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Somewhat (γ, β)-semicontinuous functions

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abstract: In this paper, some new classes of functions are introduce and study by making use of γ-semiopen sets and γ-semiclosed sets.

Key Words: Topological spaces, γ-open set, γ-semiopen set.

Contents

1. Introduction

Generalized open sets play a very important role in General Topology and they are now the research topics of many topologists worldwide. Indeed a significant theme in General Topology and Real analysis concerns the various modified forms of continuity, separation axioms etc. by utilizing generalized open sets. Andrijevic [1] introduce a class of generalized open sets in a topological space, so called b-open sets. Kasahara [4] defined the concept of an operation on topological spaces and introduced the concept of γ-closed graphs of a function. Ogata [5] introduced the notion of γ-open sets in a topological space (X, τ). Gentry and Hoyle [3] introduced the concepts of somewhat continuous functions and Santhileela and Balasubramanian [7] introduced the concepts of somewhat semicontinuous functions and somewhat semiopen functions. In this paper, some new classes of functions are introduce and study by making use of γ-semiopen sets and γ-semiclosed sets.

2. Preliminaries

Definition 2.1 *[4] Let (X, τ) be a topological space. An operation γ on the topology τ is a function from τ on to power set $\mathcal{P}(X)$ of X such that $V \subset V^\gamma$ for each $V \in \tau$, where V^γ denotes the value of γ at V. It is denoted by $\gamma : \tau \rightarrow \mathcal{P}(X)$.*

Definition 2.2 *A subset A of a topological space (X, τ) is said to be γ-open set [5] if for each $x \in A$ there exists an open neighborhood U of x such that $U^\gamma \subset A$. The complement of a γ-open set is called a γ-closed set. τ_γ denotes the set of all γ-open sets in (X, τ).*

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Definition 2.3 *[5] Let A be subset of a topological space (X, τ). Then*

(i) the τ_γ-closure of A is defined as intersection of all γ-closed sets containing A. That is, $\tau_\gamma\text{-Cl}(A) = \cap\{F : F$ is γ-closed and $A \subset F\}$.

(ii) the τ_γ-interior of A is defined as union of all γ-open sets contained in A. That is, $\tau_\gamma\text{-Int}(A) = \cup\{U : U$ is γ-open and $U \subset A\}$.

Definition 2.4 *Let (X, τ) be a topological space. A subset A of X is said to be γ-semiopen [6] if $A \subset \tau_\gamma\text{-Cl}(\tau_\gamma\text{-Int}(A))$. γ-$SO(X, \tau)$ denotes the set of all $\gamma-$semi open subsets in (X, τ).*

Definition 2.5 *[6] Let A be subset of a topological space (X, τ). Then*

(i) the τ_γ-semiclosure of A is defined as intersection of all γ-semiclosed sets containing A. That is, $\tau_\gamma\text{-s}\,\text{Cl}(A) = \cap\{F : F$ is γ-semiclosed and $A \subset F\}$.

(ii) the τ_γ-semiinterior of A is defined as union of all γ-semiopen sets contained in A. That is, $\tau_\gamma\text{-s}\,\text{Int}(A) = \cup\{U : U$ is γ-semiopen and $U \subset A\}$.

3. Somewhat (γ, β)-continuous functions

Throughout this paper (X, τ), (Y, σ) and (Z, η) are three topological spaces and $\gamma : \tau \to \mathcal{P}(X)$, $\beta : \sigma \to \mathcal{P}(Y)$ and $\alpha : \eta \to \mathcal{P}(Z)$ be operation on τ, σ and η, respectively. Also, the sets $\tau_\gamma, \sigma_\beta$, γ-$SO(X, \tau)$ and β-$SO(Y, \sigma)$ are fundamental in order to define a new class of functions.

