



## A New Sort of Separation Axioms

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ABSTRACT: This paper introduces  $\tilde{g}_\alpha$ - $D_i$  and  $\tilde{g}_\alpha$ - $R_i$ -spaces using  $\tilde{g}_\alpha$ -sets and discusses their properties comprehensively analyzing their relationship. It also derives their characterizations in terms of  $\tilde{g}_\alpha$ -continuous and  $\tilde{g}_\alpha$ -irresolute functions.

Key Words:  $\tilde{g}_\alpha$ -closed,  $\tilde{g}_\alpha$ -open sets and  $\tilde{g}_\alpha$ -closure.

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### 1. Introduction

Levine [6] offered a new and useful notion in General Topology called generalized closed set. The investigation of generalized closed sets has led to several new separation axioms weaker than  $T_1$ . Some of these separation axioms have been found to be useful in computer science and digital topology. After the introduction of generalized closed sets there are many research papers which deal with different types of generalized closed sets. Jafari et al. [4] have introduced  $\tilde{g}_\alpha$ -closed set and studied their properties using  $\#$ gs-open sets [10]. In this paper we have introduced  $\tilde{g}_\alpha$ D-sets and associated separation axioms. We also introduced  $\tilde{g}_\alpha$ - $R_0$  and  $\tilde{g}_\alpha$ - $R_1$ -spaces and discussed their properties.

### 2. Preliminaries

We list some definitions which are useful in the following sections. The interior and the closure of a subset  $A$  of  $(X, \tau)$  are denoted by  $Int(A)$  and  $Cl(A)$ , respectively. Throughout the present paper  $(X, \tau)$  and  $(Y, \sigma)$  (or  $X$  and  $Y$ ) represent topological spaces on which no separation axiom is defined, unless otherwise mentioned.

**Definition 2.1.** A subset  $A$  of a topological space  $(X, \tau)$  is called

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- (i) an  $\omega$ -closed set [7] ( $= \widehat{g}$ -closed ) if  $Cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is semi-open in  $(X, \tau)$ ,
- (ii) a  $*g$ -closed set [9] if  $Cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\omega$ -open in  $(X, \tau)$ ,
- (iii) a  $\#g$ -semi-closed set [10](briefly  $\#gs$ -closed) if  $sCl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $*g$ -open in  $(X, \tau)$  and
- (iv) a  $\widetilde{g}_\alpha$ -closed set [4] if  $\alpha Cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\#gs$ -open in  $X$ .

The complement of  $*g$ -closed (resp  $\omega$ -closed,  $\#gs$ -closed and  $\widetilde{g}_\alpha$ -closed)set is said to be  $*g$ -open(resp  $\omega$ -open,  $\#gs$ -open and  $\widetilde{g}_\alpha$ -open)respectively.

**Definition 2.2.** A function  $f : (X, \tau) \rightarrow (Y, \sigma)$  is called

- (i)  $\widetilde{g}_\alpha$ -continuous[8] if  $f^{-1}(V)$  is  $\widetilde{g}_\alpha$ -closed in  $(X, \tau)$  for every closed set  $V$  in  $(Y, \sigma)$ .
- (ii)  $\widetilde{g}_\alpha$ -irresolute[8] if  $f^{-1}(V)$  is  $\widetilde{g}_\alpha$ -closed in  $(X, \tau)$  for every  $\widetilde{g}_\alpha$ -closed set  $V$  in  $(Y, \sigma)$

**Definition 2.3** (4). Let  $(X, \tau)$  be a topological space and  $E$  be subset of  $X$ . We define  $\widetilde{g}_\alpha$ -Interior of  $E$  denoted by  $\widetilde{g}_\alpha\text{-Int}(E)$  to be the union of all  $\widetilde{g}_\alpha$ -open sets contained in  $E$ . Similarly the  $\widetilde{g}_\alpha$ -Closure of  $E$  is defined to be the intersection of all  $\widetilde{g}_\alpha$ -closed set containing  $E$  and is denoted as  $\widetilde{g}_\alpha\text{-Cl}(E)$ . The class of all  $\widetilde{g}_\alpha$ -open sets and  $\widetilde{g}_\alpha$ -closed sets are denoted by  $\widetilde{G}_\alpha O(X)$  and  $\widetilde{G}_\alpha C(X)$  respectively.

### 3. $\widetilde{g}_\alpha$ D-sets and associated separation axioms

In this section we introduce  $\widetilde{g}_\alpha$ D-sets and  $\widetilde{g}_\alpha$ - $D_i$ -spaces.

**Definition 3.1.** A subset  $A$  of a topological space  $X$  is said to be a  $\widetilde{g}_\alpha$ D-set if there are two  $\widetilde{g}_\alpha$ -open sets  $U, V$  such that  $U \neq X$  and  $A = U - V$ .

**Remark 3.2.** Every  $\widetilde{g}_\alpha$ -open set  $U$  other than  $X$  is a  $\widetilde{g}_\alpha$ D-set if  $A = U$  and  $V = \phi$ .

**Example 3.3.** Let  $X = \{a, b, c\}$ ,  $\tau = \{\phi, \{a\}, X\}$ .  $\widetilde{G}_\alpha O(X) = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$ . Then  $\widetilde{D}_\alpha D(X) = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}\}$  where  $\widetilde{D}_\alpha D(X)$  represents the collection of all  $\widetilde{g}_\alpha$ D-sets.

**Definition 3.4.** A topological space  $(X, \tau)$  is called

- (i) a  $\widetilde{g}_\alpha$ - $D_0$ -space if for any distinct pair of points  $x$  and  $y$  of  $X$  there exist a  $\widetilde{g}_\alpha$ D-set of  $X$  containing  $x$  but not  $y$  or a  $\widetilde{g}_\alpha$ D set of  $X$  containing  $y$  but not  $x$ .
- (ii) a  $\widetilde{g}_\alpha$ - $D_1$ -space if for any distinct pair of points  $x$  and  $y$  of  $X$  there exist a  $\widetilde{g}_\alpha$ D set of  $X$  containing  $x$  but not  $y$  and a  $\widetilde{g}_\alpha$ D set of  $X$  containing  $y$  but not  $x$ .
- (iii) a  $\widetilde{g}_\alpha$ - $D_2$ -space if for any distinct pair of points  $x$  and  $y$  of  $X$  there exist disjoint  $\widetilde{g}_\alpha$ D sets  $D$  and  $E$  of  $X$  containing  $x$  and  $y$  respectively.

