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Functions *almost* contra-super-continuity in m-spaces

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ABSTRACT: In this article, we study a generalizations of some class of functions that are in relation with the notions of continuity when we use the notions of minimal structures also its are characterized. Moreover we show that the notion of m-e^*-$T_{1/2}$ spaces, given by Ekici [6], is a particular case of the m-(e^*)-$T_{1/2}$ spaces when its are defined using the notion of m-generalized closed sets.

Key Words: m-e^*-$T_{1/2}$ spaces, m-e-open set, m-e^*-open set, m-a-open set.

Contents

1. Introduction

The concepts of δ-open sets was introduced and studied by Velicko [27] in 1968, which is a stronger notion of open set. The notion of generalized closed (briefly g-closed) sets was introduced by Levine [13] in 1970. In 1987, P. Bhartacharyya et al. [2] introduced the notion of semi-generalized closed sets in Topology. Furthermore, the notion of quasi θ-continuous functions [11](resp. semi generalized continuous maps and semi-$T_{1/2}$ spaces [26], α-continuous and α-open mappings [16]) is introduced and studied. Later, in [21] and [22] Popa and Noiri introduced the notions of minimal structures. After this work, various mathematicians turned their attention in introducing and studying diverse classes of sets and functions defined on an structure, because this notions are a natural generalization of many well known results related with generalized sets and several weaker forms of Continuity. Each one of these classes of sets is, in turn, used in order to obtain different separation properties and new form of continuity (see [3], [4], [10], [11], [17], [20], [21] for details). E. Ekici [5] in 2004, studied the $(\delta\text{-}pre, s)$-continuous functions on topological spaces and defined the m-e^*-$T_{1/2}$ spaces if every m-e^*-closed set is m-δ-closed. In this article we introduce and study the (m, m')-*almost* contra-super-continuous, (m, m')(a,s)-continuous, (m, m')-(δ-semi,s)continuous, (m, m')-

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(δ-pre,s)continuous, (m, m')-(e,s)continuous, (m, m')-(e^*,s)-continuous functions between spaces with minimal structure and study its relations with other class of functions and prove that the definition of m-e^*-$T_{1/2}$ spaces given by Ekici is a particular case of the m-(e^*)-$T_{1/2}$ spaces when it is defined in terms that each m-e^*-generalized closed set is m-e^*-closed.

2. Preliminaries

Let X be a nonempty set and let $m \subseteq P(X)$, where $P(X)$ denote the set of power of X. We say that m is an minimal structure on X (see [21] and [22]) if $\emptyset$ and X belong to m. The members of the minimal structure m are called m-open sets, and the pair (X, m) is called an m-space. The complement of an m-open set is called m-closed set.

Definition 2.1 *[14]* Let (X, m) be an m-space and $A \subset X$, the *m-interior* of A and the *m-closure* of A are defined, respectively, as

$$m\text{-}int(A) = \bigcup\{W : W \in m, W \subseteq A\},$$

$$m\text{-}cl(A) = \bigcap\{F : A \subseteq F, X \setminus F \in m\}$$

Theorem 2.2 *Let (X, m) be an m-space, A and B subsets of X. Then $x \in m\text{-}cl(A)$ if and only if $U \cap A \neq \emptyset$ for all $U \in m$ such that $x \in m$. And satisfy the following properties:*

1. $m\text{-}int(X \setminus A) = X \setminus m\text{-}cl(A)$.
2. $m\text{-}cl(X \setminus A) = X \setminus m\text{-}int(A)$.
3. *If* $A \subset B$, *then* $m\text{-}cl(A) \subset m\text{-}cl(B)$.
4. $m\text{-}cl(A) \cup m\text{-}cl(B) \subset m\text{-}cl(A \cup B)$.
5. $m\text{-}int(A \cap B) \subset m\text{-}int(A) \cap m\text{-}int(B)$.

Proof: It follows from by Lemma 3.1 [21], [22]. □

We can observe that, given a minimal structure m on a set X, if $A \subset X$, the $m\text{-}int(A)$ is not necessarily an element of m, but we assume on m the condition that is closed under arbitrary unions (this condition is called the Maki condition), then immediately, we have that $m\text{-}int(A)$ is an element of m, and hence $A \subset X$ is m-open if and only if $m\text{-}int(A) = A$ and m-closed if and only $m\text{-}cl(A) = A$.

Definition 2.3 Let (X, m) be an m-space. A subset A of X is said to be:

1. *m-regular open* if $A = m\text{-}int(m\text{-}cl(A))$.
2. *m-semiopen* if $A \subset m\text{-}cl(m\text{-}int(A))$.

3. m-α-*open* if $A \subset m\text{-}int(m\text{-}cl(m\text{-}int(A)))$.
4. m-*preopen* if $A \subset m\text{-}int(m\text{-}cl(A))$.
5. m-β-*open* if $A \subset m\text{-}cl(m\text{-}int(m\text{-}cl(A)))$.

Observe that if the minimal structure is a topology, then the above concepts are the same as the concepts of regular open [25], semiopen [12], α-open [19], preopen [15] and β-open [1]. The complement of an m-regular open (respectively m-semiopen, m-α-open, m-preopen, m-β-open) is called m-regular closed (respectively m-semiclosed, m-αclosed, m-preclosed, m-β-closed).

We denote by $m\text{-}RO(X)$, (respectively $m\text{-}RC(X)$, $m\text{-}SO(X)$, $m\text{-}\alpha O(X)$, $m\text{-}PO(X)$, $m\text{-}\beta O(X)$) the family of all m-regular open sets (respectively m-regular closed sets, m-semiopen sets, m-α-open sets, m-preopen sets, m-β-open sets) of X.