Definition 3.1 *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be somewhat (γ, β)-semicontinuous if for $U \in \sigma_\beta$ and $f^{-1}(U) \neq \emptyset$, there exists a γ-semiopen set, say, V in X such that $V \neq \emptyset$ and $V \subset f^{-1}(U)$.*

Example 3.2 *Let $X = Y = \{a, b, c\}$ and $\tau = \sigma = \{\emptyset, X, \{a\}, \{a, b\}\}$. Let $\gamma = \beta : \sigma \to \mathcal{P}(X)$ be operations defined as follows:*

$$A^\gamma = \begin{cases} A & \text{if } b \notin A, \\ \text{Cl}(A) & \text{if } b \in A. \end{cases}$$

Then the function $f : (X, \tau) \to (X, \sigma)$ defined as: $f(a) = a, f(b) = c$ and$f(c) = b$ is somewhat (γ, β)-semicontinuous.

Definition 3.3 *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be:*

(i) (γ, β)-semicontinuous [2] if for each $U \in \sigma_\beta$, there exists a γ-semiopen set V in X such that $V \subset f^{-1}(U)$.

(ii) (γ, β)-continuous [2] if $f^{-1}(V)$ is γ-closed for each β-closed subset V of Y.

Example 3.4 *Let* $X = Y = \{a, b, c\}$ *and* $\tau = \sigma = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$. *Let* $\gamma : \tau \to \mathcal{P}(X)$ *be an operation defined as follows:*

$$A^\gamma = \begin{cases} A & \text{if } b \notin A, \\ \mathrm{Cl}(A) & \text{if } b \in A, \end{cases}$$

Let $\beta : \sigma \to P(X)$ *be operations defined as follows:*

$$A^\gamma = \begin{cases} \mathrm{Cl}(A) & \text{if } b \notin A, \\ A & \text{if } b \in A. \end{cases}$$

Then the function $f : (X, \tau) \to (X, \sigma)$ *defined as:* $f(a) = c, f(b) = b$ *and* $f(c) = a$ *is* (γ, β)*-semicontinuous and* $g : (X, \tau) \to (X, \sigma)$ *defined as:* $g(a) = a, g(b) = c$ *and* $g(c) = b$ *is* (γ, β)*-continuous.*

It is clear that every (γ, β)-semicontinuous function is somewhat (γ, β)-continuous but the converse is not true in general as shown by the following example.

Example 3.5 *Let* $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ *and* $\sigma = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}\}$. *Let* $\gamma : \tau \to \mathcal{P}(X)$ *and* $\beta : \sigma \to \mathcal{P}(X)$ *be operations defined as follows:*

$$A^\gamma = \begin{cases} A & \text{if } b \notin A, \\ \mathrm{Int}(\mathrm{Cl}(A)) & \text{if } b \in A, \end{cases} \text{ and } A^\beta = \begin{cases} A & \text{if } c \in A, \\ A \cup \{c\} & \text{if } c \notin A. \end{cases}$$

Then the identity function $f : (X, \tau) \to (Y, \sigma)$ *is somewhat* (γ, β)*-semicontinuous but not* (γ, β)*-semicontinuous.*

Proposition 3.6 *If* $f : (X, \tau) \to (Y, \sigma)$ *is somewhat* (γ, β)*-continuous and* $g : (Y, \sigma) \to (Z, \eta)$ *is* (β, α)*-continuous, then* $g \circ f : (X, \tau) \to (Z, \eta)$ *is somewhat* (γ, α)*-continuous.*

Proof: Clear. □

Definition 3.7 *A subset* M *of a topological space* (X, τ) *is said to be* γ*-semidense (resp.* γ*-dense) in* X *if there is no proper* γ*-semiclosed (resp.* γ*-closed) set* C *in* X *such that* $M \subset C \subset X$.

