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Notes On Generalized Jordan $(\sigma,\tau)^*$ −Derivations Of Semiprime Rings With Involution

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Key Words: Semiprime *-ring, *-derivation, $(\sigma,\tau)^*$ −derivation, Jordan $(\sigma,\tau)^*$ -derivation, Jordan triple $(\sigma,\tau)^*$ −derivation, generalized Jordan $(\sigma,\tau)^*$ −derivation, generalized Jordan triple $(\sigma,\tau)^*$ −derivation

ABSTRACT: Let R be a 6−torsion free semiprime *-ring, τ an endomorphism of R, σ an epimorphism of R and $f : R \to R$ an additive mapping. In this paper we proved the following result: f is a generalized Jordan $(\sigma,\tau)^*$ −derivation if and only if f is a generalized Jordan triple $(\sigma,\tau)^*$ −derivation.

Contents

1. Introduction

Throughout R will represent an assosiative ring with center Z. Recall that a ring R is prime if $xRy = 0$ implies $x = 0$ or $y = 0$, and semiprime if $xRx = 0$ implies $x = 0$. An additive mapping $x \mapsto x^*$ satisfying $(xy)^* = y^*x^*$ and $(x^*)^* = x$ for all $x, y \in R$ is called an involution, or a *-ring.

An additive mapping $d : R \to R$ is called a derivation (resp. Jordan derivation) if $d(xy) = d(x)y + xd(y)$ (resp. $d(x^2) = d(x)x + xd(x)$) holds for all $x, y \in R$. Let σ and τ be two endomorphisms of R. An additive mapping $d : R \to R$ is said to be a (σ,τ) −derivation (resp. Jordan (σ,τ) −derivation) if $d(xy) = d(x)\sigma(y) + \tau(x)d(y)$ (resp. $d(x^2) = d(x)\sigma(x) + \tau(x)d(x)$) holds for all $x, y \in R$. One can easily prove that every derivation is a Jordan derivation, but converse is in general not true. A famous result due to Herstein [7, Theorem 3.3] asserts that a Jordan derivation of a 2−torsion free prime ring is a derivation. A brief proof of this result can be found in [2]. A Jordan triple derivation $d : R \to R$ is an additive mapping satisfying $d(xyx) = d(x)yx + xd(y)x + xyd(x)$, for all $x, y \in R$. In [8], Herstein showed that every Jordan derivation of 2−torsion free ring is a Jordan triple derivation. Bresar proved that every Jordan triple derivation of a 2−torsion free semiprime ring is a derivation in [4].

Recently, M. Bresar defined the following notation in [5]. An additive mapping $f : R \to R$ is called a generalized derivation if there exists a derivation $d : R \to R$ such that

$$f(xy) = f(x)y + xd(y), \quad \text{for all } x, y \in R.$$

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One may observe that the concept of generalized derivation includes the concept of derivations, also of the left multipliers when $d = 0$. Similarly, an additive mapping $f : R \to R$ is called a generalized Jordan derivation if there is a Jordan derivation $d : R \to R$ such that $f\left(x^2\right) = f(x)x + xd(x)$, for all $x \in R$ and is called a generalized Jordan triple derivation if there exists a derivation $d : R \to R$ such that $f(xyx) = f(x)yx + xd(y)x + xyd(x)$, for all $x, y \in R$.

