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p-absolutely Summable Type Fuzzy Sequence Spaces by Fuzzy Metric

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ABSTRACT: In this article we introduce the notion of p-absolutely summable class of sequences of fuzzy real numbers with fuzzy metric, ℓ_{∞}^{F}, for $1 \leq p < \infty$. We study some of its properties like completeness, solidness, symmetricity, sequence algebra and convergence free. Also we study some inclusion results.

Key Words: Fuzzy real number, solid space, symmetricity, convergence free, sequence algebra, fuzzy metric.

Contents

1. Introduction

The concept of fuzzy set, a set whose boundary is not sharp or precise has been introduced by L.A. Zadeh in 1965. It is the origin of new theory of uncertainty, distinct from the notion of probability. After the introduction of fuzzy set, the scope for studies in different branches of pure and applied mathematics increased widely. The notion of fuzzy set theory has been applied to introduce the notion of fuzzy real numbers which help in constructing the sequence of fuzzy real numbers. Different types of sequence spaces of fuzzy real numbers have been studied under classical metric by several workers in the last half century. Till now very few works have been done on fuzzy norm, which has relationship with fuzzy metric and there is a lot to be explored on sequences of fuzzy real numbers, those can be examined by fuzzy metric.

A *fuzzy real number* X is a fuzzy set on R, i.e. a mapping $X : R \to I(= [0, 1])$ associating each real number t with its grade of membership $X(t)$.

A fuzzy real number X is called *convex* if $X(t) \geq X(s) \wedge X(r) =$ min $(X(s), X(r))$, where $s < t < r$.

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If there exists $t_0 \in R$ such that $X(t_0) = 1$, then the fuzzy real number X is called *normal*.

A fuzzy real number X is said to be *upper-semi continuous* if, for each $\varepsilon > 0$, $X^{-1}([0, a+\varepsilon))$, for all $a \in I$ is open in the usual topology of R.

The set of all upper-semi continuous, normal, convex fuzzy real numbers is denoted by $R(I)$. Throughout the article, by a fuzzy real number we mean that the number belongs to $R(I)$.

The α-*level set* $[X]^\alpha$ of the fuzzy real number X, for $0 < \alpha \leq 1$, defined as $[X]^\alpha = \{t \in R : X(t) \geq \alpha\}$. If $\alpha = 0$, then it is the closure of the strong 0-cut.(The *strong* α *-cut* of the fuzzy real number X, for $0 \leq \alpha \leq 1$ is the set $\{t \in R : X(t) > \alpha\}$).

Let $X, Y \in R(I)$ and α-level sets be

$[X]^\alpha = [a_1^\alpha,\ b_1^\alpha]$, $[Y]^\alpha = [a_2^\alpha,\ b_2^\alpha]$, $\alpha \in [0,1]$.

Then the arithmetic operations on $R(I)$ in terms of α-level sets are defined as follows:

$$[X \oplus Y]^\alpha = [a_1^\alpha + a_2^\alpha,\ b_1^\alpha + b_2^\alpha]\,,$$

$$[X \ominus Y]^\alpha = [a_1^\alpha - b_2^\alpha,\ b_1^\alpha - a_2^\alpha]\,,$$

$$[X \otimes Y]^\alpha = \left[\min_{i,j\in\{1,2\}} a_i^\alpha b_j^\alpha,\ \max_{i,j\in\{1,2\}} a_i^\alpha b_j^\alpha\right]$$

and $[\bar{1} \div Y]^\alpha = \left[\frac{1}{b_2^\alpha},\ \frac{1}{a_2^\alpha}\right],\ 0 \notin Y.$

The *absolute value*, $|X|$ of $X \in R(I)$ is defined by (see for instance Kaleva and Seikkala [3])

$$|X|(t) = \begin{cases} \max(X(t), X(-t)), & for\ \ t \geq 0, \\ 0, & for\ \ t < 0. \end{cases}$$

A fuzzy real number X is called *non-negative* if $X(t) = 0$, for all $t < 0$. The set of all non-negative fuzzy real numbers is denoted by $R^*(I)$.

