Bol. Soc. Paran. Mat. (3s.) v. 33 1 (2015): 41–48.
©SPM –ISSN-2175-1188 on line ISSN-00378712 in press
SPM: www.spm.uem.br/bspm doi:10.5269/bspm.v33i1.21383

μ-μ^* Connectedness via Hereditary Classes

Shyamapada Modak

abstract: This paper is an attempt to study and introduce the notion of μ-μ^* - connected set in generalized topological spaces with a hereditary class. We have also investigate the relationships between $*$-separated sets, $*_s$ - connected sets, c_{μ^*}-I - connected sets, c_{μ^*}-c_μ - connected sets, c_μ-I - connected sets, $*$-I - connected sets. Further we give some representations of the above connected sets via (μ-μ') - continuity and (μ-μ') - openness.

Key Words: (μ-μ') - continuous, (μ-μ') - open, μ-μ^* - connected, μ-μ^* - separated

Contents

1. Introduction

The notion of ideal topological spaces was studied by Kuratowski [10] and Vaidyanathswamy [15]. The notion was further investigated by Jankovic and Hamlett [8]. Recently, the nation of $*$-connected ideal topological spaces has been introduced and studied in [7], [13], [14], [12] and [10].

Csaszar [6] introduced the notion of generalized topological space with hereditary class. This is a generalization of an ideal topological space. In this paper, we introduce a new type of connected set on a generalized topological space with a hereditary class and investigate properties of this connectedness. We interrelate this connected set with many types of connected set which have been defined by Modak and Noiri in [11].

2. Preliminaries

Let X be a nonempty set and $P(X)$ the power set of X. A subset μ of $P(X)$ is called a generalized topology (GT) [1,3,4,5] if $\emptyset \in \mu$ and the arbitrary union of members of μ is in μ. A generalized topology μ is called a quasi-topology [5] on X if $U, V \in \mu$ implies $U \cap V \in \mu$. A nonempty subset $\mathcal{H}$ of $P(X)$ is called a hereditary class [6] of X if $A \subset B$, $B \in \mathcal{H}$ implies $A \in \mathcal{H}$. For each subset A of

2000 *Mathematics Subject Classification:* 54A05, 54C05, 54D05

X, a subset $A^*(\mathcal{H})$(briefly A^*) of X is defined in [6] as follows: $A^*(\mathcal{H}) = \{x \in X : U \cap A \notin \mathcal{H}$ for every $U \in \mu$ containing $x\}$. If $c_{\mu^*}(A) = A \cup A^*$ for each subset A of X, then $\mu^* = \{A \subset X : c_{\mu^*}(X - A) = X - A\}$ is a generalized topology on X finear than μ [6].

Let us recall some properties established in [6] and [11].

Lemma 2.1. *[6]. For a subset A of X, the following properties hold:*
(1) $A \subset B$ implies $A^* \subset B^*$,
(2) A^* is a μ-closed, that is, $X - A^* \in \mu$,
(3) $A^* \subset c_\mu(A)$, where $c_\mu(A) = \bigcap\{F \subset X : A \subset F,\ X - F \in \mu\}$.

Lemma 2.2. *[6]. The family $\mathcal{B} = \{M - H : M \in \mu,\ H \in \mathcal{H}\}$ is a base for μ^*.*

In the sequel, a generalized topological space (X, μ) with a hereditary class $\mathcal{H}$ is dented by $(X, \mu, \mathcal{H})$ and is called a GTSH. Let $(X, \mu, \mathcal{H})$ be a GTSH. The closure of a subset A of X in (X, μ^*) is denoted by $c_{\mu^*}(A)$.

