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Some new generalized topologies via hereditary classes

M.Rajamani, V.Inthumathi and R.Ramesh

ABSTRACT: In this paper, we introduce and study the notions of $A^*_\kappa(\mathcal{H},\mu)$ in hereditary generalized topological spaces introduced by Csaszar. Using these notions we obtain new generalized topologies $\kappa\mu^*$ via hereditary classes.

Key Words: hereditary generalized topological space, $A^*_\kappa(\mathcal{H},\mu)$-sets, $\kappa\mu^*$-topology

Contents

1. Introduction and Preliminaries

In 1990, Jankovic and Hamlett [5] obtained a new topology τ^* from the old one via ideals. In 2002, Csaszar [1], introduced the notions of generalized topology. In 2007, Csaszar [2], showed that the construction leading from a topology τ and an ideal of sets to another topology remains valid, if topology is replaced by generalized topology and ideal by hereditary classes and he constructed the generalized topology μ^*. In this paper we continue the study of construction of such generalized topologies by using μ-semi-open, μ-α-open, μ-pre-open and μ-β-open sets via hereditary classes.

Let X be any nonempty set and μ be a GT [1] of subsets of X. A nonempty family $\mathcal{H}$ of subsets of X is said to be a *hereditary class* [2], if $A \in \mathcal{H}$ and $B \subset A$, then $B \in \mathcal{H}$. A generalized topological space (X,μ) with a hereditary class $\mathcal{H}$ is hereditary generalized topological space and denoted by $(X,\mu,\mathcal{H})$. For each $A \subseteq X$, $A^*(\mathcal{H},\mu)$=$\{x \in X : A \cap V \notin \mathcal{H}$ for every $V \in \mu$ such that $x \in V\}$ [2]. For $A \subset X$, define $c^*_\mu(A) = A \cup A^*(\mathcal{H},\mu)$ and $\mu^* = \{A \subset X : X - A = c^*_\mu(X - A)\}$. A subset A of (X,μ) is *μ-α-open* [3] (resp.*μ-semiopen* [3], *μ-preopen* [3], *μ-β-open* [3]), if $A \subset i_\mu c_\mu i_\mu(A)$ (resp.$A \subset c_\mu i_\mu(A), A \subset i_\mu c_\mu(A), A \subset c_\mu i_\mu c_\mu(A)$). We denote the family of all μ-α-open sets, μ-semiopen sets, μ-preopen sets and μ-β-open sets by $\alpha(\mu)$, $\sigma(\mu)$, $\pi(\mu)$ and $\beta(\mu)$. On GT, $\mu \subset \alpha(\mu) \subset \sigma(\mu) \subset \beta(\mu)$ and $\alpha(\mu) \subset \pi(\mu) \subset \beta(\mu)$ [3].

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2. $A^*_\kappa(\mathcal{H},\mu)$ sets via hereditary classes

Definition 2.1 *Let A be a subset of hereditary generalized topological space $(X,\mu,\mathcal{H})$ and $\kappa \in \{\alpha(\mu),\sigma(\mu),\pi(\mu),\beta(\mu)\}$. Then $A^*_\kappa(\mathcal{H},\mu) = \{x \in X : A\cap U \notin \mathcal{H}$ for every $U \in \kappa\}$.*

When there is no ambiguity, We simply write A^*, A^*_α, A^*_σ, A^*_π and A^*_β for $A^*(\mathcal{H},\mu)$, $A^*_\alpha(\mathcal{H},\mu)$, $A^*_\sigma(\mathcal{H},\mu)$, $A^*_\pi(\mathcal{H},\mu)$ and $A^*_\beta(\mathcal{H},\mu)$, respectively.

Proposition 2.2 *Let A be a subset of hereditary generalized topological space $(X,\mu,\mathcal{H})$. Then $A^*_\beta(\mathcal{H},\mu) \subseteq A^*_\pi(\mathcal{H},\mu) \subseteq A^*_\alpha(\mathcal{H},\mu) \subseteq A^*(\mathcal{H},\mu)$.*

The converse of the assertions of Proposition 2.2 need not be true as shown in the following example.

