Bol. Soc. Paran. Mat. (3s.) **v. 32** 2 (2014): 175–187.
©SPM –ISSN-2175-1188 on line ISSN-00378712 in press
SPM: www.spm.uem.br/bspm doi:10.5269/bspm.v32i2.20254

On δ-continuity in mixed fuzzy topological spaces

Binod Chandra Tripathy and Gautam Chandra Ray

abstract: In this paper we introduce and study the notion of δ-continuity on mixed fuzzy topological spaces. We have investigated this notion in the light of the notion of q-neighbourhoods, q-coincidence, fuzzy δ-closure, fuzzy δ-interior. We have established the relationship between fuzzy continuity and fuzzy δ-continuity in mixed fuzzy topological spaces.

Key Words: fuzzy closure; fuzzy δ-closure; fuzzy continuity; fuzzy δ-open sets; fuzzy q-neighbourhoods.

Contents

1. Introduction

The notion of topological space has been expanded in different directions in the recent past. The notion of bitopological space and mixed topological space has been introduced and their different properties have been investigated. Bitopological spaces has been studied by Ganguly and Singha [9], Tripathy and Sarma [25,26] and others. The notion of mixed topology lies in the theory of strict topology of the spaces of continuous functions on locally compact spaces. The concept of mixed topology is pity old. Mixed topology is a technique of mixing two topologies on a set in order to get a third topology. The works on mixed topology is due to Cooper [7], Buck [5], Das and Baishya [8], Tripathy and Ray [27,28], Wiweger [30] and others.

In 1965 L.A. Zadeh introduced the concept of fuzzy sets. Since then the notion of fuzziness has been applied for the study in all the branches of science and technology. It has been applied in mathematical analysis for introducing and investigating different classes of sequence spaces of fuzzy numbers by Tripathy and Baruah [15,16,17], Tripathy and Borgohain [18,19], Tripathy and Dutta [21,22], Tripathy and Sarma [23,24] and many others in the recent past. The notion of fuzziness has been applied in topology and the notion of fuzzy topological spaces

2000 *Mathematics Subject Classification:* 54A40, 54A05,54C08, 54C10, 54C60, 54D20, 54E55

has introduced and investigated by many researches on topological spaces. Different properties of fuzzy topological spaces have been investigated by Arya and Singal [2,3], Chang [6], Azad [4] Das and Baishya [8], Ganster et.al. [11], Ganguly and Singha [9], Ghanim et.al [12], Katsaras and Liu [13], Warren [29], Wong [31,32], Tripathy and Debnath [20] and many others. Recently mixed fuzzy topological spaces has been investigated from different aspect by Das and Baishya [8], Tripathy and Ray [27,28] and others.

The notion of mixed topology has been investigated by Alexiewicz and Semadeni [1]. They have introduced this notion via two normed spaces. Some stronger forms of fuzzy continuity and δ-continuity in fuzzy settings have been studied by Ganguly and Saha [10], in fuzzy topological spaces. In this paper we introduce the concepts of δ-continuity in mixed fuzzy topological spaces. Mixed fuzzy topological spaces have been investigated recently from different aspects by Das and Baishya [8], Rashid and Ali [14] and Tripathy and Ray [27].

2. Preliminaries and Definitions

Let X be a non-empty set and I, the unit interval [0,1]. A fuzzy set A on X is characterized by a function $\mu_A : X \to I$, where μ_A is called the membership function of A. $\mu_A(x)$ representing the membership grade of x in A. The empty fuzzy set is defined by $\mu_\emptyset(x) = 0$ for all $x \in X$. Also X can be regarded as a fuzzy set in itself defined by $\mu_X(x) = 1$ for all $x \in X$. Further, an ordinary subset A of X can be regarded as a fuzzy set in X if its membership function is taken as usual characteristic function of A that is $\mu_A(x) = 1$ for all $x \in X$ and $\mu_A(x) = 0$ for all $x \in X - A$. Two fuzzy set A and B are said to be equal if $\mu_A(x) = \mu_B(x)$. A fuzzy set A is said to be contained in a fuzzy set B, written as $A \subseteq B$, if $\mu_A \leq \mu_B$. Complement of a fuzzy set A in X is defined by $\mu_{A^c} = 1 - \mu_A$. We write $A^c = coA$. Union and intersection of a collection $\{A_i : i \in N\}$ of fuzzy sets in X, are written as $\underset{i\in N}{\cup} A_i$ and $\underset{i\in N}{\cap} A_i$ respectively. Their membership functions are defined as follows.