Let R be a ring with involution *. An additive mapping $d : R \to R$ is said to be a *-derivation (resp. Jordan *-derivation) if $d(xy) = d(x)y^* + xd(y)$ (resp. $d\left(x^2\right) = d(x)x^* + xd(x)$) holds for all $x, y \in R$. The concept of Jordan *-derivations introduced by Bresar and Vukman in [3]. Also, a Jordan triple *-derivation is an additive mapping $d : R \to R$ with the property $d(xyx) = d(x)y^*x^* + xd(y)x^* + xyd(x)$, for all $x, y \in R$. In [13], Vukman proved the following result: Let R be a $6-$torsion free semiprime *-ring and $d : R \to R$ be an additive mapping satisfiying $d(xyx) = d(x)y^*x^* + xd(y)x^* + xyd(x)$, for all $x, y \in R$, then d is a Jordan *-derivation. In [1], Shakir and Fosner introduced $(\sigma, \tau)^*$-derivation, Jordan $(\sigma, \tau)^*$-derivation and Jordan triple $(\sigma, \tau)^*$-derivation as follows: An additive mapping $d : R \to R$ is called a $(\sigma, \tau)^* -$derivation (resp. Jordan $(\sigma, \tau)^* -$derivation) if $d(xy) = d(x)\sigma(y^*) + \tau(x)d(y)$ (resp. $d\left(x^2\right) = d(x)\sigma(x^*) + \tau(x)d(x)$), for all $x, y \in R$. Also d is called a Jordan triple $(\sigma, \tau)^* -$derivation if $d(xyx) = d(x)\sigma(y^*x^*) + \tau(x)d(y)\sigma(x^*) + \tau(xy)d(x)$, for all $x, y \in R$. Shakir and Fosner extended the above mentioned Vukman's Theorem in the setting of Jordan triple $(\sigma, \tau)^* -$derivation.

We study more general concept of Jordan *-derivations. An additive mapping $f : R \to R$ is called a generalized Jordan *-derivation if there exists a Jordan *-derivation $d : R \to R$ such that $f\left(x^2\right) = f(x)x^* + xd(x)$, for all $x \in R$. An additive mapping $f : R \to R$ is called a generalized Jordan triple *-derivation if there exists a Jordan triple *-derivation $d : R \to R$ such that $f(xyx) = f(x)y^*x^* + xd(y)x^* + xyd(x)$, for all $x, y \in R$. Inspired by the above definitions, the notion of generalized Jordan $(\sigma, \tau)^*-$derivation was extended as follows: Let σ and τ be two endomorphisms of R. An additive mapping $f : R \to R$ is called a generalized Jordan $(\sigma, \tau)^*$-derivation if there exists a Jordan $(\sigma, \tau)^*$-derivation $d : R \to R$ such that $f\left(x^2\right) = f(x)\sigma(x^*) + \tau(x)d(x)$, for all $x \in R$. We call f a generalized Jordan triple $(\sigma, \tau)^* -$derivation if there exists a Jordan triple $(\sigma, \tau)^* -$derivation $d : R \to R$ such that $f(xyx) = f(x)\sigma(y^*x^*) + \tau(x)d(y)\sigma(x^*) + \tau(xy)d(x)$, for all $x, y \in R$.

In [3], Bresar and Vukman studied some algebraic properties of Jordan *-derivations. These mappings are closely connected with a question of representability of quadratic forms by bilinear forms. This is shown by Zalar in [14]. Also, Bresar and Zalar obtained a representation of Jordan *-derivations in terms of left an right centralizers on the algebra of compact operators on a Hilbert space in [6]. It turns out that the question whether each quadratic form can be represented by some bilinear form is intimately connected with the question whether every Jordan *-derivation is inner, as shown by Semrl [10]. For results concerning this theory we refer to [11], [12], [15].

The major purpose of this paper is to prove the theorem [1, Theorem 2.1] for a generalized Jordan $(\sigma,\tau)^*$ −derivation of R.