Definition 2.4 [23]Let (X, m) be an m-space and A be a subset of X. A point $x \in X$ is said to be an m-θ-semiclosure point of A if $m\text{-}cl(U) \cap A \neq \emptyset$ for all m-semiopen set U such that $x \in U$. The set of all m-θ-semiclosure points of A is denoted by $m\text{-}\theta\text{-}cl(A)$.

Definition 2.5 [23]Let (X, m) be an m-space and $A \subset X$. A is said to be an m-θ-semiclosed if $A = m\text{-}\theta\text{-}cl(A)$.

Definition 2.6 Let (X, m) be an m-space and $A \subseteq X$. The m-r-kernel of A, denoted by $m\text{-}r\text{-}ker(A)$, is defined as the intersection of all m-regular open sets that contain A, that is,

$$m\text{-}r\text{-}ker(A) = \bigcap\{U : U \in m\text{-}RO(X), A \subset U\}.$$

Definition 2.7 Let (X, m) be an m-space and $A \subseteq X$. The m-δ-*closure* and the m-δ-*interior* of the set A, are defined, respectively, as:

$$m\text{-}\delta\text{-}cl(A) = \{x \in X : A \cap m\text{-}int(m\text{-}cl(U)) \neq \emptyset, \forall\ U \in m, x \in U\}$$

$$m\text{-}\delta\text{-}int(A) = \bigcup\{W : W \in m\text{-}RO(X), W \subset A\}$$

Theorem 2.8 *Let A be a subset of an m-space X. The following statements hold:*

1. *If $A \subset B$ then $m\text{-}\delta\text{-}cl(A) \subset m\text{-}\delta\text{-}cl(B)$.*
2. *If $A \subset B$ then $m\text{-}\delta\text{-}int(A) \subset m\text{-}\delta\text{-}int(B)$.*
3. *$m\text{-}\delta\text{-}int(A) \subset m\text{-}int(A) \subset m\text{-}cl(A) \subset m\text{-}\delta\text{-}cl(A)$*
4. *$X \setminus m\text{-}\delta\text{-}int(A) = m\text{-}\delta\text{-}cl(X \setminus A)$.*
5. *$m\text{-}\delta\text{-}int(X \setminus A) = X \setminus m\text{-}\delta\text{-}cl(A)$.*

Definition 2.9 A subset A of an m-space X is said to be:

1. *m-δ-open* if $A = m\text{-}\delta\text{-}int(A)$.
2. *m-δ- preopen* if $A \subset m\text{-}int(m\text{-}\delta\text{-}cl(A))$.
3. *m-δ-semiopen* if $A \subset m\text{-}cl(m\text{-}\delta\text{-}int(A))$.

The complement of an m-δ-open set (respectively m-δ-semiopen set, m-δ-preopen set) is called m-δ-closed set (respectively m-δ-semiclosed set, m-δ-preclosed set). The family of all m-δ-open sets (respectively m-δ-preopen sets, m-δ-semiopen sets) are denoted by $m\text{-}\delta O(X)$ (respectively $m\text{-}\delta PO(X)$, $m\text{-}\delta SO(X)$).

Definition 2.10 A subset A of an m-space X is said to be:

1. *m-e-open* if $A \subset m\text{-}cl(m\text{-}\delta\text{-}int(A)) \cup m\text{-}int(m\text{-}\delta\text{-}cl(A))$.
2. *m-e^*-open* if $A \subset m\text{-}cl(m\text{-}int(m\text{-}\delta\text{-}cl(A)))$.
3. *m-a-open* if $A \subset m\text{-}int(m\text{-}cl(m\text{-}\delta\text{-}int(A)))$.

The complement of an m-e-open set (respectively m-e^*-open set, m-aopen set) is called m-e-closed set (respectively m-e^*-closed set, m-aclosed set). The family of all m-e-open sets (respectively m-e^*-open sets, m-a-open sets) are denoted by $m\text{-}eO(X)$ (respectively $m - e^*O(X)$, $m\text{-}aO(X)$).
The following diagram 1 shows the existence relation between the different sets defined above.
(M)=Maki condition

Example 2.11 *Let $\mathbb{R}$ be the set of real number, x, y two distinct points in $\mathbb{R}$ and $m = \{\mathbb{R}, \emptyset, \{x\}, \{y\}, \mathbb{R} \setminus \{x\}, \mathbb{R} \setminus \{y\}\}$. The set $A = \{x, y\}$ is an m-δ-open but is not m-regular open neither m-open.*

Example 2.12 *Let $\mathbb{R}$ be the set of real number, x, y two distinct points in $\mathbb{R}$ and $m = \{\mathbb{R}, \emptyset, \{x\}, \mathbb{R} \setminus \{y\}\}$. The set $A = \{x\}$ is an m-open but is not m-δ-open.*

Example 2.13 *Let $\mathbb{R}$ be the set of real number, x, y two distinct points in $\mathbb{R}$ and $m = \{\mathbb{R}, \emptyset, \{x\}, \{y\}\}$. Let a be a point in $\mathbb{R}$ distinct of x and y.*

1. *The set $A = \mathbb{R} \setminus \{a\}$ is m-δ-preopen but not is m-open.*
2. *The set $A = \{x, a\}$ is m-e-open but not is m-δ-preopen.*
3. *The set $A = \{x, y\}$ is m-α-open but not is m-open.*
4. *The set $A = \mathbb{R} \setminus \{x\}$ is m-semiopen but not is m-α-open.*
5. *The set $A = \mathbb{R} \setminus \{x\}$ is m-β-open but not is m-preopen.*

