference on Lubrication and Wear, 151-175.

4. Srinivasan U., (1977), *The analysis of a double-layered porous slider bearing*, Wear, 42, 205-215.

5. Gorla, R. S. R.,(1984). *Flow and thermal characteristics of a circular porous slider bearing*, Wear, 94 , 157-174.

6. Aziz A., Na T.Y.,(1984). *Perturbation Method in Heat Transfer*, Hemisphere Publishing corporation, Washington, DC.

7. Bildik N., Konuralp A., (2006). *The use of variational iteration method, differential transform method and adomian decomposition method for solving different types of nonlinear partial differential equations*, Int. J. Nonlinear Sci. Numer. Simul., 7, 65-70.

8. He J.H., (2006). *Non-perturbative methods for strongly nonlinear problems*, Dissertation, de-Verlag im Internet GmbH, Berlin.

9. Ganji D.D. and Sadighi, A., (2007). *Application of homotopy-perturbation and variational iteration methods to non- inear heat transfer and porous media equations.* Journal of Computational and Applied Mathematics. 207, 24-34.

10. He J.H., (1999a). *Variational iteration method - a kind of non-linear analytical technique: some examples.* International Journal of Non-Linear Mechanics, 34, 699-708.

11. Adomian, G., (1986). *Nonlinear Stochastic Operator Equations.* Academic Press Inc. New York.

12. He J.H., (1999b). *Homotopy perturbation technique.* Computer Methods in Applied Mechanics and Engineering. 178: 257-262.

13. Rostamiyan Y., Ganji D.D., Rahimipetroudi I., Khazayi Nejadabaei M.,(2014), *Analytical investigation of nonlinear model arising in heat transfer through the porous fin*, Thermal science, 18, 409-417.

14. Liao S. J., (2003). *Beyond Perturbation: Introduction to the Homotopy Analysis Method.* Chapman & Hall/CRC Press, Boca Raton, 2003.

15. Liao S. J., (2004). *on the homotopy analysis method for nonlinear problems Appl.* Math. Comput, 47 (2): 499-513.

16. Liao S. J.; 2012. *Homotopy Analysis Method in Nonlinear Differential Equation*, Berlin & Beijing: Springer & Higher Education Press, 2012.

17. Ghasempour M., Rokni, E., kimiaeifar, A. and Rahimpour, M., (2009). *Assessment of HAM and PEM to find nalytical solution for calculating displacement functions of geometrically nonlinear prestressed cable structures with concentrated mass.* International Journal of World Applied Science. 9, 2264-2271.

18. Abbasi M., Hamzeh Nava GH., Rahimipetroudi I., (2014a). *Analytic solution of hydrodynamic and thermal boundary layers over a flat plate in a uniform stream of fluid with convective surface boundary condition*, Indian J. Sci. Res, 1, 15-19.

19. Abbasi M., Ahmadian CHashmi A., Rahimipetroudi I., Hosseinzadeh KH.,(2014b).*Analysis of a fourth grade fluid flow in a channel by application of VIM and HAM*, Indian J.Sci.Res. 1, 389-395.

20. Sohouli A.R., Famouri, M., Kimiaeifar, A. and Domairry, G.; (2010). *Application of homotopy analysis method for natural convection of Darcian fluid about a vertical full cone embedded in pours media prescribed surface heat flux.* Communication in Nonlinear Science and Numerical Simulation. 15: 1691-1699.

21. Fooladi M., Abaspour, S.R., Kimiaeifar, A. and Rahimpour, M., (2009). *On the analytical solution of nonlinear normal mode for continuous systems by means of HAM.* World Applied Sciences Journal. 6, 297-302.

22. Shahbabaei M., Saedodin S., Soleymanibeshei M., Rahimipetroudi I.,(2014). *MHD effect on thermal performance of cylindrical spin porous fin with temperature dependent heat transfer coefficient and emissivity*, International Journal of Energy & Technology, 6, 1-10.

23. Dogonchi A. S., Ganji D. D., Nazari S., Mirzaali A. V., Gholami M.,(2013). *velocity analysis for circular porous slider*, AJMP, 4, 1-16.

24. Faraz N.,(2011). *Study of the effects of the Reynolds number on circular porous slider via variational iteration algorithm-II*, Computers and Mathematics with Applications, 61, 1991-1994.

25. Aziz A ., (2006). *Heat conduction with maple.* Philadelphia (PA): R.T. Edwards.

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