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Remarks on soft ω-closed sets in soft topological spaces

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abstract: The paper introduces soft ω-closed sets in soft topological spaces and establishes the relation between other existing generalised closed sets in soft topological spaces. It derives the basic properties of soft ω-closed sets. As an application it proves that a soft ω-closed set in a soft compact space is soft compact.

Key Words: soft open set,soft closed set, soft ω-closed set and soft ω-open set.

Contents

1. Introduction

The soft set introduced by Molodtsov [8] is applied in many fields such as economics,engineering,social science,medical science etc. It is used as a tool for dealing with uncertain objects. It laid the platform for further research involving soft sets.The topological structures of set theories dealing with uncertainties was introduced by Chang [2]. Shabir and Naz [9] defined soft topological spaces over an universe .They investigated the basic properties of soft topological spaces. Aygunoglu et.al. [1] also discussed the properties of soft topological spaces. Chen [3] introduced soft semi-open and soft semi-closed sets. Topology is considered to be one of the main branches of Mathematics along with algebra and analysis. Levine [6] has introduced generalised closed sets in topology in order to extend the properties of closed sets to a larger family. In the recent past there has been considerable research in the study of various forms of generalised closed sets. Kannan [5] has introduced soft generalised closed set in soft topological spaces. In this paper soft ω-closed sets are introduced in soft topological spaces and some of its basic properties are discussed. Soft ω-open sets are also defined and the necessary and sufficient

2000 *Mathematics Subject Classification:* 06D72,54A40 condition for a soft set to be soft ω-closed and soft ω-open are derived. The soft ω-closed set concept has been extended to subspaces.

2. Preliminaries

Definition 2.1. *[8] Let X be an initial universe and E be a set of parameters. Let P(X) denote the power set of X and A be a non-empty subset of E. A pair (F,A) is called a soft set over X, where F is a mapping given by $F : A \to P(X)$.The family of all soft sets (F,A) over X is denoted by SS(X,A) For two soft sets (F,A) and (G,A) over a common universe X, (F,A) is said to be soft subset of (G,A) if (i) $F(p) \subseteq G(p)$ for all $p \in A$. Symbolically it is written as $(F, A) \sqsubseteq (G, A)$. The pair (F,A) and (G,A) are soft equal if $(F, A) \sqsubseteq (G, A)$ and $(G, A) \sqsubseteq (F, A)$. Symbolically it is written as (F,A)=(G,A) [8]*

Definition 2.2. *[8] Let I be an arbitrary index set and $\{(F_i, A) : i \in I\} \subseteq SS(X, A)$. The soft union of these soft sets is the soft set $(F, A) \in SS(X, A)$ where the map $F : A \to P(X)$ is defined as $F(p) = \cup\{F_i(p) : i \in I\}$ for every $p \in A$ and denoted as $(F, A) = \sqcup\{(F_i, A) : i \in I\}$.*

Definition 2.3. *[8] Let I be an arbitrary index set and $\{(F_i, A) : i \in I\} \subseteq SS(X, A)$. The soft intersection of these soft sets is the soft set $(F, A) \in SS(X, A)$ where the map $F : A \to P(X)$ is defined as $F(p) = \cap\{F_i(p) : i \in I\}$ for every $p \in A$ and denoted as $(F, A) = \sqcap\{(F_i, A) : i \in I\}$*

Definition 2.4. *[7] A soft set (F,A) over X is said to be null soft set denoted by ϕ if for all $p \in A$ $F(p) = \phi$. A soft set (F,A) over X is said to be an absolute soft set denoted by $\tilde{A}$ if for all $e \in A$, $F(e) = X$.*

Definition 2.5. *[8]Let Y be a nonempty subset of X then $\tilde{Y}$ denotes the soft set (Y,E) over X for which Y(e)=Y, for all $e \in E$. In particular (X,E) will be denoted by $\tilde{X}$.*

Definition 2.6. *[8] The difference (H,E) of two soft sets (F,E) and (G,E) over X denoted by $(F, E) \setminus (G, E)$ is defined as $H(e) = F(e) \setminus G(e)$ for all $e \in E$.*

Definition 2.7. *[9] The relative complement of a soft subset (F,E)is denoted by $(F, E)^c$ and is defined by $(F, E)^c = (F^c, E)$ where $F^c : E \to P(X)$ is a mapping given by $F^c(e) = X - F(e)$ for all $e \in E$.*