Example 2.3 *Let $X = \{a,b,c,d,e\}$, $\mu = \{\emptyset,\{a\},\{b\},\{a,b\},\{b,d,e\},\{a,b,d,e\},\{a,c,d,e\},X\}$ and $\mathcal{H} = \{\emptyset,\{c\}\}$.*

1. *Let $A = \{c,e\}$. Then $A^* = \{c,d,e\} \nsubseteq \{e\} = A^*_\alpha$.*
2. *Let $A = \{a,d\}$. Then $A^*_\alpha = \{a,c,d,e\} \nsubseteq \{a,c,d\} = A^*_\pi$.*
3. *Let $A = \{a\}$. Then $A^*_\pi = \{a,c\} \nsubseteq \{a\} = A^*_\beta$.*

Proposition 2.4 *Let A be a subset of hereditary generalized topological space $(X,\mu,\mathcal{H})$. Then $A^*_\beta(\mathcal{H},\mu) \subseteq A^*_\sigma(\mathcal{H},\mu) \subseteq A^*_\alpha(\mathcal{H},\mu) \subseteq A^*(\mathcal{H},\mu)$.*

The converse of the assertions of Proposition 2.4 need not be true as shown in the following example.

Example 2.5 *Let $X = \{a,b,c,d,e\}$, $\mu = \{\emptyset,\{a\},\{c\},\{a,b\},\{a,c\},\{a,b,c\},\{a,c,d,e\},\{b,c,d,e\},X\}$ and $\mathcal{H} = \{\emptyset,\{a\},\{b\}\}$.*

1. *Let $A = \{a,b,c\}$. Then $A^* = \{b,c,d,e\} \nsubseteq \{b,c\} = A^*_\beta$.*
2. *Let $A = \{d\}$. Then $A^* = \{d,e\} \nsubseteq \{d\} = A^*_\alpha$.*
3. *Let $A = \{c\}$. Then $A^*_\alpha = \{c,d,e\} \nsubseteq \{c\} = A^*_\sigma$.*

Lemma 2.6 *Let $(X,\mu,\mathcal{H})$ be a hereditary generalized topological space, $\kappa \in \{\alpha(\mu),\sigma(\mu),\pi(\mu),\beta(\mu)\}$, $\kappa\cap A \in \mathcal{H}$ imply $\kappa \cap A^*_\kappa = \emptyset$. Hence $H^*_\kappa = X-\kappa$, if $H \in \mathcal{H}$.*

Proof: Let $x \in \kappa \cap A^*_\kappa$, implies $\kappa \cap A \notin \mathcal{H}$. Now $H \in \mathcal{H}$ implies $\kappa\cap H \in \mathcal{H}$ for $x \notin H^*_\kappa$, thus $H^*_\kappa \subset X-\kappa$. On the other hand $X-\kappa \subset H^*_\kappa$. Hence $H^*_\kappa = X-\kappa$. □

Definition 2.7 *Let $(X,\mu,\mathcal{H})$ be a hereditary generalized topological space, let $x \in X$ and $\kappa \in \{\alpha(\mu),\sigma(\mu),\pi(\mu),\beta(\mu)\}$. Then a subset $\mathcal{N}$ of X is said to be κ- neighbourhood of x, if there exists a κ-open set U such that $x \in U \subseteq \mathcal{N}$. Set of all κ-neighbourhoods of x is denoted by $\kappa\mathcal{N}(x)$.*

Proposition 2.8 *Let A be a subset of hereditary generalized topological space $(X, \mu, \mathcal{H})$ and $\kappa \in \{\alpha(\mu), \sigma(\mu), \pi(\mu), \beta(\mu)\}$. Then $A^*_\kappa \subset c_\kappa(A)$.*

Proof: Let $x \notin c_\kappa(A)$. Then there exists a $U \in \kappa\mathcal{N}(x)$ such that $U \cap A = \emptyset \in \mathcal{H}$, so that $x \notin A^*_\kappa$. Hence $A^*_\kappa \subset c_\kappa(A)$. □