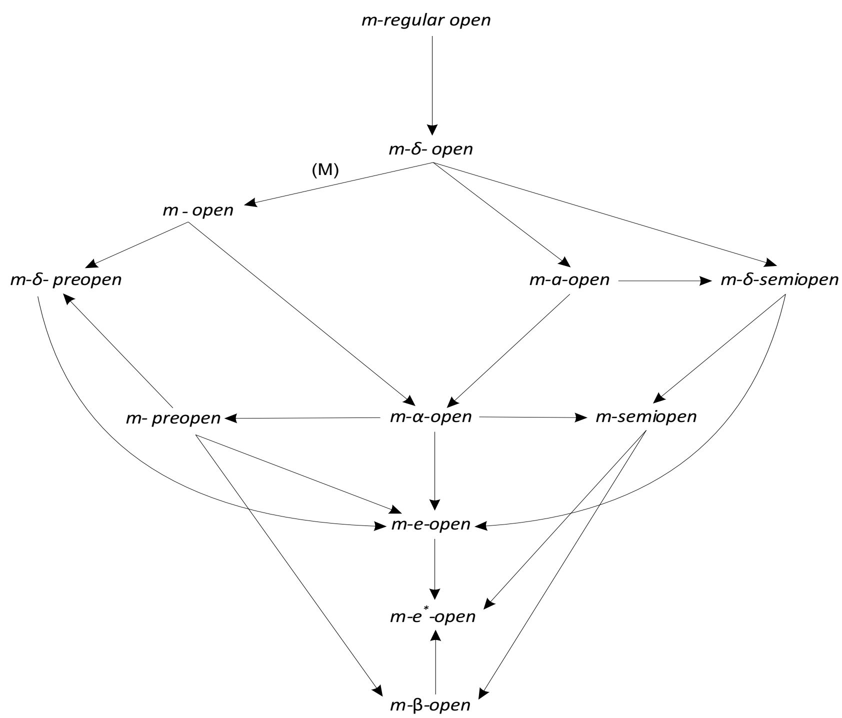

Figure 1: Relation among the sets in an m-spaces

Example 2.14 *Let* $\mathbb{R}$ *be the set of real number,* x, y, z *three distinct points in* $\mathbb{R}$ *and* $m = \{\mathbb{R}, \emptyset, \{x\}, \{y\}, \{x, y, z\}\}$.

1. *The set* $A = \{x, y, z\}$ *is* m*-a-open but is not* m*-*δ*-open.*

2. *The set* $A = \{x, z\}$ *is* m*-*δ*-semiopen but not is* m*-a-open.*

3. *The set* $A = \{x, y, z\}$ *is* m*-*δ*-semiopen but not is* m*-*δ*-open.*

Example 2.15 *Let* $\mathbb{R}$ *be the set of real number,* x, y *two distinct points in* $\mathbb{R}$ *and* $m = \{\mathbb{R}, \emptyset, \mathbb{R} \setminus \{x\}, \mathbb{R} \setminus \{y\}\}$.

1. *The set* $A = \{x\}$ *is* m*-*e^**-open but not is* m*-semiopen.*

2. *The set* $A = \{x\}$ *is* m*-*δ*-preopen but not is* m*-preopen.*

3. *The set* $A = \{x\}$ *is* m*-e-open but not is* m*-preopen.*

4. *The set* $A = \{x\}$ *is* m*-*e^**-open but not is* m*-*β*-open.*

5. *The set* $A = \{x\}$ *is* m*-e-open but not is* m*-*α*-open.*

6. *The set* $A = \mathbb{R} \setminus \{a\}$*, with* $a \notin \{x, y\}$ *is* m*-preopen but not is* m*-*α*-open.*

7. *The set* $A = \mathbb{R} \setminus \{a\}$*, with* $a \notin \{x, y\}$ *is* m*-*β*-open but not is* m*--semiopen.*

The following theorem shows that the collection of all m-e^*-open sets (respectively m-e-open sets, m-a-open sets) is an m structure that satisfy the Maki condition.

Theorem 2.16 *Let* (X, m) *be an* m*-space, the following statements hold:*

1. *The union of any collection of* m*-*e^**-open sets is an* m*-*e^**-open set.*

2. *The union of any collection of* m*-e-open sets is an* m*-e-open set.*

3. *The union of any collection of* m*-a-open sets is an* m*-a-open set.*

Proof: Let $\{U_\alpha\}_{\alpha \in J}$ any collection of m-e^*-open sets, then for each $\alpha \in J$, $U_\alpha \subset m\text{-}cl(m\text{-}int(m\text{-}\delta\text{-}cl(U_\alpha)))$, and hence $U_\alpha \subset \bigcup_{\alpha \in J} U_\alpha$
in consequence:

$$\begin{aligned} m\text{-}cl(m\text{-}int(m\text{-}\delta\text{-}cl(U_\alpha))) &\subset m\text{-}cl(m\text{-}int(m\text{-}\delta\text{-}cl(\bigcup_{\alpha \in J} U_\alpha))) \\ U_\alpha &\subset m\text{-}cl(m\text{-}int(m\text{-}\delta\text{-}cl(\bigcup_{\alpha \in J} U_\alpha))) \\ \bigcup_{\alpha \in J} U_\alpha &\subset m\text{-}cl(m\text{-}int(m\text{-}\delta\text{-}cl(\bigcup_{\alpha \in J} U_\alpha))) \end{aligned}$$

and we obtain that $\bigcup_{\alpha \in J} U_\alpha$ is an m-e^*-open set.
In analogue form follows (2) and (3).

□

We define the m-e-closure (respectively m-e^*-closure, m-a-closure) of a subset A of X, denoted by m-e-$cl(A)$ (respectively m-e^*-$cl(A)$, m-a-$cl(A)$), as the intersection of all m-e-closed sets (respectively m-e^*-closed sets, m-a-closed sets) containing A. Now using the above theorem, we obtain in a natural form that the m-e-$cl(A)$ (respectively m-e^*-$cl(A)$, m-a-$cl(A)$) is the smallest m-e-closed (respectively m-e^*-closed set, m-a-closed set) containing A.