2. Definitions and Preliminaries

The notion of fuzzy set has been applied for introducing different classes of sequences. Their different algebric and topological properties have been investigated.

In the recent years different classes of sequences of fuzzy numbers have been investigated by Esi [1], Matloka [4], Subrahmanyam [5], Syau [6], Tripathy and Baruah [7,8,9], Tripathy and Borgohain [10], Tripathy and Dutta [11,12], Trpathy and Sarma [13,14], Tripathy and Nanda [15] and many others.

A fuzzy real number sequence (X_k) is said to be *bounded* if $|X_k| \leq \mu$, for some $\mu \in R^*(I)$.

A class of sequences E^F is said to be *normal* (or *solid*) if $(Y_k) \in E^F$, whenever $|Y_k| \leq |X_k|$, for all $k \in N$ and $(X_k) \in E^F$.

Let $K = \{k_1 < k_2 < k_3 \ldots\} \subseteq N$ and E^F be a class of sequences. A *K-step set* of E^F is a class of sequences $\lambda_k^{E^F} = \{(X_{k_n}) \in w^F : (X_n) \in E^F\}$.

A *canonical pre-image of a sequence* $(X_{k_n}) \in \lambda_k^{E^F}$ is a sequence $(Y_n) \in w^F$, defined as follows:

$$Y_n = \begin{cases} X_n, & for \;\; n \in K, \\ \bar{0}, & otherwise. \end{cases}$$

A *canonical pre-image of a step set* $\lambda_k^{E^F}$ is a set of canonical pre-images of all elements in $\lambda_k^{E^F}$, i.e., Y is in canonical pre-image $\lambda_k^{E^F}$ if and only if Y is canonical pre-image of some $X \in \lambda_k^{E^F}$.

A class of sequences E^F is said to be *monotone* if E^F contains the canonical pre-images of all its step sets.

From the above definitions we have the following well known Remark.

Remark 2.1. *A class of sequences E^F is solid $\Rightarrow E^F$ is monotone.*

A class of sequences E^F is is said to be *symmetric* if $(X_{\pi(n)}) \in E^F$, whenever $(X_k) \in E^F$, where π is a permutation of N.

A class of sequences E^F is is said to be *sequence algebra* if $(X_k \otimes Y_k) \in E^F$, whenever $(X_k), (Y_k) \in E^F$.

A class of sequences E^F is is said to be *convergence free* if $(Y_k) \in E^F$, whenever $(X_k) \in E^F$ and $X_k = \bar{0}$ implies $Y_k = \bar{0}$.

Throughout the article $\overline{0}$ and $\overline{1}$ represent the additive and multiplicative identities for fuzzy numbers respectively.

Definition 2.2. *Let d be a mapping from $R(I) \times R(I)$ into $R^*(I)$ and let the mappings $L, M : [0,1] \times [0,1] \to [0,1]$ be symmetric, non-decreasing in both arguments and satisfy $L(0,0) = 0$ and $M(1,1) = 1$. Denote*

$$[d(X,Y)]_\alpha = [\lambda_\alpha(X,Y), \rho_\alpha(X,Y)], \text{ for } X, Y \in R(I) \text{ and } 0 < \alpha \leq 1.$$

The quadruple $(R(I), d, L, M)$ is called a *fuzzy metric space* and d a *fuzzy metric*, if

(1) $d(X,Y) = \overline{0}$ if and only if $X = Y$,

(2) $d(X,Y) = d(Y,X)$ for all $X, Y \in X$,

(3) for all $X, Y, Z \in R(I)$,
(i) $d(X,Y)(s+t) \geq L(d(X,Z)(s),\ d(Z,Y)(t))$ whenever $s \leq \lambda_1(X,Z),\ t \leq \lambda_1(Z,Y)$ and $(s+t) \leq \lambda_1(X,Y)$,
(ii) $d(X,Y)(s+t) \leq M(d(X,Z)(s),\ d(Z,Y)(t))$ whenever $s \geq \lambda_1(X,Z),\ t \geq \lambda_1(Z,Y)$ and $(s+t) \geq \lambda_1(X,Y)$.

