Bol. Soc. Paran. Mat.
©SPM –ISSN-2175-1188 ON LINE
SPM: www.spm.uem.br/bspm

(3s.) **v. 30** 1 (2012): 21–33.
ISSN-00378712 IN PRESS
doi:10.5269/bspm.v30i1.13013

Operation approaches on b-open sets and applications

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ABSTRACT: In this paper, we introduce the concept of an operation γ on a family of b-open sets in a topological space (X, τ). Using this operation γ, we introduce the concept of b-γ-open sets and study some of their properties.

Key Words: Topological spaces, b-open set, γ-open set, γ-b-open set

Contents

1. Introduction and Preliminaries

Generalized open sets play a very important role in General Topology and they are now the research topics of many topologists worldwide. Indeed a significant theme in General Topology and Real analysis concerns the various modified forms of continuity, separation axioms etc. by utilizing generalized open sets. Andrijevic [1] introduce a class of generalized open sets in a topological space, so- called b-open sets. The class of b-open sets is contained in the class of semi preopen sets and contains all semi open sets and preopen sets. Kasahara [4] defined the concept of an operation on topological spaces and introduce the concept of γ-closed graphs of a function. Ogata [5] introduce the notions of γ-open sets in a topological space and C. K. Bass, B. M. Uzzil Afsan and M. K. Ghosh [2] introduce the notion of γ-β-openess and investigatd its fundamental properties. In this paper, we introduce the concept of an operation γ on a family of b-open sets in a topological space (X, τ). Using this operation γ, we introduce the concept of b-γ-open sets and study some of their properties. The closure and the interior of A of a topological space (X, τ) are denoted by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively. A subset A of X is said to be b-open [1] $A \subset \mathrm{Int}(\mathrm{Cl}(A)) \cup \mathrm{Cl}(\mathrm{Int}(A))$. The complement of a b-open is called b-closed [1]. The intersection of all b-closed sets containing A is called the b-closure [1] of A and is denoted by $b\,\mathrm{Cl}(A)$. The family of all b-open sets of (X, τ) is denoted by $BO(X)$. The b-θ-closure [6] of A, denoted by $b\,\mathrm{Cl}_\theta(A)$, is defined to be the set of all $x \in X$ such that $A \cap b\,\mathrm{Cl}(U) \neq \varnothing$ for every b-open set U containing x. A subset A is called b-θ-closed [6] if and only if $A = b\,\mathrm{Cl}_\theta(A)$.

2000 *Mathematics Subject Classification:* 54A05,54A10,54A20,54A40,54D10,54D30

2. b-γ-Open sets

Definition 2.1 *Let $\gamma : BO(X,\tau) \to \mathcal{P}(X)$ be a mapping satisfying the following property: $V \subset \gamma(V)$ for any $V \in BO(X,\tau)$. We call the mapping γ an operation on $BO(X,\tau)$. We denote $V^\gamma = \gamma(V)$ for any $V \in BO(X,\tau)$.*

Definition 2.2 *Let (X,τ) be a topological space and $\gamma : BO(X,\tau) \to \mathcal{P}(X)$ an operation on $BO(X,\tau)$. A nonempty set A of X is called a b-γ-open set of (X,τ) if for each point $x \in A$, there exists a b-open set U containing x such that $U^\gamma \subset A$. The complement of a γ-open set is called γ-closed in (X,τ). We suppose that the empty set is b-γ-open for any operation $\gamma : BO(X,\tau) \to \mathcal{P}(X)$. We denote the set of all b-γ-open sets of (X,τ) by $\gamma BO(X)$ (or shortly $\gamma BO(X)$).*

Example 2.3 *A subset A is a b-id-open set of (X,τ) if and only if A is b-open in (X,τ). The operation $id : BO(X,\tau) \to \mathcal{P}(X)$ is defined by $V^{id} = V$ for any set $V \in BO(X,\tau)$; this operation is called the identity operation on $BO(X,\tau)$. Therefore, we have that $BO(X,\tau)_{id} = BO(X,\tau)$.*

Example 2.4 *The operation $b\,\mathrm{Cl} : BO(X,\tau) \to \mathcal{P}(X)$ is defined by $V^{b\,\mathrm{Cl}} = b\,\mathrm{Cl}(V)$ for any subset $V \in BO(X)$. A nonempty set A is b-$b\,\mathrm{Cl}$-open in (X,τ) if and only if, by definition, for each $x \in U$ $U^{b\,\mathrm{Cl}} \subset A$; if and only if for each point $x \in X \backslash A$, there exists a subset $V \in BO(X,\tau)$ such that $x \in V$ and $V^{b\,\mathrm{Cl}} \cap (X\backslash A) = \emptyset$; if and only if $b\,\mathrm{Cl}_\theta(X\backslash A) \subset X\backslash A$, where $b\,\mathrm{Cl}_\theta(B) = \{z \in X : b\,\mathrm{Cl}(W) \cap B \neq \emptyset$ for any subset $W \in BO(X,\tau)$ such that $z \in W\}$ for a subset B of (X,τ) and so A is b-θ-open set in X. Then we have the following: a nonempty set A is b-$b\,\mathrm{Cl}$-open in (X,τ) if and only if A is b-θ-open in (X,τ).*

Example 2.5 *Let $X = \{a,b,c\}$ and $\tau = \{\emptyset, X, \{a\}, \{a,b\}\}$. Define an operation $\gamma : BO(X,\tau) \to \mathcal{P}(X)$ by*

$$A^\gamma = \begin{cases} A & \text{if } b \in A, \\ b\,\mathrm{Cl}(A) & \text{if } b \notin A \end{cases}$$

Then we have $\gamma BO(X) = \{\emptyset, X, \{a,b\}\}$.

Theorem 2.6 *Let $\gamma : BO(X,\tau) \to \mathcal{P}(X)$ be any operation on $BO(X,\tau)$. Then*

(i) Every b-γ-open set of (X,τ) is b-open in (X,τ).

(ii) An arbitrary union of b-γ-open sets is b-γ-open.