Proposition 2.9 *Let $\kappa \in \{\alpha(\mu), \sigma(\mu), \pi(\mu), \beta(\mu)\}$ and A be a κ-closed subset of hereditary generalized topological space $(X, \mu, \mathcal{H})$. Then $A^*_\kappa \subset A$.*

Proof: Let A be κ-closed. By Proposition 2.8 we have $A^*_\kappa \subset c_\kappa(A) = A$. □

Remark 2.10 *Let A be a subset of hereditary generalized topological space $(X, \mu, \mathcal{H})$ and $\kappa \in \{\alpha(\mu), \sigma(\mu), \pi(\mu), \beta(\mu)\}$. Then*

1. $A^*_\kappa(\mathcal{H}, \mu) = A^*(\mathcal{H}, \mu)$, *if* $\kappa = \mu$.
2. *If* $\mathcal{H} = \emptyset$, *then* $A^*_\kappa(\mathcal{H}, \mu) = c_\kappa(A)$.
3. *If* $\mathcal{H} = P(X)$, *then* $A^*_\kappa(\mathcal{H}, \mu) = \emptyset$.
4. *If* $A \in \mathcal{H}$, *then* $A^*_\kappa = \emptyset$.

Theorem 2.11 *If A and B are subsets of hereditary generalized topological space $(X, \mu, \mathcal{H})$ and $\kappa \in \{\alpha(\mu), \sigma(\mu), \pi(\mu), \beta(\mu)\}$. Then*

1. $A \subseteq B$, *then* $A^*_\kappa \subseteq B^*_\kappa$.
2. $A^*_\kappa = c_\kappa(A^*_\kappa) \subseteq c_\kappa(A)$ *and* A^*_κ *is* κ*-closed set in* $(X, \mu, \mathcal{H})$.
3. $(A^*_\kappa)^*_\kappa \subseteq A^*_\kappa$.
4. $(A \cup B)^*_\kappa = A^*_\kappa \cup B^*_\kappa$.
5. $A^*_\kappa - B^*_\kappa = (A - B)^*_\kappa - B^*_\kappa \subseteq (A - B)^*_\kappa$.
6. *If* $U \in \mu$, *then* $U \cap A^*_\kappa = U \cap (U \cap A)^*_\kappa \subseteq (U \cap A)^*_\kappa$.
7. *If* $H_0 \in \mathcal{H}$, *then* $(A \cup H_0)^*_\kappa = A^*_\kappa = (A - H_0)^*_\kappa$.
8. *If* $\mathcal{H}_1 \subseteq \mathcal{H}_2$, *then* $A^*_\kappa(\mathcal{H}_2) \subseteq A^*_\kappa(\mathcal{H}_1)$.
9. $A^*_\kappa(\mathcal{H}_1 \cap \mathcal{H}_2) = A^*_\kappa(\mathcal{H}_1) \cup A^*_\kappa(\mathcal{H}_2)$.

Proof: 1. Suppose that $A \subseteq B$ and $x \notin B^*_\kappa$. Then there exists a $U \in \kappa\mathcal{N}(x)$ such that $U \cap B \in \mathcal{H}$. Since $A \subseteq B$, $U \cap A \subseteq U \cap B \in \mathcal{H}$ and $x \notin A^*_\kappa$. Hence $A^*_\kappa \subseteq B^*_\kappa$.
2. We know that $A^*_\kappa \subseteq c_\kappa(A^*_\kappa)$, Let $x \notin A^*_\kappa$. Then there exists a $U \in \kappa\mathcal{N}(x)$ such that $U \cap A \in \mathcal{H}$. By Lemma 2.6, $\kappa \cap A^*_\kappa = \emptyset$ involving $x \notin c_\kappa(A^*_\kappa)$, $c_\kappa(A^*_\kappa) \subset A^*_\kappa \subset c_\kappa(A^*_\kappa)$ and $A^*_\kappa = c_\kappa(A^*_\kappa)$.
3. By 2, A^*_κ is κ-closed and by Proposition 2.9, $(A^*_\kappa)^*_\kappa \subseteq A^*_\kappa$.