Definition 2.8. *[9] Let $\tilde{\tau}$ be the collection of soft sets over X, then $\tilde{\tau}$ is said to be a soft topology on X if*

(i) $\tilde{\phi}, \tilde{X} \in \tilde{\tau}$

(ii)If $(F, E), (G, E) \in \tilde{\tau}$ *then* $(F, E) \sqcap (G, E) \in \tilde{\tau}$

(iii) If $\{(F_i, E)\}_{i\in I} \in \tilde{\tau}, for\ all\ i \in I\}$, *then* $\sqcup_{i\in I}(F_i, E) \in \tilde{\tau}$

The triplet $(X,\tilde{\tau},E)$ is called a soft topological space over X. Every member of $\tilde{\tau}$ is called a soft open set. A soft set (F,E) is called soft closed in X if $(F,E)^c \in \tilde{\tau}$. The soft closure of a soft set over X is defined as the intersection of all soft closed supersets of (F,E) and is denoted as $\overline{(F,E)}$ and it is the smallest soft closed set over X containing (F,E). The soft interior of the soft set (F,E) is defined as union of all soft open subsets of (F,E) and is denoted as $(F,E)^0$ and it is the largest soft open set over X which is contained in (F,E).

Theorem 2.9. *[9] Let $(X,\tilde{\tau},E)$ be a soft topological space over X, (F,E) and (G,E) be soft sets over X.Then*

(i) $\overline{\tilde{\phi}} = \tilde{\phi}$ *and* $\overline{\tilde{X}} = \tilde{X}$

(ii) $(F,E) \sqsubseteq \overline{(F,E)}$

(iii) (F,E) is a soft closed set if and only if $(F,E) = \overline{(F,E)}$

(iv) $\overline{\overline{(F,E)}} = \overline{(F,E)}$

(v) $(F,E) \sqsubseteq (G,E)$ *implies* $\overline{(F,E)} \sqsubseteq \overline{(G,E)}$

(vi) $\overline{(F,E) \sqcup (G,E)} = \overline{(F,E)} \sqcup \overline{(G,E)}$

(vii) $\overline{(F,E) \sqcap (G,E)} \sqsubseteq \overline{(F,E)} \sqcap \overline{(G,E)}$

Theorem 2.10. *[3] Let $(X,\tilde{\tau},E)$ be a soft topological space over X, (F,E) and (G,E) be soft sets over X.Then*

(i) $\tilde{\phi}^0 = \tilde{\phi}$ *and* $\tilde{X}^0 = \tilde{X}$

(ii) $(F,E)^0 \sqsubseteq (F,E)$

(iii) (F,E) is a soft open set if and only if $(F,E) = (F,E)^0$

(iv) $((F,E)^0)^0 = (F,E)^0$

(v) $(F,E) \sqsubseteq (G,E)$ *implies* $(F,E)^0 \sqsubseteq (G,E)^0$

(vi) $((F,E) \sqcap (G,E))^0 = (F,E)^0 \sqcap (G,E)^0$

(vii) $((F,E) \sqcup (G,E))^0 \sqsupseteq F,E)^0 \sqcup (G,E)^0$

Theorem 2.11. *[3] Let $(X,\tilde{\tau},E)$ be a soft topological space over X, (F,E) and (G,E) are soft sets over X.Then*

(i)$((F,E)^c)^0 = (\overline{(F,E)})^c$

(ii)$\overline{(F,E)^c} = ((F,E)^0)^c$

Definition 2.12. *[8] Let (F,E) be a soft set over X and Y be non-empty subset of X. Then the soft subset of (F,E) over Y denoted by (Y_F,E) is defined as $Y_{F(e)} = Y \cap F(e)$ for all $e \in E$. In other words $(Y_F,E) = \tilde{Y} \cap F(e)$*

Definition 2.13. *[8] Let $(X, \tilde{\tau}, E)$ be a soft topological space over X and Y be a nonempty subset of X. Then $\tilde{\tau}_Y = \{(Y_F, E) : (F, E) \in \tilde{\tau}\}$ is said to be the soft relative topology on Y and $(Y, \tilde{\tau}_Y)$ is called a soft subspace of $(X, \tilde{\tau})$.In fact $\tilde{\tau}_Y$ is a soft topology on Y*

Theorem 2.14. *[9] Let $(Y, \tilde{\tau}_Y)$be a soft subspace of a soft topological space $(X, \tilde{\tau})$ and (F,E) be a soft set over X then*

(i)(F,E) is soft open in Y if and only if $(F, E) = \tilde{Y} \sqcap (G, E)$ for some $(G, E) \in \tilde{\tau}$

(ii)(F,E) is soft closed in Y if and only if $(F, E) = \tilde{Y} \sqcap (G, E)$ for some soft closed set(G,E) in X.