Proposition 3.8 *For a surjective function* $f : (X, \tau) \to (Y, \sigma)$*, the following statements are equivalent:*

(i) f *is somewhat* (γ, β)*-semicontinuous.*

(ii) If C *is a* β*-closed subset of* Y *such that* $f^{-1}(C) \neq X$*, then there is a proper* γ*-semiclosed subset* D *of* X *such that* $D \supset f^{-1}(C)$.

(iii) If A *is a* γ*-semiopen subset of* Y *such that* $f^{-1}(A) \neq X$*, then there is a proper* γ*-semiopen subset* B *of* X *such that* $f^{-1}(A) = B$.

(iv) If M is a γ-semidense subset of X, then $f(M)$ is a β-dense subset of $f(X)$.

Proof: (i)⇒(ii): Let C be a β-closed subset of Y such that $f^{-1}(C) \neq X$. Then $Y \backslash C$ is a β-open set in Y such that $f^{-1}(Y \backslash C) = X \backslash f^{-1}(C) \neq \emptyset$. By (i), there exists a γ-semiopen set V in X such that $V \neq \emptyset$ and $V \subset f^{-1}(Y \backslash C) = X \backslash f^{-1}(C)$. This means that $X \backslash V \supset f^{-1}(C)$ and $X \backslash V = D$ is a proper γ-semiclosed set in X.
(ii)⇒(i): Let $U \in \sigma_\beta$ and $f^{-1}(U) \neq X$. Then $Y \backslash U$ is γ-closed and $f^{-1}(Y \backslash U) = X \backslash f^{-1}(U) \neq \emptyset$. By (ii), there exists a proper γ-semiclosed set D such that $D \supset f^{-1}(Y \backslash U)$. This implies that $X \backslash D \subset f^{-1}(U)$ and $X \backslash D$ is γ-semiopen and $X \backslash D \neq \emptyset$.
(ii)⇔(iii): Clear.
(ii)⇒(iv): Let M be a γ-semidense set in X. Suppose that $f(M)$ is not γ-dense in Y. Then there exists a proper β-closed set C in Y such that $f(M) \subset C \subset Y$. Clearly $f^{-1}(C) \neq X$. By (ii), there exists a proper γ-semiclosed set D such that $M \subset f^{-1}(C) \subset D \subset X$. This is a contradiction to the fact that M is β-dense in X.
(iv)⇒(ii): Suppose (ii) is not true. This means that there exists a β-closed set C in Y such that $f^{-1}(C) \neq X$ but there is no proper γ-semiclosed set D in X such that $f^{-1}(C) \subset D$. This means that $f^{-1}(C)$ is γ-semidense in X. But by (iv), $f(f^{-1}(C)) = C$ must be β-dense in Y, which is a contradiction to the choice of C. □

Definition 3.9 *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be somewhat (γ, β)-semiopen provided that if $U \in \tau_\gamma$ and $U \neq \emptyset$, then there exists β-semiopen set V in Y such that $V \neq \emptyset$ and $V \subset f(U)$.*

Example 3.10 *Let $X = Y = \{a, b, c\}$ and $\tau = \sigma = \{\emptyset, X, \{a\}, \{a, b\}\}$. Let $\gamma = \beta : \sigma \to \mathcal{P}(X)$ be operations defined as follows:*

$$A^\gamma = \begin{cases} A & \text{if } b \notin A, \\ \mathrm{Cl}(A) & \text{if } b \in A. \end{cases}$$

Then the function $f : (X, \tau) \to (X, \sigma)$ defined as: $f(a) = a, f(b) = c$ and $f(c) = b$ is somewhat (γ, β)-semiopen.

Definition 3.11 *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be (γ, β)-semiopen [2] provided that if $U \in \tau_\gamma$, then there exists a β-semiopen set V in Y such that $V \subset f(U)$.*

Clearly every (γ, β)-semiopen function is somewhat (γ, β)-open but the converse is not true in general as the following example shows.