Proof: (i). Suppose that $A \in \gamma BO(X)$. Let $x \in A$. Then, there exists a b-open set $U(x)$ containing x such that $U(x)^\gamma \subset A$. Then, $\cup\{U(x) : x \in A\} \subset \cup\{U(x)^\gamma : x \in A\} \subset A$ and so $A = \cup\{U(x) : x \in A\} \in BO(X,\tau)$. (ii). Let $x \in \cup\{A_i : i \in J\}$, where J is any index set, then $x \in A_i$ for some $i \in J$. Since A_i is b-γ-open set, there exists a b-open set U containing x such that $U^\gamma \subset A_i \subset \cup\{A_i : i \in J\}$. Hence $\cup\{A_i : i \in J\}$ is a b-γ-open set. □

Remark 2.7 *The following example shows that the converse of Theorem 2.6 (i) and (ii) is not true in general.*

Example 2.8 *Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, X, \{a\}\}$. Let $\gamma : BO(X, \tau) \to \mathcal{P}(X)$ be an operation defined as follows:*

$$A^\gamma = \begin{cases} A & \text{if } b \in A, \\ A \cup \{b\} & \text{if } b \notin A, \end{cases}$$

Then the set $\{a\}$ is b-open but not b-γ-open in (X, τ).

The following example shows that the intersection of two b-γ-open sets need not be b-γ-open.

Example 2.9 *Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$. Define an operation $\gamma : BO(X, \tau) \to \mathcal{P}(X)$ by*

$$A^\gamma = \begin{cases} A & \text{if } A \neq \{a\}, \\ \{a, b\} & \text{if } A = \{a\} \end{cases}$$

Then $\{a, b\}$ and $\{a, c\}$ are b-γ-open sets but their intersection $\{a\}$ is not b-γ-open.

Definition 2.10 *Let (X, τ) be a topological space and $\gamma : BO(X, \tau) \to \mathcal{P}(X)$ an operation on $BO(X, \tau)$. Then (X, τ) is said to be b-γ-regular if for each point $x \in X$ and for every b-open set V containing x, there exist a b-open set U containing x such that $U^\gamma \subset V$.*

Theorem 2.11 *Let (X, τ) be a topological space and $\gamma : BO(X, \tau) \to \mathcal{P}(X)$ an operation on $BO(X, \tau)$. Then the following statements are equivalent:*

(i) $BO(X, \tau) = \gamma BO(X)$.

(ii) (X, τ) *is a b-γ-regular space.*

(iii) *For every $x \in X$ and for every b-open set U of (X, τ) containing x, there exists a b-γ-open set W of (X, τ) such that $x \in W$ and $W \subset U$.*

Proof: (i)⇒(ii): Let $x \in X$ and V a b-open set containing x. It follows from assumption that V is a b-γ-open set. This implies that there exists a b-open set U containing x such that $U^\gamma \subset V$. Hence, (X, τ) is b-γ-regular. (ii)⇒(iii): Let $x \in X$ and U a b-open set containing x. Then by (ii), there is a b-open set W containing x and $W \subset W^\gamma \subset U$. By using (ii) for the set W, it is shown that W is b-γ-open. Hence W is a b-γ-open set containing x such that $W \subset U$. (iii)⇒(i): By (iii) and Theorem 2.6 (iii), it follows that every b-open set is b-γ-open, that is, $BO(X, \tau) \subset \gamma BO(X)$. It follows from Theorem 2.6 (i) that the converse inclusion $\gamma BO(X) \subset BO(X, \tau)$ holds. □

Definition 2.12 *An operation $\gamma : BO(X,\tau) \to \mathcal{P}(X)$ is called b-regular if for each point $x \in X$ and for every pair of b-open sets say, U and V containing $x \in X$, there exists a b-open set W such that $x \in W$ and $W^\gamma \subset U^\gamma \cap V^\gamma$.*

Example 2.13 *The operation γ defined in Example 2.5 is b-regular.*

Theorem 2.14 *For an operation $\gamma : BO(X,\tau) \to \mathcal{P}(X)$, the following properties holds:*

(i) Let γ be a b-regular operation. If A and B are b-γ-open in (X,τ), then $A \cap B$ is also b-γ-open in (X,τ).

(ii) If γ is a b-regular operation, then $\gamma BO(X)$ is a topology on X.

Proof: Let $x \in A \cap B$. Since A and B are b-γ-open sets, there exist b-open sets U, V such that $x \in U$, $x \in V$ and $U^\gamma \subset A$ and $V^\gamma \subset B$. By b-regularity of γ, there exists a b-open set W containing x such that $W^\gamma \subset U^\gamma \cap V^\gamma \subset A \cap B$. Therefore, $A \cap B$ is a b-γ-open set. (ii). It is proved by (i) above and Theorem 2.6 (iii). □

Definition 2.15 *Let A be subset of a topological space (X,τ) and $\gamma : BO(X,\tau) \to \mathcal{P}(X)$ an operation on $BO(X,\tau)$. Then the τ_γ-b-closure of A is defined as the intersection of all b-γ-closed sets containing A. That is, $\tau_\gamma\text{-}b\,\mathrm{Cl}(A) = \cap\{F : F$ is b-γ-closed and $A \subset F\}$.*

Definition 2.16 *Let A be subset of a topological space (X,τ) and $\gamma : BO(X,\tau) \to \mathcal{P}(X)$ an operation on $BO(X,\tau)$. A point $x \in X$ is in the $b\,\mathrm{Cl}_\gamma$-closure of a set A if $U^\gamma \cap A \neq \emptyset$ for each b-open set U containing x. The $b\,\mathrm{Cl}_\gamma$-closure of A is denoted by $b\,\mathrm{Cl}_\gamma(A)$.*

Theorem 2.17 *Let A be subset of a topological space (X,τ) and $\gamma : BO(X,\tau) \to \mathcal{P}(X)$ an operation on $BO(X,\tau)$. Then for a point $x \in X$, $x \in \tau_\gamma\text{-}b\,\mathrm{Cl}(A)$ if and only if there exists a b-γ-open set V of X containing x such that $V \cap A \neq \emptyset$.*