**Definition 3.5.** A topological space  $(X, \tau)$  is called

- (i) a  $\tilde{g}_\alpha$ - $T_0$ -space if for any distinct pair of points  $x$  and  $y$  of  $X$  there is a  $\tilde{g}_\alpha$ -open set of  $X$  containing one of the points but not the other.
- (ii) a  $\tilde{g}_\alpha$ - $T_1$ -space if for any distinct pair of points  $x$  and  $y$  of  $X$  there exist a  $\tilde{g}_\alpha$ -open set of  $X$  containing  $x$  but not  $y$  and a  $\tilde{g}_\alpha$ -open set of  $X$  containing  $y$  but not  $x$ .
- (iii) a  $\tilde{g}_\alpha$ - $T_2$ -space if for any distinct pair of points  $x$  and  $y$  of  $X$  there exist disjoint  $\tilde{g}_\alpha$ -open sets  $U$  and  $V$  of  $X$  containing  $x$  and  $y$  respectively.

**Remark 3.6.** (i) If  $(X, \tau)$  is  $\tilde{g}_\alpha$ - $T_i$  then  $(X, \tau)$  is  $\tilde{g}_\alpha$ - $D_i$  for  $i = 0, 1, 2$ .  
(ii) If  $(X, \tau)$  is  $\tilde{g}_\alpha$ - $D_i$  then  $(X, \tau)$  is  $\tilde{g}_\alpha$ - $D_{i-1}$  for  $i = 1, 2$ .

**Theorem 3.7.** For a topological space  $(X, \tau)$  the following hold.

- (i)  $(X, \tau)$  is  $\tilde{g}_\alpha$ - $D_0$  if and only if it is  $\tilde{g}_\alpha$ - $T_0$
- (ii)  $(X, \tau)$  is  $\tilde{g}_\alpha$ - $D_1$  if and only if it is  $\tilde{g}_\alpha$ - $D_2$

**Proof:** i) Necessity. Suppose that  $(X, \tau)$  is  $\tilde{g}_\alpha$ - $T_0$ . Then for any pair of points  $x, y \in X, x \neq y$  there exist a  $\tilde{g}_\alpha$ -open set of  $X$  containing any one of the points but not the other. Since any  $\tilde{g}_\alpha$ -open set is a  $\tilde{g}_\alpha$ - $D$ -set,  $(X, \tau)$  is a  $\tilde{g}_\alpha$ - $D_0$ -space.

Sufficiency. Let  $(X, \tau)$  be a  $\tilde{g}_\alpha$ - $D_0$ -space. Then for each distinct pair  $x, y \in X$ , atleast one of  $x, y$  say  $x$  belongs to a  $\tilde{g}_\alpha$ - $D$ -set  $G$  where  $y \notin G$ . Let  $G = U - V$  such that  $U \neq X$  and  $U, V \in \tilde{G}_\alpha O(X)$ . Then  $x \in U$ . For  $y \notin G$  we have two cases:

- a)  $y \notin U$
- b)  $y \in U$  and  $y \in V$ . In case (a)  $x \in U$  but  $y \notin U$ .

In case (b)  $y \in V$  but  $x \notin V$ . Hence  $X$  is  $\tilde{g}_\alpha$ - $T_0$ .

(ii) Sufficiency. Let  $(X, \tau)$  be a  $\tilde{g}_\alpha$ - $D_2$ -space. Then for any pair of distinct points  $x, y \in X$  there exist disjoint  $\tilde{g}_\alpha$ - $D$ -sets containing  $x$  and  $y$  respectively. Hence  $X$  is  $\tilde{g}_\alpha$ - $D_1$

Necessity. Suppose that  $(X, \tau)$  is a  $\tilde{g}_\alpha$ - $D_1$ -space. Then for each pair of distinct points  $x, y \in X$  we have  $\tilde{g}_\alpha$ - $D$  sets  $G, H$  such that  $x \in G, y \notin G; y \in H, x \notin H$ . Let  $G = U_1 - U_2, H = U_3 - U_4$ , where  $U_1, U_2, U_3, U_4 \in \tilde{G}_\alpha O(X)$

By  $x \notin H$ , it follows that either  $x \notin U_3$  or  $x \in U_3$  and  $x \in U_4$ . We have two cases.

- a)  $x \notin U_3$ . By  $y \notin G$ . We have two subcases: (a<sub>1</sub>)  $y \notin U_1$ . By  $x \in U_1 - U_2$ , it follows that  $x \in U_1 - (U_2 \cup U_3)$  and by  $y \in U_3 - U_4$  we have  $y \in U_3 - (U_1 \cup U_4)$ . Hence  $U_1 - (U_2 \cup U_3) \cap U_3 - (U_1 \cup U_4) = \phi$

(a<sub>2</sub>)  $y \in U_1$  and  $y \in U_2$ . We have  $x \in (U_1 - U_2, y \in U_2)$ .  $(U_1 - U_2) \cap U_2 = \phi$ .

- b)  $x \in U_3$  and  $x \in U_4$ . We have  $y \in U_3 - U_4, x \in U_4$ .  $(U_3 - U_4) \cap U_4 = \phi$ . Therefore  $X$  is  $\tilde{g}_\alpha$ - $D_2$ . □

**Theorem 3.8.** If  $(X, \tau)$  is  $\tilde{g}_\alpha$ - $D_1$ , then it is  $\tilde{g}_\alpha$ - $T_0$

**Proof:** It follows from the Remark 3.6 and Theorem 3.7. □

**Example 3.9.** Let  $X = \{a, b, c\}$ ,  $\tau = \{\phi, \{a, b\}, X\}$ .  $\tilde{G}_\alpha O(X) = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$   
The space  $X$  is  $\tilde{g}_\alpha$ - $T_0$  but not  $\tilde{g}_\alpha$ - $D_1$ .

**Definition 3.10.** A point  $x \in X$  which has  $X$  as a  $\tilde{g}_\alpha$ -neighbourhood is called a  $\tilde{g}_\alpha$ -neat point.