2. Results

Theorem 2.1. *Let R be a 6−torsion free semiprime *-ring, τ an endomorphism of R, σ an epimorphism of R and $f : R \to R$ an additive mapping. Then f is a generalized Jordan $(\sigma,\tau)^*$ −derivation if and only if f is a generalized Jordan triple $(\sigma,\tau)^*$ −derivation.*

Proof: Assume that f is a generalized Jordan $(\sigma,\tau)^*$ −derivation. That is

$$f\left(x^2\right) = f(x)\,\sigma(x^*) + \tau(x)\,d(x), \text{ for all } x \in R. \tag{2.1}$$

The linearization of the relation below

$$f\left(x^2 + xoy + y^2\right) = f(x+y)\,\sigma(x^*+y^*) + \tau(x+y)\,d(x+y).$$

where $xoy = xy + yx$. By the additive mapping of f in the above relation, we see that

$$\begin{aligned} f\left(x^2\right) + f(xoy) + f\left(y^2\right) &= f(x)\,\sigma(x^*) + \tau(x)\,d(x) + f(x)\,\sigma(y^*) + f(y)\,\sigma(x^*) \\ &+ \tau(x)\,d(y) + \tau(y)\,d(x) + f(y)\,\sigma(y^*) + \tau(y)\,d(y). \end{aligned}$$

By the equation (2.1), we obtain that

$$f(xoy) = f(x)\,\sigma(y^*) + f(y)\,\sigma(x^*) + \tau(x)\,d(y) + \tau(y)\,d(x), \text{ for all } x,y \in R. \tag{2.2}$$

Using d is a Jordan $(\sigma,\tau)^*$ −derivation, we get

$$d(xoy) = d(x)\,\sigma(y^*) + d(y)\,\sigma(x^*) + \tau(x)\,d(y) + \tau(y)\,d(x), \text{ for all } x,y \in R. \tag{2.3}$$

Also, $x^2oy + 2xyx = xo(xoy)$, we arrive at

$$f\left(x^2oy + 2xyx\right) = f(xo(xoy)), \text{ for all } x,y \in R.$$

Using (2.2), we see that

$$\begin{aligned} f\left(x^2oy + 2xyx\right) &= f\left(x^2\right)\sigma(y^*) + f(y)\,\sigma(\left(x^2\right)^*) + \tau\left(x^2\right)d(y) \\ &+ \tau(y)\,d\left(x^2\right) + f(2xyx) \\ &= f(x)\,\sigma(x^*)\,\sigma(y^*) + \tau(x)\,d(x)\,\sigma(y^*) + f(y)\,\sigma(\left(x^2\right)^*) \\ &+ \tau\left(x^2\right)d(y) + \tau(y)\,d(x)\,\sigma(x^*) + \tau(y)\,\tau(x)\,d(x) + f(2xyx). \end{aligned}$$

Appliying (2.3) in the last equation, we have

$$\begin{aligned} f(xo(xoy)) &= f(x)\,\sigma\left((xoy)^*\right) + f(xoy)\,\sigma(x^*) + \tau(x)\,d(xoy) + \tau(xoy)\,d(x) \\ &= f(x)\,\sigma(x^*y^*) + f(x)\,\sigma(y^*x^*) + f(x)\,\sigma(y^*)\,\sigma(x^*) \\ &+ f(y)\,\sigma(x^*)\,\sigma(x^*) + \tau(x)\,d(y)\,\sigma(x^*) + \tau(y)\,d(x)\,\sigma(x^*) \\ &+ \tau(x)\,d(x)\,\sigma(y^*) + \tau(x)\,d(y)\,\sigma(x^*) + \tau(x)\,\tau(x)\,d(y) \\ &+ \tau(x)\,\tau(y)\,d(x) + \tau(xy)\,d(x) + \tau(yx)\,d(x). \end{aligned}$$

Last two equations imply that

$$2f\left(xyx\right)=2f\left(x\right)\sigma\left(y^{*}x^{*}\right)+2\tau\left(x\right)d\left(y\right)\sigma\left(x^{*}\right)+2\tau\left(xy\right)d\left(x\right),\text{ for all }x,y\in R.$$

Since R is $2-$torsion free, we get

$$f\left(xyx\right)=f\left(x\right)\sigma\left(y^{*}x^{*}\right)+\tau\left(x\right)d\left(y\right)\sigma\left(x^{*}\right)+\tau\left(xy\right)d\left(x\right),\text{ for all }x,y\in R.$$

Hence, we obtain that f is a generalized Jordan triple $(\sigma,\tau)^{*}-$derivation.