In 2007 Salas, M. et. al. [24] studied and generalized the separation axioms using minimal structure. Now we define the notions of m-T_1 spaces and m-T_2 spaces given in [23].

Definition 2.17 *[23] Let* (X, m) *be an* m*-space,* X *is said to be:*

1. m*-*T_1 *if for each pair of different points* x, y *of* X*, there exist* m*-open sets* M *and* N *such that* $x \in M$*,* $y \in N$ *and* $y \notin M$ *and* $x \notin N$*.*

2. *m-T_2 If for each pair of different points x, y of X there exist m-open sets M and N such that $x \in M$, $y \in N$ and $M \cap N = \emptyset$.*

If (X, m) is an m space and consider the m spaces $(X, m\text{-}e^*O(X))$, $(X, m\text{-}eO(X))$ and $(X, m\text{-}aO(X)$. We obtain the concepts of m-e^*-T_1, m-e^*-T_2 spaces (respectively m-e-T_1, m-e-T_2, m-a-T_1, m-a-T_2) spaces, that are a natural generalizations of the definitions given by Ekici [9] when the minimal structure m is a topology.

3. Functions almost contra-super-continuous in m-spaces

Using the sets described in the above section, we define a new class of continuous functions between m spaces and we give some characterizations.

Definition 3.1 Let (X, m), (Y, m') two m-spaces and $f : X \to Y$ be a function between m-spaces, f is said to be:

1. (m, m')-*contra R-map* if $f^{-1}(A)$ is an m-regular closed set in X for all m'-regular open set A in Y.
2. (m, m')-*almost contra-super-continuous* if $f^{-1}(A)$ is an m-δclosed set in X for all m'-regular open set A in Y.
3. (m, m')-*(δ-semi,s)-continuous* if $f^{-1}(A)$ is an m-δ-semiclosed set in X for all m'-regular open set A in Y.
4. (m, m')-*(δ-pre,s)-continuous* if $f^{-1}(A)$ is an m-δ-preclosed set in X for all m'-regular open set A in Y.
5. (m, m')-*(e^*,s)-continuous* if the inverse image of each m'-regular open set in Y is an m-e^*-closed set in X.
6. (m, m')-*(e,s)-continuous* if the inverse image of each m'-regular open set in Y is an m-e-closed set in X.
7. (m, m')-*(a,s)-continuous* if the inverse image of each m'-regular open set in Y is an m-a-closed set in X.

Example 3.2 *Let $X = \{a, b, c\}$ and $m = \{\emptyset, X, \{a\}\}$. Let $f : (X, m) \to (X, m)$ the identity function, then f is (m, m)-almost contrasuper-continuous.*

Example 3.3 *Let $X = \{a, b, c, d\}$ and $m = \{\emptyset, X, \{a, b\}, \{c, d\}\}$. Let $f : (X, m) \to (X, m)$ the identity function, then f is (m, m)-contra R-map.*

Example 3.4 *Let $X = Y = \{a, b, c\}$ and $m = \{\emptyset, X, \{a\}, \{b, c\}\}$ Consider the function $f : (X, m) \to (X, m)$ defined as f(a)=a, f(b)=c, f(c)=a. Then, f is (m, m)-e^*-continuous and (m, m)-e-continuous.*

Example 3.5 *Let* $X = Y = \{a,b,c,d\}$ *and* $m = \{\emptyset, X, \{a\}, \{c\}, \{a,b\}, \{a,c\}, \{a,b,c\}, \{a,c,d\}\}$ *Consider the function* $f : (X,m) \to (X,m)$ *defined as* $f(a) = d$, $f(b) = c$, $f(c) = b$ *and* $f(d) = a$. *Then,* f *is* (m,m)*-almost* e^**-continuous.*

If in the above definitions the minimal structures m and m' are topologies on X and Y respectively we obtain the classical concepts of function contra R-map [8], *almost* contra-super-continuous [6], (δ-semi, s)-continuous [7], (δ-pre, s)-continuous [5], (e^*,s)-continuous [9], (e, s)-continuous [9] and (a, s)-continuous [9] respectively. The following theorem shows the existent relations between the different class of functions defined above.

Theorem 3.6 *Let* $f : X \to Y$ *be a function between* m*-spaces. The following statements hold:*

1. *If* f *is* (m,m')*-almost contra-super-continuous then* f *is* (m,m')*(a,s)-continuous.*
2. *If* f *is* (m,m')*-(a,s)-continuous then* f *is* (m,m')*-(*δ*-semi,s)continuous.*
3. *If* f *is* (m,m')*-(a,s)-continuous then* f *is* (m,m')*-(*δ*-pre,s)-continuous.*
4. *If* f *is* (m,m')*-(*δ*-semi,s)-continuous then* f *is* (m,m')*-(e,s)continuous.*
5. *If* f *is* (m,m')*-(*δ*-pre,s)-continuous then* f *is* (m,m')*-(e,s)-continuous.*
6. *If* f *is* (m,m')*-(e,s)-continuous then* f *is* (m,m')*-(*e^**,s)-continuous.*

The converse implications of the Theorem 3.6, in general are not true, as we can see in the following examples:

Example 3.7 *Let* $X = Y = \{a,b,c,d\}$ *and* $m = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}, \{a,b,d\}\}$ *and* $m' = \{\emptyset, X, \{a\}, \{c\}, \{a,b\}, \{a,c\}, \{a,b,c\}, \{a,c,d\}\}$. *Consider the function* $f : (X,m) \to (Y,m')$ *defined as f(a)=d, f(b)=d, f(c)=d, f(d)=c. Then,* f *is* (m,m')*-(a,s)-continuous but not is* (m,m')*-almost contra-super-continuous.*