$\mu_{\underset{i\in N}{\cup} A_i}(x) = sup\{\mu_{A_i}(x) : i \in N\}$, for all $x \in X$ and $\mu_{\underset{i\in N}{\cap} A_i}(x) = sup\{\mu_{A_i}(x) : i \in N\}$, for all $x \in X$.

Definition 2.1. *A fuzzy topology τ on X is a collection of fuzzy sets in X such that $\overline{0}$, $\overline{1} \in \tau$; if $A_i \in \tau$, $i \in N$, then $\cup A_i \in \tau$; if $A, B \in \tau$ then $A \cap B \in \tau$. The pair (X, τ) is called a fuzzy topological space (fts). Members of τ are called open fuzzy sets and the complement of an open fuzzy set is called a closed fuzzy set.*

Definition 2.2. *If (X, τ) is a fuzzy topological space, then the closure and interior of a fuzzy set A in X, denoted by clA and $intA$ respectively, are defined by $clA = \cap\{B : B$ is a closed fuzzy set in X and $A \subseteq B\}$ and $intA = \cap\{V : V$ is an open fuzzy set in X and $V \subseteq A\}$. Clearly, clA (respectively $intA$) is the smallest (respectively largest) closed(respectively open) fuzzy set in X containing (respectively contained in) A. If there are more than one topologies on X, then the* *closure and interior of A with respect to a fuzzy topology τ on X will be denoted by $\tau - clA$ and $\tau - intA$.*

Definition 2.3. *A collection $\mathcal{B}$ of open fuzzy sets in a fts X is said to be an open base for X if every open fuzzy set in X is a union of members of $\mathcal{B}$.*

Definition 2.4. *If A is a fuzzy set in X and B is a fuzzy set in Y then, $A \times B$ is a fuzzy set on $X \times Y$ defined by $\mu_{A\times B}(x, y) = min\{\mu_A(x), \mu_B(x)\}$ for all $x \in X$ and for all $y \in Y$. Let f be a function from X into Y. Then, for each fuzzy set B in Y, the inverse image of B under f, written as $f^{-1}[B]$, is a fuzzy set on X defined by $\mu_{f^{-1}[B]}(x) = \mu_B(f(x))$ for all $x \in X$.*

Definition 2.5. *A fuzzy set A in a fuzzy topological space (X, τ) is called a neighborhood of a point $x \in X$ if and only if there exists $B \in \tau$ such that $B \subseteq A$ and $A(x) = B(x) > 0$.*

Definition 2.6. *A fuzzy point x_α is said to be quasi-coincident with A, denoted by $x_\alpha qA$, if and only if $\alpha + A(x) > 1$ or $\alpha > (A(x))^c$.*

Definition 2.7. *A fuzzy set A is said to be quasi-coincident with B and is denoted by AqB, if and only if there exists a $x \in X$ such that $A(x) + B(x) > 1$.*

Remark 2.8. *It is clear that if A and B are quasi-coincident at x both $A(x)$ and $B(x)$ are not zero at x and hence A and B intersect at x.*

Definition 2.9. *A fuzzy set A in a fts (X, τ) is called a quasi-neighborhood of x_λ if and only if $A_1 \in \tau$ such that $A_1 \subseteq A$ and $x_\lambda qA_1$. The family of all Q-neighborhoods of x_λ is called the system of Q-neighborhood of x_λ. Intersection of two quasi-neighborhoods of x_λ is a quasi-neighborhood of x_λ. Let (X, τ_1) and (X, τ_2) be two fuzzy topological spaces and let $\tau_1(\tau_2)$ be defined as follows.*

$\tau_1(\tau_2) = \{A \in I^X$: for every fuzzy set B in X with AqB, there exists a τ_2-open set A_α, such that $A_\alpha qB$ and τ_1-closure, $\overline{A_\alpha} \subseteq B\}$.

Then $\tau_1(\tau_2)$ is a topology on X and this is called mixed fuzzy topology and the space $(X, \tau_1(\tau_2))$ is called mixed fuzzy topological space is introduced and studied by Tripathy and Ray [27].