Proof: Let F be the set of all $y \in X$ such that $V \cap A \neq \emptyset$ for every $V \in \gamma BO(X)$ and $y \in V$. Now to prove the theorem it is enough to prove that $F = \tau_\gamma\text{-}b\,\mathrm{Cl}(A)$. Let $x \notin F$. Then there exists a b-γ-open set V containing x such that $V \cap A = \emptyset$. This implies $A \subset X \backslash V$. Hence $\tau_\gamma\text{-}b\,\mathrm{Cl}(A) \subset X \backslash V$. It follows that $x \notin \tau_\gamma\text{-}b\,\mathrm{Cl}(A)$. Thus, we have that $\tau_\gamma\text{-}b\,\mathrm{Cl}(A) \subset F$. Conversely, let $x \notin \tau_\gamma\text{-}b\,\mathrm{Cl}(A)$. Then there exists a b-γ-closed set E such that $A \subset E$ and $x \notin E$. Then we have that $x \in X \backslash E$, $X \backslash E \in \gamma BO(X)$ and $(X \backslash E) \cap A = \emptyset$. This implies that $x \notin F$. Hence $F \subset \tau_\gamma\text{-}b\,\mathrm{Cl}(A)$. Therefore, we have that $F = \tau_\gamma\text{-}b\,\mathrm{Cl}(A)$. □

Theorem 2.18 *Let A and B be subsets of a topological space (X,τ) and $\gamma : BO(X,\tau) \to \mathcal{P}(X)$ an operation on $BO(X,\tau)$. Then we have the following properties:*

(i) The set $b\,\mathrm{Cl}_\gamma(A)$ *is a b-closed set of* (X,τ) *and* $A \subset b\,\mathrm{Cl}_\gamma(A)$.

(ii) $b\,\mathrm{Cl}_\gamma(\emptyset) = \emptyset$ *and* $b\,\mathrm{Cl}_\gamma(X) = X$.

(iii) A is b-closed if and only if $b\,\mathrm{Cl}_\gamma(A) = A$.

(iv) If $A \subset B$*, then* $b\,\mathrm{Cl}_\gamma(A) \subset b\,\mathrm{Cl}_\gamma(B)$.

(v) $b\,\mathrm{Cl}_\gamma(A) \cup b\,\mathrm{Cl}_\gamma(B) \subset b\,\mathrm{Cl}_\gamma(A \cup B)$.

(vi) If $\gamma : BO(X,\tau) \to \mathcal{P}(X)$ *is b-regular, then* $b\,\mathrm{Cl}_\gamma(A \cup B) = b\,\mathrm{Cl}_\gamma(A) \cup b\,\mathrm{Cl}_\gamma(B)$ *holds.*

(vii) $b\,\mathrm{Cl}_\gamma(A \cap B) \subset b\,\mathrm{Cl}_\gamma(A) \cap b\,\mathrm{Cl}_\gamma(B)$ *holds.*

Proof: (i). For each point $x \in X \backslash b\,\mathrm{Cl}_\gamma(A)$, then $x \notin b\,\mathrm{Cl}_\gamma(A)$. By Definition 2.16, there exists a b-open set $U(x)$ containing x such that $U(x)^\gamma \cap A = \emptyset$. We let $V = \cup\{U(x) : U(x) \in BO(X,\tau), x \in X \backslash b\,\mathrm{Cl}_\gamma(A)\}$. Then it is shown that $V = X \backslash b\,\mathrm{Cl}_\gamma(A)$ holds. Indeed, for a point $y \in V$, there exists $U(x) \in BO(X,\tau)$ such that $y \in U(x)$ and $U(x)^\gamma \cap A = \emptyset$. This shows that $y \notin b\,\mathrm{Cl}_\gamma(A)$ and so $V \subseteq X \backslash b\,\mathrm{Cl}_\gamma(A)$. Conversely, let $y \in X \backslash b\,\mathrm{Cl}_\gamma(A)$, then $y \notin b\,\mathrm{Cl}_\gamma(A)$. Then there exists $U(y) \in BO(X,\tau)$ such that $y \in U(y)$ and $U(y)^\gamma \cap A = \emptyset$ and so $y \in U(y) \subseteq V$. Thus, we conclude that $X \backslash b\,\mathrm{Cl}_\gamma(A) \subseteq V$; follows that $V = X \backslash b\,\mathrm{Cl}_\gamma(A)$. Therefore, $b\,\mathrm{Cl}_\gamma(A)$ is b-closed in (X,τ), because $V \in BO(X,\tau)$. Obviously, by Definition 2.16, we have that $A \subseteq b\,\mathrm{Cl}_\gamma(A)$. (ii), (iv). They are obtained from Definition 2.16. (iii). Suppose that A is b-closed. Then $X \backslash A$ is b-open in (X,τ). We claim that $b\,\mathrm{Cl}_\gamma(A) \subseteq A$. Let $x \notin A$. There exists a b-open set U containing x such that $U^\gamma \subseteq X \backslash A$, that is, $U^\gamma \cap A = \emptyset$. Hence by Definition 2.16, we have that $x \notin b\,\mathrm{Cl}_\gamma(A)$ and so $b\,\mathrm{Cl}_\gamma(A) \subseteq A$. By (i), it is proved that $b\,\mathrm{Cl}_\gamma(A) = A$. Conversely, suppose that $b\,\mathrm{Cl}_\gamma(A) = A$. Let $x \in X \backslash A$. Since $x \notin b\,\mathrm{Cl}_\gamma(A)$, there exists a b-open set U containing x such that $U^\gamma \cap A = \emptyset$, that is $U^\gamma \subseteq X \backslash A$. In consequence, A is b-closed. (v), (vi). They are obtained from (iv). (vi). Let $x \notin b\,\mathrm{Cl}_\gamma \cup b\,\mathrm{Cl}_\gamma(B)$. Then there exist two b-open sets U and V containing x such that $U^\gamma \cap A = \emptyset$ and $V^\gamma \cap B = \emptyset$. Since γ is a regular operator, by Definition 2.12, there exists a b-open set W containing x such that $W^\gamma \subset U^\gamma \cap V^\gamma$. Thus, we have $W^\gamma \cap (A \cup B) \subseteq (U^\gamma \cap V^\gamma) \cap (A \cup B) \subset (U^\gamma \cap A) \cup (V^\gamma \cap A) = \emptyset$, that is, $W^\gamma \cap (A \cup B) = \emptyset$. Hence, $x \notin b\,\mathrm{Cl}_\gamma(A \cup B)$. This shows that $b\,\mathrm{Cl}_\gamma(A) \cup b\,\mathrm{Cl}_\gamma(B) \supset b\,\mathrm{Cl}_\gamma(A \cup B)$. □

Theorem 2.19 *Let A and B be subsets of a topological space* (X,τ) *and* $\gamma : BO(X,\tau) \to \mathcal{P}(X)$ *an operation on* $BO(X,\tau)$*. Then we have the following properties:*

(i) The set τ_γ-$b\,\mathrm{Cl}(A)$ *is a b-*γ*-closed set of* (X,τ) *and* $A \subset \tau_\gamma$-$b\,\mathrm{Cl}(A)$.