Example 3.8 *Let* $X = \{a,b,c\}$ *and* $m = \{\emptyset, X, \{a\}, \{c\}, \{a,c\}, \{b,c\}\}$. *Let* $f : (X,m) \to (X,m)$ *the function defined as* $f(a) = b, f(b) = a, f(c) = c$. *Then* f *is* (m,m')*-(e,s)-continuous but not is* (m,m')*-(*δ*-semi,s)-continuous.*

Example 3.9 *Let* $X = \{a,b,c,d\}$ *and* $m = \{\emptyset, X, \{a\}, \{c\}, \{a,b\}, \{a,c\}, \{a,b,c\}, \{a,c,d\}\}$. *Then the identity function* $i : (X,m) \to (X,m)$ *is* (m,m')*-(*δ*-semi,s)-continuous but not is* (m,m')*-(a,s)-continuous.*

Example 3.10 *Let* $X = \{a,b,c,d\}$ *and* $m = \{\emptyset, X, \{a\}, \{c\}, \{a,b\}, \{a,c\}, \{a,b,c\}, \{a,c,d\}\}$. *The function* $f : (X,m) \to (X,m)$ *defined as* $f(a) = a$, $f(b) = c$, $f(c) = a$, $f(d) = c$ *is* (m,m')*-(*e^**,s)-continuous but not is* (m,m')*-(e,s)-continuous.*

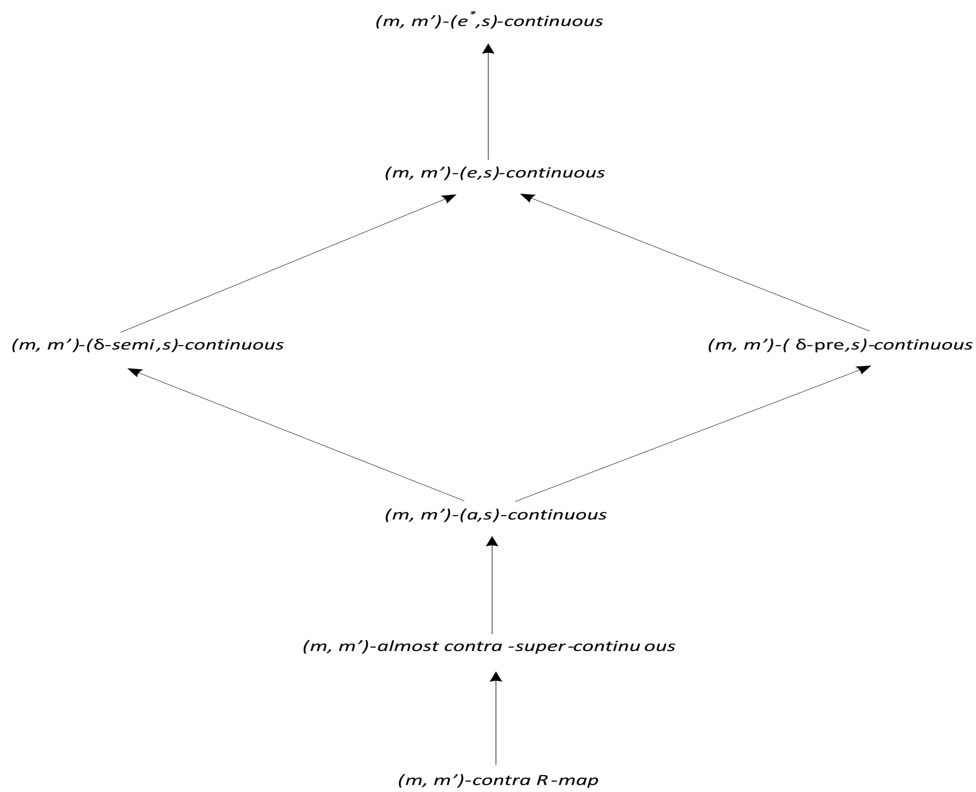

Figure 2: Relation among continuous function in m-spaces

Example 3.11 *Let* $X = \{a, b, c, d\}$ *and* $m = \{\emptyset, X, \{a, d\}, \{c\}, \{a, c, d\}\}$. *The function* $f : (X, m) \to (X, m)$ *defined as* $f(a) = d$, $f(b) = a$, $f(c) = b$, $f(d) = c$. *Is* (m, m')*-(*δ*-pre,s)-continuous but not is* (m, m')*-(a,s)-continuous.*

Example 3.12 *Let* $X = \{a, b, c\}$ *and* $m = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$. *Then the identity function* $i : (X, m) \to (X, m)$ *is* (m, m')*-(e,s)-continuous but not is* (m, m')*-(*δ*-pre,s)-continuous.*

Definition 3.13 Let (X, m), (Y, m') two m-spaces and $f : X \to Y$ be a function between m-spaces, f is said to be:

1. (m, m')-e^**-continuous* if $f^{-1}(A)$ is an m-e^*-open set for all m'-open set A .

2. (m, m')*-almost* e^**-continuous* if $f^{-1}(A)$ is an m-e^*-open set for all m'-regular open set A.

3. (m, m')*-almost e-continuous* if $f^{-1}(A)$ is an m-e-open set for all m'-regular open set A.

4. (m, m')-*almost a-continuous* if $f^{-1}(A)$ is an m-a-open set for all m'-regular open set A.

Definition 3.14 An m-space (X, m) is said to be *m-extremely disconnected* if the m-closure of all m-open set of X is m-open.