Using the concept of fuzzy metric, we introduce the following class of sequences.

$$\ell_p^F = \left\{ X = (X_k) \in w^F : \sum_{k=1}^{\infty} \left\{ \lambda(X_k, \overline{0}) \right\}^p < \infty \text{ and } \sum_{k=1}^{\infty} \left\{ \rho(X_k, \overline{0}) \right\}^p < \infty \right\}.$$

We procure the following classes of sequences of fuzzy numbers defined with respect to the concept of fuzzy metric,those will be used in this article.

$$\ell_\infty^F = \left\{ X = (X_k) \in w^F : \sup_k \lambda(X_k, \overline{0}) < \infty \text{ and } \sup_k \lambda(X_k, \overline{0}) < \infty \right\}.$$

$$c^F = \left\{ X = (X_k) \in w^F : \lambda(X_k, T) \to \overline{0} \text{ and } \rho(X_k, T) \to \overline{0}, \text{ as } k \to \infty, \right.$$

for some $T \in R(I) \}$.

$$c_0^F = \left\{ X = (X_k) \in w^F : \lambda(X_k, \overline{0}) \to \overline{0} \text{ and } \rho(X_k, \overline{0}) \to \overline{0}, \text{ as } k \to \infty \right\}.$$

Throughout $w^F,\ \ell_\infty^F,\ \ell_p^F,\ c^F$ and c_0^F denote the classes of *all*, *bounded*, *p-absolutely summable*, *convergent* and *null* sequences of fuzzy real numbers respectively.

3. Main Results

Theorem 3.1. *The class of sequences* ℓ_p^F, $1 \le p < \infty$ *is a complete metric space with the fuzzy metric* d^* *defined by*

$$[d^*(X,Y)]_\alpha = \left[\left\{\sum_{k=1}^{\infty}\{\lambda_\alpha(X_k,Y_k)\}^p\right\}^{\frac{1}{p}},\ \left\{\sum_{k=1}^{\infty}\{\rho_\alpha(X_k,Y_k)\}^p\right\}^{\frac{1}{p}}\right],$$

where $X=(X_k),\ Y=(Y_k)\in \ell_p^F$ and $0<\alpha\le 1$.

Proof: Let $(X^{(n)})$ be a Cauchy sequence in ℓ_p^F,
where $X^{(n)} = \left(X_k^{(n)}\right) = \left(X_1^{(n)},\ X_2^{(n)},\ X_3^{(n)},\ \ldots\right) \in \ell_p^F$, for all $n \in N$

and $\sum_{k=1}^{\infty}\left\{\lambda\left(X_k^{(n)},\overline{0}\right)\right\}^p < \infty$ and $\sum_{k=1}^{\infty}\left\{\rho\left(X_k^{(n)},\overline{0}\right)\right\}^p < \infty$.

Then, for each $\varepsilon > 0$, there exists a positive integer n_0 such that for all $m,\ n \ge n_0$,

$d^*(X^{(n)},X^{(m)}) < \overline{\varepsilon}$.

i.e., $\left\{\sum_{k=1}^{\infty}\left\{\lambda\left(X_k^{(n)},X_k^{(m)}\right)\right\}^p\right\}^{\frac{1}{p}} < \varepsilon$ and $\left\{\sum_{k=1}^{\infty}\left\{\rho\left(X_k^{(n)},X_k^{(m)}\right)\right\}^p\right\}^{\frac{1}{p}} < \varepsilon \ldots(1)$
$\Rightarrow\ \lambda\left(X_k^{(n)},X_k^{(m)}\right) < \varepsilon$ and $\rho\left(X_k^{(n)},X_k^{(m)}\right) < \varepsilon$.