Definition 2.10. *A fuzzy point x_λ is called a δ-cluster point of a fuzzy set S in a fuzzy topological spaces (X, τ) if and only if every regular open Q-neighborhood of x_λ is quasi-coincident with S.*

The set of δ-cluster points of S is denoted by $[S]_\delta$ or $\delta - clS$. If $S = [S]_\delta$ then S is said to be δ-closed. The complement of a δ-closed set is said to δ-open.

Now, we introduce the notion of δ-cluster point in a mixed fuzzy topological space $(X, \tau_1(\tau_2))$ as follows:

Definition 2.11. *A fuzzy point x_λ in a mixed fuzzy topological space is said to be δ-cluster point of a fuzzy set A if and only if, every regularly open Q-neighborhood of x_λ is quasi-coincident with A.*

The set of all δ-cluster points of A is denoted by $[A]_\delta$ and $[A] = A \cup A^\delta$, where A^δ denote the set of all points of $[A]_\delta$ which are not in A.

3. Main Results

Definition 3.1. *A fuzzy set A in a mixed fuzzy topological space $(X, \tau_1(\tau_2))$ is said to be regularly open set in X if and only if $int(clA) = A$ (closure with respect to τ_2 and interior with respect to τ_1).*

Example 3.2. *Let $X = \{x, y\}$ be any non-empty set. Let $A = \{(x, 0.5), (y, 0.6)\}$ and $B = \{(x, 0.6), (y, 0.5)\}$ be two fuzzy sets in X. Then $\tau_1 = \{\overline{0}, \overline{1}, B\}$ and $\tau_2 = \{\overline{0}, \overline{1}, A\}$ are two fuzzy topologies on X.*

Now, we construct a mixed topology from these two fuzzy topologies. Since the mixed topology $\tau_1(\tau_2)$ is coarser than τ_2, and it is obvious that $\overline{0}, \overline{1}$ belongs to the mixed topology $\tau_1(\tau_2)$, so we have to verify whether $A \in \tau_1(\tau_2)$ or $A \notin \tau_1(\tau_2)$. If $A \in \tau_1(\tau_2)$ and the fuzzy set B in X be such that AqB, so by definition there exists a τ_2-open set such that $Bq\overline{1}$ and tau_1-closure $cl(\overline{1}) \subseteq A$.

But, τ_1-closure $cl(\overline{1}) = \overline{1}$ and $A \supseteq \overline{1}$.

Therefore $A \notin \tau_1(\tau_2)$ and consequently $\tau_1(\tau_2) = \{\overline{0}, \overline{1}\}$. In this mixed fuzzy topology the only regularly open set is $\overline{1}$, because $int(cl\overline{1}) = \overline{1}$ and $int(cl\overline{0}) = \overline{1}$.

Lemma 3.3. ***(Das and Baishya [8])*** *Let τ_1 and τ_2 be two topologies on a set X. If every τ_1-quasi neighbourhood of x_λ is τ_2-quasi neighbourhood of x_λ, for all fuzzy points x_λ, then τ_1 is coarser than τ_2.*

Theorem 3.4. *Every δ-closed set is closed in a mixed fuzzy topological space $(X, \tau_1(\tau_2))$.*

Proof: Let A be any δ-closed set in X.

Then $A = [A]_\delta$. We show that A is closed in X i.e. $A = \overline{A}$, for this it is sufficient to show that $\overline{A} \subseteq [A]_\delta$.

Suppose, $\overline{A} \notin [A]_\delta$.

Then there exists $x \in X$, such that $\overline{A}(x) > [A]_\delta(x)$.

$\Rightarrow \overline{A(x)} = min\{F(x)\colon F$ is fuzzy closed in X and $A \subseteq F\} > max\{A(x), A^\delta(x)\}$.

Let F_1 be the smallest such fuzzy closed set in X containing A, then $F_1(x) > A(x)$ and $F_1(x) > A^\delta(x)$. Which contradicts the fact that $A^\delta \supseteq F_1$.

Thus, we must have $\overline{A} \subseteq [A]_\delta$, and so $A \subseteq \overline{A} \subseteq [A]_\delta = A \Rightarrow A = \overline{A}$.

Hence A is a closed fuzzy set in mixed fuzzy topological space X. □

Theorem 3.5. *Every δ-open set is open in a mixed fuzzy topological space $(X, \tau_1(\tau_2))$.*

Proof: Let S be any δ-open set in X.