Let us prove the reverse. We have

$$f\left(xyx\right)=f\left(x\right)\sigma\left(y^{*}x^{*}\right)+\tau\left(x\right)d\left(y\right)\sigma\left(x^{*}\right)+\tau\left(xy\right)d\left(x\right),\text{ for all }x,y\in R. \quad (2.4)$$

Taking y by xyx in (2.4) and using (2.4), [1, Theorem 2.1], we obtain

$$\begin{aligned}&f\left(x^{2}\right)\sigma(y^{*}\left(x^{*}\right)^{2})+\tau\left(x^{2}\right)d\left(y\right)\sigma((x^{*})^{2})+\tau\left(x^{2}y\right)d\left(x\right)\sigma\left(x^{*}\right)+\tau\left(x^{2}yx\right)d\left(x\right)\\&=f\left(x\right)\sigma(x^{*}y^{*}\left(x^{*}\right)^{2})+\tau\left(x\right)d\left(x\right)\sigma(y^{*}\left(x^{*}\right)^{2})+\tau\left(x^{2}\right)d\left(y\right)\sigma((x^{*})^{2})\\&+\tau\left(x^{2}y\right)d\left(x\right)\sigma\left(x^{*}\right)+\tau\left(x^{2}yx\right)d\left(x\right)\end{aligned}$$

and so,

$$\left(f\left(x^{2}\right)-f\left(x\right)\sigma(x^{*})-\tau\left(x\right)d\left(x\right)\right)\sigma(y^{*}\left(x^{*}\right)^{2})=0.$$

The relation above reduces to

$$A\left(x\right)\sigma(y^{*})\sigma((x^{*})^{2})=0,\text{ for all }x,y\in R,$$

where $A\left(x\right)$ stands for $f\left(x^{2}\right)-f\left(x\right)\sigma(x^{*})-\tau\left(x\right)d\left(x\right)$. Since σ is an epimorphism of R, we find that

$$A\left(x\right)R\sigma((x^{*})^{2})=0,\text{ for all }x\in R. \quad (2.5)$$

If we multiplying (2.4) from the left side by $\sigma((x^{*})^{2})$ and from the right side by $A\left(x\right)$, we get

$$\sigma((x^{*})^{2})A\left(x\right)R\sigma((x^{*})^{2})A\left(x\right)=0,\text{ for all }x\in R.$$

Since R is semiprime ring, it follows that

$$\sigma((x^{*})^{2})A\left(x\right)=0,\text{ for all }x\in R. \quad (2.6)$$

Similarly, we see that

$$A\left(x\right)\sigma((x^{*})^{2})=0,\text{ for all }x\in R. \quad (2.7)$$

Writing x by $x+y$ in (2.7), we have

$$\begin{aligned}0&=A\left(x+y\right)\sigma((x^{*}+y^{*})^{2})\\&=(f\left(x^{2}+xoy+y^{2}\right)-f\left(x+y\right)\sigma(x^{*}+y^{*})-\tau\left(x+y\right)d\left(x+y\right))\sigma((x^{*}+y^{*})^{2})\\&=(f\left(x^{2}\right)-f\left(x\right)\sigma(x^{*})-\tau\left(x\right)d\left(x\right)+f\left(y^{2}\right)-f\left(y\right)\sigma(y^{*})-\tau\left(y\right)d\left(y\right)\\&+f\left(xoy\right)-f\left(x\right)\sigma(y^{*})-f\left(y\right)\sigma(x^{*})-\tau\left(x\right)d\left(y\right)-\tau\left(y\right)d\left(x\right))\sigma((x^{*}+y^{*})^{2}).\end{aligned}$$