Example 3.15 *Consider* $X = \{a, b, c, d\}$ *and* $m = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{d\}, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, \{a, b, c\}\}$. *The m-space (X, m) is extremely disconnected.*

If the range of a function is m-extremely disconnected, then the concepts of functions (m, m')-(e^*,s)-continuous and (m, m')-*almost* e^*-continuous are equivalents. This fact we can see in the following theorem.

Theorem 3.16 *Let (X, m), (Y, m') two m-spaces and $f : X \to Y$ be a function. If (Y, m') is an m'-extremely disconnected, then f is (m, m')-(e^*,s)-continuous if and only if f is (m, m')-almost e^*-continuous.*

Proof: Suppose that f is (m, m')-(e^*,s)-continuous and let U be a subset of Y that is m' regular open, we want to prove that $f^{-1}(U)$ is m-e^*-open. Since Y is m'-extremaly disconnected then U es m'-open and m'-closed. In effect, $U = m'$-$int(m'$-$cl(U))$. But the m'-closure of U is an m'-open, follows that the m'-closure is m'-open and m'-closed. In consequence, $U = m'$-$cl(U)$, This proof that U is m'-open and m'-closed. By hypothesis U is m'-regular closed, then $f^{-1}(U)$ is m-e^*-open. Therefore, f is (m, m')-*almost* e^*-continuous.

Conversely, suppose that f is (m, m')-*almost* e^*-continuous and let $W \in m'$-$RC(Y)$. Since Y is m'-extremely disconnected, W is m'-regular open. Since f is (m, m')-*almost* e^*-continuous then $f^{-1}(W)$ is m-e^*-open. Therefore, f is (m, m')-(e^*,s)-continuous. □

The concept of e^*-$T_{1/2}$ spaces was introduced by Ekici [10] in the case of topological spaces, this concept characterize some classes of functions. Now we define a new class of spaces, the m-e^*-$T_{1/2}$ spaces and characterize some class of functions.

Definition 3.17 An m-space X is called m-e^*-$T_{1/2}$ if all m-e^*-closed set is m-δ-closed.

Example 3.18 *Let X={a,b,c} and* $m = \{\emptyset, X, \{a\}, \{b\}, \{c\}\}$. *$X$ is m-e^*-$T_{1/2}$ space.*

Theorem 3.19 *Let (X, m), (Y, m') m-spaces and $f : X \to Y$ be a function. If X is an m-e*-$T_{1/2}$ space, the following propositions are equivalent:*

1. *f is (m, m')-(e^*,s)-continuous.*
2. *f is (m, m')-(e,s)-continuous.*

3. *f is (m, m')-(δ-semi,s)-continuous.*
4. *f is (m, m')-(δ-pre,s)-continuous.*
5. *f is (m, m')-(a,s)-continuous.*
6. *f is (m, m')-almost contra-super-continuous.*

Proof: (6) → (5) Suppose that f is (m, m')-*almost* contra-super-continuous. Let W an m'-regular open set in Y, by hypothesis $f^{-1}(W)$ is m-δ-closed, that is,

$$f^{-1}(W) = m\text{-}\delta\text{-}cl(f^{-1}(W))$$

Since

$$\begin{aligned} m\text{-}cl(m\text{-}int(m\text{-}\delta\text{-}cl(f^{-1}(W))) &\subset m\text{-}cl(m\text{-}\delta\text{-}cl(f^{-1}(W))) \\ &\subset m\text{-}\delta\text{-}cl(m\text{-}\delta\text{-}cl(f^{-1}(W))) \\ &= m\text{-}\delta\text{-}cl(f^{-1}(W)) \\ &= f^{-1}(W) \end{aligned}$$

Then $f^{-1}(W)$ is an m-a-closed set. Therefore f is (m, m')-(a,s)continuous.

(5) → (3) Suppose that f is (m, m')-(a,s)-continuous. Let W an m'-regular open set in Y, by hypothesis $f^{-1}(W)$ is m-a-closed. $X \setminus f^{-1}(W)$ is m-a-open. Since

$$\begin{aligned} X \setminus f^{-1}(W) &\subset m\text{-}int(m\text{-}cl(m\text{-}\delta\text{-}int(X \setminus f^{-1}(W))) \\ &\subset m\text{-}cl(m\text{-}\delta\text{-}int(X \setminus f^{-1}(W)) \end{aligned}$$

Then $X \setminus f^{-1}(W)$ is an m-δ-semiopen. $f^{-1}(W)$ is m-δ-semiclosed. Therefore f is (m, m')-(δ-semi,s)-continuous.

(3) → (2) Suppose that f is (m, m')-(δ-semi,s)-continuous. Let W be an m'-regular open set in Y, by hypothesis $f^{-1}(W)$ is m-δ-semiclosed, using the fact that all m_X-δ-semiclosed is m_X-e-closed then $f^{-1}(W)$ is m_X-e-closed. Therefore f is (m, m')-(e,s)-continuous.

(2) → (1) Suppose that f is (m, m')-(e,s)-continuous. Let W be an m'-regular open set in Y, by hypothesis $f^{-1}(W)$ is m-e-closed, since all m-e-closed set is m-e^*-closed,then $f^{-1}(W)$ is m-e^*-closed. Therefore f is (m, m')-(e^*,s)-continuous.

(1) → (4) Suppose that f is (m, m')-(e^*,s)-continuous. Let W be an m'-regular open set in Y, by hypothesis $f^{-1}(W)$ is m-e^*-closed, since X is an m-e^*-$T_{1/2}$ space, $f^{-1}(W)$ is m-δ-closed. Now using the fact that all m-δ-closed set is m-δ-preclosed follows that $f^{-1}(W)$ is m-δ-preclosed. And hence f is (m, m')-(δ-pre,s)-continuous.