(ii) τ_γ-$b\,\mathrm{Cl}(\emptyset) = \emptyset$ *and* τ_γ-$b\,\mathrm{Cl}(X) = X$.

(iii) A is b-γ*-closed if and only if* τ_γ-$b\,\mathrm{Cl}(A) = A$.

(iv) *If* $A \subset B$, *then* τ_γ-$b\,\mathrm{Cl}(A) \subset \tau_\gamma$-$b\,\mathrm{Cl}(B)$.

(v) τ_γ-$b\,\mathrm{Cl}(A) \cup \tau_\gamma$-$b\,\mathrm{Cl}(B) \subset \tau_\gamma$-$b\,\mathrm{Cl}(A \cup B)$.

(vi) *If* $\gamma : BO(X,\tau) \to \mathcal{P}(X)$ *is b-regular, then* τ_γ-$b\,\mathrm{Cl}(A \cup B) = \tau_\gamma$-$b\,\mathrm{Cl}(A) \cup \tau_\gamma$-$b\,\mathrm{Cl}(B)$ *holds.*

(vii) τ_γ-$b\,\mathrm{Cl}(A \cap B) \subset \tau_\gamma$-$b\,\mathrm{Cl}(A) \cap \tau_\gamma$-$b\,\mathrm{Cl}(B)$ *holds.*

The proof of the following theorems are obvious and hence omitted.

Theorem 2.20 *For a subset* A *of a topological space* (X,τ) *and any operation* $\gamma : BO(X,\tau) \to \mathcal{P}(X)$, *the following relations holds.*

(i) $b\,\mathrm{Cl}(A) \subset b\,\mathrm{Cl}_\gamma(A) \subset \tau_\gamma$-$b\,\mathrm{Cl}(A) \subset \tau_\gamma$-$\mathrm{Cl}(A)$.

(ii) $b\,\mathrm{Cl}(A) \subset \mathrm{Cl}(A) \subset \mathrm{Cl}_\gamma(A) \subset \tau_\gamma$-$\mathrm{Cl}(A)$.

Theorem 2.21 *Let* A *be a subset of a topological space* (X,τ) *and* $\gamma : BO(X,\tau) \to \mathcal{P}(X)$ *an operation on* $BO(X,\tau)$. *The following properties are equivalent:*

(i) *A subset* A *is b-*γ*-open in* (X,τ);

(ii) $b\,\mathrm{Cl}_\gamma(X \backslash A) = X \backslash A$;

(iii) τ_γ-$b\,\mathrm{Cl}(X \backslash A) = X \backslash A$;

(iv) $X \backslash A$ *is b-*γ*-closed in* (X,τ).

Proof: Clear. □

Corollary 2.22 *Let* A *be a subset of a topological space* (X,τ) *and* $\gamma : BO(X,\tau) \to \mathcal{P}(X)$ *an operation on* $BO(X,\tau)$. *If* (X,τ) *is b-regular, then* $b\,\mathrm{Cl}(A) = b\,\mathrm{Cl}_\gamma(A) = \tau_\gamma$-$b\,\mathrm{Cl}(A)$.

In order to find the relationship between τ_γ-$b\,\mathrm{Cl}(A)$, $b\,\mathrm{Cl}_\gamma(A)$ and τ_γ-$\mathrm{Cl}(A)$, we introduce the following notions of b-open operations:

Definition 2.23 *An operation* $\gamma : BO(X,\tau) \to \mathcal{P}(X)$ *is called b-open if for each point* $x \in X$ *and for every b-open set* U *containing* x, *there exists a b-*γ*-open set* V *containing* x *such that* $V \subset U^\gamma$.

Theorem 2.24 *Let* $\gamma : BO(X,\tau) \to \mathcal{P}(X)$ *be an operation on* $BO(X,\tau)$ *and* A *be a subset of a topological space* (X,τ). *If* γ *is a b-open operation, then* $b\,\mathrm{Cl}_\gamma(A) = \tau_\gamma$-$b\,\mathrm{Cl}(A)$ *and* $b\,\mathrm{Cl}_\gamma(b\,\mathrm{Cl}_\gamma(A)) = b\,\mathrm{Cl}_\gamma(A)$ *hold and* $b\,\mathrm{Cl}_\gamma(A)$ *is b-*γ*-closed in* (X,τ).

Proof: By Theorem 2.20 (i) we have $b\,\mathrm{Cl}_\gamma(A) \subset \tau_\gamma - b\,\mathrm{Cl}(A)$. Suppose that $x \notin b\,\mathrm{Cl}_\gamma(A)$. Then there exists a b-open set U containing x such that $U^\gamma \cap A = \emptyset$. Since γ is b-open, by Definition 2.23, there exists a b-γ-open set V such that $x \in V \subset U^\gamma$ and so $V \cap A = \emptyset$. By Theorem 2.17(i). $x \notin \tau_\gamma$-$b\,\mathrm{Cl}(A)$. Hence we have that $b\,\mathrm{Cl}_\gamma(A) = \tau_\gamma$-$b\,\mathrm{Cl}(A)$. Furthermore, using the above result and Theorem 2.19, we have that $b\,\mathrm{Cl}_\gamma(b\,\mathrm{Cl}_\gamma(A)) = \tau_\gamma$-$b\,\mathrm{Cl}(\tau_\gamma$-$b\,\mathrm{Cl}(A)) = \tau_\gamma$-$b\,\mathrm{Cl}(A) = b\,\mathrm{Cl}_\gamma(A)$ and so $b\,\mathrm{Cl}_\gamma(A)$ is b-γ-closed in (X,τ). □

Definition 2.25 *A subset A of a topological space (X,τ) is said to be b-γ-generalized closed (for shortly, b-γ-g-closed) in (X,τ) if $b\,\mathrm{Cl}_\gamma(A) \subset U$ whenever $A \subset U$ and U is a b-γ-open set of (X,τ).*

The complement of a b-γ-g-closed set is called a b-γ-g-open set.

