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On faintly πg-continuous functions

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ABSTRACT: A new class of functions, called faintly πg-continuous functions, has been defined and studied. The relationships among faintly πg-continuous functions and πg-connected spaces, strongly πg-normal spaces and πg-compact spaces are investigated. Furthermore, the relationships between faintly πg-continuous functions and graphs are investigated.

Key Words: πg-open sets, πg-continuity, faintly πg-continuity

Contents

1. Introduction

Generalized open sets play a very important role in General Topology and they are now the research topics of many topologists worldwide. Indeed a significant theme in General Topology and Real analysis concerns the variously modified forms of continuity, separation axioms etc. by utiliaing generalized closed sets. In 1970, Levine [9] initiated the study of so-called g-closed sets, that is, a subset A of a topological space (X, τ) is g-closed if the closure of A included in every open superset of A and defined a $T_{1/2}$ space to be one in which the closed sets and the g-closed sets coincide. Zaitsev [17] defined the concept of π-closed sets and a class of topological spaces called quasi-normal spaces. Recently, Dontchev and Noiri [1] defined the notion of πg-closed sets and used this notion to obtain a characterization and some preservation theorems for quasi-normal spaces. In this paper, faintly πg-continuity is introduced and studied. Moreover, basic properties and preservation theorems of faintly πg-continuous functions are investigated and relationships between faintly πg-continuous functions and graphs are investigated.

2. Preliminaries

In the present paper, (X, τ) and (Y, σ) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), $\mathrm{Cl}(A)$, $\mathrm{Int}(A)$ and A^c denote the closure of A, the interior of A and

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the complement of A in X, respectively. A subset A of a space (X, τ) is said to be regular open [15] (resp. semiopen [8], preopen [11]) if $A = \mathrm{Int}(\mathrm{Cl}(A))$ (resp. $A \subset \mathrm{Cl}(\mathrm{Int}(A))$, $A \subset \mathrm{Int}(\mathrm{Cl}(A))$). The complement of a regular open (resp. preopen) set is called a regular closed (resp. preclosed) set. The finite union of regular open sets is said to be π-open [1]. The complement of π-open set is said to be π-closed. A subset A of a topological space (X, τ) is said to be πg-closed [17] if $\mathrm{Cl}(A) \subset U$ whenever $A \subset U$ and U is π-open. The complement of πg-closed set is called πg-open. The family of all πg-open (resp. πg-closed, πg-clopen) sets of (X, τ) is denoted by $\pi GO(X)$ (resp. $\pi GC(X), \pi GCO(X)$). Assume throughout this paper $\pi GO(X)$ is closed under arbitrary unions. The intersection (resp. union) of all πg-closed (resp. πg-open) sets of X containing (resp. contained in) $A \subset X$ is called the πg-closure [6] (resp. πg-interior [6]) of A and is denoted by πg-$\mathrm{Cl}(A)$ (resp. πg-$\mathrm{Int}(A)$). A function $f : (X, \tau) \to (Y, \sigma)$ is called πg-continuous [6] if $f^{-1}(V)$ is πg-open set in X for each open set V of Y. A point $x \in X$ is called a θ-cluster point of A if $\mathrm{Cl}(V) \cap A \neq \varnothing$ for every open set V of X containing x. The set of all θ-cluster points of A is called the θ-closure of A and is denoted by $\mathrm{Cl}_\theta(A)$. If $A = \mathrm{Cl}_\theta(A)$, then A is said to be θ-closed. The complement of θ-closed set is said to be θ-open. The union of all θ-open sets contained in a subset A is called the θ-interior of A and is denoted by $\mathrm{Int}_\theta(A)$. It follows from [16] that the collection of θ-open sets in a topological space (X, τ) forms a topology τ_θ on X. A subset $A \subset X$ is said to be δ-open [16] if it is the union of regular open sets of X. The complement of a δ-open set is called a δ-closed set. The intersection of all δ-closed sets containing A is called the δ-closure [16] of A and is denoted by $\mathrm{Cl}_\delta(A)$.