Definition 2.3. *[11]. Let $(X, \mu, \mathcal{H})$ be a generalized topology with a hereditary class $\mathcal{H}$. Nonempty disjoint subsets A, B of $(X, \mu, \mathcal{H})$ are said to be*
(1) c_μ-c_{μ^*}-separated if $c_\mu(A) \cap c_{\mu^*}(B) = \emptyset = c_{\mu^*}(A) \cap c_\mu(B)$,
(2) c_μ-I-separated if $c_\mu(A) \cap B = \emptyset = A \cap c_\mu(B)$,
(3) c_{μ^*}-I-separated if $c_{\mu^*}(A) \cap B = \emptyset = A \cap c_{\mu^*}(B)$,
(4) c_μ-$*$-separated if $c_\mu(A) \cap B^* = \emptyset = A^* \cap c_\mu(B)$,
(5) c_{μ^*}-$*$-separated if $c_{\mu^*}(A) \cap B^* = \emptyset = A^* \cap c_{\mu^*}(B)$,
(6) $*$-I-separated if $A^* \cap B = \emptyset = A \cap B^*$.

Theorem 2.4. *[11]. For a subset of $(X, \mu, \mathcal{H})$, the following implications hold:*

$$\begin{array}{ccccc}
c_\mu\text{-}c_{\mu^*}\text{- separated} & \Longrightarrow & c_\mu\text{-}I\text{- separated} & \Longrightarrow & c_{\mu^*}\text{-}I\text{- separated} \\
\Downarrow & & & & \Downarrow \\
c_\mu\text{-}*\text{- separated} & \Longrightarrow & c_{\mu^*}\text{-}*\text{- separated} & \Longrightarrow & *\text{-}I\text{- separated}
\end{array}$$

The authors Modak and Noiri in [11] have shown that the converses of the above implications need not hold in general.

3. $\mu - \mu^*$-connectedness

Definition 3.1. *Nonempty disjoint subsets A, B of a GTSH $(X, \mu, \mathcal{H})$ are called $\mu - \mu^*$ - separated if $c_\mu(A) \cap B = \emptyset = A \cap c_{\mu^*}(B)$.*

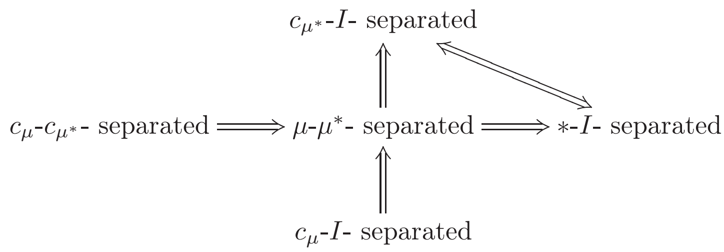
If $c_\mu(A) \cap B = \emptyset$ implies $c_{\mu^*}(A) \cap B = \emptyset$. Then every μ-μ^* - separated sets are c_{μ^*}-I - separated sets [11]. Again $A \cap c_{\mu^*}(B) = \emptyset$ does not necessarily imply $A \cap c_\mu(B) = \emptyset$. So they are not separated sets in (X, μ^*).

Theorem 3.2. *Let $(X, \mu, \mathcal{H})$ be a GTSH. Then*
(1) every μ-μ^* - separated sets is $*$-I - separated.
(2) every c_μ-c_{μ^*} - separated sets is μ-μ^* - separated.

(3) every c_μ-I - separated sets is $\mu - \mu^*$ - separated.
(4) every μ-μ^* - separated sets is c_{μ^*}-I - separated.

Proof: The proof is obvious from the following fact.
(1) $A^* \cap B \subset c_\mu(A) \cap B$ and $A \cap B^* \subset A \cap c_{\mu^*}(B)$.
(2) $c_\mu(A) \cap B \subset c_\mu(A) \cap c_{\mu^*}(B)$ and $A \cap c_{\mu^*}(B) \subset c_\mu(A) \cap c_{\mu^*}(B)$.
(3) $A \cap c_{\mu^*}(B) \subset A \cap c_\mu(B)$.
(4) $c_{\mu^*}(A) \cap B \subset c_\mu(A) \cap B$.