Proposition 2.15. *[8] Let (A,E) and (G,E) are soft sets over X then*

(i) $((A, E) \sqcup (G, E))^c = (F, E)^c \sqcap (G, E)^c$

(ii) $((A, E) \sqcap (G, E))^c = (F, E)^c \sqcup (G, E)^c$

Definition 2.16. *[9] Let $(X, \tilde{\tau}, E)$ be a soft topological space.*

(i) A family $\mathfrak{C} = \{(F_i, E) : i \in I\}$ of soft open sets in X is called a soft open cover of X, if $\sqcup_{i \in I}(F_i, E) = \tilde{X}$. A finite subfamily of soft open cover of $\mathfrak{C}$ of X is called a finite sub cover of X,it it is also a soft open cover of X.

(ii)X is called soft compact if every soft open cover of X has a finite subcover.

Definition 2.17. *[1] Let $(X, \tilde{\tau}, E)$ be a soft topological space, (F,E) and (G,E) are soft closed sets in X such that $(F, E) \sqcap (G, E) = \tilde{\phi}$ If there exists soft open sets (A,E) and (B,E) such that $(F, E) \sqsubseteq (A, E), (G, E) \sqsubseteq (B, E)$ and $(A, E) \sqcap (B, E) = \tilde{\phi}$, then X is called a soft normal space.*

Theorem 2.18. *[10] Let$(X, \tilde{\tau}, E)$ be a soft topological space If (F,E) is a soft closed set in X, then (F,E) is soft compact.*

Definition 2.19. *[5] Let $(X, \tilde{\tau}, E)$ be a soft topological space over X.A soft set (F,E) is called a soft generalized closed if $\overline{(F, E)} \sqsubseteq (G, E)$ whenever $(F, E) \sqsubseteq (G, E)$ and (G,E) is soft open in X.*

Theorem 2.20. *[3] A soft subset (A,E) in a soft topological space is soft semi-open if and only if $(A, E) \sqsubseteq \overline{(A, E)^0}$.*

Theorem 2.21. *[3] Let $\{(A_\alpha, E)\}_{\alpha \in \Delta}$ be a collection of soft semi open sets in a soft topological space. Then $\bigcup_{\alpha \in \Delta}(A_\alpha, E)$ is soft semi open.*

Definition 2.22. *[3] A soft set in a soft topological space is said to be soft semi closed if its relative complement is soft semi open.*

Theorem 2.23. *[3] A soft subset (B,E) in a soft topological space is soft semi closed if and only if $(\overline{(B, E)}^0) \sqsubseteq (B, E)$*

Remark 2.24. *[3] Every soft closed set in a soft topological space is soft semi closed. The converse is not true.*

Theorem 2.25. *[3] Let $\{(B_\alpha, E)\}_{\alpha \in \Delta}$ be a collection of soft semi closed sets in a soft topological space. Then $\bigcap_{\alpha \in \Delta}(B_\alpha, E)$ is soft semi closed.*

3. soft ω-closed sets

In this section soft ω-closed set is introduced and some of its basic properties are derived. The necessary and sufficient condition for a soft set to be soft ω-closed is stated and proved.

Definition 3.1. *A soft set (A,E) is called soft ω-closed in a soft topological space $(X, \tilde{\tau}, E)$, if $\overline{(A,E)} \sqsubseteq (G,E)$ whenever $(A,E) \sqsubseteq (G,E)$ and (G,E) is soft semi-open in X.*

Proposition 3.2. *Every soft closed is soft ω-closed.*

Proof: Let (F,E) be a soft closed set and (G,E) be a soft semi-open set in X containing (F,E). Then $\overline{(F,E)} = (F,E) \sqsubseteq (A,E)$. Hence (F,E) is soft ω-closed. □