Definition 2.1 *The intersection of all preclosed sets containing the set A in a space X is called the preclosure of A and is denoted by $p\,\mathrm{Cl}(A))$ [11].*

Definition 2.2 *A subset A of a topological space (X, τ) is said to be πgp-closed [12] if $p\,\mathrm{Cl}(A) \subset U$ whenever $A \subset U$ and U is π-open in (X, τ).*

Definition 2.3 *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be faintly continuous [10] if $f^{-1}(V)$ is open in (X, τ) for every θ-open set V of (Y, σ).*

Definition 2.4 *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be slightly πg-continuous [13] if for each $x \in X$ and each clopen set V of Y containing $f(x)$, there exists $U \in \pi GO(X, x)$ such that $f(U) \subset V$.*

3. Faintly πg-continuous functions

Definition 3.1 *A function $f : (X, \tau) \to (Y, \sigma)$ is called faintly πg-continuous at a point $x \in X$ if for each θ-open set V of Y containing $f(x)$, there exists $U \in \pi GO(X, x)$ such that $f(U) \subset V$. If f has this property at each point of X, then it is said to be faintly πg-continuous.*

Theorem 3.2 *For a function $f : (X, \tau) \to (Y, \sigma)$, the following statements are equivalent:*

(i) f is faintly πg-continuous;

(ii) $f^{-1}(V)$ *is* πg*-open in* X *for every* θ*-open set* V *of* Y*;*

(iii) $f^{-1}(F)$ *is* πg*-closed in* X *for every* θ*-closed subset* F *of* Y*;*

(iv) $f : (X, \tau) \to (Y, \sigma_\theta)$ *is* πg*-continuous.*

(v) $\pi g\text{-Cl}(f^{-1}(B)) \subseteq f^{-1}(\text{Cl}_\theta(B))$ *for every subset* B *of* Y*;*

(vi) $f^{-1}(\text{Int}_\theta(G)) \subseteq \pi g\text{-Int}(f^{-1}(G))$ *for every subset* G *of* Y*.*

Proof: (i)⇒(ii): Let V be an θ-open set of Y and $x \in f^{-1}(V)$. Since $f(x) \in V$ and f is faintly πg-continuous, there exists $U \in \pi GO(X, x)$ such that $f(U) \subset V$. It follows that $x \in U \subset f^{-1}(V)$. Hence $f^{-1}(V)$ is πg-open in X.
(ii)⇒(i): Let $x \in X$ and V be an θ-open set of Y containing $f(x)$. By (ii), $f^{-1}(V)$ is a πg-open set containing x. Take $U = f^{-1}(V)$. Then $f(U) \subset V$. This shows that f is faintly πg-continuous.
(ii)⇒(iii): Let V be any θ-closed set of Y. Since $Y \setminus V$ is an θ-open set, by (ii), it follows that $f^{-1}(Y \backslash V) = X \backslash f^{-1}(V)$ is πg-open. This shows that $f^{-1}(V)$ is πg-closed in X.
(iii)⇒(ii): Let V be an θ-open set of Y. Then $Y \backslash V$ is θ-closed in Y. By (iii), $f^{-1}(Y \backslash V) = X \backslash f^{-1}(V)$ is πg-closed and thus $f^{-1}(V)$ is πg-open.
The other equivalances are Obvious. □

Definition 3.3 *A function* $f : (X, \tau) \to (Y, \sigma)$ *is said to be faintly* πgp*-continuous [4] if* $f^{-1}(V)$ *is* πgp*-open in* (X, τ) *for every* θ*-open set* V *of* (Y, σ)*.*

Remark 3.4 *Every faintly* πg*-continuous function is faintly* πgp*-continuous but not conversely as shown by the following example.*

Example 3.5 *Let* $X = \{a, b, c, d, e\}$*,* $Y = \{a, b, c\}$*,* $\tau = \{\varnothing, \{a, b\}, \{c, d\}, \{a, b, c, d\}, X\}$ *and* $\sigma = \{\varnothing, \{a\}, \{b, c\}, Y\}$*. Define a function* $f : (X, \tau) \to (Y, \sigma)$ *by* $f(a) = a$*,* $f(b) = f(c) = f(d) = f(e) = b$*. Then* f *is faintly* πgp*-continuous but not faintly* πg*-continuous.*