Since S is δ-open so by definition, its complement, $S^c = 1 - S$ is δ-closed. But we know that every δ-closed set is closed in mixed fuzzy topological spaces by Theorem 3.4.

Therefore S^c is closed in X and consequently S is open in X. □

Theorem 3.6. *For any two fuzzy topologies τ_1 and τ_2 on a set X, the mixed fuzzy topology $\tau_1(\tau_2)$ is coarser than τ_2.*

Proof: Let A be any $\tau_1(\tau_2)$ quasi-neighborhood of a fuzzy point x_λ. Then there exists a $\tau_1(\tau_2)$-open set B such that $x_\lambda qB$ and $B \subseteq A$.

A fuzzy point can be considered as a fuzzy singleton set.

Therefore $x_\lambda qB$ and $B \in \tau_1(\tau_2)$ implies that there exists τ_2-open set S such that Sqx_λ and τ_1-closure of S, such that $S \subseteq \overline{S} \subseteq B \subseteq A$.

$\Rightarrow A$ is τ_2-quasi neighborhood of the fuzzy point x_λ.

Hence $\tau_1(\tau_2)$ is coarser than τ_2. □

Theorem 3.7. *Every fuzzy regularly open set in a mixed fuzzy topological space is fuzzy δ-open.*

Proof: Let A be any fuzzy regularly open set in a mixed fuzzy topological space $(X, \tau_1(\tau_2))$. We show that A is fuzzy δ-open. It is sufficient to show that A^c is δ-closed i.e. $[A^c]_\delta = A^c$.

Clearly, $A^c \subset [A^c]_\delta \Rightarrow (A^c)^c \supset [A^c]_\delta^c$

$$\Rightarrow [A^c]_\delta^c \subset A. \tag{3.1}$$

Now, we show that $[A^c]^c_\delta \supset A$.

Suppose, $A \supseteq [A^c]^c_\delta$. Then there exists an element $x \in X$, such that $A(x) > [A^c]^c_\delta(x)$. If $A(x) = \alpha$, then $0 < \alpha < 1$ and $A(x) > [A^c]^c_\delta(x) \Rightarrow x_\alpha \notin [A^c]^c_\delta$.

Now, $\alpha > [A^c]^c_\delta(x)$.

$\Rightarrow [A^c]_\delta(x) > 1 - \alpha$.

$$\Rightarrow x_{1-\alpha} \in [A^c]_\delta. \tag{3.2}$$

Also, we have $A(x) - \alpha + 1 > 1$.

$\Rightarrow A(x) + (1 - \alpha) > 1$.

$$\Rightarrow x_{1-\alpha} q A. \tag{3.3}$$

Hence, from (3.2) and (3.3) we get $Aq[A^c]_\delta$.

Since, A is regularly open set, therefore, $Aq[A^c]_\delta \Rightarrow AqA^c$. Which leads to a contradiction.

Therefore,

$$A \subseteq [A^c]^c_\delta. \tag{3.4}$$

Hence from (3.1) and (3.4) we can conclude that $[A^c]_\delta = A^c$ .i.e. A^c is δ-closed i.e. A is δ-open. □

Theorem 3.8. *In a mixed fuzzy topological space a δ-open set need not be regularly open.*

Proof: The results follows from the following example. □

Example 3.9. *Let us consider a non-empty set $X = \{x, y\}$ and consider the following fuzzy sets in X.*
$A = \{(x, .7), (y, .3)\}$ and $B = \{(x, .3), (y, .7)\}$. Then the collection of fuzzy sets $\tau_1 = \{\overline{0}, \overline{1}, B\}$ and $\tau_2 = \{\overline{0}, \overline{1}, A\}$ are two fuzzy topologies on X.

Now, we construct a mixed fuzzy topology on X from these two fuzzy topologies τ_1 and τ_2.

Since the mixed fuzzy topology is coarser than τ_2, so we need only to verify whether the fuzzy set A is in $\tau_1(\tau_2)$ or not ?

Let us consider a fuzzy set S in X such that AqS.