**Example 3.11.** Let  $X = \{a, b, c\}$ ,  $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ .  
 $\tilde{G}_\alpha O(X) = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$   
The point  $\{c\}$  is a  $\tilde{g}_\alpha$ -neat point.

**Theorem 3.12.** For a  $\tilde{g}_\alpha$ - $T_0$  space  $(X, \tau)$  the following are equivalent.

- (i)  $(X, \tau)$  is  $\tilde{g}_\alpha$ - $D_1$ .
- (ii)  $(X, \tau)$  has no  $\tilde{g}_\alpha$ -neat point.

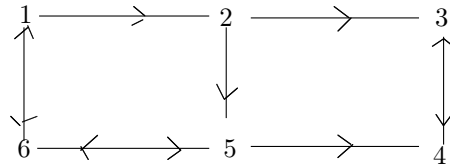
**Proof:** (i)  $\Rightarrow$  (ii). Since  $(X, \tau)$  is  $\tilde{g}_\alpha$ - $D_1$ , then each point  $x$  of  $X$  is contained in a  $\tilde{g}_\alpha D$  set  $G = U - V$  thus contained in  $U$ . By definition  $U \neq X$ . This implies that  $x$  is not a  $\tilde{g}_\alpha$ -neat point.

(ii)  $\Rightarrow$  (i). If  $(X, \tau)$  is  $\tilde{g}_\alpha$ - $T_0$ , then for each pair of points  $x, y \in X$ , atleast one of them  $x$  (say) has  $\tilde{g}_\alpha$ -neighbourhood  $U$  containing  $x$  and not  $y$ . Thus  $U$  which is different from  $X$  is a  $\tilde{g}_\alpha D$  set. If  $X$  has no  $\tilde{g}_\alpha$ -neat point, then  $y$  is not a  $\tilde{g}_\alpha$ -neat point. This means that there exists a  $\tilde{g}_\alpha$ -neighbourhood  $V$  of  $y$  such that  $V \neq X$ . Thus  $y \in (V - U)$  but not  $x$  and  $V - U$  is a  $\tilde{g}_\alpha D$ -set. Hence  $X$  is  $\tilde{g}_\alpha$ - $D_1$ .  $\square$

**Remark 3.13.** A  $\tilde{g}_\alpha$ - $T_0$  topological space is not  $\tilde{g}_\alpha$ - $D_1$  if and only if there is a unique  $\tilde{g}_\alpha$ -neat point in  $X$ . It is unique if  $x$  and  $y$  are both  $\tilde{g}_\alpha$ -neat points in  $X$  then atleast one of them say  $x$  has a  $\tilde{g}_\alpha$ -neighbourhood  $U$  containing  $x$  but not  $y$ . This is a contradiction since  $U \neq X$ .

**Remark 3.14.** From the above discussions we have the following figure which gives the relationship between the different spaces.

1.  $\tilde{g}_\alpha$ - $T_2$  2.  $\tilde{g}_\alpha$ - $T_1$  3.  $\tilde{g}_\alpha$ - $T_0$  4.  $\tilde{g}_\alpha$ - $D_0$  5.  $\tilde{g}_\alpha$ - $D_1$  6.  $\tilde{g}_\alpha$ - $D_2$



#### 4. $\tilde{g}_\alpha$ - $R_i$ -spaces

**Definition 4.1.** For a point  $x$  of a space  $(X, \tau)$  the  $\tilde{g}_\alpha$ -Kernel of  $x$  is defined to be the intersection of all  $\tilde{g}_\alpha$ -open sets containing the point  $x$  and is denoted by  $\tilde{g}_\alpha\text{-Ker}(\{x\})$

**Definition 4.2.** For a subset  $A$  of a space  $(X, \tau)$  the  $\tilde{g}_\alpha$ -Kernel of  $A$  is defined to be the intersection of all  $\tilde{g}_\alpha$ -open sets containing  $A$  and is denoted by  $\tilde{g}_\alpha\text{-Ker}(A)$ .

**Definition 4.3.** A topological space  $(X, \tau)$  is said to be a  $\tilde{g}_\alpha$ - $R_0$  space if every  $\tilde{g}_\alpha$ -open set contains the  $\tilde{g}_\alpha$ -closure of each of its singletons.

**Lemma 4.4.** Let  $(X, \tau)$  be a topological space and  $x \in X$ . Then  $\tilde{g}_\alpha\text{-Ker}(\{A\}) = \{x \in X : \tilde{g}_\alpha\text{-Cl}(\{x\}) \cap A \neq \phi\}$

**Proof:** Let  $x \in \tilde{g}_\alpha\text{-Ker}(\{A\})$  and  $\tilde{g}_\alpha\text{-Cl}(\{x\}) \cap A = \phi$ . Hence  $x \notin X - \tilde{g}_\alpha\text{-Cl}(\{x\})$  which is a  $\tilde{g}_\alpha$ -open set containing  $A$ . This is not possible since  $x \in \tilde{g}_\alpha\text{-Ker}(\{A\})$ . Hence  $\tilde{g}_\alpha\text{-Cl}(\{x\}) \cap A \neq \phi$ . Let  $\tilde{g}_\alpha\text{-Cl}(\{x\}) \cap A \neq \phi$  and  $x \notin \tilde{g}_\alpha\text{-Ker}(\{A\})$ . Then there exist a  $\tilde{g}_\alpha$ -open set  $U$  containing  $A$  and  $x \notin U$ . Let  $y \in \tilde{g}_\alpha\text{-Cl}(\{x\}) \cap A$ . Hence  $U$  is a  $\tilde{g}_\alpha$  neighbourhood of  $y$  with  $x \notin U$  which is a contradiction. Hence  $x \in \tilde{g}_\alpha\text{-Ker}(\{A\})$ . Hence the result.  $\square$

**Lemma 4.5.** Let  $(X, \tau)$  be a topological space and  $x \in X$ . Then  $y \in \tilde{g}_\alpha\text{-Ker}(\{x\})$  if and only if  $x \in \tilde{g}_\alpha\text{-Cl}(\{y\})$