Definition 3.6 *A function* $f : (X, \tau) \to (Y, \sigma)$ *is called*

(i) $(\pi g, s)$*-continuous [5] if* $f^{-1}(V)$ *is* πgp*-closed in* (X, τ) *for every regular open set* V *of* (Y, σ)*.*

(ii) almost contra-super-continuous [2] if $f^{-1}(V)$ *is* δ*-closed in* (X, τ) *for every regular open set* V *of* (Y, σ)*.*

(iii) contra R-map [3] if $f^{-1}(V)$ *is regular closed in* (X, τ) *for every regular open set* V *of* (Y, σ)*.*

(iv) contra πg*-continuous [7] if* $f^{-1}(V)$ *is* πgp*-closed in* (X, τ) *for every open set* V *of* (Y, σ)*.*

Remark 3.7 *Every almost contra-super-continuous function, every contra πg-continuous function and every contra-R-map is $(\pi g, s)$-continuous function.*

Theorem 3.8 *Suppose that the collection of πg-closed sets of X is closed under arbitrary intersections. Then if $f : (X, \tau) \to (Y, \sigma)$ is $(\pi g, s)$-continuous, then f is faintly πg-continuous.*

Proof: It follows from Theorem 11 of [5] and Theorem 3.2. □

Theorem 3.9 *Every πg-continuous function is faintly πg-continuous.*

Proof: Clear. □

Remark 3.10 *The converse of Theorem 3.9 is not true in general as can be seen from the following example.*

Example 3.11 *Let $X = \{a, b, c\}$ with topologies $\tau = \{\varnothing, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\varnothing, \{a\}, \{b\}, \{a, b\}, \{b, c\}, X\}$. Then the identity function $f : (X, \tau) \to (X, \sigma)$ is faintly πg-continuous but not πg-continuous.*

Definition 3.12 *A topological space (X, τ) is said to be a πg-$T_{1/2}$-space [6] if every πg-closed subset of (X, τ) is in closed.*

Theorem 3.13 *Let (X, τ) be a πg-$T_{1/2}$-space. Then a function $f : (X, \tau) \to (Y, \sigma)$ is faintly πg-continuous if and only if it is faintly continuous.*

Proof: Follows from the Definition 3.12. □

Theorem 3.14 *If a function $f : (X, \tau) \to (Y, \sigma)$ is faintly πg-continuous and (Y, σ) is a regular space, then f is πg-continuous.*

Proof: Let V be any open set of Y. Since Y is regular, V is θ-open in Y. Since f is faintly πg-continuous, by Theorem 3.2, we have $f^{-1}(V)$ is πg-open and hence f is πg-continuous. □

Definition 3.15 *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be almost πgp-continuous [4] if $f^{-1}(\mathrm{Int}(\mathrm{Cl}(V)))$ is πgp-open in (X, τ) for every open set V of (Y, σ).*

Remark 3.16 *[4] Every πg-continuous function is almost πgp-continuous.*

Theorem 3.17 *If a function $f : (X, \tau) \to (Y, \sigma)$ is faintly πg-continuous and (Y, σ) is regular, then f is almost πg-continuous.*

Proof: It follows from Remark 3.16 and Theorem 3.14. □

Theorem 3.18 *If a function $f : (X, \tau) \to (Y, \sigma)$ is faintly πg-continuous, then it is slightly πg-continuous.*

Proof: Let $x \in X$ and V be any clopen subset of (Y, σ) containing $f(x)$. Then V is θ-open in Y. Since f is faintly πg-continuous, there exists $U \in \pi GO(X, x)$ containing x such that $f(U) \subset V$. This shows that f is slightly πg-continuous. $\square$

Definition 3.19 *Let (X, τ) be a topological space. Since the intersection of two clopen sets of (X, τ) is clopen, the clopen sets of (X, τ) may be use as a base for a topology for X. This topology is called the ultra-regularization of τ [8] and is denoted by τ_u. A topological space (X, τ) is said to be ultra-regular if $\tau = \tau_u$.*

Theorem 3.20 *Let (Y, σ) be an ultra-regular space. Then, for a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(i) f is πg-continuous;

(ii) f is faintly πg-continuous;

(iii) f is slightly πg-continuous.