Example 3.12 *Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}\}$ and $\sigma = \{\emptyset, X, \{c\}, \{a, b\}\}$. Let $\gamma : \tau \to \mathcal{P}(X)$ and $\beta : \sigma \to \mathcal{P}(X)$ be operations defined as follows:*

$$A^\gamma = \begin{cases} A & \text{if } c \in A, \\ A \cup \{c\} & \text{if } c \notin A, \end{cases} \text{ and } A^\beta = \begin{cases} A & \text{if } a \in A, \\ A \cup \{c\} & \text{if } a \notin A. \end{cases}$$

Then the function $f:(X,\tau)\to(Y,\sigma)$ defined by $f(a)=a$, $f(b)=f(c)=c$ is somewhat (γ,β)-semiopen but not (γ,β)-semiopen.

Proposition 3.13 *For a bijective function $f:(X,\tau)\to(Y,\sigma)$, the following statements are equivalent:*

(i) f is somewhat (γ,β)-semiopen.

(ii) If C is a γ-closed subset of X, such that $f(C)\neq Y$, then there is a β-semiclosed subset D of Y such that $D\neq Y$ and $D\supset f(C)$.

Proof: (i)⇒(ii): Let C be any γ-closed subset of X such that $f(C)\neq Y$. Then $X\backslash C$ is γ-open in X and $X\backslash C\neq\emptyset$. Since f is somewhat (γ,β)-open, there exists β-open set $V\neq\emptyset$ in Y such that $V\subset f(X\backslash C)$. Put $D=Y\backslash V$. Clearly D is β-semiclosed in Y and we claim $D\neq Y$. If $D=Y$, then $V=\emptyset$, which is a contradiction. Since $V\subset f(X\backslash C)$, $D=Y\backslash V\supset(Y\backslash f(X\backslash C))=f(C)$.
(ii)⇒(i): Let U be any nonempty γ-open subset of X. Then $C=X\backslash U$ is a γ-closed set in X and $f(X\backslash U)=f(C)=Y\backslash f(U)$ implies $f(C)\neq Y$. Therefore, by (ii), there is a β-semiclosed set D of Y such that $D\neq Y$ and $f(C)\subset D$. Clearly $V=Y\backslash D$ is a β-semiopen set and $V\neq\varnothing$. Also, $V=Y\backslash D\subset Y\backslash f(C)=Y\backslash f(X\backslash U)=f(U)$. □

Proposition 3.14 *For a function $f:(X,\tau)\to(Y,\sigma)$, the following statements are equivalent:*

(i) f is somewhat (γ,β)-semiopen.

(ii) If A is a β-semidense subset of Y, Then $f^{-1}(A)$ is a γ-dense subset of X.

Proof: (i)⇒(ii): Suppose A is a β-semidense set in Y. We want to show that $f^{-1}(A)$ is a γ-dense subset of X. Suppose not, then there exists a γ-closed set B in X such that $f^{-1}(A)\subset B\subset X$. Since f is somewhat (γ,β)-semiopen and $X\backslash B$ is γ-open, there exists a nonempty β-semiopen set C in Y such that $C\subset f(X\backslash B)$. Therefore, $C\subset f(X\backslash B)\subset f(f^{-1}(X\backslash A)\subseteq X\backslash A$. That is, $A\subset X\backslash C\subset X$. Now, $X\backslash C$ is a β-semiclosed set and $A\subset X\backslash C\subset X$. This implies that A is not a γ-semidense set in X, which is a contradiction. Therefore, $f^{-1}(A)$ must be γ-dense set in X.
(ii)⇒(i): Suppose A be a nonempty γ-open subset of X. We want to show that $\sigma_\beta\text{-}s\,\mathrm{Int}(f(A))\neq\emptyset$. Suppose $\sigma_\beta\text{-}s\,\mathrm{Int}(f(A))=\emptyset$. Then, $\sigma_\beta\text{-}s\,\mathrm{Cl}(Y\backslash f(A))=Y$. Therefore, by (ii), $f^{-1}(Y\backslash f(A))$ is γ-dense in X. But $f^{-1}(Y\backslash f(A))\subseteq X\backslash A$. Now, $X\backslash A$ is γ-closed. Therefore, $f^{-1}(Y\backslash f(A))\subseteq X\backslash A$ gives $\sigma_\gamma\text{-}\mathrm{Cl}(f^{-1}(Y\backslash f(A)))\subseteq X\backslash A$. This implies that $A=\emptyset$, which is contradiction to $A\neq\emptyset$. Therefore, $\sigma_\beta\text{-}s\,\mathrm{Int}(f(A))\neq\emptyset$. This proves that f is somewhat (γ,β)-semiopen. □