Remark 3.3. *The converse of the proposition 3.2 is not true.*

Example 3.4. *Let $X = \{x,y,z\}, E = \{a,b\}$ The soft set (F,E) is defined as $F(a) = \{x\}, F(b) = \{y\}$ and the soft set (G,E) is defined as $G(a) = \{x,y\}, G(b) = \{y,z\}$ and $\tilde{\tau} = \{\tilde{\phi}, \tilde{X}, (F,E), (G,E)\}$. The soft set (H,E) defined by $H(a) = \{z\}, H(b) = \{\phi\}$ is soft ω-closed but not soft closed.*

Proposition 3.5. *Every soft ω-closed set is soft g-closed.*

Proof: Let (H,E) be a soft ω-closed set and (U,E) be a soft open set containing(H,E).Since every soft open set is soft semi-open, $\overline{(H,E)} \sqsubseteq (U,E)$. Hence (H,E) is soft ω-closed. □

Remark 3.6. *The converse of the proposition 3.5 is not true.*

Example 3.7. *Let $X = \{x,y,z\}, E = \{a,b\}$ The soft set (F,E) is defined as $F(a) = \{x\}, F(b) = \{y\}$ and the soft set (G,E) is defined as $G(a) = \{x,y\}, G(b) = \{y,z\}$ and $\tilde{\tau} = \{\tilde{\phi}, \tilde{X}, (F,E), (G,E)\}$. The soft set (H,E) defined by $H(a) = \{x,z\}, H(b) = \{x,y\}$ is soft g-closed but not soft ω-closed.*

Theorem 3.8. *If (A,E) and (B,E) are soft ω-closed sets then $(A,E) \sqcup (B,E)$ is also soft ω-closed.*

Proof: Let (U,E) be a soft semi-open set containing $(A,E) \sqcup (B,E)$. Then $(A,E) \sqsubseteq (U,E)$ and $(B,E) \sqsubseteq (U,E)$. Since (A,E) and (B,E) are soft ω-closed sets $\overline{(A,E)} \sqsubseteq (U,E)$, $\overline{(B,E)} \sqsubseteq (U,E)$. Hence $\overline{(A,E) \sqcup (B,E)} = \overline{(A,E)} \sqcup \overline{(B,E)} \sqsubseteq (U,E)$. □

Proposition 3.9. *If (A,E) is soft ω-closed and $(A,E) \sqsubseteq (B,E) \sqsubseteq \overline{(A,E)}$ then (B,E) is also soft ω-closed.*

Proof: Suppose (A,E) is soft ω-closed and $(A,E) \sqsubseteq (B,E) \sqsubseteq \overline{(A,E)}$. Let $(B,E) \sqsubseteq (U,E)$ and (U,E) be soft semi-open, then $(A,E) \sqsubseteq (U,E)$. Since (A,E) is soft ω-closed, $\overline{(A,E)} \sqsubseteq (U,E)$ and $\overline{(B,E)} \sqsubseteq \overline{(A,E)} \sqsubseteq (U,E)$. Hence (B,E) is soft ω-closed. □

Theorem 3.10. *If a set (A,E) is soft ω-closed in X then $\overline{(A,E)} - (A,E)$ contains only null soft closed set.*

Proof: Suppose (A,E) is soft ω-closed in X and (F,E) be a soft closed set such that $(F,E) \sqsubseteq \overline{(A,E)} - (A,E)$ Since (F,E) is soft closed its relative complement is soft open ,$(F,E) \sqsubseteq (A,E)^c$. Thus $(A,E) \sqsubseteq (F,E)^c$. Consequently $\overline{(A,E)} \sqsubseteq (F,E)^c$. Therefore $(F,E) \sqsubseteq (\overline{(A,E)})^c$. Hence $(F,E) = \phi$ and thus $\overline{(A,E)} - (A,E)$ contains only null soft closed set. □

Theorem 3.11. *A soft set (A,E) is soft ω-closed if and only if $\overline{(A,E)} - (A,E)$ contains only null soft semi-closed set.*

Proof: Suppose that (A,E) is soft ω-closed in X let (F,E) be a soft semi-closed set such that $(F,E) \sqsubseteq \overline{(A,E)} - (A,E)$. Since (F,E) is soft semi-closed its relative complement is soft semi-open with $(F,E) \sqsubseteq (A,E)^c$. Thus $(A,E) \sqsubseteq (F,E)^c$. Consequently $\overline{(A,E)} \sqsubseteq (F,E)^c$. Therefore $(F,E) \sqsubseteq (\overline{(A,E)})^c$. Hence $(F,E) = \phi$ and thus $\overline{(A,E)} - (A,E)$ contains only null soft semi-closed set..
Conversely suppose that $\overline{(A,E)} - (A,E)$ contains only null soft semi-closed set. Let $(A,E) \sqsubseteq (G,E)$ and (G,E) be soft semi-open. If $\overline{(A,E)}$ is not a subset of (G,E) then $\overline{(A,E)} \sqcap (G,E)^c$ is a non null soft semi-closed subset of $\overline{(A,E)} - (A,E)$(since any soft closed set is soft semi-closed and arbitrary intersection of soft semi-closed sets is soft semi-closed set [3]) which is a contradiction. Thus $\overline{(A,E)} \sqsubseteq (G,E)$ and hence (A,E) is soft ω-closed. □