4. Let $A, B \subseteq A \cup B$. By (1), we have $A^*_\kappa \subseteq (A \cup B)^*_\kappa$ and $B^*_\kappa \subseteq (A \cup B)^*_\kappa$. Therefore, $A^*_\kappa \cup B^*_\kappa \subseteq (A \cup B)^*_\kappa$, let $x \in (A \cup B)^*_\kappa$. Then for every $U \in \kappa\mathcal{N}(x)$, $(U \cap A) \cup (U \cap B) = U \cap (A \cup B) \notin \mathcal{H}$. Therefore, $U \cap A \notin \mathcal{H}$ or $U \cap B \notin \mathcal{H}$. This implies that $x \in A^*_\kappa$ or $x \in B^*_\kappa$, that is $x \in A^*_\kappa \cup B^*_\kappa$. Therefore, we have $(A \cup B)^*_\kappa \subseteq A^*_\kappa \cup B^*_\kappa$. Consequently, we obtain $(A \cup B)^*_\kappa = A^*_\kappa \cup B^*_\kappa$.
5. Since $A = (A - B) \cup (B \cap A)$, by (4) we have $A^*_\kappa = (A - B)^*_\kappa \cup (B \cap A)^*_\kappa$ and hence $A^*_\kappa - B^*_\kappa = A^*_\kappa \cap (X - B^*_\kappa) = [(A - B)^*_\kappa \cup (B \cap A)^*_\kappa] \cap (X - B^*_\kappa) = [(A - B)^*_\kappa \cap (X - B^*_\kappa)] \cup [(B \cap A)^*_\kappa \cap (X - B^*_\kappa)] = [(A - B)^*_\kappa - B^*_\kappa] \cup \emptyset \subseteq (A - B)^*_\kappa$.
6. Suppose that $U \in \mu$ and $x \in U \cap A^*_\kappa$. Then $x \in U$ and $x \in A^*_\kappa$. Let V be any κ-open set containing x. Then $V \cap U \in \kappa\mathcal{N}(x)$ and $V \cap (U \cap A) = (V \cap U) \cap A \notin \mathcal{H}$. This shows that $x \in (U \cap A)^*_\kappa$ and hence we obtain $U \cap A^*_\kappa \subseteq (U \cap A)^*_\kappa$. Moreover, $U \cap A \subseteq A$ and by (1), $(U \cap A)^*_\kappa \subseteq A^*_\kappa$ and $U \cap (U \cap A)^*_\kappa \subseteq U \cap A^*_\kappa$. Therefore, $U \cap A^*_\kappa = U \cap (U \cap A)^*_\kappa$.
7. Since $\mathcal{H}_0 \in \mathcal{H}$, $(A \cup \mathcal{H}_0)^*_\kappa = A^*_\kappa \cup (\mathcal{H}_0)^*_\kappa = A^*_\kappa$. Also $A \cup \mathcal{H}_0 = (A - \mathcal{H}_0) \cup \mathcal{H}_0$ implies that $(A \cup \mathcal{H}_0)^*_\kappa = (A - \mathcal{H}_0)^*_\kappa \cup (\mathcal{H}_0)^*_\kappa = (A - \mathcal{H}_0)^*_\kappa$.
8. Let $\mathcal{H}_1 \subseteq \mathcal{H}_2$ and $x \in A^*_\kappa(\mathcal{H}_2)$. Then $U \cap A \notin \mathcal{H}_2$, for every $U \in \kappa\mathcal{N}(x)$. This implies that $U \cap A \notin \mathcal{H}_1$, for every $U \in \kappa\mathcal{N}(x)$. Hence $A^*_\kappa(\mathcal{H}_2) \subseteq A^*_\kappa(\mathcal{H}_1)$.
9. Let $x \notin (A^*_\kappa(\mathcal{H}_1) \cup A^*_\kappa(\mathcal{H}_2))$. Then $x \notin$ both $A^*_\kappa(\mathcal{H}_1)$ and $A^*_\kappa(\mathcal{H}_2)$. Now, $x \notin A^*_\kappa(\mathcal{H}_1)$ implies that there exist atleast one $U \in \kappa\mathcal{N}(x)$ such that $U \cap A \in \mathcal{H}_1$. Again $x \notin A^*_\kappa(\mathcal{H}_2)$ implies that there exists atleast one $U \in \kappa\mathcal{N}(x)$ such that $U \cap A \in \mathcal{H}_2$. Therefore, there exists atleast one $U \in \kappa\mathcal{N}(x)$ such that $U \cap A \in \mathcal{H}_1 \cap \mathcal{H}_2$, which implies that $x \notin A^*_\kappa(\mathcal{H}_1 \cap \mathcal{H}_2)$. Thus, $(A^*_\kappa(\mathcal{H}_1) \cup A^*_\kappa(\mathcal{H}_2)) \subseteq A^*_\kappa(\mathcal{H}_1 \cap \mathcal{H}_2)$. Let $x \in A^*_\kappa(\mathcal{H}_1 \cap \mathcal{H}_2)$. Then for each $U \in \kappa\mathcal{N}(x)$, $U \cap A \notin (\mathcal{H}_1 \cap \mathcal{H}_2)$. Since $U \cap A \notin \mathcal{H}_1 \cap \mathcal{H}_2$, either $U \cap A \notin \mathcal{H}_1$ or $U \cap A \notin \mathcal{H}_2$. If $U \cap A \notin \mathcal{H}_1$, then $x \in A^*_\kappa(\mathcal{H}_1)$, otherwise, $x \in A^*_\kappa(\mathcal{H}_2)$. Thus $x \in A^*_\kappa(\mathcal{H}_1) \cup A^*_\kappa(\mathcal{H}_2)$. Therefore, $A^*_\kappa(\mathcal{H}_1 \cap \mathcal{H}_2) \subseteq A^*_\kappa(\mathcal{H}_1) \cup A^*_\kappa(\mathcal{H}_2)$. Consequently, we have $A^*_\kappa(\mathcal{H}_1 \cap \mathcal{H}_2) = A^*_\kappa(\mathcal{H}_1) \cup A^*_\kappa(\mathcal{H}_2))$.