Now $\left(X_k^{(n)}\right)$, for all $k \in N$ is a Cauchy sequences in $R(I)$. Since $R(I)$ is complete, so $\left(X_k^{(n)}\right)$, for all $k \in N$ is convergent in $R(I)$.

Let $\lim_{n\to\infty} X_k^{(n)} = X_k$, for all $k \in N$.

We shall prove that,

$X^{(n)} \to X$, where $X = (X_k)$
and $X \in \ell_p^F$.

Now fix $n \ge n_0$ and let $m \to \infty$ in (1), we have

$\left\{\sum_{k=1}^{\infty}\left\{\lambda\left(X_k^{(n)},X_k\right)\right\}^p\right\}^{\frac{1}{p}} < \varepsilon$ and $\left\{\sum_{k=1}^{\infty}\left\{\rho\left(X_k^{(n)},X_k\right)\right\}^p\right\}^{\frac{1}{p}} < \varepsilon \ldots(2)$
$\Rightarrow d^*(X^{(n)},X) < \overline{\varepsilon}$, for all $n \ge n_0$

$\Rightarrow X^{(n)} \to X$ *as* $n \to \infty$.

Again, we have from (2)

$$\sum_{k=1}^{\infty}\left\{\lambda\left(X_k^{(n)},X_k\right)\right\}^p<\varepsilon \text{ and } \sum_{k=1}^{\infty}\left\{\rho\left(X_k^{(n)},X_k\right)\right\}^p<\varepsilon.$$

Since $X^{(n)}=(X_k^{(n)})\in\ell_p^F$, so $\sum_{k=1}^{\infty}\left\{\lambda\left(X_k^{(n)},\overline{0}\right)\right\}^p<\infty$ and $\sum_{k=1}^{\infty}\left\{\rho\left(X_k^{(n)},\overline{0}\right)\right\}^p<\infty$.

Now for all $n\geq n_0$ we have,

$$\sum_{k=1}^{\infty}\left\{\lambda\left(X_k,\overline{0}\right)\right\}^p=\sum_{k=1}^{\infty}\left\{\lambda\left(X_k,X_k^{(n)}\right)\right\}^p+\sum_{k=1}^{\infty}\left\{\lambda\left(X_k^{(n)},\overline{0}\right)\right\}^p<\varepsilon$$

and

$$\sum_{k=1}^{\infty}\left\{\rho\left(X_k,\overline{0}\right)\right\}^p=\sum_{k=1}^{\infty}\left\{\rho\left(X_k,X_k^{(n)}\right)\right\}^p+\sum_{k=1}^{\infty}\left\{\rho\left(X_k^{(n)},\overline{0}\right)\right\}^p<\varepsilon.$$

$\Rightarrow$ $(X_k)=X\in\ell_p^F$. This proves the completeness of ℓ_p^F. □

Theorem 3.2. *The class of sequences ℓ_p^F is solid and as such is monotone.*

Proof: Consider two sequences (X_k) and (Y_k) such that $|X_k|\leq|Y_k|$, for all $k\in N$ and $Y_k\in\ell_p^F$.

We have,

$$\sum_{k=1}^{\infty}\left\{\lambda\left(X_k,\overline{0}\right)\right\}^p<\sum_{k=1}^{\infty}\left\{\lambda\left(Y_k,\overline{0}\right)\right\}^p<\infty$$

and $\sum_{k=1}^{\infty}\left\{\rho\left(X_k,\overline{0}\right)\right\}^p<\sum_{k=1}^{\infty}\left\{\rho\left(Y_k,\overline{0}\right)\right\}^p<\infty$.

$\Rightarrow$ $(X_k)\in\ell_p^F$. Thus the class ℓ_p^F is solid.