Theorem 3.12. *If (A,E) is soft semi-open and soft ω-closed then (A,E) is soft closed.*

Proof: Since (A,E) is soft semi-open and soft ω-closed,$\overline{(A,E)} \sqsubseteq (A,E)$. Hence (A,E) is soft closed. □

Definition 3.13. *The intersection of all soft semi open sets containing (A,E) is called the semi-kernel of (A,E) and is denoted as sker(A,E).*

Theorem 3.14. *A soft set (A,E) of a soft topological space X is soft ω-closed if and only if $\overline{(A,E)} \sqsubseteq sker(A,E)$.*

Proof: The first part follows from the definition of sker(A,E). Conversely let $\overline{(A,E)} \sqsubseteq sker(A,E)$. If (U,E) is any soft semi-open set containing (A,E), then $\overline{(A,E)} \sqsubseteq sker(A,E) \sqsubseteq (U,E)$. Therefore (A,E) is soft ω-closed. □

Theorem 3.15. *Let (A,E) be a soft ω-closed set in X. Then (A,E) is soft closed if and only if $\overline{(A,E)} - (A,E)$ is soft semi-closed.*

Proof: Suppose (A,E) is soft ω-closed which is also soft closed. Then $\overline{(A,E)} = (A,E)$ and so $\overline{(A,E)} - (A,E) = \tilde{\phi}$ which is soft semi-closed.

Conversely since (A,E) is soft ω-closed the by theorem 3.11 $\overline{(A,E)}-(A,E)$ contains no non null soft semi-closed. But $\overline{(A,E)}-(A,E)$ is soft semi-closed set. This implies that $\overline{(A,E)}-(A,E)=\tilde{\phi}$. That is $\overline{(A,E)}=(A,E)$. Hence (A,E) is soft closed. □

Theorem 3.16. *Let $(X,\tilde{\tau},E)$ be a soft topological space and $Y \subseteq Z \subseteq X$ be non-empty subsets of X. If $\tilde{Y}$ is a soft ω-closed set relative to $(Z,\tilde{\tau}_Z)$ and $\tilde{Z}$ is a soft ω-closed set relative to $(X,\tilde{\tau})$, then $\tilde{Y}$ is soft ω-closed relative to $(X,\tilde{\tau})$*

Proof: Let $\tilde{Y} \sqsubseteq (F,E)$, (F,E) is soft semi-open in X. Since Y is a subset of Z , $\tilde{Y} \sqsubseteq \tilde{Z}$ then $\tilde{Y} \sqsubseteq \tilde{Z} \sqcap (F,E)$. Since $\tilde{Y}$ is soft ω-closed relative to $(Z,\tilde{\tau}_Z)$ and $\tilde{Z} \sqcap (F,E)$ is a soft semi-open set in $(Z,\tilde{\tau}_Z)$, $\overline{\tilde{Y}}_Z \sqsubseteq \tilde{Z} \sqcap (F,E)$ where $\overline{\tilde{Y}}_Z$ represents the soft closure of $\tilde{Y}$ with respect to the relative topology $(Z,\tilde{\tau}_Z)$. It follows that $\overline{\tilde{Y}} \sqcap \tilde{Z} \sqsubseteq \tilde{Z} \sqcap (F,E)$ and $\overline{\tilde{Y}} \sqcap \tilde{Z} \sqsubseteq (F,E)$. Hence $\tilde{Z} \sqcap [\overline{\tilde{Y}} \sqcup (\overline{\tilde{Y}})^c] \sqsubseteq (F,E) \sqcap (\overline{\tilde{Y}})^c$ i.e $\tilde{Z} \sqcap \tilde{X} \sqsubseteq (F,E) \sqcup (\overline{\tilde{Y}})^c$. Since Z is subset of X, $\tilde{Z} \sqsubseteq \tilde{X}$.So $\tilde{Z} \sqsubseteq (F,E) \sqcup (\overline{\tilde{Y}})^c$ and $(F,E) \sqcup (\overline{\tilde{Y}})^c$ is soft semi-open in X. Since $\tilde{Z}$ is soft ω-closed set relative to X and $\overline{\tilde{Y}} \sqsubseteq \overline{\tilde{Z}}, \overline{\tilde{Y}} \sqsubseteq (F,E) \sqcup ((\overline{\tilde{Y}})^c$. Therefore $\overline{\tilde{Y}} \sqsubseteq (F,E)$ since $\overline{\tilde{Y}} \sqcap (\overline{\tilde{Y}})^c = \tilde{\phi}$. □