Proof: The proof follows from definitions and Theorems 3.9, and 3.18. $\square$

Definition 3.21 *A πg-frontier [7] of a subset A of (X, τ) is πg-$Fr(A) = \pi g$-$\mathrm{Cl}(A) \cap \pi g$-$\mathrm{Cl}(X \backslash A)$.*

Theorem 3.22 *The set of all points $x \in X$ in which a function $f : (X, \tau) \to (Y, \sigma)$ is not faintly πg-continuous is the union of πg-frontier of the inverse images of θ-open sets containing $f(x)$.*

Proof: Suppose that f is not faintly πg-continuous at $x \in X$. Then there exists an θ-open set V of Y containing $f(x)$ such that $f(U)$ is not contained in V for each $U \in \pi GO(X, x)$ and hence $x \in \theta$-$\mathrm{Cl}(X \backslash f^{-1}(V))$. On the otherhand, $x \in f^{-1}(V) \subset \pi g$-$\mathrm{Cl}(f^{-1}(V))$ and hence $x \in \pi g$-$Fr(f^{-1}(V))$. Conversely, suppose that f is faintly πg-continuous at $x \in X$ and let V be a θ-open set of Y containing $f(x)$. Then there exists $U \in \pi GO(X, x)$ such that $U \subset f^{-1}(V)$. Hence $x \in \pi g$-$\mathrm{Int}(f^{-1}(V))$. Therefore, $x \notin \pi g$-$Fr(f^{-1}(V))$ for each θ-open set V of Y containing $f(x)$. $\square$

Theorem 3.23 *Let $f : (X, \tau) \to (Y, \sigma)$ be a function and $g : (X, \tau) \to (X \times Y, \tau \times \sigma)$ the graph function of f, defined by $g(x) = (x, f(x))$ for every $x \in X$. If g is faintly πg-continuous, then f is faintly πg-continuous.*

Proof: Let U be an θ-open set in (Y, σ), then $X \times U$ is a θ-open set in $X \times Y$. It follows that $f^{-1}(U) = g^{-1}(X \times U) \in \pi GO(X)$. This shows that f is faintly πg-continuous. $\square$

Definition 3.24 *A space (X, τ) is said to be πg-connected [[7], [5]] if X cannot be written as a disjoint union of two nonempty πg-open sets.*

Theorem 3.25 *If $f : (X, \tau) \to (Y, \sigma)$ is a faintly πg-continuous function and (X, τ) is a πg-connected space, then Y is a connected space.*

Proof: Assume that (Y, σ) is not connected. Then there exist nonempty open sets V_1 and V_2 such that $V_1 \cap V_2 = \varnothing$ and $V_1 \cup V_2 = Y$. Hence we have $f^{-1}(V_1) \cap f^{-1}(V_2) = \varnothing$ and $f^{-1}(V_1) \cup f^{-1}(V_2) = X$. Since f is surjective, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are nonempty subsets of X. Since V_i is open and closed, V_i is θ-open for each $i = 1, 2$. Since f is faintly πg-continuous, $f^{-1}(V_i) \in \pi GO(X)$. Therefore, (X, τ) is not πg-connected. This is a contradiction and hence (Y, σ) is connected. □

Definition 3.26 *A space (X, τ) is said to be πg-compact [6] (resp. θ-compact [14]) if each πg-open (resp. θ-open) cover of X has a finite subcover.*

Theorem 3.27 *The surjective faintly πg-continuous image of a πg-compact space is θ-compact.*

Proof: Let $f : (X, \tau) \to (Y, \sigma)$ be a faintly πg-continuous function from a πg-compact space X onto a space Y. Let $\{G_\alpha\colon \alpha \in I\}$ be any θ-open cover of Y. Since f is faintly πg-continuous, $\{f^{-1}(G_\alpha)\colon \alpha \in I\}$ is a πg-open cover of X. Since X is πg-compact, there exists a finit subcover $\{f^{-1}(G_i) : i = 1, 2, \ldots.\ n\}$ of X. Then it follows that $\{G_i\colon i = 1, 2, \ldots.\ n\}$ is a finite subfamily which cover Y. Hence Y is θ-compact. □

Definition 3.28 *A space (X, τ) is said to be:*

(i) countably πg-compact [7] (resp. countably θ-compact) if every πg-open [7] (resp. θ-open) countably cover of X has a finite subcover;

(ii) πg-Lindelof [7] (resp. θ-Lindelof) if every πg-open (resp. θ-open) cover of X has a countable subocver

Theorem 3.29 *Let $f : (X, \tau) \to (Y, \sigma)$ be a faintly πg-continuous surjective function. Then the following hold:*

(i) If X is πg-Lindelof, then Y is θ-Lindelof;

(ii) If X is countably πg-compact, then Y is countably θ-compact.