Example 2.26 *Let $X = \{a,b,c\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}\}$. Let $\gamma : BO(X,\tau) \to P(X)$ be operations defined as follows:*

$$A^\gamma = \begin{cases} A & \text{if } b \notin A, \\ cl(A) & \text{if } b \in A, \end{cases}$$

The collection of b-γ-closed sets in (X,τ) are $\{\emptyset, X, \{a\}, \{b\}, \{c\}\{a,c\}, \{b,c\}\}$.

Theorem 2.27 *Let $\gamma : BO(X,\tau) \to \mathcal{P}(X)$ be an operation on $BO(X,\tau)$ and A a subset of a topological space (X,τ). Then the following statements are equivalent:*

(i) A subset A is said to be, b-γ-g-closed in (X,τ));

(ii) τ_γ-$b\,\mathrm{Cl}(\{x\}) \cap A \neq \emptyset$ for every $x \in b\,\mathrm{Cl}_{\gamma b}(A)$;

(iii) $b\,\mathrm{Cl}_\gamma(A) \subset b - \tau_\gamma$-$Ker(A)$ holds, where b-τ_γ-$Ker(A) = \cap\{V : A \subset V, V \in \gamma BO(X)\}$ for any subset A of (X,τ).

Proof: (i)⇒(ii): Let A be a b-γ-g-closed set of (X,τ). Suppose that there exists a point $x \in b\,\mathrm{Cl}_\gamma(A)$ such that τ_γ-$b\,\mathrm{Cl}(\{x\}) \cap A = \emptyset$. By Theorem 2.19(i), τ_γ-$b\,\mathrm{Cl}(\{x\})$ is a b-γ-closed. Put $U = X \backslash \tau_\gamma$-$b\,\mathrm{Cl}(\{x\})$. Then, we have that $A \subset U$, $x \in U$ and U is a b-γ-open set of (X,τ). Since A is a b-γ-g-closed set, $b\,\mathrm{Cl}_\gamma(A) \subset U$. Thus, we have $x \notin b\,\mathrm{Cl}_\gamma(A)$. This is a contradicition. (ii)⇒(iii): Let $x \notin b\,\mathrm{Cl}_\gamma(A)$. By (ii), there exists a point z such that $z \in \tau_\gamma$-$b\,\mathrm{Cl}(\{x\})$ and $z \in A$. Let $U \in \gamma BO(X)$ be a subset of X such that $A \subset U$. Since $z \in U$ and $z \in \tau_\gamma$-$b\,\mathrm{Cl}(\{x\})$, we have that $U \cap \{x\} \neq \emptyset$. Namely, we show that $x \in b - \tau_\gamma$-$Ker(A)$. Therefore, we prove that $b\,\mathrm{Cl}_\gamma(A) \subset b$-$\tau_\gamma$-$Ker(A)$. (iii) ⇒ (i): Let U be any b-γ-open set such that $A \subset U$. Let x be a point such that $x \in b\,\mathrm{Cl}_\gamma(A)$. By (iii), $x \in b - \tau_\gamma - Ker(A)$ holds. Namely, we have that $x \in U$, because $A \subset U$ and $U \in \gamma BO(X)$. □

Theorem 2.28 *Let A be a subset of a topological space (X,τ). Then*

(i) A subset A is b-γ-g-closed in (X,τ), then $b\,\mathrm{Cl}_\gamma(A) \backslash A$ does not contain any nonempty b-γ-closed set.

(ii) *If* $\gamma : BO(X,\tau) \to P(X)$ *be a b-open operation on* (X,τ)*, then the converse of (i) is true.*

Proof: (i). Suppose that there exists a b-γ-g-closed set F such that $F \subset b\,\mathrm{Cl}_\gamma(A)\backslash A$. Then, we have that $A \subset X\backslash F$ and $X\backslash F$ is b-γ-open. It follows from assumption that $b\,\mathrm{Cl}_\gamma(A) \subset X\backslash F$ and so $F \subset (b\,\mathrm{Cl}_\gamma(A)\backslash A) \cap (X\backslash b\,\mathrm{Cl}_\gamma(A))$. Therefore, we have that $F = \emptyset$. (ii). Let U be a b-γ-open set such that $A \subset U$. Since γ is a b-open operation, it follows from Theorem 2.24, that $b\,\mathrm{Cl}_\gamma(A)$ is b-γ-closed in (X,τ). Thus using Theorem 2.6 (iii), we have that $b\,\mathrm{Cl}_\gamma(A) \cap X\backslash U$, say, F, is a b-γ-closed in (X,τ). Since $X\backslash U \subset X\backslash$, $F \subset b\,\mathrm{Cl}_\gamma(A)\backslash A$. Using the assumption of the converse of (i) above, $F = \emptyset$ and hence $b\,\mathrm{Cl}_\gamma(A) \subset U$. □

Definition 2.29 *A topological space* (X,τ) *is said to be a* b-γ-$T_{1/2}$ *space if every* b-γ-*g-closed set of* (X,τ) *is* b-γ-*closed.*

Theorem 2.30 *Let* A *be a subset of a topological space* (X,τ) *and* $\gamma : BO(X,\tau) \to P(X)$ *be an operation on* (X,τ)*, then for each point* $x \in X$*,* $\{x\}$ *is* b-γ-*g-closed or* $\{x\})$ *is* b-γ-*g-open set of* (X,τ)*.*

Proof: Suppose that $\{x\}$ is not b-γ-closed set. Then $\{x\}$ is not b-γ-closed set. Let U be any b-γ-open set such that $X\backslash\{x\} \subset U$. Then $U = X$ and so we have that $b\,\mathrm{Cl}_\gamma(X\backslash\{x\} \subset U$. Therefore, $X\backslash\{x\}$ is a b-γ-g-closed set in (X,τ). □