Corollary 3.17. *If (A,E) is a soft ω-closed set and (F,E) is a soft closed set in X then $(A,E) \sqcap (F,E)$ is soft ω-closed set in X.*

Proof: $(A,E) \sqcap (F,E)$ is a soft closed set in (A,E).
By the Theorem 3.16 $(A,E) \sqcap (F,E)$ is soft ω closed in X.

□

Theorem 3.18. *Let $(X,\tilde{\tau},E)$ be a soft topological space and $Y \subseteq X$, (F,E) be a soft set in Y such that it is ω-closed in X. Then (F,E) is soft ω-closed relative to $(Y,\tilde{\tau}_Y)$*

Proof: Let $(F,E) \sqsubseteq \tilde{Y} \sqcap (G,E)$ and (G,E) is soft semi-open in X. Then $(F,E) \sqsubseteq (G,E)$ and hence $\overline{(F,E)} \sqsubseteq (G,E)$ Hence $\tilde{Y} \sqcap \overline{(F,E)} \sqsubseteq \tilde{Y} \sqcap (G,E)$ □

Theorem 3.19. *In a soft topological space $SSO(X) = SC(X)$ if and only if evry soft set over X is a soft ω-closed set in X. SSO(X) represents the collection of all soft semi-open sets in X and SC(X) represents the collection of all soft closed sets in X.*

Proof: Suppose that SSO(X)=SC(X). Let (A,E) be a soft set of X such that $(A,E) \sqsubseteq (G,E)$ where $(G,E) \in SSO(X)$.Then $\overline{(G,E)} = (G,E)$. Also $\overline{(A,E)} \sqsubseteq \overline{(G,E)} \subseteq (G,E)$. Hence (A,E) is soft ω-closed. Conversely suppose that every subset of X is soft ω-closed.Let $(G,E) \in SSO(X)$ Since $(G,E) \sqsubseteq (G,E)$, $\overline{(G,E)} \sqsubseteq (G,E)$ Thus $\overline{(G,E)} = (G,E)$ and $(G,E) \in SC(X)$. Therefore $SSO(X) \sqsubseteq SC(X)$. If $(G,E) \in SC(X)$ then $(G,E)^c$ is soft open and hence soft semi-open. Therefore $(G,E)^c \in (SC(X))^c \sqsubseteq SC(X)$ and hence $(G,E) \in SSO(X)$. Thus $SSO(X) = SC(X)$ □

4. soft ω-open sets

Definition 4.1. *A soft set (A,E) is called a soft ω-open in a soft topological space $(X, \tilde{\tau}, E)$ if the relative complement of (A,E) is soft ω-closed in X.*

Theorem 4.2. *A soft set (A,E) is soft ω-open if and only if $(F, E) \sqsubseteq (A, E)^0$ whenever (F,E) is soft semi-closed and $(F, E) \sqsubseteq (A, E)$*

Proof: Let (A,E) be a soft ω-open set in X. Let (F,E) be soft semi-closed set such that $(F, E) \sqsubseteq (A, E)$. Then $(A, E)^c \sqsubseteq (F, E)^c$ where $(F, E)^c$ is soft semi-open.$(A, E)^c$ is soft ω-closed implies that $\overline{(A, E)^c} \sqsubseteq (F, E)^c$ i.e. $((A, E)^0)^c \sqsubseteq (F, E)^c$. That is $(F, E) \sqsubseteq (A, E)^0$.
Conversely Suppose (F,E) is soft semi-closed and $(F, E) \sqsubseteq (A, E)$. Also $(F, E) \sqsubseteq (A, E)^0$. Let $(U, E)^c \sqsubseteq (A, E)$ where $(U, E)^c$ is soft semi-closed. By hypothesis $(U, E)^c \sqsubseteq (A, E)^0$. That is $((A, E)^0)^c \sqsubseteq (U, E)$. i.e.$\overline{(A, E)^c} \sqsubseteq (U, E)$. This implies that $(A, E)^c$ is soft ω-closed. Hence (A,E) is soft ω-open. □