□

Corollary 2.12 *Let A be a subset of hereditary generalized topological space $(X, \kappa, \mathcal{H})$ and $\kappa \in \{\alpha(\mu), \sigma(\mu), \pi(\mu), \beta(\mu)\}$. Then $(A^*_\kappa)^*_\kappa \subseteq A^*_\kappa = c_\kappa(A^*_\kappa) \subseteq c_\kappa(A)$.*

3. Some new generalized topologies via hereditary classes

Let $(X, \mu, \mathcal{H})$ be a hereditary generalized topological space. For $A \subset X$, define $c^*_\kappa A = A \cup A^*_\kappa$, $\kappa \in \{\alpha(\mu), \sigma(\mu), \pi(\mu), \beta(\mu)\}$. According to [2], Corollary 3.4, c^*_κ is enlarging, monotone and idempotent. Thus according to [3], Lemma 1.4, there exist generalized topologies $\kappa\mu^*$, where $\kappa \in \{\alpha(\mu), \sigma(\mu), \pi(\mu), \beta(\mu)\}$ such that $c^*_\kappa A$ is the intersection of all $\kappa\mu^*$-closed super sets of M; $M \in \kappa\mu^*$ iff $X - M = c^*_\kappa(X - M)$. Also $c^*_\kappa(A) = A \cup c_\kappa(A^*_\kappa) \subseteq c_\kappa(A)$.

Definition 3.1 *A subset A of a hereditary generalized topological space $(X, \mu, \mathcal{H})$ is said to be $\kappa\mu^*$-closed if $A^*_\kappa \subseteq A$.*

Lemma 3.2 *The sets $\{M - H : M \in \kappa\mu, H \in \mathcal{H}\}$ constitute a base $\mathcal{B}$ for $\kappa\mu^*$.*