Proof: The proof is similar to Theorem 3.27. □

4. Separation Axioms

Definition 4.1 *A topological space (X, τ) is said to be:*

(i) πg-T_1 [7] (resp. θ-T_1) if for each pair of distinct points x and y of X, there exists πg-open (resp. θ-open) sets U and V containing x and y, respectively such that $y \notin U$ and $x \notin V$.

(ii) πg-T_2 [7] (resp. θ-T_2 [14]) if for each pair of distinct points x and y in X, there exists disjoint πg-open (resp. θ-open) sets U and V in X such that $x \in U$ and $y \in V$.

Theorem 4.2 *If $f : (X, \tau) \to (Y, \sigma)$ is faintly πg-continuous injection and Y is a θ-T_1 space, then X is a πg-T_1 space.*

Proof: Suppose that Y is θ-T_1. For any distinct points x and y in X, there exist $V, W \in \sigma_\theta$ such that $f(x) \in V$, $f(y) \notin V$, $f(x) \notin W$ and $f(y) \in W$. Since f is faintly πg-continuous, $f^{-1}(V)$ and $f^{-1}(W)$ are πg-open subsets of (X, τ) such that $x \in f^{-1}(V)$, $y \notin f^{-1}(V)$, $x \notin f^{-1}(W)$ and $y \in f^{-1}(W)$. This shows that X is πg-T_1. □

Theorem 4.3 *If $f : (X, \tau) \to (Y, \sigma)$ is faintly πg-continuous injection and Y is a θ-T_2 space, then X is a πg-T_2 space.*

Proof: Suppose that Y is θ-T_2. For any pair of distinct points x and y in X, there exist disjoint θ-open sets U and V in Y such that $f(x) \in U$ and $f(y) \in V$. Since f is faintly πg-continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are πg-open in X containing x and y, respectively. Therefore, $f^{-1}(U) \cap f^{-1}(V) = \varnothing$ because $U \cap V = \varnothing$. This shows that X is πg-T_2. □

Definition 4.4 *A space (X, τ) is called strongly θ-regular (resp. strongly πg-regular) if for each θ-closed (resp. πg-closed) set F and each point $x \notin F$, there exist disjoint θ-open (resp. πg-open) sets U and V such that $F \subset U$ and $x \in V$.*

Definition 4.5 *A space (X, τ) is said to be strongly θ-normal (resp. strongly πg-normal) if for any pair of disjoint θ-closed (resp. πg-closed) subsets F_1 and F_2 of X, there exist disjoint θ-open (resp. πg-open) sets U and V such that $F_1 \subset U$ and $F_2 \subset V$.*

Definition 4.6 *A function $f : (X, \tau) \to (Y, \sigma)$ is called:*

(i) πg-θ-open if $f(V) \in \sigma_\theta$ for each $V \in \pi GO(X)$.

(ii) πg-θ-closed if $f(V)$ is θ-closed in Y for each $V \in \pi GC(X)$.