Theorem 4.3. *If $(A, E)^0 \sqsubseteq (B, E) \sqsubseteq (A, E)$ and (A,E) is soft ω-open then B is soft ω-open.*

Proof: $(A, E)^0 \sqsubseteq (B, E) \sqsubseteq (A, E)$ implies $(A, E)^c \sqsubseteq (B, E)^c \sqsubseteq \overline{(A, E)^c}$ and $(A, E)^c$ is soft ω-closed. By the proposition 3.9 $(B, E)^c$ is soft ω-closed. Hence (B,E) is soft ω-open. □

Theorem 4.4. *If (A,E) and (B,E) are soft ω-open in X then $(A, E) \sqcap (B, E)$ is also soft ω-open.*

Proof: Since (A,E) and (B,E) are soft ω-open their relative complements are soft ω-closed sets and by the Theorem 3.8$(A, E)^c \sqcup (B, E)^c$ is soft ω-closed. Hence by The proposition 2.15 $(A, E) \sqcap (B, E)$ is soft ω-open. □

Theorem 4.5. *A soft set (A,E) is soft ω-open in X if and only if $(G, E) = \tilde{X}$ whenever (G,E) is soft semi-open and $(A, E)^0 \sqcup (A, E)^c \sqsubseteq (G, E)$.*

Proof: Let (A,E) is soft ω-open and (G,E) is soft semi-open with $(A, E)^0 \sqcup (A, E)^c \sqsubseteq (G, E)$. Therefore $(G, E)^c \sqsubseteq ((A, , E)^0)^c \sqcap ((A, E)^c)^c = \overline{(A, E)^c} - (A, E)^c$. Since $(A, E)^c$ is soft ω-closed and $(G, E)^c$ is soft semi-closed by the Theorem 3.11 $(G, E)^c = \tilde{\phi}$. Therefore $(G, E) = \tilde{X}$.
Conversely suppose that (F,E) is soft semi-closed and $(F, E) \sqsubseteq (A, E)$. Then $(A, E)^0 \sqcup (A, E)^c \sqsubseteq (A, E)^0 \sqcup (F, E)^c = \tilde{X}$. It follows that $(F, E) \sqsubseteq (A, E)^0$. Therefore (A,E) is soft ω-open by The theorem 4.2. □

Theorem 4.6. *Let $(X, \tilde{\tau}, E)$ be a soft topological space and $Y \subseteq Z \subseteq X$ are non empty subsets of X. If $\tilde{Y}$ is is a soft ω-open set relative to $(Z, \tilde{\tau}_Z)$ and $\tilde{Z}$ is a soft ω-open set relative to X, then $\tilde{Y}$ is soft ω-open relative to X.*

Proof: Let (F,E) be a soft semi-closed set in X and $(F,E) \sqsubseteq \tilde{Y}$. Then $(F,E) \sqcap \tilde{Z}$ is soft semi-closed set relative to $(Z, \tilde{\tau}_Z)$. But in $\tilde{Y}$ is is a soft ω-open set relative to $(Z, \tilde{\tau}_Z)$, then $(F,E) \sqsubseteq (A,E)^0_Z$, where $(A,E)^0_{\tilde{Z}}$ is the soft open set relative to $(Z, \tilde{\tau}_Z)$,$(F,E) \sqsubseteq (G,E) \sqcap \tilde{Z} \sqsubseteq (A,E)$. Since $\tilde{Z}$ is soft ω-open relative to X, $(F,E) \sqsubseteq (\tilde{Z})^0 \sqsubseteq \tilde{Z}$. Therefore $(F,E) \sqsubseteq (\tilde{Z})^0 \sqcap (G,E) \sqsubseteq \tilde{Z} \sqcap (G,E) \sqsubseteq \tilde{Y}$. It follows that $(F,E) \sqsubseteq (\tilde{Y})^0$. Hence then $\tilde{Y}$ is soft ω-open relative to X. □

Theorem 4.7. *A soft set (A,E) is soft ω-closed if and only if $\overline{(A,E)} - (A,E)$ is soft ω-open.*