**Proof:** Suppose that  $y \notin \tilde{g}_\alpha\text{-Ker}(\{x\})$ . Then there exists a  $\tilde{g}_\alpha$ -open set  $V$  containing  $x$  but not  $y$ . Therefore we have  $x \notin \tilde{g}_\alpha\text{-Cl}(\{y\})$ . Conversely suppose that  $x \notin \tilde{g}_\alpha\text{-Cl}(\{y\})$ . Then there exists a  $\tilde{g}_\alpha$ -closed set  $F$  containing  $y$  but not  $x$ . Therefore we have  $y \notin \tilde{g}_\alpha\text{-Ker}(\{x\})$ .  $\square$

**Lemma 4.6.** For any points  $x$  and  $y$  in a topological space  $(X, \tau)$  the following are equivalent.

$$(i) \tilde{g}_\alpha\text{-Ker}(\{x\}) \neq \tilde{g}_\alpha\text{-Ker}(\{y\}).$$

$$(ii) \tilde{g}_\alpha\text{-Cl}(\{x\}) \neq \tilde{g}_\alpha\text{-Cl}(\{y\}).$$

**Proof:** (i)  $\Rightarrow$  (ii). Suppose that  $\tilde{g}_\alpha\text{-Ker}(\{x\}) \neq \tilde{g}_\alpha\text{-Ker}(\{y\})$  then there exists a point  $z$  in  $X$  such that  $z \in \tilde{g}_\alpha\text{-Ker}(\{x\})$  and  $z \notin \tilde{g}_\alpha\text{-Ker}(\{y\})$ . It follows from  $z \in \tilde{g}_\alpha\text{-Ker}(\{x\})$  that  $\{x\} \cap \tilde{g}_\alpha\text{-Cl}(\{z\}) \neq \phi$ . This implies that  $x \in \tilde{g}_\alpha\text{-Cl}(\{z\})$ . By  $z \notin \tilde{g}_\alpha\text{-Ker}(\{y\})$ , we have  $\{y\} \cap \tilde{g}_\alpha\text{-Cl}(\{z\}) = \phi$ . Since  $x \in \tilde{g}_\alpha\text{-Cl}(\{z\})$ ,  $\tilde{g}_\alpha\text{-Cl}(\{x\}) \subseteq \tilde{g}_\alpha\text{-Cl}(\{z\})$  and  $\{y\} \cap \tilde{g}_\alpha\text{-Cl}(\{x\}) = \phi$ . Therefore  $\tilde{g}_\alpha\text{-Cl}(\{x\}) \neq \tilde{g}_\alpha\text{-Cl}(\{y\})$ .

(ii)  $\Rightarrow$  (i) Suppose that  $\tilde{g}_\alpha\text{-Cl}(\{x\}) \neq \tilde{g}_\alpha\text{-Cl}(\{y\})$ . Then there exist a point  $z \in X$  such that  $z \in \tilde{g}_\alpha\text{-Cl}(\{x\})$  and  $z \notin \tilde{g}_\alpha\text{-Cl}(\{y\})$ . Then there exist a  $\tilde{g}_\alpha$ -open set containing  $z$  and therefore  $x$  but not  $y$ . i.e  $y \notin \tilde{g}_\alpha\text{-Ker}(\{x\})$ . Hence  $\tilde{g}_\alpha\text{-Ker}(\{x\}) \neq \tilde{g}_\alpha\text{-Ker}(\{y\})$ .  $\square$

**Theorem 4.7.** *A topological space  $(X, \tau)$  is a  $\tilde{g}_\alpha$ - $R_0$  space if and only if for any  $x$  and  $y$  in  $X$ ,  $\tilde{g}_\alpha$ - $Cl(\{x\}) \neq \tilde{g}_\alpha$ - $Cl(\{y\})$  implies  $\tilde{g}_\alpha$ - $Cl(\{x\}) \cap \tilde{g}_\alpha$ - $Cl(\{y\}) = \phi$*

**Proof:** Necessity. Suppose that  $(X, \tau)$  is  $\tilde{g}_\alpha$ - $R_0$  and  $x, y \in X$  such that  $\tilde{g}_\alpha$ - $Cl(\{x\}) \neq \tilde{g}_\alpha$ - $Cl(\{y\})$ . Then there exist  $z \in \tilde{g}_\alpha$ - $Cl(\{x\})$  such that  $z \notin \tilde{g}_\alpha$ - $Cl(\{y\})$  (or  $z \in \tilde{g}_\alpha$ - $Cl(\{y\})$  such that  $z \notin \tilde{g}_\alpha$ - $Cl(\{x\})$ ). There exist  $V \in \tilde{G}_\alpha O(X)$  such that  $y \notin V$  and  $z \in V$ , hence  $x \in V$ . Therefore, we have  $x \notin \tilde{g}_\alpha$ - $Cl(\{y\})$ . Thus  $x \in X - \tilde{g}_\alpha$ - $Cl(\{y\}) \in \tilde{G}_\alpha O(X)$ , which implies that  $\tilde{g}_\alpha$ - $Cl(\{x\}) \subseteq X - \tilde{g}_\alpha$ - $Cl(\{y\})$  and  $\tilde{g}_\alpha$ - $Cl(\{x\}) \cap \tilde{g}_\alpha$ - $Cl(\{y\}) = \phi$ . The proof for other part is similar.

Sufficiency. Let  $V \in \tilde{G}_\alpha O(X)$  and let  $x \in V$ . We prove that  $\tilde{g}_\alpha$ - $Cl(\{x\}) \subseteq V$ . i.e  $y \notin V \implies y \in X - V$ . Then  $x \neq y$  and  $x \notin \tilde{g}_\alpha$ - $Cl(\{y\})$ . This shows that  $\tilde{g}_\alpha$ - $Cl(\{x\}) \neq \tilde{g}_\alpha$ - $Cl(\{y\})$ . By assumption  $\tilde{g}_\alpha$ - $Cl(\{x\}) \cap \tilde{g}_\alpha$ - $Cl(\{y\}) = \phi$ . Hence  $y \notin \tilde{g}_\alpha$ - $Cl(\{x\})$  and therefore  $\tilde{g}_\alpha$ - $Cl(\{x\}) \subseteq V$ . Hence  $X$  is  $\tilde{g}_\alpha$ - $R_0$ .  $\square$