The class of sequences ℓ_p^F is monotone follows from the Remark 2.1. □

Theorem 3.3. *The class of sequences ℓ_p^F is not convergence free.*

Proof: Consider a sequence $(X_k)\in\ell_p^F$ defined as follows:

$$\text{For } k \textit{ even},\ X_k(t)=\begin{cases}1+k^{\frac{2}{p}}t, & for\ -k^{-\frac{2}{p}}\leq t\leq 0,\\ 1-k^{\frac{2}{p}}t, & for\ \ 0<t\leq k^{-\frac{2}{p}},\\ 0, & otherwise\end{cases}$$

and for k *odd*, $X_k=\overline{0}$

Now for $\alpha \in (0,1]$, $[X_k]^\alpha = \begin{cases} \left[(\alpha-1)k^{-\frac{2}{p}}, (1-\alpha)k^{-\frac{2}{p}}\right], & for\ k\ even, \\ [0,0], & for\ k\ odd. \end{cases}$

Then, $\sum\limits_{k=1}^{\infty} \left\{\lambda_\alpha\left(X_k, \overline{0}\right)\right\}^p = (\alpha-1)^p \sum\limits_{k=1}^{\infty} k^{-2} < \infty$

and $\sum\limits_{k=1}^{\infty} \left\{\rho_\alpha\left(X_k, \overline{0}\right)\right\}^p = (1-\alpha)^p \sum\limits_{k=1}^{\infty} k^{-2} < \infty$.

Thus, $(X_k) \in \ell_p^F$.

Let us define a sequence (Y_k) as follows:

For $k\ odd$, $Y_k = \overline{0}$

and for $k\ even$, $Y_k(t) = \begin{cases} 1 + k^{\frac{1}{p}}t, & for\ -k^{-\frac{1}{p}} \le t \le 0, \\ 1 - k^{\frac{1}{p}}t, & for\ 0 < t \le k^{-\frac{1}{p}}, \\ 0, & otherwise \end{cases}$

Now for $\alpha \in (0,1]$, $[Y_k]^\alpha = \begin{cases} [0,0], & for\ k\ odd, \\ \left[(\alpha-1)k^{-\frac{1}{p}}, (1-\alpha)k^{-\frac{1}{p}}\right], & for\ k\ even. \end{cases}$

Then, $\sum\limits_{k=1}^{\infty} \left\{\lambda_\alpha\left(Y_k, \overline{0}\right)\right\}^p = (\alpha-1)^p \sum\limits_{k=1}^{\infty} k^{-1} = \infty$

and
$\sum\limits_{k=1}^{\infty} \left\{\rho_\alpha\left(Y_k, \overline{0}\right)\right\}^p = (1-\alpha)^p \sum\limits_{k=1}^{\infty} k^{-1} = \infty$.

Thus, $(y_k) \notin \ell_p^F$. Hence ℓ_p^F is not convergence free. □

Theorem 3.4. *The class of sequences* ℓ_p^F *is symmetric.*

Proof: Let $(X_k) \in \ell_p^F$.

Let (Y_k) be a arrangement of the sequence (X_k) such that $X_k = Y_{m_k}$ for each $k \in N$.

Then, $\sum\limits_{k=1}^{\infty} \left\{\lambda\left(X_k, \overline{0}\right)\right\}^p = \sum\limits_{k=1}^{\infty} \left\{\lambda\left(Y_{m_k}, \overline{0}\right)\right\}^p$

and
$\sum\limits_{k=1}^{\infty} \left\{\rho\left(X_k, \overline{0}\right)\right\}^p = \sum\limits_{k=1}^{\infty} \left\{\rho\left(Y_{m_k}, \overline{0}\right)\right\}^p$

$\Rightarrow \sum\limits_{k=1}^{\infty} \left\{\lambda\left(Y_{m_k}, \overline{0}\right)\right\}^p < \infty$ and $\sum\limits_{k=1}^{\infty} \left\{\rho\left(Y_{m_k}, \overline{0}\right)\right\}^p < \infty$