Theorem 4.7 *If f is faintly πg-continuous πg-θ-open injective function from a strongly πg-regular space (X, τ) onto a space (Y, σ), then (Y, σ) is strongly θ-regular.*

Proof: Let F be an θ-closed subset of Y and $y \notin F$. Take $y = f(x)$. Since f is faintly πg-continuous, $f^{-1}(F)$ is πg-closed in X such that $f^{-1}(y) = x \notin f^{-1}(F)$. Take $G = f^{-1}(F)$. We have $x \notin G$. Since X is strongly πg-regular, then there exist disjoint πg-open sets U and V in X such that $G \subset U$ and $x \in V$. We obtain that $F = f(G) \subset f(U)$ and $y = f(x) \in f(U)$ such that $f(U)$ and $f(V)$ are disjoint θ-open sets. This shows that Y is strongly θ-regular. □

Theorem 4.8 *If f is faintly πg-continuous πg-θ–open injective function from a strongly πg-normal space (X, τ) onto a space (Y, σ), then Y is strongly θ-normal.*

Proof: Let F_1 and F_2 be disjoint θ-closed subsets of Y. Since f is faintly πg-continuous, $f^{-1}(F_1)$ and $f^{-1}(F_2)$ are πg-closed sets. Take $U = f^{-1}(F_1)$ and $V = f^{-1}(F_2)$. We have $U \cap V = \varnothing$. Since X is strongly πg-normal, there exist disjoint πg-open sets A and B such that $U \subset A$ and $V \subset B$. We obtain that $F_1 = f(U) \subset f(A)$ and $F_2 = f(V) \subset f(B)$ such that $f(A)$ and $f(B)$ are disjoint θ-open sets. Thus, Y is strongly θ-normal. □

Recall that for a function $f : (X, \tau) \to (Y, \sigma)$, the subset $\{(x, f(x)) : x \in X\} \subset X \times Y$ is called the graph of f and is denoted by $G(f)$.

Definition 4.9 *A graph $G(f)$ of a function $f : (X, \tau) \to (Y, \sigma)$ is said to be θ-πg-closed if for each $(x, y) \in (X \times Y)\backslash G(f)$, there exist $U \in \pi GO(X, x)$ and $V \in \sigma_\theta$ containing y such that $(U \times V) \cap G(f) = \varnothing$.*

Lemma 4.10 *A graph $G(f)$ of a function $f : (X, \tau) \to (Y, \sigma)$ is θ-πg-closed in $X \times Y$ if and only if for each $(x, y) \in (X \times Y) \setminus G(f)$, there exist $U \in \pi GO(X, x)$ and $V \in \sigma_\theta$ containing y such that $f(U) \cap V = \varnothing$.*

Proof: It is an immediate consequence of Definition 4.9. □

Theorem 4.11 *If $f : (X, \tau) \to (Y, \sigma)$ is faintly πg-continuous function and (Y, σ) is θ-T_2, then $G(f)$ is θ-πg-closed.*

Proof: Let $(x, y) \in (X \times Y)\backslash G(f)$, then $f(x) \neq y$. Since Y is θ-T_2, there exist θ-open sets V and W in Y such that $f(x) \in V$, $y \in W$ and $V \cap W = \varnothing$. Since f is faintly πg-continuous, $f^{-1}(V) \in \pi GO(X, x)$. Take $U = f^{-1}(V)$. We have $f(U) \subset V$. Therefore, we obtain $f(U) \cap W = \varnothing$. This shows that $G(f)$ is θ-πg closed. □

Theorem 4.12 *Let $f : (X, \tau) \to (Y, \sigma)$ has θ-πg-closed graph $G(f)$. If f is a faintly πg-continuous injection, then (X, τ) is πg-T_2.*

Proof: Let x and y be any two distinct points of X. Then since f is injective, we have $f(x) \neq f(y)$. Then, we have $(x, f(y)) \in (X \times Y) \backslash G(f)$. By Lemma 4.10, $U \in \pi GO(X)$ and $V \in \sigma_\theta$ such that $(x, f(y)) \in U \times V$ and $f(U) \cap V = \varnothing$. Hence $U \cap f^{-1}(V) = \varnothing$ and $y \notin U$. Since f is faintly πg-continuous, there exists $W \in \pi GO(X, y)$ such that $f(W) \subset V$. Therefore, we have $f(U) \cap f(W) = \varnothing$. Since f is injective, we obtain $U \cap W = \varnothing$. This implies that (X, τ) is πg-T_2. □

Definition 4.13 *A function $f : (X, \tau) \to (Y, \sigma)$ is said to have a $\widetilde{g}c$-closed graph if for each $(x, y) \in (X \times Y) \backslash G(f)$, there exists $U \in \pi GO(X, x)$ and an open set V of Y containing y such that $(U \times \mathrm{Cl}(V)) \cap G(f) = \varnothing$.*