Now, the only τ_2-open sets are $\overline{1}$ and A such that AqS and $\overline{1}qS$.
Again, τ_1-closure of $A = \wedge\{F : F$ is τ_1-closed and $A \subseteq F\}$
$= \overline{1} \wedge A = A \subseteq A$.
Hence, $A \in \tau_1(\tau_2)$ and so $\tau_1(\tau_2) = \{\overline{0}, \overline{1}, A\}$.
In this mixed fuzzy topology, the fuzzy regularly open sets are $\overline{0}, \overline{1}$, but A is not regularly open because $\tau_1 - Int(\tau_2 - cl(A)) = \overline{1}$.
Now, we shall show that A is δ-open.
It is sufficient to prove that $A^c = F$ is δ-closed.

Consider the fuzzy point $x_{.3}$ of B, Then there exists a regularly open Q- neighborhood $\overline{1}$ of $x_{.3}$ such that $\overline{1}qB$ and so $x_{.3}$ is a δ-cluster point of B. Similarly we can show that the fuzzy point $y_{.7}$ in B is also δ-cluster point of B. Therefore, B is δ-closed and so A is δ-open.
Thus we conclude that a δ-open set need not be regularly open.

4. δ-continuity and continuity

In this section we introduce the notion of fuzzy δ-continuity and continuity between two mixed fuzzy topological spaces and investigate some properties of fuzzy δ-continuity.

Definition 4.1. *A mapping $f : (X, \tau_1(\tau_2)) \longrightarrow (Y, \tau_1(\tau_2))$ is said to be fuzzy δ-continuous mapping if for any regularly open q-neighborhood U of a fuzzy point y_λ of Y, there exists a regularly open q-neighborhood V of x_λ of X such that $f(V) \subset U$, where $f(x) = y$.*

Definition 4.2. *A mapping $f : (X, \tau_1(\tau_2))) \longrightarrow (Y, \tau_1(\tau_2)))$ is said to be fuzzy continuous mapping if for any open q-neighbourhood U of a fuzzy point y_λ of Y, there exists a open q-neighbourhood V of x_λ of X such that $f(V) \subset U$, where $f(x) = y$.*

The following is an alternative for the above definition.

Definition 4.3. *A mapping $f : (X, \tau_1(\tau_2))) \longrightarrow (Y, \tau_1(\tau_2)))$ is said to be fuzzy continuous mapping if for any open fuzzy set U in Y, there exists an open fuzzy set V in X such that $f(V) \subset U$, where $f(x) = y$.*

Theorem 4.4. *If $f : X \longrightarrow Y$ and $g : Y \longrightarrow Z$ be two fuzzy δ-continuous mapping from a mixed fuzzy topological spaces X into another mixed fuzzy topological space Y and from a mixed fuzzy topological spaces Y into another mixed fuzzy topological space Z respectively, then the mapping $g \circ f : X \longrightarrow Z$ is also fuzzy δ-continuous.*

Proof: Let U be any fuzzy regularly open q-neighbourhood of a fuzzy point z_λ of Z. Since g is fuzzy δ-continuous mapping, so by definition of δ-continuity, there exists a fuzzy regularly open q-neighbourhood V of a fuzzy point y_λ of Y such that $g(V) \subset U$, where $g(y) = z$.

Since f is fuzzy δ-continuous, so for the fuzzy regularly open q-neighbourhood V of a fuzzy point y_λ of Y, there exists a fuzzy regularly open q-neighbourhood W of x_λ of X such that $f(W) \subset V$, where $f(x) = y$.

Thus, for any fuzzy regularly open q-neighbourhood U of a fuzzy point of Z, there exists a fuzzy regularly open q-neighbourhood W of the fuzzy point x_λ of X such that

$f(W) \subset V$

$\Rightarrow g(f(W)) \subset g(V)$

$\Rightarrow (g \circ f)(W) \subset U$, where $(g \circ f)(x) = z$.

Hence, $(g \circ f)$ is a fuzzy δ-continuous mapping. □

Theorem 4.5. *Let $f : (X, \tau_1(\tau_2)) \longrightarrow (Y, \tau_3(\tau_4))$ be two mappings, then the following conditions are equivalent:*

(i) f is δ-continuous.

(ii) $f([A]_\delta) \subset (f[A]_\delta)$,for any fuzzy set A in X.

(iii) For every fuzzy δ-closed set A in Y, $f^{-1}(A)$ is fuzzy δ-closed in X.

(iv) For every fuzzy δ-open set A in Y, $f^{-1}(A)$ is fuzzy δ-open in X.

(v) For every fuzzy regularly open set A in Y, $f^{-1}(A)$ is fuzzy δ-open in X.