From this Theorem and the Theorem 3.2 of [11] we get following diagram:



It is obvious that every c_{μ^*}-I - separated set is a $*$-I - separated set. Again if A and B are $*$-I - separated sets, then $A^* \cap B = \emptyset = A \cap B^*$. Now $(A \cup A^*) \cap B = (A \cap B) \cup (A \cap B) = \emptyset$. Similarly $c_{\mu^*} \cap A = \emptyset$. □

Converse implications are not true in general.

Example 3.3. *(1) Let $X = \{a, b, c, d\}$, $\mu = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c, d\}\}$ and $\mathcal{H} = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$. Then $(\{a\})^* = \emptyset$ and $(\{b, c, d\})^* = \{c, d\}$. Now $c_{\mu^*}(\{a\}) \cap (\{b, c, d\}) = \emptyset = (\{a\}) \cap c_{\mu^*}(\{b, c, d\})$. So $\{b, c, d\}$ and $\{a\}$ are c_{μ^*}-I - separated but not μ-μ^* - separated, because $c_\mu(\{a\}) = X$.*

(2) $X = \{a, b\}$, $\mu = \{\emptyset, \{a\}, X\}$ and $\mathcal{H} = \{\emptyset, \{a\}\}$. Here $\mu^* = \{\emptyset, \{a\}, \{b\}, X\}$, $(\{a\})^* = \emptyset$ and $(\{b\})^* = \{b\}$. Therefore $\{b\}$ and $\{a\}$ are μ-μ^* - separated sets but they are not c_μ-I - separated, because $c_\mu(\{a\}) = X$. Again $c_{\mu^*}(\{b\}) \cap c_\mu(\{a\}) \neq \emptyset$. Therefore $\{b\}$ and $\{a\}$ are not c_μ-c_{μ^*} - separated sets.

Theorem 3.4. *Let A and B are two μ-μ^* - separated If $C \subset A$ and $D \subset B$ then C and D are also μ-μ^* - separated.*

Proof: Proof is obvious from the fact $c_\mu(C) \subset c_\mu(A)$ and $c_{\mu^*}(D) \subset c_{\mu^*}(B)$. □

Theorem 3.5. *Let $(X, \mu, \mathcal{H})$ be a GTSH. If A and B are nonempty disjoint μ-open sets, then A and B are $\mu - \mu^*$ - separated.*

Proof: Proof is obvious from the fact that $c_\mu(A) \subset c_\mu(X - B) = (X - B)$ and $c_\mu(A) \cap B = \emptyset$ and $c_{\mu^*}(B) \subset c_{\mu^*}(X - A) = (X - A)$ and $c_{\mu^*}(B) \cap A = \emptyset$. □

Lemma 3.6. *For U, $V \subset X$, the following statements are equivalent:*
(a) U and V are μ-μ^* - separated;
(b) There are μ - closed sets F_U and F_V such that $U \subset F_U \subset X - V$ and

$V \subset F_V \subset X - U$;

(c) There are μ - open sets G_U and G_V such that $U \subset G_U \subset X - V$ and $V \subset G_V \subset X - U$.

Proof: (a)$\Rightarrow$ (b). Put $F_U = c_\mu(U)$, $F_V = c_{\mu^*}(V)$. Then F_U and F_V are μ - closed sets and $U \subset F_U \subset X - V$ (since $c_\mu(U) \cap V = \emptyset$), $V \subset F_V \subset X - U$ (since $c_{\mu^*}(V) \cap U = \emptyset$).
(b)$\Rightarrow$ (c). Put $G_U = X - F_V$, $G_V = X - F_U$. Now from complement, we have $V \subset X - F_U \subset X - U$ and $U \subset X - F_V \subset X - V$. Therefore, $V \subset G_V \subset X - U$ and $U \subset G_U \subset X - V$.
(c)$\Rightarrow$ (b). Put $F_U = X - G_V$, $F_V = X - G_U$. Then $X - F_U = G_V$ i.e. $V \subset F_U \subset X - V$. Similarly $V \subset F_V \subset X - U$.
(b)$\Rightarrow$ (a). Clearly $c_\mu(U) \subset F_U$ and $c_{\mu^*}(V) \subset F_V$. Then $c_\mu(U) \cap V \subset (X - V) \cap V = \emptyset$ and similarly $c_{\mu^*}(V) \cap U = \emptyset$. □

Now we recall the following definition and a theorem from [11].