(4) → (6) Suppose that f is (m, m')-(δ-pre,s)-continuous. Let W be an m'-regular open set in Y, by hypothesis $f^{-1}(W)$ is m-δ-preclosed, follows that $f^{-1}(W)$ is

m-e^*-closed. Since X is an m-e^*-$T_{1/2}$ space, $f^{-1}(W)$ is m-δ-closed. Therefore f is (m, m')-*almost* contra-super-continuous. □

Theorem 3.20 *Let (Y, m') be an m'-regular space and $f : X \to Y$ be a function. If f is (m, m')-(e^*,s)-continuous then f is (m, m')-e^*-continuous.*

Proof: Let $x \in X$ an A be an m'-open set in Y such that $f(x) \in A$. Since Y is an m'-regular space, there exists an m'-open set G in Y that contain $f(x)$ such that $f(x) \in m'\text{-}cl(G) \subset A$. Since f is (m, m')-(e^*,s)-continuous, there exists $U \in m\text{-}e^*O(X, x)$ such that $f(U) \subset m'\text{-}cl(G)$. Follows that f is (m, m')-e^*-continuous. □

Theorem 3.21 *Let $f : X \to Y$ be a function between m-spaces. The following statements are equivalent:*

1. *f is (m, m')-(e^*,s)-continuous.*
2. *The inverse image of any m'-regular closed set in Y is m-e^*-open.*
3. *$f^{-1}(m\text{-}e^*\text{-}cl(U)) \subset m'\text{-}r\text{-}ker(f(U))$ for all $U \subset X$.*
4. *$m\text{-}e^*\text{-}cl(f^{-1}(A)) \subset f^{-1}(m'\text{-}r\text{-}ker(A))$ for all $A \subset Y$.*
5. *For each $x \in X$ and each $A \in m'\text{-}SO(Y)$, $f(x) \in A$ there exists an m-e^*-open set U in X, $x \in U$ such that $f(U) \subset m'\text{-}cl(A)$.*
6. *$f(m\text{-}e^*\text{-}cl(P)) \subset m'\text{-}\theta\text{-}s\text{-}cl(f(P))$ for all $P \subset X$.*
7. *$m\text{-}e^*\text{-}cl(f^{-1}(R)) \subset f^{-1}(m'\text{-}\theta\text{-}s\text{-}cl(R))$ for all $R \subset Y$.*
8. *$m\text{-}e^*\text{-}cl(f^{-1}(A)) \subset f^{-1}(m'\text{-}\theta\text{-}s\text{-}cl(A))$ for all m'-open subset A of Y.*
9. *$m\text{-}e^*\text{-}cl(f^{-1}(A)) \subset f^{-1}(m'\text{-}s\text{-}cl(A))$ for all m'-open set A in Y.*
10. *$m\text{-}e^*\text{-}cl(f^{-1}(A)) \subset f^{-1}(m'\text{-}int(m'\text{-}cl(A)))$ for all m'-open set A in Y.*
11. *The inverse image of any m'-θ-semi-open set in Y is m-e^*-open.*
12. *$f^{-1}(A) \subset m\text{-}e^*\text{-}int(f^{-1}(m'\text{-}cl(A)))$ for all $A \in m'\text{-}SO(Y)$.*
13. *The inverse image of any m'-θ-semi-closed set in Y is m-e^*-closed.*
14. *$f^{-1}(m'\text{-}int(m'\text{-}cl(A)))$ is m-e^*-closed for all m'-open set A in Y.*
15. *$f^{-1}(m'\text{-}cl(m'\text{-}int(F)))$ is m-e^*-open for all m'-closed set F in Y.*
16. *$f^{-1}(m'\text{-}cl(U))$ is m-e^*-open for all $U \in m'\text{-}\beta O(Y)$.*
17. *$f^{-1}(m'\text{-}cl(U))$ is m-e^*-open for all $U \in m'\text{-}SO(Y)$.*
18. *$f^{-1}(m'\text{-}int(m'\text{-}cl(U)))$ is m-e^*-closed for all $U \in m'\text{-}PO(Y)$.*

Proof: (1) $\rightarrow$ (2) Let W be an m'-regular closed set in Y, $Y \setminus W$ is an m'-regular open set in Y. Since f is (m, m')-(e^*,s)-continuous then $f^{-1}(Y \setminus W)$ is m-e^*-closed. But

$$f^{-1}(Y \setminus W) = X \setminus f^{-1}(W).$$

Follows that $f^{-1}(W)$ is m-e^*-open.

(2) $\rightarrow$ (1) Let W be an m'-regular open in Y, $Y \setminus W$ is m'-regular closed in Y. By hypothesis, $f^{-1}(Y \setminus W)$ is m-e^*-open. Follows that

$$f^{-1}(Y \setminus W) = X \setminus f^{-1}(W)$$

and $f^{-1}(W)$ is m-e^*-closed.

(2) $\rightarrow$ (3) Let $U \subset X$, and suppose that $y \notin m'$-r-$ker(f(U))$. Then there exists an m'-regular closed set F, with $y \in F$ such that $f(U) \cap F = \emptyset$. Follows that, $U \cap f^{-1}(F) = \emptyset$ and m-e^*-$cl(U) \cap f^{-1}(F) = \emptyset$. Therefore, $f(m\text{-}e^*\text{-}cl(U)) \cap F = \emptyset$ and $y \notin f(m\text{-}e^*\text{-}cl(U))$. We obtain that $f^{-1}(m\text{-}e^*\text{-}cl(U)) \subset m'\text{-}r\text{-}ker(f(U))$.

(3) $\rightarrow$ (4) Let $A \subset Y$. By (3), $f(m\text{-}e^*\text{-}cl(f^{-1}(A))) \subset m'\text{-}r\text{-}ker(A)$. Follows that, $m\text{-}e^*\text{-}cl(f^{-1}(A)) \subset f^{-1}(m'\text{-}r\text{-}ker(A))$.