Proof: Let $M \in \kappa\mu$, $H \in \mathcal{H}$ implies $M - H \in \kappa\mu^*$ since $F = X - (M - H) = X - (M \cap (X - H)) = (X - M) \cup H$ is $\kappa\mu^*$-closed by Definition 3.1, $x \notin F$ iff $x \in M - H$, hence $x \in M$ and $M \cap F = M \cap ((X - M) \cup H) = (M \cap (X - M)) \cup (M \cap H) = M \cap H \in \mathcal{H}$ so that $x \notin F^*_\kappa$, $F^*_\kappa \subset F$. Thus $\mathcal{B} \subset \kappa\mu^*$. If $A \in \kappa\mu^*$ then $B = X - A$ is $\kappa\mu^*$-closed, hence $B^*_\kappa \subset B$. Thus $x \in A$ implies $x \notin B$ and $x \notin B^*_\kappa$ so that there exists $M \in \kappa\mu$ such that $x \in M$ and $H = M \cap B \in \mathcal{H}$, therefore $x \in M - H \subset X - B = A$. Hence A is the union of sets in $\mathcal{B}$. □

Proposition 3.3 *Let A be a subset of a generalized topological space $(X, \mu, \mathcal{H})$. Then*

1. $A^*_\beta(\mathcal{H}, \mu^*) \subseteq A^*_\sigma(\mathcal{H}, \mu^*) \subseteq A^*_\alpha(\mathcal{H}, \mu^*) \subseteq A^*(\mathcal{H}, \mu^*)$.
2. $A^*_\beta(\mathcal{H}, \mu^*) \subseteq A^*_\pi(\mathcal{H}, \mu^*) \subseteq A^*_\alpha(\mathcal{H}, \mu^*) \subseteq A^*(\mathcal{H}, \mu^*)$.

The converse of the assertions of Proposition 3.3 need not be true as shown in the following examples.

Example 3.4 *Let $X = \{a, b, c, d, e\}$, $\mu = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, d, e\}, \{a, b, d, e\}, \{a, c, d, e\}, X\}$ and $\mathcal{H} = \{\emptyset, \{c\}\}$.*

1. Let $A = \{d\}$. Then $A^*(\mathcal{H}, \mu^*) = \{c, d, e\} \nsubseteq \{d\} = A^*_\alpha(\mathcal{H}, \mu^*)$.
2. Let $A = \{b, d\}$. Then $A^*_\alpha(\mathcal{H}, \mu^*) = \{b, c, d, e\} \nsubseteq \{b, d\} = A^*_\sigma(\mathcal{H}, \mu^*)$.
3. Let $A = \{a, c, d\}$. Then $A^*_\sigma(\mathcal{H}, \mu^*) = \{a, c, d, e\} \nsubseteq \{a, d\} = A^*_\beta(\mathcal{H}, \mu^*)$.

Example 3.5 *Let $X = \{a, b, c, d, e\}$, $\mu = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, d, e\}, \{a, b, d, e\}, \{a, c, d, e\}, X\}$ and $\mathcal{H} = \{\emptyset, \{c\}\}$.*

1. Let $A = \{e\}$. Then $A^*(\mathcal{H}, \mu^*) = \{c, d, e\} \nsubseteq \{e\} = A^*_\alpha(\mathcal{H}, \mu^*)$.
2. Let $A = \{b, e\}$. Then $A^*_\alpha(\mathcal{H}, \mu^*) = \{b, c, d, e\} \nsubseteq \{b, e\} = A^*_\pi(\mathcal{H}, \mu^*)$.
3. Let $A = \{a, e\}$. Then $A^*_\pi(\mathcal{H}, \mu^*) = \{a, c, e\} \nsubseteq \{a, e\} = A^*_\beta(\mathcal{H}, \mu^*)$.

Remark 3.6 *Let $(X, \mu^*, \mathcal{H})$ be a hereditary generalized topological space. Then $\mu^* \subseteq \alpha(\mu^*) \subseteq \sigma(\mu^*) \subseteq \beta(\mu^*)$ and $\mu^* \subseteq \alpha(\mu^*) \subseteq \pi(\mu^*) \subseteq \beta(\mu^*)$.*

Theorem 3.7 *Let $(X, \mu, \mathcal{H})$ be a hereditary generalized topological space, $\kappa \in \{\alpha(\mu), \sigma(\mu), \pi(\mu), \beta(\mu)\}$ and $A \subset A^*_\kappa$. Then*

1. $A^*_\kappa = c_\kappa(A^*_\kappa) = c_\kappa(A)$.
2. $c_\kappa(A) = c^*_\kappa(A)$
3. $i_\kappa(X - A) = i^*_\kappa(X - A)$.