That is,

$$(A(x)+A(y)+B(x,y))\,\sigma((x^*+y^*)^2)=0,\ \text{for all } x,y\in R,$$

where $B(x,y)$ stands for $f(xoy)-f(x)\sigma(y^*)-f(y)\sigma(x^*)-\tau(x)d(y)-\tau(y)d(x)$. Appliying equation (2.7), we get

$$\begin{aligned}0&=A(y)\sigma((x^*)^2)+A(x)\sigma((y^*)^2)+A(x)\sigma(x^*oy^*)+A(y)\sigma(x^*oy^*)\\&\quad+B(x,y)\sigma((x^*)^2)+B(x,y)\sigma((y^*)^2)+B(x,y)\sigma(x^*oy^*).\end{aligned}\tag{2.8}$$

Putting $-x$ for x in (2.8) and using $A(-x)=A(x)$ and $B(-x,y)=-B(x,y)$, we obtain that

$$\begin{aligned}0&=A(y)\sigma((x^*)^2)+A(x)\sigma((y^*)^2)-A(x)\sigma(x^*oy^*)-A(y)\sigma(x^*oy^*)\\&\quad-B(x,y)\sigma((x^*)^2)-B(x,y)\sigma((y^*)^2)+B(x,y)\sigma(x^*oy^*).\end{aligned}\tag{2.9}$$

By comparing (2.8) and (2.9), we arrive at

$$2A(x)\sigma(x^*oy^*)+2A(y)\sigma(x^*oy^*)+2B(x,y)\sigma((x^*)^2)+2B(x,y)\sigma((y^*)^2)=0.$$

Since R is $2-$torsion free, we have

$$A(x)\sigma(x^*oy^*)+A(y)\sigma(x^*oy^*)+B(x,y)\sigma((x^*)^2)+B(x,y)\sigma((y^*)^2)=0.\tag{2.10}$$

Replacing x by $2x$ in (2.8), we see that

$$\begin{aligned}0&=4A(y)\sigma((x^*)^2)+4A(x)\sigma((y^*)^2)+8A(x)\sigma(x^*oy^*)+2A(y)\sigma(x^*oy^*)\\&\quad+8B(x,y)\sigma((x^*)^2)+2B(x,y)\sigma((y^*)^2)+4B(x,y)\sigma(x^*oy^*).\end{aligned}$$

Using (2.8) and (2.10) in the last equation, we get

$$6A(y)\sigma(x^*oy^*)+6B(x,y)\sigma((y^*)^2)=0,\ \text{for all } x,y\in R.$$

Since R is $6-$torsion free, we have

$$A(y)\sigma(x^*oy^*)+B(x,y)\sigma((y^*)^2)=0,\ \text{for all } x,y\in R.$$

Again appliying equation (2.10), we find that

$$A(x)\sigma(x^*oy^*)+B(x,y)\sigma((x^*)^2)=0,\ \text{for all } x,y\in R.\tag{2.11}$$

Right multiplication of (2.11) by $A(x)$, we obtain that

$$A(x)\sigma(x^*oy^*)A(x)+B(x,y)\sigma((x^*)^2)A(x)=0.$$

Using (2.6) gives that

$$A(x)\sigma(x^*y^*)A(x)+A(x)\sigma(y^*x^*)A(x)=0,\ \text{for all } x,y\in R.$$

By the surjectivity of σ, it follows that

$$A(x)\,\sigma(x^*)yA(x) + A(x)\,y\sigma(x^*)A(x) = 0, \text{ for all } x, y \in R. \tag{2.12}$$

Replacing y by $y\sigma(x^*)$ in the above relation, we get

$$A(x)\,\sigma(x^*)y\sigma(x^*)A(x) + A(x)\,y\sigma((x^*)^2)A(x) = 0, \text{ for all } x, y \in R.$$

Again using (2.6) implies that

$$A(x)\,\sigma(x^*)y\sigma(x^*)A(x) = 0, \text{ for all } x, y \in R,$$

and so

$$\sigma(x^*)A(x)\,\sigma(x^*)y\sigma(x^*)A(x)\,\sigma(x^*) = 0, \text{ for all } x, y \in R.$$