Definition 3.7. *[11]. A subset A of a GTSH $(X, \mu, \mathcal{H})$ is said to be $P - Q$-connected if A is not the union of two $P - Q$-separated sets in $(X, \mu, \mathcal{H})$, where P and Q denote the operations in Definition 3.1.*

For example, in case $P = c_{\mu^*}$ and $Q = *$, $(X, \mu, \mathcal{H})$ is said to be c_{μ^*}-$*$-connected if X cannot be written as the disjoint union of two nonempty c_{μ^*}-$*$ - separated sets.

Theorem 3.8. *[11]. For a subset of $(X, \mu, \mathcal{H})$, the following implications hold:*

$$\begin{array}{ccccc}
c_\mu\text{-}c_{\mu^*}\text{- connected} & \Longleftarrow & c_\mu\text{-}I\text{- connected} & \Longleftarrow & c_{\mu^*}\text{-}I\text{- connected} \\
\Uparrow & & & & \Updownarrow \\
c_\mu\text{-}*\text{- connected} & \Longleftarrow & c_{\mu^*}\text{-}*\text{- connected} & \Longleftarrow & *\text{-}I\text{- connected}
\end{array}$$

Definition 3.9. *A subset A of a GTSH $(X, \mu, \mathcal{H})$ is called μ-μ^*-connected if A is not the union of two μ-μ^* - separated sets in $(X, \mu, \mathcal{H})$.*

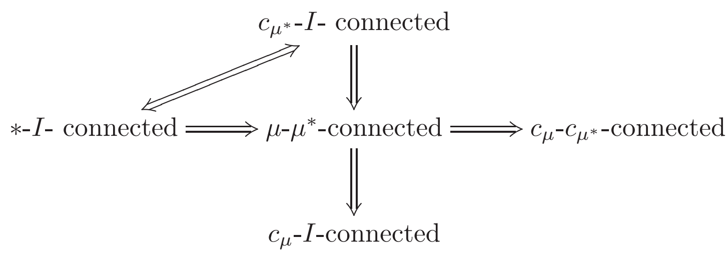
μ-μ^* - connectedness neither a c_μ-c_{μ^*} - connectedness nor a connectedness of (X, μ^*).

Theorem 3.10. *Let $(X, \mu, \mathcal{H})$ be a GTSH. Then*

(1) every $*$-I - connected set is μ-μ^* - connected.
(2) every μ-μ^* - connected set is c_μ-c_{μ^*} - connected.
(3) every μ-μ^* - connected set is c_μ-I - connected.
(4) every c_{μ^*}-I - connected set is μ-μ^* - connected.

Proof: Proof is obvious from Theorem 3.2.

From this theorem and the Theorem 3.8, we get following diagram:



From Example 3.3(1), we have X is not a c_{μ^*}-I - connected set but it is μ-μ^* - connected. Again from Example 3.3(2), we get X is c_μ-I - connected but not μ-μ^* - connected although it is c_μ-c_{μ^*} - connected. □

Lemma 3.11. *Let $(X, \mu, \mathcal{H})$ be a GTSH. If A is a $\mu - \mu^*$-connected of X and H, G are μ-μ^* - separated subsets of X with $A \subset H \cup G$, then either $A \subset H$ or $A \subset G$.*

Proof: Let $A \subset H \cup G$. Since $A = (A \cap H) \cup (A \cap G)$, then $c_\mu(A \cap G) \cap (A \cap H) \subset c_\mu(G) \cap H = \emptyset$. By similar way, we have $c_{\mu^*}(A \cap H) \cap (A \cap G) = \emptyset$. Suppose $A \cap H$ and $A \cap G$ are nonempty. Then A is not a μ-μ^* - connected. This is a contradiction. Thus, either $A \cap H = \phi$ or $A \cap G = \phi$. This implies that $A \subset H$ or $A \subset G$. □