Theorem 2.31 *A topological space* (X,τ) *is said to be a* b-γ-$T_{1/2}$ *space if and only if for each point* $x \in X$*,* $\{x\}$ *is* b-γ-*open or* b-γ-*closed in* (X,τ)*. Then for each point* $x \in X$*,* $\{x\}$ *is b-open or b-closed in* (X,τ)*.*

Proof: Suppose that $\{x\}$ is not b-γ-closed set, by Theorem 2.30, $X\backslash\{x\}$ is a b-γ-g-closed. Since (X,τ) is a b-γ-$T_{1/2}$ space, $X\backslash\{x\}$ is b-γ-closed. Hence $\{x\}$ is a b-γ-open set. Conversely, let F be a b-γ-g-closed set in (X,τ). We shall prove that $b\,\mathrm{Cl}_\gamma(F) = F$. It is sufficient to show that $b\,\mathrm{Cl}_\gamma(F) \subset F$. Assume that there exists a point x such that $x \in b\,\mathrm{Cl}_\gamma(F)\backslash F$. Then by assumption, $\{x\}$ is b-γ-closed or b-γ-open. Case(i): $\{x\}$ is b-γ-closed set. For this case, we have a b-γ-closed set $\{x\}$ such that $\{x\} \subset b\,\mathrm{Cl}_\gamma(F)\backslash F$. This is a contradiction to Theorem 2.28 (i). Case(ii): $\{x\}$ is b-γ-open set. Then we have $x \in b\,\mathrm{Cl}_\gamma(F)$. Since $\{x\}$ is b-γ-open, $\{x\} \cap F = \emptyset$. This is a contradiction. Thus, we have that, $b\,\mathrm{Cl}_\gamma(F) = F$ and so F is b-γ-closed set. □

3. Separation axioms

Definition 3.1 *A topological space* (X,τ) *is said to be*

(i) b-γ-T_0 *if for any two distinct points* $x, y \in X$ *there exists b-open set* U *such that either* $x \in U$ *and* $y \notin U^\gamma$ *or* $y \in U$ *and* $x \notin U^\gamma$*.*

(ii) b-γ-T_1 *if for any two distinct points* $x, y \in X$ *there exist two* b*-open sets* U *and* V *containing* x *and* y *respectively such that* $y \notin U^\gamma$ *and* $x \notin V^\gamma$*.*

(iii) b-γ-T_2 *if for any two distinct points* $x, y \in X$ *there exist two* b*-open sets* U *and* V *containing* x *and* y *respectively such that* $U^\gamma \cap V^\gamma = \emptyset$*.*

Theorem 3.2 *Let* A *be a subset of a topological space* (X, τ) *and* $\gamma : BO(X, \tau) \to P(X)$ *be a* b*-open operation on* $BO(X, \tau)$*. Then* (X, τ) *is a* b-γ-T_0 *space if and only if for each pair* $x, y \in X$ *with* $x \neq y$*,* $b\,\mathrm{Cl}_\gamma(\{x\}) = b\,\mathrm{Cl}_\gamma(\{y\})$ *holds.*

Proof: Let x and y be any two distinct points of a γ-T_0' space. Then by definition, we assume that there exists a b-open set U such that $x \in U$ and $y \notin U^\gamma$. It follows from assumption that there exists a b-open set S such that $x \in S$ and $S \subset U^\gamma$. Hence, $y \in X \backslash U^\gamma \subset X \backslash S$. Because $X \backslash S$ is a b-γ-closed set, we obtain that $b\,\mathrm{Cl}_\gamma(\{y\}) \subset X \backslash S$ and so $b\,\mathrm{Cl}_\gamma(\{x\}) \neq b\,\mathrm{Cl}_\gamma(\{y\})$. Conversely, Suppose that $x \neq y$ for any $x, y \in X$. Then we have that, $b\,\mathrm{Cl}_\gamma(\{x\})) \neq b\,\mathrm{Cl}_\gamma(\{y\})$. Thus, we assume that there exists $z \in b\,\mathrm{Cl}_\gamma(\{x\})$ but $z \notin b\,\mathrm{Cl}_\gamma(\{y\})$. If $x \in b\,\mathrm{Cl}_\gamma(\{y\})$, then we get $b\,\mathrm{Cl}_\gamma(\{x\}) \subset b\,\mathrm{Cl}_\gamma(\{y\})$. This implies that $z \in b\,\mathrm{Cl}_\gamma(\{y\})$. This contradiction shows that $x \in b\,\mathrm{Cl}_\gamma(\{y\})$. Then there exists a b-open set W such that $x \in W$ and $W^\gamma \cap \{y\} = \emptyset$. Thus, we have that $x \in W$ and $y \notin W^\gamma$. Hence, (X, τ) is a b-γ-T_0 space. □

Theorem 3.3 *A space* (X, τ) *is* b-γ-T_1 *if and only if every singleton set of* X *is* b-γ*-closed.*

Proof: The proof follows from the Definition 3.1 (ii). □

Remark 3.4 *It is clear that* b-γ-$T_2 \Rightarrow b$-γ-$T_1 \Rightarrow b$-γ-$T_{1/2} \Rightarrow b$-γ-T_0*. But the reverse implications are not true in general.*

Example 3.5 *Let* $X = \{a, b, c\}$ *and* $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$*. Define an operation* $\gamma : BO(X, \tau) \to \mathcal{P}(X)$ *by*

$$A^\gamma = \begin{cases} A & \text{if } b \in A, \\ \mathrm{Cl}(A) & \text{if } b \notin A \end{cases}$$

Then this space is b-γ-T_0 *but not* b-γ-$T_{1/2}$*.*

Example 3.6 *Let* $X = \{a, b, c\}$ *and* $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$*. Define an operation* $\gamma : BO(X, \tau) \to \mathcal{P}(X)$ *by*

$$A^\gamma = \begin{cases} A & \text{if } b \in A, \\ \mathrm{Cl}(A) & \text{if } b \notin A \end{cases}$$

Then this space is b-γ-$T_{1/2}$ *but not* b-γ-T_1*.*

Example 3.7 *Let $X = \{a, b, c\}$ and τ be the discrete topology on X. Define an operation $\gamma : BO(X, \tau) \to \mathcal{P}(X)$ by*

$$A^\gamma = \begin{cases} A \cup \{c\} & \text{if } A = \{a\} \text{ or } \{b\}, \\ A \cup \{a\} & \text{if } A = \{c\}, \\ A & \text{if } A \neq \{a\}, \{b\}, \{c\} \end{cases}$$

Then this space is b-γ-T_1 but not b-γ-T_2.