(vi) For every fuzzy regularly closed set A in Y, $f^{-1}(A)$ is fuzzy δ-closed in X.

Proof: $(i) \Rightarrow (ii)$ Suppose $f : X \to Y$ is fuzzy δ-continuous. Consider a fuzzy point y_α in $f([A]_\delta)$, then $f(x) = y \Rightarrow x_\alpha \in [A]_\delta$. Let U be a fuzzy regularly open q-neighbourhood of the fuzzy point y_α. Since f is fuzzy δ-continuous so there exists a fuzzy regularly open q-nieghbourhood V of x_α such that $f(V) \subset U$, where $f(x) = y$.

Now, $x_\alpha \in [A]_\delta$ and V is a neighbourhood of x_α, therefore VqA.

Next we show that $f(V)qf(A)$, which will imply that $Uqf(A)$ and U is a fuzzy regularly open q-neighbourhood of y_α, so we conclude that $y_\alpha \in [f(A)]_\delta$.

Suppose, $f(V)$ is not quasi-coincident with $f(A)$

$\Rightarrow$ For all $y \in Y$ such that $f(V)(y) + f(A)(y) \leq 1$.

$\Rightarrow \sup_{z \in f^{-1}(y)} V(z) + \sup_{z \in f^{-1}(y)} A(z) \leq 1$.

$\Rightarrow V(z) + A(z) \leq 1$, for all $z \in f^{-1}(y)$.

In particular, $V(x) + A(x) \leq 1$, which implies V is not quasi-coincident with A. Which leads to a contradiction. Hence, we must have $f(V)qf(A) \Rightarrow Uqf(A)$.

Therefore, $y_\alpha \in [f(A)]_\delta$, and so $[f(A)]_\delta \subset (f[A])_\delta$.

(ii) $\Rightarrow$ *(iii)* Let A be any fuzzy δ-closed set in Y.

Then $[A]_\delta = A$. Next we show that $[f^{-1}(A)]_\delta = f^{-1}(A)$.

For any fuzzy set A in Y, $f^{-1}(A)$ is a fuzzy set in X.

Hence, by the given condition, $f([f^{-1}(A)]_\delta \subset [f(f^{-1}[A])_\delta)$.

$\Rightarrow f([f^{-1}(A)]_\delta \subset [A]_\delta$.

$\Rightarrow f([f^{-1}(A)]_\delta \subset A$ (since $[A]_\delta = A$).

$\Rightarrow [f^{-1}(A)]_\delta \subset f^{-1}(A)$.

Further from definition of δ-closure we have $[f^{-1}(A)]_\delta \supset f^{-1}(A)$. Therefore, $f^{-1}(A)$ is fuzzy δ-closed set in X.

(iii) $\Rightarrow$ *(iv)* Easy, so omitted.

(iv) $\Rightarrow$ *(v)*, This implication follows directly from the fact that every regularly open set in a mixed fuzzy topological space is fuzzy δ-open (see Theorem 3.7).

(v) $\Rightarrow$ *(vi)* Easy, so omitted. □

Theorem 4.6. *Every fuzzy δ-continuous function is fuzzy continuous between two mixed fuzzy topological spaces.*

Proof: Let $f : (X, \tau_1(\tau_2)) \to (Y, \tau_3(\tau_4))$ be a fuzzy δ-continuous function from a mixed fuzzy topological space $(X, \tau_1(\tau_2))$ into another mixed fuzzy topological space $(Y, \tau_3(\tau_4))$. Then for every fuzzy δ-closed set A in Y, $f^{-1}(A)$ is fuzzy δ-closed in X.

We know that every fuzzy δ-closed set is fuzzy closed set in mixed fuzzy topological space (see Theorem 3.4). Since A is fuzzy closed set in Y such that $f^{-1}(A)$ is fuzzy closed se in X, so by the definition of fuzzy continuity between two mixed fuzzy topological spaces f is fuzzy continuous function.