**Theorem 4.8.** *A topological space  $(X, \tau)$  is a  $\tilde{g}_\alpha$ - $R_0$  space if and only if for any points  $x$  and  $y$  in  $X$ ,  $\tilde{g}_\alpha$ - $Ker(\{x\}) \neq \tilde{g}_\alpha$ - $Ker(\{y\})$  implies that  $\tilde{g}_\alpha$ - $Ker(\{x\}) \cap \tilde{g}_\alpha$ - $Ker(\{y\}) = \phi$ .*

**Proof:** Suppose that  $(X, \tau)$  is a  $\tilde{g}_\alpha$ - $R_0$  space. Then by Lemma 4.5 for any points  $x$  and  $y$  in  $X$  if  $\tilde{g}_\alpha$ - $Ker(\{x\}) \neq \tilde{g}_\alpha$ - $Ker(\{y\})$  then  $\tilde{g}_\alpha$ - $Cl(\{x\}) \neq \tilde{g}_\alpha$ - $Cl(\{y\})$ . We prove that  $\tilde{g}_\alpha$ - $Ker(\{x\}) \cap \tilde{g}_\alpha$ - $Ker(\{y\}) = \phi$ . Assume that  $z \in \tilde{g}_\alpha$ - $Ker(\{x\}) \cap \tilde{g}_\alpha$ - $Ker(\{y\})$ . By  $z \in \tilde{g}_\alpha$ - $Ker(\{x\})$  and Lemma 4.6 it follows that  $x \in \tilde{g}_\alpha$ - $Cl(\{z\})$ . By Theorem 4.7  $\tilde{g}_\alpha$ - $Cl(\{x\}) = \tilde{g}_\alpha$ - $Cl(\{z\})$ . Similarly we have  $\tilde{g}_\alpha$ - $Cl(\{y\}) = \tilde{g}_\alpha$ - $Cl(\{z\}) = \tilde{g}_\alpha$ - $Cl(\{x\})$ . This is a contradiction. Therefore we have  $\tilde{g}_\alpha$ - $Ker(\{x\}) \cap \tilde{g}_\alpha$ - $Ker(\{y\}) = \phi$ .

Conversely let  $X$  be a space such that for any points  $x$  and  $y$  in  $X$ ,  $\tilde{g}_\alpha$ - $Ker(\{x\}) \neq \tilde{g}_\alpha$ - $Ker(\{y\})$  implies  $\tilde{g}_\alpha$ - $Ker(\{x\}) \cap \tilde{g}_\alpha$ - $Ker(\{y\}) = \phi$ . If  $\tilde{g}_\alpha$ - $Cl(\{x\}) \neq \tilde{g}_\alpha$ - $Cl(\{y\})$ , then by Lemma 4.6  $\tilde{g}_\alpha$ - $Ker(\{x\}) \neq \tilde{g}_\alpha$ - $Ker(\{y\})$ . Hence  $\tilde{g}_\alpha$ - $Ker(\{x\}) \cap \tilde{g}_\alpha$ - $Ker(\{y\}) = \phi$  implies  $\tilde{g}_\alpha$ - $Cl(\{x\}) \cap \tilde{g}_\alpha$ - $Cl(\{y\}) = \phi$ . Because  $z \in \tilde{g}_\alpha$ - $Cl(\{x\})$  implies that  $x \in \tilde{g}_\alpha$ - $Ker(\{z\})$ . Therefore  $\tilde{g}_\alpha$ - $Ker(\{x\}) \cap \tilde{g}_\alpha$ - $Ker(\{z\}) \neq \phi$ . By hypothesis, we have  $\tilde{g}_\alpha$ - $Ker(\{x\}) = \tilde{g}_\alpha$ - $Ker(\{z\})$ . Then  $z \in \tilde{g}_\alpha$ - $Cl(\{x\}) \cap \tilde{g}_\alpha$ - $Cl(\{y\})$  implies that  $\tilde{g}_\alpha$ - $Ker(\{x\}) = \tilde{g}_\alpha$ - $Ker(\{z\}) = \tilde{g}_\alpha$ - $Ker(\{y\})$ . This is a contradiction. Hence  $\tilde{g}_\alpha$ - $Cl(\{x\}) \cap \tilde{g}_\alpha$ - $Cl(\{y\}) = \phi$ . By Theorem 4.7  $(X, \tau)$  is a  $\tilde{g}_\alpha$ - $R_0$  space.  $\square$

**Theorem 4.9.** *For a topological space  $(X, \tau)$  the following are equivalent.*

(i)  $(X, \tau)$  is a  $\tilde{g}_\alpha$ - $R_0$  space.

(ii) For any  $A \neq \phi$  and  $G \in \tilde{G}_\alpha O(X)$  such that  $A \cap G \neq \phi$  there exists  $F \in \tilde{G}_\alpha C(X)$  such that  $A \cap F \neq \phi$  and  $F \subseteq G$ .

(iii) Any  $G \in \tilde{G}_\alpha O(X)$ ,  $G = \bigcup \{F \in \tilde{G}_\alpha C(X) : F \subseteq G\}$ .

(iv) Any  $F \in \tilde{G}_\alpha C(X)$ ,  $F = \bigcap \{G \in \tilde{G}_\alpha O(X) : F \subseteq G\}$ .

(v) For any  $x \in X, \tilde{g}_\alpha\text{-Cl}(\{x\}) \subseteq \tilde{g}_\alpha\text{-Ker}(\{y\})$ .

**Proof:** (i)  $\Rightarrow$  (ii) Let  $A$  be a nonempty set of  $X$  and  $G \in \tilde{G}_\alpha O(X)$  such that  $A \cap G \neq \phi$ . There exists  $x \in A \cap G$ . Since  $x \in G \in \tilde{G}_\alpha O(X), \tilde{g}_\alpha\text{-Cl}\{x\} \subseteq G$ . Let  $F = \tilde{g}_\alpha\text{-Cl}(\{x\})$ . Then  $F \in \tilde{G}_\alpha C(X), F \subseteq G$ , and  $A \cap F \neq \phi$ .

(ii)  $\Rightarrow$  (iii). Let  $G \in \tilde{G}_\alpha O(X)$ . Then  $G \supseteq \bigcup\{F \in \tilde{G}_\alpha C(X) : F \subseteq G\}$ . Let  $x$  be any point of  $G$ . There exists  $F \in \tilde{G}_\alpha C(X)$  such that  $x \in F$  and  $F \subseteq G$ . Therefore we have  $x \in F \subseteq \bigcup\{F \in \tilde{G}_\alpha C(X) : F \subseteq G\}$  and hence  $G = \bigcup\{F \in \tilde{G}_\alpha C(X) : F \subseteq G\}$ .