Theorem 3.12. *Let $(X, \mu, \mathcal{H})$ be a QTSH. If A and B are μ-μ^* - separated of X and $A \cup B \in \mu$, then A, $B \in \mu^*$.*

Proof: Since A and B are μ-μ^* - separated, then $B = (A \cup B) \cap (X - c_\mu(A))$. Since $A \cup B \in \mu$ and $c_\mu(A)$ is μ-closed in X, then B is μ-open. By the similar way A is μ^*-open. □

Theorem 3.13. *If A and B are μ-μ^*-connected sets of a GTSH such that none of them is μ-μ^* - separated, then $A \cup B$ is μ-μ^* - connected.*

Proof: Let A and B be μ-μ^* - connected in X. Suppose $A \cup B$ is not μ-μ^* - connected. Then, there exist two nonempty disjoint μ-μ^* - separated sets G and H such that $A \cup B = G \cup H$. Since A and B are μ-μ^* - connected, by Lemma 3.11, either $A \subset G$ and $B \subset H$ or $B \subset G$ and $A \subset H$. Now if $A \subset G$ and $B \subset H$, then $A \cap H = B \cap G = \emptyset$. Therefore, $(A \cup B) \cap G = (A \cap G) \cup (B \cap G) = (A \cap G) \cup \emptyset = A \cap G = A$. Also, $(A \cup B) \cap H = (A \cap H) \cup (B \cap H) = B \cap H = B$. Now, $c_\mu((A \cup B) \cap H) \cap ((A \cup B) \cap G) \subset c_\mu(A \cup B) \cap c_\mu(H) \cap (A \cup B) \cap G = (A \cup B) \cap c_\mu(H) \cap G = \emptyset$ and $((A \cup B) \cap H) \cap c_{\mu^*}((A \cup B) \cap G) \subset (A \cup B) \cap (H) \cap c_{\mu^*}(A \cup B) \cap c_{\mu^*}(G) = (A \cup B) \cap H \cap c_{\mu^*}(G) = \emptyset$. Therefore $(A \cup B) \cap G$ and $(A \cup B) \cap H$ are μ-μ^* - separated sets. Thus, A and B are μ-μ^*-separated, which is a contradiction. Hence, $A \cup B$ is μ-μ^* - connected. □

Theorem 3.14. *If $\{M_i : i \in I\}$ is a nonempty family of μ-μ^* - connected sets of a GTSH, then $\cup_{i \in I} M_i$ is $\mu - \mu^*$ - connected.*

Proof: Suppose $\cup_{i\in I}M_i$ is not μ-μ^* - connected. Then we have $\cup_{i\in I}M_i = H \cup G$, where H and G are μ-μ^* - separated sets in X. Since $\cap_{i\in I}M_i \neq \emptyset$, we have a point $x \in \cap_{i\in I}M_i$. Since $x \in \cup_{i\in I}M_i$, either $x \in H$ or $x \in G$. Suppose that $x \in H$. Since $x \in M_i$ for each $i \in I$, then M_i and H intersect for each $i \in I$. By Lemma 3.11, $M_i \subset H$ or $M_i \subset G$. Since H and G are disjoint, $M_i \subset H$ for all $i \in I$ and hence $\cup_{i\in I}M_i \subset H$. This implies that G is empty. This is a contradiction. Suppose that $x \in G$. By the similar way, we have that H is empty. This is a contradiction. Thus, $\cup_{i\in I}M_i$ is μ-μ^* - connected. □

Theorem 3.15. *Let (X, μ) be a GTS, $\{A_\alpha : \alpha \in \triangle\}$ be a family of $\mu - \mu^*$-connected. If $A \cap A_\alpha \neq \emptyset$ for every α, then $A \cup (\cup A_\alpha)$ is μ-μ^* - connected.*