But the converse is not necessarily true in general. We give here one example that ensures that fuzzy continuity does not implies δ-continuity. □

Example 4.7. *Consider the identity function $E : (X, \tau_1(\tau_2)) \to (X, \tau_3(\tau_4))$ from a mixed fuzzy topological space X into another mixed fuzzy topological space X.*

Where the mixed fuzzy topological space $(X, \tau_1(\tau_2))$ is defined as follows:

Let $X = x, y$ and the fuzzy sets are defined by

$A_1 = \{(x, 0.2), (y, 0.8)\}$, $A_2 = \{(x, 0.2), (y, 0.2)\}$,

$A_3 = \{(x, 0.8), (y, 0.2)\}$, $A_4 = \{(x, 0.8), (y, 0.8)\}$.

Then the collection $\tau_1 = \{\overline{0}, \overline{1}, A_1, A_2, A_3, A_4\}$ will form a fuzzy topology on X.

Consider the following fuzzy sets in X

$B_1 = \{(x, 0.3), (y, 0.7)\}$, $B_2 = \{(x, 0.7), (y, 0.3)\}$,

$B_3 = \{(x, 0.3), (y, 0.3)\}$, $B_4 = \{(x, 0.7), (y, 0.7)\}$.

Then the collection of fuzzy sets $\tau_2 = \{\overline{0}, \overline{1}, B_1, B_2, B_3, B_4\}$ will form a fuzzy topology on X.

We construct a mixed fuzzy topology $\tau_1(\tau_2)$ on X from these two fuzzy topologies τ_1 and τ_2. Since the mixed fuzzy topology is coarser than τ_2, so we need only to verify whether the fuzzy sets B_1, B_2, B_3, B_4 are in $\tau_1(\tau_2)$ or not.

Consider any fuzzy set $U = \{(x, 0.6), (y, 0.5)\}$ in X. Then $B_1 q U$.

Now, $B_1 \tau_1(\tau_2)$ if there exists τ_2-open set B_2 such that $B_2 q U$ and τ_1-closure $\overline{B_2} = \wedge\{F : F$ is $\tau_1 -$ closed and $B_2 \subseteq F\} = \overline{1} \wedge A_4$. But, $A_4 \supseteq B_1$ and so $B_1 \notin \tau_1(\tau_2)$.

Similarly we can show that $B_2, B_3, B_4 \notin \tau_1(\tau_2)$.

Hence $\tau_1(\tau_2) = \{\overline{0}, \overline{1}\}$.

Another topology on X *is defined as in Example 3.8. i.e.* $\tau_3(\tau_4) = \{\overline{0}, \overline{1}, A\}$.

Then the identity function is fuzzy continuous because inverse images of open set is open. But this function is not δ*-continuous because the inverse image of* δ*-open set* A *is not* δ*-open.*

Conclusion : In this paper we introduce the notions of fuzzy δ-open set, fuzzy δ-closed set and fuzzy δ-continuity in mixed fuzzy topological spaces. We have investigated some of their properties. These notions can be applied for further investigations from different aspects.

References

1. A. Alexiewicz and Z. Semadeni : A generalization of two norm spaces, *Bull. Pol. Acad. Sci.*; 6(1958),135-139.

2. S.P. Arya and N. Singal : Fuzzy subweakly continuous functions, *Math. Student*; 70(1-4)(2001), 231-240.

3. S.P. Arya and N. Singal : Fuzzy subweakly α-continuous functions, *Math. Student*; 70(1-4)(2001), 241-246.

4. K.K. Azad : On fuzzy semi-continuity, fuzzy almost continuity and fuzzy weakly continuity, *J. Math. Anal. Appl.*; 82(1981), 14-32.

5. R.C. Buck : Operator algebras and dual spaces, *Proc. Amer Math. Soc.*; 3(1952), 681-687.

6. C.L. Chang : Fuzzy topological spaces, *J. Math. Anal. Appl.*; 24(1968), 182-190.

7. J.B. Cooper : The strict topology and spaces with mixed topologies, *Proc. Amer. Math. Soc.*; 30(1971), 583-592.

8. N.R. Das and P.B. Baishya : Mixed fuzzy topological spaces, *J. Fuzzy Math.*; 3(4)(1995), 777-784.

9. S. Ganguly and D. Singha : Mixed topology for a bi-topological spaces, *Bull. Cal. Math. Soc.*; 76(1984), 304-314.

10. S. Ganguly and S. Saha : A note on δ-continuity and δ-connected sets in fuzzy set theory, *Simon Stevin*; 62(2)(1988), 127-141.

11. M. Ganster, D.N. Georgiou, S. Jafari and S. Mosokhoa : On some application of fuzzy points, *Appl. General Topology*; 6(2)(2005), 119-133.