By the semiprimeness of R, we have

$$\sigma(x^*)A(x)\,\sigma(x^*) = 0, \text{ for all } x \in R.$$

Multiplying (2.12) by $\sigma(x^*)$ from right and using the last equation, we see that

$$A(x)\,\sigma(x^*)yA(x)\,\sigma(x^*) = 0, \text{ for all } x, y \in R,$$

and so

$$A(x)\,\sigma(x^*) = 0, \text{ for all } x \in R. \tag{2.13}$$

Substitution $x + y$ for x, we have

$$\begin{aligned} 0 &= A(x+y)\,\sigma(x^* + y^*) \\ &= (A(x) + A(y) + B(x,y))\,\sigma(x^* + y^*). \end{aligned}$$

In view of equation (2.13) the last equation reduces to

$$A(x)\,\sigma(y^*) + A(y)\,\sigma(x^*) + B(x,y)\,\sigma(x^*) + B(x,y)\,\sigma(y^*) = 0$$

Replacing x by $-x$ in the above relation and comparing the relation so obtained with the above relation we get

$$A(x)\,\sigma(y^*) + B(x,y)\,\sigma(x^*) = 0, \text{ for all } x, y \in R. \tag{2.14}$$

Right multiplication of (2.14) by $\sigma(x^*)A(x)$, we conclude that

$$A(x)\,\sigma(y^*)\sigma(x^*)A(x) + B(x,y)\,\sigma((x^*)^2)A(x) = 0.$$

Using (2.6), we obtain that

$$A(x)\,\sigma(y^*)\sigma(x^*)A(x) = 0$$

and so

$$\sigma(x^*)A(x)\,y\sigma(x^*)A(x) = 0.$$

Since R is semiprime ring, we have

$$\sigma(x^*)A(x) = 0, \text{ for all } x \in R.$$

We right multiple in relation (2.14) by $A(x)$ and then use the above relation to get

$$A(x)\sigma(y^*)A(x) = 0, \text{ for all } x, y \in R.$$

By the surjectivity of σ and semiprimeness of R, we arrive at $A(x) = 0$, for all $x \in R$. This means that $f(x^2) = f(x)\sigma(x^*) + \tau(x)d(x)$, for all $x \in R$, and so f is a generalized Jordan $(\sigma,\tau)^*$ –derivation, which completes the proof. □

Corollary 2.2. *Let R be a 6–torsion free semiprime *-ring and $f : R \to R$ an additive mapping. Then f is a generalized Jordan*-derivation if and only if f is a generalized Jordan triple-derivation*

The following corollary which is proved Vukman in [13], is a direct consequence of Theorem 2.1.

Corollary 2.3. *Let R be a 6–torsion free semiprime *-ring and $d : R \to R$ an additive mapping. Then d is a Jordan*-derivation if and only if*

$$d(xyx) = d(x)y * x^* + xd(y)x^* + xyd(x), \text{ for all } x, y \in R.$$

In particular, if we take $f = d$ in Theorem 2.1, then we have the following result which is a generalization of [1, Theorem 2.1] even without σ an automorphism assumption on ring.

Corollary 2.4. *Let R be a 6–torsion free semiprime *-ring, τ an endomorphism of R, σ an epimorphism of R and $d : R \to R$ an additive mapping. Then d is a Jordan $(\sigma,\tau)^*$ –derivation if and only if*

$$d(xyx) = d(x)\sigma(y^*x^*) + \tau(x)d(y)\sigma(x^*) + \tau(xy)d(x), \text{ for all } x, y \in R.$$

We can give a following corollary in view of Corollary 2.4, which is a generalization of [9, Theorem 2].

Corollary 2.5. *Let R be a 6–torsion free semiprime ∗–ring, θ an epimorphism of R. An additive mapping $T : R \to R$ is a Jordan left θ^*–centralizer on R if and only if $T(xyx) = T(x)\theta(y^*x^*)$, for all $x, y \in R$.*

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