(iii)  $\Rightarrow$  (iv) Let  $F \in \tilde{G}_\alpha C(X)$  then  $F^c \in \tilde{G}_\alpha O(X)$  By (iii)  $F^c = \bigcup\{H \in \tilde{G}_\alpha C(X) : H \subseteq F^c\}$  then  $F = (\bigcup\{H \in \tilde{G}_\alpha C(X) : H \subseteq F^c\})^c = \bigcap\{H^c \in \tilde{G}_\alpha O(X) : H^c \supseteq F\} = \bigcap\{G \in \tilde{G}_\alpha O(X) : F \subseteq G\}$ .

(iv)  $\Rightarrow$  (v). Let  $x$  be any point of  $X$  and  $y \notin \tilde{g}_\alpha\text{-Ker}(\{x\})$ . There exists  $V \in \tilde{G}_\alpha O(X)$  such that  $x \in V$  and  $y \notin V$ . Hence  $\tilde{g}_\alpha\text{-Cl}(\{y\}) \cap V = \phi$ . By (iv)  $(\bigcap\{G \in \tilde{G}_\alpha O(X) : \tilde{g}_\alpha\text{-Cl}(\{y\}) \subseteq G\}) \cap V = \phi$ . There exists  $G \in \tilde{G}_\alpha O(X)$  such that  $x \notin G$  and  $\tilde{g}_\alpha\text{-Cl}(\{y\}) \subseteq G$ . Therefore  $\tilde{g}_\alpha\text{-Cl}(\{x\}) \cap G = \phi$  and  $y \notin \tilde{g}_\alpha\text{-Cl}(\{x\})$ . Consequently we get  $\tilde{g}_\alpha\text{-Cl}(\{x\}) \subseteq \tilde{g}_\alpha\text{-Ker}(\{x\})$ .

(v)  $\Rightarrow$  (i). Let  $G \in \tilde{G}_\alpha O(X)$  and  $x \in G$ . Suppose  $y \in \tilde{g}_\alpha\text{-Ker}(\{x\})$ , then  $x \in \tilde{g}_\alpha\text{-Cl}(\{y\})$  and  $y \in G$ . This implies that  $\tilde{g}_\alpha\text{-Cl}(\{x\}) \subseteq \tilde{g}_\alpha\text{-Ker}(\{x\}) \subseteq G$ . Therefore  $(X, \tau)$  is a  $\tilde{g}_\alpha\text{-R}_0$ -space.  $\square$

**Theorem 4.10.** For a topological space  $(X, \tau)$  the following are equivalent.

(i)  $(X, \tau)$  is a  $\tilde{g}_\alpha\text{-R}_0$  space.

(ii)  $x \in \tilde{g}_\alpha\text{-Cl}\{y\}$  if and only if  $y \in \tilde{g}_\alpha\text{-Cl}(\{x\})$ , for any points  $x$  and  $y$  in  $X$ .

**Proof:** (i)  $\Rightarrow$  (ii). Assume that  $X$  is  $\tilde{g}_\alpha\text{-R}_0$ . Let  $x \in \tilde{g}_\alpha\text{-Cl}\{y\}$  and  $D$  be any  $\tilde{g}_\alpha$ -open set such that  $y \in D$ . By hypothesis  $x \in D$ . Therefore every  $\tilde{g}_\alpha$ -open set containing  $y$  contains  $x$ . Hence  $y \in \tilde{g}_\alpha\text{-Cl}\{x\}$ .

(ii)  $\Rightarrow$  (i). Let  $U$  be a  $\tilde{g}_\alpha$ -open set and  $x \in U$ . If  $y \notin U$  then  $x \notin \tilde{g}_\alpha\text{-Cl}\{y\}$  and hence  $y \notin \tilde{g}_\alpha\text{-Cl}(\{x\})$ . This implies that  $\tilde{g}_\alpha\text{-Cl}\{x\} \subseteq U$ . Hence  $(X, \tau)$  is a  $\tilde{g}_\alpha\text{-R}_0$  space.  $\square$

**Theorem 4.11.** For a topological space  $(X, \tau)$  the following are equivalent.

(i)  $(X, \tau)$  is a  $\tilde{g}_\alpha\text{-R}_0$  space.

(ii) If  $F$  is  $\tilde{g}_\alpha$ -closed then  $F = \tilde{g}_\alpha\text{-Ker}(F)$ .

(iii) If  $F$  is  $\tilde{g}_\alpha$ -closed and  $x \in F$  then  $\tilde{g}_\alpha\text{-Ker}(\{x\}) \subseteq F$ .

(iv) If  $x \in X$  then  $\tilde{g}_\alpha\text{-Ker}(\{x\}) \subseteq \tilde{g}_\alpha\text{-Cl}(\{x\})$ .