12. M.H. Ghanim, E.E. Kerre and A.S. Mashhour : Separation axioms, subspaces and sums in fuzzy topology, *J. Math. Anal. Appl.*, 102(1984), 189-202.

13. A.K. Katsaras and D.B. Liu : Fuzzy vector spaces and fuzzy topological vector spaces, *J. Math. Anal. Appl.*; 58(1977), 135-146.

14. M.H. Rashid and D.M. Ali : Separation axioms in mixed fuzzy topological spaces, *Jour. Bangladesh Acad. Sci.*; 32(2)(2008), 211-220.

15. B.C. Tripathy and Baruah A. : New type of difference sequence spaces of fuzzy real numbers, *Math. Modell. Anal.*; 14(3)(2009), 391-397.

16. B.C. Tripathy and Baruah A. : Nörlund and Riesz mean of sequences of fuzzy real numbers, *Appl. Math. Lett.*; 23(2010), 651-655.

17. B.C. Tripathy and Baruah A. : Lacunary statistically convergent and lacunary strongly convergent generalized difference sequences of fuzzy real numbers, *Kyungpook Math J.*; 50(2010), 565-574.

18. B.C. Tripathy and S. Borgogain : On a class of n-normed sequences related to the -space, *Bol. Soc. Paran. Mat.*; 31(1)(2013), 167-173.

19. B.C. Tripathy and S. Borgogain : Some classes of difference sequence spaces of fuzzy real numbers defined by Orlicz function, *Advances Fuzzy Syst.*; v.2011,(2011) Article ID216414, 6 pages.

20. B.C. Tripathy and S. Debnath : γ-open sets and γ-continuous mappings in fuzzy bitopological spaces, *Jour. Intell. Fuzzy Syst.*; 24(3)(2013), 631-635.

21. B.C. Tripathy and A.J. Dutta : On fuzzy real-valued double sequence spaces ${}_2\ell_F^p$, *Math. Comput. Modell.*; 46(9-10)(2007), 1294-1299.

22. B.C. Tripathy and A.J. Dutta : Bounded variation double sequence space of fuzzy real numbers, *Comput Math Appl.*; 59(2)(2010), 1031-1037.

23. B.C. Tripathy and B. Sarma : Sequence spaces of fuzzy real numbers defined by Orlicz functions, *Math Slovaca*; 58(5)(2008), 621-628.

24. B.C. Tripathy and B. Sarma : Double sequence spaces of fuzzy numbers defined by Orlicz function, *Acta Math Sci.*; 31B(1)(2011), 134-140.

25. B.C. Tripathy and D.J. Sarma : On b-locally open sets in bitopological spaces, *Kyungpook Math. J.*; 51(4)(2011), 429-433.

26. B.C. Tripathy and D.J. Sarma : On pairwise b-locally open and pairwise b-locally closed functions in bitopological spaces; *Tamkang Jour. Math.*; 43(4)(2012), 533-539.

27. B.C. Tripathy and G.C. Ray : On Mixed fuzzy topological spaces and countability, *Soft Computing*; 16(10)(2012), 1691-1695.

28. B.C. Tripathy and G.C. Ray : Weakly continuous functions on mixed fuzzy topological spaces, *Acta Scientiarum Technology*;(2013) (In press).

29. R.H. Warren : Neighbourhood bases and continuity in fuzzy topological spaces, *Rocky Mount. J. Math.*; 8(1978), 459-470.

30. A. Wiweger : Linear spaces with mixed topology, *Studia Math.*; 20(1961), 47-68.

31. C.K. Wong : Fuzzy topology, Product and quotient theorems, *J. Math. Anal. Appl.*; 45(1974), 512-521.

32. C.K. Wong : Fuzzy points and local properties of fuzzy topology, *J. Math.Anal. Appl.*; 46(1974), 512-521.

Binod Chandra Tripathy
Mathematical Sciences Division,
Institute of Advanced Study in Science and Technology,
Pashim Boragaon, Guwahati - 781035, Assam, India.
E-mail address: `tripathybc@yahoo.com; tripathybc@rediffmail.com`

and

Gautam Chandra Ray
Department of Mathematics;
Central Institute of Technology, Kokrajhar - 783370, Assam, India.
E-mail address: `gautam_math@yahoo.com; gautomofcit@gmail.com`