Thus, $(Y_k) \in \ell_p^F$. Hence the class ℓ_p^F is symmetric. □

Theorem 3.5. *The class of sequences* ℓ_p^F *is a sequence algebra.*

Proof: Let (X_K), $(Y_K) \in \ell_p^F$, then we have

$\sum\limits_{k=1}^{\infty} \{\lambda(X_k,\overline{0})\}^p < \infty$ and $\sum\limits_{k=1}^{\infty} \{\lambda(Y_k,\overline{0})\}^p < \infty$;

$\sum\limits_{k=1}^{\infty} \{\rho(X_k,\overline{0})\}^p < \infty$ and $\sum\limits_{k=1}^{\infty} \{\rho(Y_k,\overline{0})\}^p < \infty$.

The result follows from the following inequalities:

$$\sum_{k=1}^{\infty} \{\lambda(X_k \bigotimes Y_k,\overline{0})\}^p \le \left[\sum_{k=1}^{\infty} \{\lambda(X_k,\overline{0})\}^p\right]\left[\sum_{k=1}^{\infty} \{\lambda(Y_k,\overline{0})\}^p\right] < \infty$$

and

$$\sum_{k=1}^{\infty} \{\lambda(X_k \bigotimes Y_k,\overline{0})\}^p \le \left[\sum_{k=1}^{\infty} \{\rho(X_k,\overline{0})\}^p\right]\left[\sum_{k=1}^{\infty} \{\rho(Y_k,\overline{0})\}^p\right] < \infty.$$

Thus, $(X_k \bigotimes Y_k) \in \ell_p^F$. Hence the class ℓ_p^F is sequence algebra. □

Theorem 3.6. $\ell_p^F \subseteq \ell_q^F$, $1 \le p \le q < \infty$.

Proof: Let $(X_k) \in \ell_p^F$, then we have

$\sum\limits_{k=1}^{\infty} \{\lambda(X_k,\overline{0})\}^p < \infty$ and $\sum\limits_{k=1}^{\infty} \{\rho(X_k,\overline{0})\}^p < \infty$.

Since $X_k \to \overline{0}$ whenever $k \to \infty$, so there exists a positive integer n_0 such that

$\lambda(X_k,\overline{0}) \le 1$ and $\rho(X_k,\overline{0}) \le 1$, for all $k \ge n_0$.

We have, $\sum\limits_{k=1}^{\infty} \{\lambda(X_k,\overline{0})\}^q = \sum\limits_{k=1}^{n_0-1} \{\lambda(X_k,\overline{0})\}^q + \sum\limits_{k=n_0}^{\infty} \{\lambda(X_k,\overline{0})\}^q$

and

$\sum\limits_{k=1}^{\infty} \{\rho(X_k,\overline{0})\}^q = \sum\limits_{k=1}^{n_0-1} \{\rho(X_k,\overline{0})\}^q + \sum\limits_{k=n_0}^{\infty} \{\rho(X_k,\overline{0})\}^q$

Clearly, $\sum\limits_{k=n_0}^{\infty} \{\lambda(X_k,\overline{0})\}^q \le \sum\limits_{k=n_0}^{\infty} \{\lambda(X_k,\overline{0})\}^p < \infty$

and $\sum\limits_{k=n_0}^{\infty} \{\rho(X_k,\overline{0})\}^q \le \sum\limits_{k=n_0}^{\infty} \{\rho(X_k,\overline{0})\}^p < \infty$.

Further we have, $\sum\limits_{k=1}^{n_0-1} \{\lambda(X_k,\overline{0})\}^q$ and $\sum\limits_{k=1}^{n_0-1} \{\rho(X_k,\overline{0})\}^q$ are finite sum.

Thus, $\sum\limits_{k=1}^{\infty} \{\lambda(X_k,\overline{0})\}^q < \infty$ and $\sum\limits_{k=1}^{\infty} \{\rho(X_k,\overline{0})\}^q < \infty$.

$\Rightarrow (X_k) \in \ell_q^F$. Hence the result. □

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