4. γ-semiresolvable spaces and γ-semiirresolvable spaces

In this section we define and characterize the spaces γ-semiresolvable and γ-resolvable using the notions of γ-semidense and γ-dense set.

Definition 4.1 *A topological space (X, τ) is said to be:*

(i) γ-semiresolvable if there exists a γ-semidense set A in (X, τ) such that $X \backslash A$ is also γ-semidense in (X, τ). Otherwise, (X, τ) is called γ-semiirresolvable.

(ii) γ-resolvable if there exists a γ-dense set A in (X, τ) such that $X \backslash A$ is also γ-dense in (X, τ). Otherwise, (X, τ) is called γ-irresolvable.

Example 4.2 *Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$. Let $\gamma : \tau \to (\mathcal{P}X)$ be an operation defined as follows:*

$$A^{\gamma} = \begin{cases} A & \text{if } b \notin A, \\ \mathrm{Cl}(A) & \text{if } b \in A. \end{cases}$$

Then the space (X, τ) is γ-semiirresolvable and γ-resolvable.

Proposition 4.3 *For a topological space (X, τ), the following statements are equivalent:*

(i) (X, τ) is γ-semiresolvable;

(ii) (X, τ) has a pair of γ-semidense sets A and B such that $A \subseteq X \backslash B$.

Proof: (i)$\Rightarrow$(ii): Suppose that $X \backslash B \subset A$ for all γ-semidense sets A and B in X. Then τ_γ-$\mathrm{Cl}(X \backslash B) \subset \tau_\gamma$-$s\,\mathrm{Cl}(A)$. Since A is γ-dense, τ_γ-$s\,\mathrm{Cl}(X \backslash B) \subset X$. That is, τ_γ-$s\,\mathrm{Cl}(X \backslash B) \neq X$. In a similar manner, we have τ_γ-$s\,\mathrm{Cl}(X \backslash A) \neq X$ for all γ-semidense sets A in X, which is a contradiction to X being a γ-semiresolvable space. Therefore, (X, τ) has a pair of γ-semidense sets A and B such that $A \subseteq X \backslash B$.
(ii)$\Rightarrow$(i): Suppose that (X, τ) is a γ-semiirresolvable space. Then for all γ-semidense sets, say, A_i in (X, τ), we have τ_γ-$s\,\mathrm{Cl}(X \backslash A_i) \neq X$. In particular, τ_γ-$\mathrm{Cl}(X \backslash A_2) \neq X$. That is, there exists a γ-closed set B in (X, τ) such that $X \backslash B \subset C \subset X$. Then $A \subset X \backslash B \subset C \subset X$, which is a contradiction to A being a γ-semidense set in X; hence (X, τ) is γ-semiresolvable. $\square$

Proposition 4.4 *For a topological space (X, τ), the following statements are equivalent:*

(i) (X, τ) is γ-irresolvable;

(ii) For all γ-dense sets A in X, τ_γ-$\mathrm{Int}(A) \neq \emptyset$.