Proof: 1.For every subset A of X, we have $A^*_\kappa = c_\kappa(A^*_\kappa) \subseteq c_\kappa(A)$ by Theorem 2.11. $A \subset A^*_\kappa$ implies $c_\kappa(A) \subset c_\kappa(A^*_\kappa)$ and so $A^*_\kappa = c_\kappa(A^*_\kappa) = c_\kappa(A)$.
2.Clearly for every subset A of X, $c^*_\kappa(A) \subset c_\kappa(A)$. Let $x \notin c^*_\kappa(A)$, then there exists a $\kappa\mu^*$ open set U containing x such that $U \cap A = \emptyset$. By Lemma 3.2 there exists a $M \in \kappa\mu$ and $H \in \mathcal{H}$ such that $x \in M - H \subset U$. $U \cap A = \emptyset \Rightarrow (M - H) \cap A = \emptyset$. By Theorem 2.11 $(M \cap A) - H = \emptyset \Rightarrow ((M \cap A) - H)^*_\kappa = \emptyset \Rightarrow (M \cap A)^*_\kappa - H^*_\kappa = \emptyset \Rightarrow (M \cap A)^*_\kappa = \emptyset \Rightarrow M \cap A^*_\kappa \Rightarrow M \cap A = \emptyset$. Since M is $\kappa\mu$-open set containing x, $x \notin c_\kappa(A)$ and $c_\kappa(A) \subset c^*_\kappa(A)$. Hence $c_\kappa(A) = c^*_\kappa(A)$ which proves (2).
3. If $A \subset A^*_\kappa$, then $c_\kappa(A) = c^*_\kappa(A)$ by (2) and so $X - c_\kappa(A) = X - c^*_\kappa(A)$ implies $i_\kappa(X - A) = i^*_\kappa(X - A)$.

□

Theorem 3.8 *Let $(X, \mu, \mathcal{H})$ be a hereditary generalized topological space. Then,*

1. $A^*_\sigma(\mathcal{H}, \sigma(\mu)) = A^*_\alpha(\mathcal{H}, \sigma(\mu)) = A^*_\sigma(\mathcal{H}, \mu)$.
2. $A^*_\pi(\mathcal{H}, \sigma(\mu)) = A^*_\beta(\mathcal{H}, \sigma(\mu)) = A^*_\beta(\mathcal{H}, \mu)$.
3. $A^*_\pi(\mathcal{H}, \beta(\mu)) = A^*_\beta(\mathcal{H}, \beta(\mu)) = A^*_\beta(\mathcal{H}, \mu)$.
4. $A^*_\alpha(\mathcal{H}, \beta(\mu)) = A^*_\sigma(\mathcal{H}, \beta(\mu)) = A^*_\beta(\mathcal{H}, \mu)$.
5. $A^*_\pi(\mathcal{H}, \pi(\mu)) = A^*_\alpha(\mathcal{H}, \pi(\mu)) = A^*_\pi(\mathcal{H}, \mu)$.

Proof: 1. Let $x \in A^*_\sigma(\mathcal{H}, \sigma(\mu))$. Then there exists a $x \in U \in \sigma(\sigma)$ such that $A \cap U \notin \mathcal{H}$. By Theorem 2.3, of [3], $\sigma(\sigma) = \alpha(\sigma) = \sigma$. Hence $x \in A^*_\alpha(\mathcal{H}, \sigma(\mu))$ and $x \in A^*_\sigma(\mathcal{H}, \mu)$. Thus $A^*_\sigma(\mathcal{H}, \sigma(\mu)) \subseteq A^*_\alpha(\mathcal{H}, \sigma(\mu))$ and $A^*_\sigma(\mathcal{H}, \sigma(\mu)) \subseteq A^*_\sigma(\mathcal{H}, \mu)$. Similarly $A^*_\alpha(\mathcal{H}, \sigma(\mu)) \subseteq A^*_\sigma(\mathcal{H}, \sigma(\mu))$ and $A^*_\sigma(\mathcal{H}, \mu) \subseteq A^*_\sigma(\mathcal{H}, \sigma(\mu))$. Hence $A^*_\sigma(\mathcal{H}, \sigma(\mu)) = A^*_\alpha(\mathcal{H}, \sigma(\mu)) = A^*_\sigma(\mathcal{H}, \mu)$.
Proof of 2, 3, 4 and 5 are similar as in proof 1. □