**Proof:** (i)  $\Rightarrow$  (ii) Let  $F$  be a  $\tilde{g}_\alpha$ -closed set and  $x \notin F$ . Thus  $X - F$  is  $\tilde{g}_\alpha$ -open and contains  $x$ . Since  $(X, \tau)$  is a  $\tilde{g}_\alpha$ - $R_0$  space,  $\tilde{g}_\alpha\text{-Cl}(\{x\}) \subseteq X - F$ . Thus  $\tilde{g}_\alpha\text{-Cl}(\{x\}) \cap F = \phi$  and by Lemma 4.4  $x \notin \tilde{g}_\alpha\text{-Ker}(F)$ . Therefore  $\tilde{g}_\alpha\text{-Ker}(F) = F$ .  
(ii)  $\Rightarrow$  (iii). If  $A \subseteq B$  implies  $\tilde{g}_\alpha\text{-Ker}(A) \subseteq \tilde{g}_\alpha\text{-Ker}(B)$ . Therefore by (ii) it follows that  $\tilde{g}_\alpha\text{-Ker}(\{x\}) \subseteq \tilde{g}_\alpha\text{-Ker}(F) = F$ .  
(iii)  $\Rightarrow$  (iv). Since  $x \in \tilde{g}_\alpha\text{-Cl}(\{x\})$  and  $\tilde{g}_\alpha\text{-Cl}(\{x\})$  is  $\tilde{g}_\alpha$ -closed by (iii)  $\tilde{g}_\alpha\text{-Ker}(\{x\}) \subseteq \tilde{g}_\alpha\text{-Cl}(\{x\})$ .  
(iv)  $\Rightarrow$  (i). We prove the result using Theorem 4.9. Let  $x \in \tilde{g}_\alpha\text{-Cl}(\{y\})$ . Then by Lemma 4.5  $y \in \tilde{g}_\alpha\text{-Ker}(\{x\})$ . Since  $x \in \tilde{g}_\alpha\text{-Cl}(\{y\})$  and  $\tilde{g}_\alpha\text{-Cl}(\{x\})$  is  $\tilde{g}_\alpha$ -closed by (iv) we get  $y \in \tilde{g}_\alpha\text{-Ker}(\{x\}) \subseteq \tilde{g}_\alpha\text{-Cl}(\{x\})$ . Therefore  $x \in \tilde{g}_\alpha\text{-Cl}(\{y\})$  implies  $y \in \tilde{g}_\alpha\text{-Cl}(\{x\})$ . Conversely let  $y \in \tilde{g}_\alpha\text{-Cl}(\{x\})$ . By Lemma 4.5  $x \in \tilde{g}_\alpha\text{-Ker}(\{y\})$ . Since  $y \in \tilde{g}_\alpha\text{-Cl}(\{x\})$  and  $\tilde{g}_\alpha\text{-Cl}(\{y\})$  is a  $\tilde{g}_\alpha$ -closed set by (iv) we get  $x \in \tilde{g}_\alpha\text{-Ker}(\{y\}) \subseteq \tilde{g}_\alpha\text{-Cl}(\{y\})$ . Thus  $y \in \tilde{g}_\alpha\text{-Cl}(\{x\})$  implies  $x \in \tilde{g}_\alpha\text{-Cl}(\{y\})$ . Therefore by theorem 4.10 we prove that  $(X, \tau)$  is a  $\tilde{g}_\alpha$ - $R_0$  space.  $\square$

**Theorem 4.12.** For a topological space  $(X, \tau)$  the following are equivalent.

- (i)  $(X, \tau)$  is  $\tilde{g}_\alpha$ - $R_0$ .
- (ii) For any  $F \in \tilde{G}_\alpha C(X)$ ,  $x \notin F$  implies  $F \subseteq U$  and  $x \notin U$  for some  $U \in \tilde{G}_\alpha O(X)$ .
- (iii) For any  $F \in \tilde{G}_\alpha C(X)$ ,  $x \notin F$  implies  $F \cap \tilde{g}_\alpha\text{-Cl}(\{x\}) = \phi$ .
- (iv) For any distinct points  $x$  and  $y$  in  $X$ , either  $\tilde{g}_\alpha\text{-Cl}(\{x\}) = \tilde{g}_\alpha\text{-Cl}(\{y\})$  or  $\tilde{g}_\alpha\text{-Cl}(\{x\}) \cap \tilde{g}_\alpha\text{-Cl}(\{y\}) = \phi$ .

**Proof:** (i)  $\Rightarrow$  (ii) Let  $F \in \tilde{G}_\alpha C(X)$ ,  $x \notin F$ . By (i)  $\tilde{g}_\alpha\text{-Cl}(\{x\}) \subseteq X - F$ . Set  $U = X - \tilde{g}_\alpha\text{-Cl}(\{x\})$  then  $U \in \tilde{G}_\alpha O(X)$ ,  $F \subseteq U$  and  $x \notin U$ .  
(ii)  $\Rightarrow$  (iii) Let  $F \in \tilde{G}_\alpha C(X)$ ,  $x \notin F$ . There exist  $U \in \tilde{G}_\alpha O(X)$  such that  $F \subseteq U$  and  $x \notin U$ . Since  $U \in \tilde{G}_\alpha O(X)$ ,  $U \cap \tilde{g}_\alpha\text{-Cl}(\{x\}) = \phi$ ,  $F \cap \tilde{g}_\alpha\text{-Cl}(\{x\}) = \phi$ .  
(iii)  $\Rightarrow$  (iv) Suppose that  $\tilde{g}_\alpha\text{-Cl}(\{x\}) \neq \tilde{g}_\alpha\text{-Cl}(\{y\})$  for distinct points  $x, y \in X$ . There exists  $z \in \tilde{g}_\alpha\text{-Cl}(\{x\})$  such that  $z \notin \tilde{g}_\alpha\text{-Cl}(\{y\})$  (or  $z \in \tilde{g}_\alpha\text{-Cl}(\{y\})$  such that  $z \notin \tilde{g}_\alpha\text{-Cl}(\{x\})$ ). There exists  $V \in \tilde{G}_\alpha O(X)$  such that  $y \notin V$  and  $z \in V$ . Hence  $x \in V$ . Therefore we have  $x \notin \tilde{g}_\alpha\text{-Cl}(\{y\})$ . By (iii) we get  $\tilde{g}_\alpha\text{-Cl}(\{x\}) \cap \tilde{g}_\alpha\text{-Cl}(\{y\}) = \phi$ . The proof of the other part is similar.  
(iv)  $\Rightarrow$  (i) Let  $V \in \tilde{G}_\alpha O(X)$  and  $x \in V$ . For each  $y \notin V$ ,  $x \neq y$ ,  $x \notin \tilde{g}_\alpha\text{-Cl}(\{y\})$ . This shows that  $\tilde{g}_\alpha\text{-Cl}(\{y\}) \neq \tilde{g}_\alpha\text{-Cl}(\{x\})$ . By (iv)  $\tilde{g}_\alpha\text{-Cl}(\{x\}) \cap \tilde{g}_\alpha\text{-Cl}(\{y\}) = \phi$  for each  $y \in X - V$  and hence  $\tilde{g}_\alpha\text{-Cl}(\{x\}) \cap (\bigcup_{y \in X - V} \tilde{g}_\alpha\text{-Cl}(\{y\})) = \phi$ . On the other hand since  $V \in \tilde{G}_\alpha O(X)$  and  $y \in X - V$ , we have  $\tilde{g}_\alpha\text{-Cl}(\{y\}) \subseteq X - V$  and  $X - V = (\bigcup_{y \in X - V} \tilde{g}_\alpha\text{-Cl}(\{y\}))$ . Therefore we get  $(X - V) \cap \tilde{g}_\alpha\text{-Cl}(\{x\}) = \phi$  and hence  $\tilde{g}_\alpha\text{-Cl}(\{x\}) \subseteq V$ . This shows that  $(X, \tau)$  is a  $\tilde{g}_\alpha$ - $R_0$ .  $\square$