4. b-(γ, β)-continuous functions

Definition 4.1 *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be b-(γ, β)-continuous if for each $x \in X$ and each b-open set V containing $f(x)$ there exists a b-open set U such that $x \in U$ and $f(U^\gamma) \subset V^\beta$.*

Example 4.2 *Let $X = Y = \{a, b, c\}$, $\tau = \sigma = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$. Let $\gamma : BO(X, \tau) \to P(X)$ be operations defined as follows:*

$$A^\gamma = \begin{cases} A & \text{if } b \notin A, \\ cl(A) & \text{if } b \in A, \end{cases}$$

Let $\beta : BO(Y, \sigma) \to P(Y)$ be operations defined as follows:

$$A^\gamma = \begin{cases} cl(A) & \text{if } b \notin A, \\ A & \text{if } b \in A, \end{cases}$$

Then the function $f : (X, \tau) \to (X, \sigma)$ defined as: $f(a) = b, f(b) = c$ and $f(c) = a$ is b-(γ, β)-continuous.

Theorem 4.3 *Let $f : (X, \tau) \to (Y, \sigma)$ be a b-(γ, β)-continuous function. Then the following hold:*

(i) $f(b\,\mathrm{Cl}_\gamma(A)) \subset b\,\mathrm{Cl}_\beta(f(A))$ holds for every subset A of (X, τ).

(ii) for every b-β-open set B of (Y, σ), $f^{-1}(B)$ is b-γ-open in (X, τ).

Proof: (i). Let $y \in f(b\,\mathrm{Cl}_\gamma(A))$ and V any b-β-open set containing y. Then there exists $x \in X$ and b-γ-open set U such that $f(x) = y$, $x \in U$ and $f(U^\gamma) \subset V^\beta$. Since $x \in b\,\mathrm{Cl}_\gamma(A)$, we have $U^\gamma \cap A \neq \emptyset$ and hence $\emptyset \neq f(U^\gamma \cap A) \subset f(U^\gamma) \cap f(A) \subset V^\beta \cap f(A)$. This implies $x \in b\,\mathrm{Cl}_\beta(f(A))$. Therefore, we have $f(b\,\mathrm{Cl}_\gamma(A)) \subset b\,\mathrm{Cl}_\beta(f(A))$.
(ii). Let B be a b-β-closed set in (Y, σ). By using (i) we have $f(b\,\mathrm{Cl}_\gamma(f^{-1}(B)) \subset b\,\mathrm{Cl}_\gamma(B) = B$. Therefore, we have $b\,\mathrm{Cl}_\gamma(f^{-1}(B)) = f^{-1}(B)$. Hence $f^{-1}(B)$ is b-γ-open in (X, τ). □

Theorem 4.4 *In the above theorem, suppose that (Y, σ) is b-β-regular space. Then $f : (X, \tau) \to (Y, \sigma)$ is b-(γ, β) continuous function. This implies (ii)⇒(i).*

Proof: Let $x \in X$ be any point in X and let V be b-open set contain x. Since (Y, σ) is a b-γ-regular space, $V \in \gamma BO(X)$, then $f^{-1}(V) \subset \gamma BO(X)$. Implies there exists a b-open set U such that $U^\gamma \subset f^{-1}(V)$ implies $f(U^\gamma) \subset V^\beta$. Therefore f is b-(γ, β)-continuous. Hence (i) and (ii) are equivalent to each other. □

Definition 4.5 *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be b-(γ, β)-closed if for any b-γ-closed set A of (X, τ), $f(A)$ is b-β-closed in (Y, β).*

Definition 4.6 *If f is b-(id, β)-closed, then $f(F)$ is b-β-closed for every b-closed set F of (X, τ).*

Remark 4.7 *If f is bijective and $f : (X, \tau) \to (Y, \sigma)$ is b-(id, β)-closed.*

Theorem 4.8 *If f is b-(γ, β)-continuous and b-(id, β)-closed then,*

(i) For every b-γ-g-closed set A of (X, τ), the image $f(A)$ is b-β-g-closed.

(ii) For every b-β-g-closed set B of (Y, σ), $f^{-1}(B)$ is b-γ-g-closed.

Proof: (i) Let V be b-β-open set in (Y, σ) such that $f(A) \subset V$ then by the Theorem **??** (ii), $f^{-1}(V)$ b-γ-open. Since A is a b-γ-g-closed and $A \subset f^{-1}(V)$ implies $b\,\mathrm{Cl}_\gamma(A) \subset f^{-1}(V)$. Hence $f(b\,\mathrm{Cl}_\gamma(A)) \subset V$. Therefore, $f(b\,\mathrm{Cl}_\gamma(A))$ b-β-closed and f is b-(id, β)-closed. Therefore, $b\,\mathrm{Cl}_\beta(f(A)) \subset b\,\mathrm{Cl}_\beta(f(b\,\mathrm{Cl}_\gamma(A))) = f(b\,\mathrm{Cl}_\gamma(A)) \subset V$. Hence $f(A)$ is b-β-g-closed. (ii) Let U be b-γ-open set in (X, τ) such that $f^{-1}(B) \subset U$. Let $F = b\,\mathrm{Cl}_\gamma(f^{-1}(B)) \cap U$, then F is b-closed in (X, τ). Since f is b-(id, β)-closed. Therefore $b\,\mathrm{Cl}_\beta(f(A)) \subset U$. Therefore $f^{-1}(B)$ is b-γ-g-closed in (X, τ). □

Theorem 4.9 *If $f : (X, \tau) \to (Y, \sigma)$ is b-(γ, β)-continuous and b-(id, β)-closed. Then*

(i) If f is injective and (Y, σ) is b-β-$T_{1/2}$, then (X, τ) is b-γ-$T_{1/2}$.

(ii) If f is surjective and (X, τ) is b-γ-$T_{1/2}$, then (Y, σ) is b-β-$T_{1/2}$.