Theorem 3.9 *Let $(X, \mu, \mathcal{H})$ be a hereditary generalized topological space and $\kappa \in \{\alpha(\mu), \sigma(\mu), \pi(\mu), \beta(\mu)\}$. Then $A^*_\kappa(\mathcal{H}, \alpha(\mu)) = A^*_\kappa(\mathcal{H}, \mu)$.*

Proof: Let $x \in A^*_\kappa(\mathcal{H}, \alpha(\mu))$ then for any $U \in \kappa(\alpha)$ such that $A \cap U \notin \mathcal{H}$. By Theorem 2.5, of [3], $\kappa(\alpha) = \kappa(\mu)$. Hence $x \in A^*_\kappa(\mathcal{H}, \mu)$. Thus $A^*_\kappa(\mathcal{H}, \alpha(\mu)) \subseteq A^*_\kappa(X, \mu)$. Similarly $A^*_\kappa(\mathcal{H}, \mu) \subseteq A^*_\kappa(\mathcal{H}, \alpha(\mu))$. Hence $A^*_\kappa(\mathcal{H}, \alpha(\mu)) = A^*_\kappa(\mathcal{H}, \mu)$. □

Corollary 3.10 *Let $(X, \mu, \mathcal{H})$ be a hereditary generalized topological space and $\kappa \in \{\alpha(\mu), \sigma(\mu), \pi(\mu), \beta(\mu)\}$. Then $A^*_\kappa(\mathcal{H}, \beta(\mu)) = A^*_\beta(\mathcal{H}, \mu)$.*

Theorem 3.11 *Let μ and μ' are GT's on X and $\mathcal{H}$ be a hereditary class.*

1. *If $\mu \subset \mu' \subset \beta(\mu)$, then $A^*_\alpha(\mathcal{H}, \mu') \subseteq A^*_\alpha(\mathcal{H}, \mu)$.*
2. *If $\mu \subset \mu' \subset \alpha(\mu)$, then $A^*_\alpha(\mathcal{H}, \mu) = A^*_\alpha(\mathcal{H}, \mu')$.*

Proof: (1). Let $x \in A^*_\alpha(\mathcal{H}, \mu')$. Then there exists a $U \in \alpha(\mu')$ such that $U \cap A \in \mathcal{H}$. By Theorem 2.2, of [4], $\alpha(\mu) \subset \alpha(\mu')$. We have $U \cap A \in \mathcal{H}$ for any $U \in \alpha(\mu)$ and hence $x \in A^*_\alpha(\mathcal{H}, \mu)$. Thus $A^*_\alpha(\mathcal{H}, \mu') \subseteq A^*_\alpha(\mathcal{H}, \mu)$.
(2). Let $x \in A^*_\alpha(\mathcal{H}, \mu)$. Then for any $U \in \alpha(\mu)$, $U \cap A \notin \mathcal{H}$. By Corollary 2.3, of [4], $\alpha(\mu) = \alpha(\mu')$. We have $U \cap A \in \mathcal{H}$ when $U \in \alpha(\mu')$ and hence $x \in A^*_\alpha(\mathcal{H}, \mu')$. Thus $A^*_\alpha(\mathcal{H}, \mu) \subseteq A^*_\alpha(\mathcal{H}, \mu')$. Similarly $A^*_\alpha(\mathcal{H}, \mu') \subseteq A^*_\alpha(\mathcal{H}, \mu)$. Hence $A^*_\alpha(\mathcal{H}, \mu) = A^*_\alpha(\mathcal{H}, \mu')$. □

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M.Rajamani, V.Inthumathi and R.Ramesh
Department of Mathematics,
N G M College, Pollachi - 642 001,
Tamil Nadu, India.
E-mail address: rajkarthy@yahoo.com