Proof: Since $A \cap A_\alpha \neq \emptyset$ for each $\alpha \in \triangle$, by Theorem 3.14, $A \cup A_\alpha$ is $\mu - \mu^*$-connected for each $\alpha \in \triangle$. Moreover, $A \cup (\cup A_\alpha) = \cup(A \cup A_\alpha)$ and $\cap(A \cup A_\alpha) \supset A \neq \emptyset$. Thus by Theorem 3.14, $A \cup (\cup A_\alpha)$ is μ-μ^*-connected. □

Theorem 3.16. *If A is a μ-μ^* - connected subset of $(X, \mu, \mathcal{H})$ and $A \subset B \subset c_\mu(A)$, then B is also a μ-μ^* - connected.*

Proof: Suppose B is not a μ-μ^* - connected subset of $(X, \mu, \mathcal{H})$ then there exist μ-μ^* - separated sets H and G such that $B = H \cup G$. This implies that H and G are nonempty and $c_\mu(G) \cap H = \emptyset = G \cap c_{\mu^*}(H)$. By Lemma 3.11, we have that either $A \subset H$ or $A \subset G$. Suppose that $A \subset H$. Then $c_\mu(A) \subset c_\mu(H)$ and $G \cap c_\mu(A) = \emptyset$ (from above). This implies that $G \subset B \subset c_\mu(A)$ and $G = c_\mu(A) \cap G = \emptyset$. Thus G is an empty set. Since G is nonempty, this is a contradiction. Hence, B is μ-μ^* - connected. □

Corollary 3.17. *If A is a μ-μ^* - connected subset of $(X, \mu, \mathcal{H})$ then $c_\mu(A)$ is also a μ-μ^* - connected subset of X.*

Definition 3.18. *Let $(X, \mu, \mathcal{H})$ be a GTSH and $x \in X$. Then union of all μ-μ^* - connected subsets of X containing x is called the μ-μ^* - component of X containing x.*

Theorem 3.19. *Each μ-μ^* - component of a GTSH $(X, \mu, \mathcal{H})$ is a maximal μ-μ^* - connected set of X.*

Theorem 3.20. *The set of all distinct μ-μ^*-component forms a partition of X.*

Theorem 3.21. *Each μ-μ^* - component of a GTSH is μ - closed in X.*

Proof: Suppose A is a μ-μ^* - component in X. Again $c_\mu(A)$ is also a μ-μ^* - connected set in X. So $A = c_\mu(A)$, therefore A is μ-closed. □

4. Images and preimages of μ-connected sets

Let $f : (X, \mu) \to (Y, \mu')$ be a function. We say that f is (μ, μ') - continuous [1] if and only if $f^{-1}(A)$ is μ-open whenever A is μ'-open.

Lemma 4.1. *If f is (μ, μ') - continuous and U', $V' \subset X'$ are μ-μ^* - separated in X', then $f^{-1}(U')$, $f^{-1}(V')$ are μ-μ^* - separated in X.*

Proof: By Lemma 3.6, there exists μ' - open sets $G_{U'}$, $G_{V'}$ such that $U' \subset G_{U'} \subset X' - V'$ and $V' \subset G_{V'} \subset X' - U'$. Then $f^{-1}(U') \subset f^{-1}(G_{U'}) \subset X - f^{-1}(V')$ and $f^{-1}(V') \subset f^{-1}(G_{V'}) \subset X - f^{-1}(U')$, with μ-open sets $f^{-1}(G_{U'})$, $f^{-1}(G_{V'})$. Hence by Theorem 3.5, $f^{-1}(U')$ and $f^{-1}(V')$ are μ-μ^* - separated. □

Theorem 4.2. *If $S \subset X$ if μ-μ^* - connected and f is (μ, μ') - continuous, then $f(S)$ is μ-μ^* - connected in Y.*

Proof: Suppose $f(S) = U' \cup V'$ with μ-μ^* - separated sets U', V'. Then by Lemma 4.1, we have $S \subset f^{-1}(U') \cup f^{-1}(V')$ and $f^{-1}(U')$, $f^{-1}(V')$ are μ-μ^* - separated so that, by Lemma 3.11, either $S \subset f^{-1}(U')$ or $S \subset f^{-1}(V')$, i.e. $f(S) \subset U'$ or $f(S) \subset V'$ and $V' = \emptyset$ or $U' = \emptyset$.