Proof: Suppose that (A,E) is soft ω-closed. Let $(F,E) \sqsubseteq \overline{(A,E)} - (A,E)$ where (F,E) is soft semi-closed. By the Theorem 3.11 $(F,E) = \tilde{\phi}$. Therefore $(F,E) \sqsubseteq (\overline{(A,E)} - (A,E))^0$. By the Theorem 4.2 $\overline{(A,E)} - (A,E)$ is soft ω-open.
Conversely let $(A,E) \sqsubseteq (G,E)$ where (G,E) is a soft semi-open set. Then $\overline{(A,E)} \sqcap (G,E)^c \sqsubseteq \overline{(A,E)} \sqcap (A,E)^c = \overline{(A,E)} - (A,E)$. Since $\overline{(A,E)} \sqcap (G,E)^c$ is soft semi closed and $\overline{(A,E)} - (A,E)$ is soft ω-open, it follows by the Theorem 4.2 $\overline{(A,E)} \sqcap (G,E)^c \sqsubseteq (\overline{(A,E)} \sqcap (A,E)^c)^0 = (\overline{(A,E)} - (A,E))^0 = \tilde{\phi}$. Hence (A,E) is soft ω-closed. □

Theorem 4.8. *For a soft subset of a soft topological space the following are equivalent.*

(i) (A,E) is soft ω-closed.

(ii)$\overline{(A,E)} - (A,E)$ contains only null soft semi-closed set.

(iii) $\overline{(A,E)} - (A,E)$ is soft ω-open.

Proof: It follows from the theorems 3.11 and 4.7 □

5. Applications

Theorem 5.1. *Let $(X, \tilde{\tau}, E)$ be a soft compact topological space .If(A,E) is a soft ω-closed set in X then (A,E) is soft compact.*

Proof: Let $\mathfrak{C} = \{(F_i, E) : i \in I\}$ be a soft open cover of (A,E). Since (A,E) is soft ω-closed, $\overline{(A,E)} \sqsubseteq \sqcup_{i \in I}(F_i, E)$. From the Theorem 2.18 $\overline{(A,E)}$ is soft compact and hence $(A,E) \sqsubseteq \overline{(A,E)} \sqsubseteq ((F_1,E) \sqcup (F_2,E)...(F_n,E))$ where $(F_i,E) \in \mathfrak{C}$ for $i = 1, 2, ..n$. Hence (A,E) is soft compact. □

Theorem 5.2. *Let $(X, \tilde{\tau}, E)$ be a soft topological space . Y be a nonempty subset of X and if $\tilde{Y}$ be a soft ω-closed set in X then $(Y, \tilde{\tau_Y})$ is soft normal.*

Proof: Let (A,E) and (B,E) be soft closed sets in X and $(\tilde{Y} \sqcap (A,E)) \sqcap (\tilde{Y} \sqcap (B,E)) = \tilde{\phi}$ This implies that $\tilde{Y} \sqsubseteq ((A,E) \sqcap (B,E))^c \in \tilde{\tau}$ and hence $\overline{\tilde{Y}} \sqsubseteq ((A,E) \sqcap (B,E))^c$. Thus $(\overline{\tilde{Y}} \sqcap (A,E)) \sqcap (\overline{\tilde{Y}} \sqcap (B,E)) = \tilde{\phi}$. Since X is soft normal there exists disjoint soft open sets (G,E) and (U,E) such that $(\overline{\tilde{Y}} \sqcap (A, E)) \sqsubseteq (G, E)$ and $(\overline{\tilde{Y}} \sqcap (B, E)) \sqsubseteq (U, E)$. Hence it follows that

$(\overline{\tilde{Y}} \sqcap (A, E)) \sqsubseteq \tilde{Y} \sqcap (G, E)$ and $(\overline{\tilde{Y}} \sqcap (B, E)) \sqsubseteq \tilde{Y} \sqcap (U, E)$. Hence $(Y, \tilde{\tau_Y})$ is soft normal. □

6. Conclusion

The class of soft ω-closed sets lies between the the class of soft closed sets and the class of soft g-closed sets. The union of two soft ω-closed sets is soft ω-closed. The necessary and sufficient condition for a soft set to be soft ω-closed are derived.The soft ω-closed set concept has been extended to subspaces. As an application it has been proved that a soft ω-closed set in a soft compact space is also soft compact.

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