Proof: (i) Let A be a b-γ-g-closed set of (X, τ). Then by assumption $f(A)$ is b-β-g-closed. Since (X, τ) is b-γ-$T_{1/2}$, $f(A)$ is b-β-closed. Therefore, $f^{-1}(f(A))$ is b-γ-closed. This implies that A is b-γ-closed. (ii) Let B is a b-β-g-closed set in (Y, σ). Then $f^{-1}(B)$ is b-γ-closed. Since (X, τ) is b-γ-$T_{1/2}$, $f^{-1}(B)$ is b-γ-closed. Therefore, $B(= f(f^{-1}(B))$ is b-β-closed in (Y, σ). Hence (Y, σ) is a b-β-$T_{1/2}$ space. □

Definition 4.10 *A graph $G(f)$ of $f : (X, \tau) \to (Y, \sigma)$ is called b-γ-closed if for each $(x, y) \in (X \times Y) \backslash G(f)$ there exists b-open sets U and V contains x and y, respectively such that $(U \times V^\gamma) \cap G(f) = \emptyset$.*

Definition 4.11 *An operation $\rho : BO(X \times Y, \tau \times \sigma) \to P(X \times Y)$ is said to be b-associated with γ and β, if $(U \times V)^{\rho} = U^{\gamma} \times V^{\beta}$ holds for each b-open set U in (X, τ) and V in (Y, σ).*

Remark 4.12 *In [3] we can find examples of operators that satisfying the above Definition.*

Definition 4.13 *The operation $\rho : BO(X \times Y, \tau \times \sigma) \to P(X \times Y)$ is said to be b-regular with respect to γ and β, if for each point $(x, y) \in X \times Y$ and each b-open set W containing (x, y), there exist b-open sets U in (X, τ) and V in (Y, σ) such that $x \in U$, $y \in V$ and $U^{\gamma} \times V^{\beta} \subset W^{\rho}$.*

Theorem 4.14 *Let $\rho : BO(X \times X) \to P(X \times X)$ be a b-associated operation with γ and β. If $f : (X, \tau) \to (Y, \sigma)$ is a b-(γ, β)-continuous and (Y, σ) is b-γ-T_2 space, then the set $A = \{(x, y) \in X \times Y : f(x) = f(y)\}$ is a b-ρ-closed set of $(X \times X, \tau \times \sigma)$.*

Proof: We have to show that $b\,\mathrm{Cl}_{\rho}(A) \subset A$. Let $(x; y) \in (X \times X)\backslash A$. Then, there exist two b-open set $U, V \in BO(Y)$ such that $f(x) \in U$, $f(y) \in V$ and $U^{\beta} \cap V^{\beta} = \emptyset$. Moreover, for U and V there exist W, $S \in BO(X)$ such that $x \in W$, $y \in S$ and $f(W^{\gamma} \subset U^{\beta}$ and $f(S^{\gamma} \subset V^{\beta}$. Therefore, we have $(x, y) \in W^{\gamma} \times S^{\gamma} = (W \times S)^{\rho} \cap A = \emptyset$. This shows that $(x, y) \notin b\,\mathrm{Cl}_{\gamma}(A)$. □

Theorem 4.15 *Let $\rho : BO(X \times Y, \tau \times \sigma) \to P(X \times Y)$ be an associated with γ and β. If $f : (X, \tau) \to (Y, \sigma)$ is b-(γ, β)-continuous and (Y, σ) is $b\tilde{U}\beta$-T_2, then the graph of f, $G(f) = \{(x, f(x)) \in X \times Y\}$ is a b-ρ-closed set of $(X \times Y, \tau \times \sigma)$.*

Proof: The proof is similar to that of Theorem 4.14. □

Definition 4.16 *Let (X, τ) be a topological space and γ be an operation on $BO(X)$. A subset K of X is said to be b-γ-compact, if for every b-open cover $\{G_i : i \in N\}$ of K there exists a finite subfamily $\{G_1, G_2, ..., G_n\}$ such that $K \subset G_1^{\gamma} \cup G_2^{\gamma} \cup .... \cup G_n^{\gamma}$.*

Theorem 4.17 *Suppose that γ is b-regular and $\rho : BO(X \times Y) \to P(X \times Y)$ is b-regular with respect to γ and β. Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping whose graph $G(f)$ is b-ρ-closed in $(X \times Y, \tau \times \sigma)$. If a subset B is b-γ-compact in (Y, σ), then $f^{-1}(B)$ is b-γ-closed in (X, τ).*

Proof: Suppose that $f^{-1}(B)$ is not b-γ-closed, then there exist a point x such that $x \in b\,\mathrm{Cl}_{\gamma}(f^{-1}(B))$ and $x \notin f^{-1}(B)$. Since $(x, b) \notin G(f)$ and each $b \in B$ and $b\,\mathrm{Cl}^{\gamma}(G(f)) \subset G(f)$, there exist a b-open set W of $(X \times Y, \tau \times \sigma)$ such that $(x, b) \in W$ and $W^{\rho} \cap G(f) = \emptyset$. By b-regularity of ρ, for each $b \in B$ we can take two b-open sets $U(b)$ and $V(b)$ in (Y, σ) such that $x \in U(b)$, $b \in V(b)$ and $U(b)^{\gamma} \times V(b)^{\beta} \subset W^{\rho}$. Then we have $f(U(b)^{\gamma}) \cap V(b)^{\beta} = \emptyset$. Since $\{V(b) : b \in B\}$ is b-open cover of B, then by b-γ-compactness there exists a finite number $b_1, b_2, ...b_n \in B$ such that

$B \subset V(b_1)^{\beta} \cup V(b_2)^{\beta} \cup .... \cup V(b_n)^{\beta}$. By the b-regularity of γ, there exist a b-open set U such that $x \in U$, $U^{\gamma} \subset U(b_1)^{\gamma} \cap U(b_2)^{\gamma} \cap ..... \cap U(b_n)^{\gamma}$. Therefore, we have $U^{\gamma} \cap f^{-1}(B) \subset U(b_i) \subset \bigcap_{i=1}^{n} U(b_i) \cap f^{-1}(V(b_i))^{\beta} = \emptyset$. This shows that $x \notin b\,\mathrm{Cl}_{\gamma}(f^{-1}(B))$. This a contradiction. Therefore, $f^{-1}(B)$ is b-γ-closed. □

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