Recall that f is (μ, μ') - open if $f(S)$ is μ' - open, for each μ - open set S [2].□

Theorem 4.3. *If f is $(\mu, \mu\prime)$ - open and injective, U, $V \subset X$ are $\mu - \mu^*$ - separated then $f(U)$, $f(V)$ are μ-μ^* - separated in Y.*

Proof: By Lemma 3.6, there exists μ - open sets G_U, G_V such that $U \subset G_U \subset X - V$ and $V \subset G_V \subset X - U$. Then we have $f(U) \subset f(G_U) \subset f(X - V)$ and $f(V) \subset f(G_V) \subset f(X - U)$ for the μ' - open sets $f(G_U)$, $f(G_V)$. By hypothesis $f(X - U) \subset X' - f(U)$ and $f(X - V) \subset X' - f(V)$ so that Lemma 3.6 shows that μ-μ^* - separated property of $f(U)$ and $f(V)$. □

Conclusion

If μ and $\mathcal{H}$ in GTSH $(X, \mu, \mathcal{H})$ are replaced by the topology τ and the ideal I in an ideal topological space (X, τ, I), then we obtain the following:
(1) μ-μ^* - separated coincides with $*$ - separated in [7].
(2) μ-μ^* - connectedness coincides with $*_s$ - connectedness in [7].
(3) μ-μ^* - component coincides with $*$ - component in [7].

References

1. A. Csaszar, Generalized topology, generalized continuity, Acta Math. Hungar.,**96 (4)** (2002), 351 - 357.

2. A. Csaszar, γ - connected sets, Acta Math. Hungar **101(4)** (2003), 273 - 279.

3. A. Csaszar, Generalized open sets in generalized topologies, Acta Math. Hungar., **106** (2005), 53 - 66.

4. A. Csaszar, Futher remarks on the formula for γ-interior, Acta Math. Hungar., **113** (2006), 325 - 332.

5. A. Csaszar, Remarks on quasi topologies, continuity, Acta Math. Hungar., **119** (2008), 197 - 200.

6. A. Csaszar, Modification of generalized topologies via hereditary classes, Acta Math. Hungar., **115 (1-2)** (2008), 197 - 200.

7. E. Ekici and T. Noiri, Connectedness in ideal topological spaces, Novi Sad J. Math. 38 (2) (2008), 65 - 70.

8. D. Jankovic and T. R. Hamlett, New topologies from old via ideals, Amer. Math. Monthly 97 (1990), 295 - 310.

9. Y. K. Kim and W. K. Min, On operations induced by heredutary classes on generalized topological spaces, Acta Math. Hungar., **137 (1-2)** (2012), 130 - 138.

10. K. Kuratowski, Topology, Vol I, Academic Press, New York, 1966.

11. S. Modak and T. Noiri, Mixed connectedness in GTS via hereditary classes, (submitted).

12. V. Renukadevi and K. Karuppayi, On modifications of generalized topologies via hereditary, J. Adv. Res. Pure Math., **2(2)** (2010), 14 - 20.

13. V. Renukadevi and P. Vimaladevi, Note on generalized topological spaces with hereditary calsses, Bol. Soc. Paran. Mat., 3s **32** (2014), 89 - 97.

14. N. Sathiyasundari and V. Renukadevi, Note on $*$-connected ideal spaces, Novi Sad J. Math. 42 (1) (2012), 15 - 20.

15. R. Vaidyanathaswamy, Set Topology, Chelsea Publishing Company, 1960.

Shyamapada Modak
Department of Mathematics,
University of Gour Banga, P.O. Mokdumpur,
Malda - 732103, India
E-mail address: `spmodak2000@yahoo.co.in`