Lemma 4.14 *Let $f : (X, \tau) \to (Y, \sigma)$ be a function. Then its graph $G(f)$ is $\widetilde{g}c$-closed in $X \times Y$ if and only if for each point $(x, y) \in (X \times Y) \setminus G(f)$, there exist $U \in \pi GO(X)$ and $V \in \sigma$ containing x and y, respectively, such that $f(U) \cap \mathrm{Cl}(V) = \varnothing$.*

Proof: It is an immediate consequence of Definition 4.13. □

Theorem 4.15 *If $f : (X, \tau) \to (Y, \sigma)$ is a surjective function with a $\widetilde{g}c$-closed graph, then (Y, σ) is Hausdorff.*

Proof: Let y_1 and y_2 be any distinct points of Y. Then since f is surjective, there exists $x_1 \in X$ such that $f(x_1) = y_1$; hence $(x_1, y_2) \in (X \times Y) \setminus G(f)$. Since $G(f)$ is $\widetilde{g}c$-closed, there exist $U \in \pi GO(X, x_1)$ and an open set V of Y containing y_2 such that $f(U) \cap \mathrm{Cl}(V) = \varnothing$. Therefore, we have $y_1 = f(x_1) \in f(U) \subset Y \setminus \mathrm{Cl}(V)$. Then there exists an open set H of Y such that $y_1 \in H$ and $H \cap V = \varnothing$. Moreoever, we have $y_2 \in V$ and V is open in Y. This shows that Y is Hausdorff. □

Theorem 4.16 *If $f : (X, \tau) \to (Y, \sigma)$ has an θ-πg-closed graph, it has a $\widetilde{g}c$-closed graph.*

Proof: Let $x \in X$ and $y \neq f(x)$, then $(x, y) \in (X \times Y) \backslash G(f)$. By Lemma 4.10, there exist $U \in \pi GO(X, x)$ and a θ-open set V containing y such that $f(U) \cap V = \varnothing$. Since V is θ-open, there exists an open set V_0 such that $y \in V_0 \subset \mathrm{Cl}(V_0) \subset V$ so that $f(U) \cap \mathrm{Cl}(V_0) = \varnothing$. It follows from Lemma 4.14 that the graph of f is $\widetilde{g}c$-closed. □

Corollary 4.17 *If $f : (X, \tau) \to (Y, \sigma)$ is a faintly πg-continuous and (Y, σ) is θ-T_2, then f has a $\widetilde{g}c$-closed graph.*

Proof: The proof follows from Theorems 4.11 and 4.16. □

Theorem 4.18 *If $f : (X, \tau) \to (Y, \sigma)$ has the θ-πg-closed graph , then $f(K)$ is θ-closed in (Y, σ) for each subset K which is πg-compact relative to X.*

Proof: Suppose that $y \notin f(K)$. Then $(x, y) \notin G(f)$ for each $x \in K$. Since $G(f)$ is θ-πg-closed, there exist $U_x \in \pi GO(X, x)$ and a θ-open set V_x of Y containing y such that $f(U_x) \cap V_x = \varnothing$ by Lemma 4.10. The family $\{U_x : x \in K\}$ is a cover of K by πg-open sets. Since K is πg-compact relative to (X, τ), there exists a finite subset K_0 of K such that $K \subset \cup\{U_x : x \in K_0\}$. Set $V = \cap\{V_x : x \in K_0\}$. Then V is a θ-open set in Y containing y. Therefore, we have $f(K) \cap V \subset [\cup_{x\in K_0} f(U_x)] \cap V \subset \cup_{x\in K_0}[f(U_x) \cap V] = \varnothing$. It follows that $y \notin \mathrm{Cl}_\theta(f(K))$. Therefore, $f(K)$ is θ-closed in (Y, σ). □

Corollary 4.19 *If $f : (X, \tau) \to (Y, \sigma)$ is faintly πg-continuous and (Y, σ) is θ-T_2, then $f(K)$ is θ-closed in (Y, σ) for each subset K which is πg-compact relative to (X, τ).*

Proof: The proof follows from Theorems 4.11 and 4.18. □

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