**Definition 4.13.** A topological space  $(X, \tau)$  is said to be  $\tilde{g}_\alpha$ - $R_1$  if for  $x, y$  in  $X$  with  $\tilde{g}_\alpha\text{-Cl}(\{x\}) \neq \tilde{g}_\alpha\text{-Cl}(\{y\})$ , there exist disjoint  $\tilde{g}_\alpha$ -open sets  $U$  and  $V$  such that  $\tilde{g}_\alpha\text{-Cl}(\{x\})$  is a subset of  $U$  and  $\tilde{g}_\alpha\text{-Cl}(\{y\})$  is a subset of  $V$ .



**Remark 4.14.** Every  $\tilde{g}_\alpha$ - $R_1$ -space is  $\tilde{g}_\alpha$ - $R_0$ . Let  $U$  be a  $\tilde{g}_\alpha$ -open set such that  $x \in U$ . By the definition of  $\tilde{g}_\alpha$ - $R_1$ -space there exist disjoint  $\tilde{g}_\alpha$ -open sets  $U$  and  $V$  such that  $\tilde{g}_\alpha$ -Cl( $\{x\}$ )  $\subseteq U$ ,  $\tilde{g}_\alpha$ -Cl( $\{y\}$ )  $\subseteq V$ . Hence by Theorem 4.7  $X$  is  $\tilde{g}_\alpha$ - $R_0$ .

**Theorem 4.15.** A topological space  $(X, \tau)$  is  $\tilde{g}_\alpha$ - $R_1$  if and only if for  $x, y \in X$ ,  $\tilde{g}_\alpha$ -Ker( $\{x\}$ )  $\neq \tilde{g}_\alpha$ -Ker( $\{y\}$ ), there exist disjoint  $\tilde{g}_\alpha$ -open sets  $U$  and  $V$  such that  $\tilde{g}_\alpha$ -Cl( $\{x\}$ )  $\subseteq U$  and  $\tilde{g}_\alpha$ -Cl( $\{y\}$ )  $\subseteq V$ .

**Proof:** It follows from Lemma 4.5 □

## 5. Applications

**Theorem 5.1.** If  $f : (X, \tau) \rightarrow (Y, \sigma)$  is  $\tilde{g}_\alpha$ -irresolute surjective function and  $E$  is a  $\tilde{g}_\alpha$ - $D$  set in  $Y$  then the inverse image of  $E$  is a  $\tilde{g}_\alpha$ - $D$ -set in  $X$ .

**Proof:** Let  $E$  be a  $\tilde{g}_\alpha$ - $D$  set in  $Y$ . Then there are  $\tilde{g}_\alpha$ -open sets  $U$  and  $V$  in  $Y$  such that  $E = U - V$  and  $U \neq Y$ . Since  $f$  is  $\tilde{g}_\alpha$ -irresolute,  $f^{-1}(U), f^{-1}(V)$  are  $\tilde{g}_\alpha$ -open sets in  $X$ . Since  $U \neq Y$ . We have  $f^{-1}(U) \neq X$ . Hence  $f^{-1}(E) = f^{-1}(U) - f^{-1}(V)$  is a  $\tilde{g}_\alpha$ - $D$ -set. □

**Theorem 5.2.** If  $f : (X, \tau) \rightarrow (Y, \sigma)$  is  $\tilde{g}_\alpha$ -irresolute bijective and  $Y$  is  $\tilde{g}_\alpha - D_1$  then  $X$  is  $\tilde{g}_\alpha - D_1$ .

**Proof:** Suppose that  $Y$  is a  $\tilde{g}_\alpha - D_1$  space. Let  $x$  and  $y$  be any pair of distinct points in  $X$ . Since  $f$  is injective and  $Y$  is  $\tilde{g}_\alpha - D_1$ , there exist  $\tilde{g}_\alpha$ - $D$  sets  $G$  and  $H$  of  $Y$  containing  $f(x)$  and  $f(y)$  respectively such that  $f(x) \notin G$  and  $f(x) \notin H$ . By Theorem 5.1  $f^{-1}(G), f^{-1}(H)$  are  $\tilde{g}_\alpha$ - $D$ -sets in  $X$  containing  $x$  and  $y$  respectively. This implies that  $X$  is a  $\tilde{g}_\alpha - D_1$ . □

**Theorem 5.3.** A topological space  $(X, \tau)$  is  $\tilde{g}_\alpha - D_1$  if and only if for each pair of distinct points  $x, y \in X$ , there exist a  $\tilde{g}_\alpha$ -irresolute surjective function  $f : (X, \tau) \rightarrow (Y, \sigma)$  where  $Y$  is a  $\tilde{g}_\alpha$ - $D_1$  space such that  $f(x)$  and  $f(y)$  are distinct.

**Proof:** Necessity. For any pair of distinct points of  $X$  if we consider the identity function then it satisfies the required condition.

Sufficiency. Let  $x$  and  $y$  be any pair of distinct points in  $X$ . By hypothesis there exists a  $\tilde{g}_\alpha$ -irresolute, surjective function  $f$  of a space  $(X, \tau)$  onto a  $\tilde{g}_\alpha$ - $D_1$ -space  $(Y, \sigma)$  such that  $f(x) \neq f(y)$ . Therefore, there exist disjoint  $\tilde{g}_\alpha$ - $D$ -sets  $G$  and  $H$  in  $Y$  such that  $f(x) \in G$  and  $f(y) \in H$ . Since  $f$  is  $\tilde{g}_\alpha$ -irresolute and surjective by Theorem 5.1  $f^{-1}(G), f^{-1}(H)$  are disjoint  $\tilde{g}_\alpha$ - $D$ -sets in  $X$  containing  $x$  and  $y$  respectively. Therefore  $X$  is a  $\tilde{g}_\alpha$ - $D_1$  space. □

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