Proof: (i)$\Rightarrow$(ii): Let A be any γ-dense subset of X. Since (X, τ) is γ-irresolvable, τ_γ-Cl$(X \backslash A) \neq X$; follows τ_γ-Cl$(X \backslash A) = X \backslash \tau_\gamma$-Int$(A) \neq X$. And therefore, τ_γ-Int$(A) \neq \emptyset$.
(ii)$\Rightarrow$(i): Suppose that (X, τ) is a γ-resolvable space. Then by Definition 4.1 (ii), there exists a γ-dense set A in (X, τ) such that $X \backslash A$ is also γ-dense in X. It follows that, τ_γ-Cl$(X \backslash A) = X = X \backslash \tau_\gamma$-Int$(A)$, and therefore, τ_γ-Int$(A) = \emptyset$, which is a contradiction; hence (X, τ) is γ-irresolvable. □

Proposition 4.5 *If $\bigcup_{i=1}^{n} A_i = X$, where A_i's are subsets of X such that τ_γ-Int$(A_i) = \emptyset$, then (X, τ) is a γ-resolvable space.*

Proof: By hypothesis, we have $\bigcap_{i=1}^{n} (X \backslash A_i) = \emptyset$. Then, there must be at least two nonempty disjoint subsets $X \backslash A_i$ and $X \backslash A_j$ in X. That is, $X \backslash A_i \cup X \backslash A_j \emptyset$. Then $v A_i \subseteq A_j$; follows that, $X = \tau_\gamma$-Cl$(X \backslash A_i) \subseteq \tau_\gamma$-Cl$(A_j)$. Hence, τ_γ-Cl$(A_j) = X$. Therefore, (X, τ) has a γ-dense set A_j such that τ_γ-Cl$(X \backslash A_j) = X$. Hence (X, τ) is a γ-resolvable space. □

Proposition 4.6 *If $f : (X, \tau) \to (Y, \sigma)$ is a somewhat (γ, β)-semiopen function and σ_β-$s\,$Int$(A) = \emptyset$ for a nonempty set A in Y, then τ_γ-Int$(f^{-1}(A))$ $= \emptyset$.*

Proof: Let A be a nonempty set in Y such that σ_β-$s\,$Int$(A) = \emptyset$. Then σ_β-$s\,$Cl$(Y \backslash A) = Y \backslash \sigma_\beta$-$s\,Int(A) = Y$. Since f is somewhat (γ, β)-semiopen and $Y \backslash A$ is β-dense in Y, using Proposition 3.14, $f^{-1}(Y \backslash A)$ is γ-dense in X. Then, τ_γ-Cl$(f^{-1}(Y \backslash A)) = \tau_\gamma$-Cl$(X \backslash f^{-1}(A)) = X \backslash \tau_\gamma$-Int$(f^{-1}(A)) = X$; hence τ_γ-Int$(f^{-1}(A))$ $= \emptyset$. □

Proposition 4.7 *Let $f : (X, \tau) \to (Y, \sigma)$ be a somewhat (γ, β)-semiopen function. If X is γ-irresolvable, then Y is β-semiirresolvable.*

Proof: Let A be a nonempty set in Y such that σ_β-$s\,$Cl$(A) = Y$. We show that σ_β-$s\,$Int$(A) \neq \emptyset$. Suppose not, then σ_β-$s\,$Cl$(Y \backslash A) = Y$. Since f is somewhat (γ, β)-semiopen and $Y \backslash A$ is β-semidense in Y, we have $f^{-1}(Y \backslash A)$ is γ-dense in X. Then τ_γ-Int$(f^{-1}(A)) = \emptyset$. Now, A is β-semidense in Y, $f^{-1}(A)$ is γ-dense in X. Therefore, for the γ-dense set $f^{-1}(A)$, we have σ_γ-Int$(f^{-1}(A)) = \emptyset$, which is a contradiction to Proposition 4.4. Hence we must have σ_β-$s\,$Int$(A) \neq \emptyset$ for all β-semidense sets A in Y. Hence by Proposition 4.4, Y is β-semiirresolvable. □

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