(4) $\rightarrow$ (1) Let $A \subset m'$-$RO(Y)$. By hypothesis

$$m\text{-}e^*\text{-}cl(f^{-1}(A)) \subset f^{-1}(m'\text{-}r\text{-}ker(A)) = f^{-1}(A)$$

since $f^{-1}(A) \subset m\text{-}e^*\text{-}cl(f^{-1}(A))$ we obtain that $m\text{-}e^*\text{-}cl(f^{-1}(A)) = f^{-1}(A)$. Therefore, $f^{-1}(A)$ is m-e^*-closed in X.

(5) $\rightarrow$ (6) Let $P \subset X$, $x \in m\text{-}e^*\text{-}cl(P)$ and $G \in m'\text{-}SO(Y)$, $f(x) \in G$. Using (5) there exists $U \in m\text{-}e^*O(X)$, $x \in U$ such that $f(U) \subset m'\text{-}cl(G)$. Since $x \in m\text{-}e^*\text{-}cl(P)$, $U \cap P \neq \emptyset$ and $\emptyset \neq f(U) \cap f(P) \subset m'\text{-}cl(G) \cap f(P)$. Follows that $f(x) \in m'\text{-}\theta\text{-}s\text{-}cl(f(P))$ and therefore $f(m\text{-}e^*\text{-}cl(P)) \subset m'\text{-}\theta\text{-}s\text{-}cl(f(P))$.

(6) $\rightarrow$ (7) Let $R \subset Y$. Using hypothesis, we obtain that

$$f(m\text{-}e^*\text{-}cl(f^{-1}(R))) \subset m'\text{-}\theta\text{-}s\text{-}cl(f(f^{-1}(R))) \subset m'\text{-}\theta\text{-}s\text{-}cl(R)$$

and $m\text{-}e^*\text{-}cl(f^{-1}(R)) \subset f^{-1}(m'\text{-}\theta\text{-}s\text{-}cl(R))$.

(7) $\rightarrow$ (5) Let $A \in m'\text{-}SO(Y)$, $f(x) \in A$. Since $m'\text{-}cl(A) \cap (Y \setminus m'\text{-}cl(A)) = \emptyset$, we obtain that $f(x) \notin m'\text{-}\theta\text{-}s\text{-}cl(Y \setminus m'\text{-}cl(A))$ and $x \notin f^{-1}(m'\text{-}\theta\text{-}s\text{-}cl(Y \setminus m'\text{-}cl(A)))$. By (7) $x \notin m\text{-}e^*\text{-}cl(f^{-1}(Y \setminus m'\text{-}cl(A)))$ and therefore there exists $U \in m\text{-}e^*O(X)$, $x \in U$ such that $U \cap f^{-1}(Y \setminus m'\text{-}cl(A)) = \emptyset$ and $f(U) \cap (Y \setminus m'\text{-}cl(A)) = \emptyset$. Follows that $f(U) \subset m'\text{-}cl(A)$.

(7) → (8) Since m-e^*-$cl(f^{-1}(R)) \subset f^{-1}(m'$-$\theta$-$s$-$cl(R))$ for all $R \subset Y$, in particular,

$$m\text{-}e^*\text{-}cl(f^{-1}(A)) \subset f^{-1}(m'\text{-}\theta\text{-}s\text{-}cl(A)).$$

for all m'-open subset A in Y.

(8) → (9) Since m-θ-s-$cl(A) = m$-s-$cl(A)$ for all m-open set A, then m-e^*-$cl(f^{-1}(A)) \subset f^{-1}(m$-$s$-$cl(A))$.

(9) → (10) Since m'-s-$cl(A) = m'$-$int(m'$-$cl(A))$ for all m'-open set A in Y, then m-e^*-$cl(f^{-1}(A)) \subset f^{-1}(m'$-$int(m'$-$cl(A)))$.

(10) → (1) Let $A \in m'$-$RO(Y)$. By (10) we have m-e^*-$cl(f^{-1}(A)) \subset f^{-1}(m'$-$int(m'$-$cl(A))) = f^{-1}(A)$. Therefore, $f^{-1}(A)$ is m-e^*-closed and then f is (m, m')-(e^*,s)-continuous.

(2) → (11) Since any m-θ-semi open set is the union of m-regular closed and the result follows.

(11) → (5) Let $x \in X$ and $A \in m'$-$SO(Y)$, $f(x) \in A$. Since the m'-$cl(A)$ is m'-θ-semi open in Y, then there exists an m-e^*-open set U in X, such that $x \in U \subset f^{-1}(m'$-$cl(A))$. In consequence $f(U) \subset m'$-$cl(A)$.

(5) → (12) Let $A \in m'$-$SO(Y)$ and $x \in f^{-1}(A)$. then $f(x) \in A$, using hypothesis, there exists an m-e^*-open set U in X, such that $x \in U$ and $f(U) \subset m'$-$cl(A)$. Follows that $x \in U \subset f^{-1}(m'$-$cl(A))$ and therefore $x \in m$-$e^*int(f^{-1}(m'$-$cl(A)))$. Follows $f^{-1}(A) \subset m$-e^*-$int(f^{-1}(m'$-$cl(A)))$.

(12) → (2) Let F any m'-regular closed set in Y. Since $F \in m'$-$SO(Y)$, then $f^{-1}(F) \subset m$-e^*-$int(f^{-1}(F))$. Follows that $f^{-1}(F)$ is m-e^*-open in X.

(11) → (13) Let W be an m'-θ-semi closed set in Y. $Y \setminus W$ is m'-θ-semi open. By hypothesis $f^{-1}(Y \setminus W)$ is m-e^*-open. Follows that, $f^{-1}(W)$ is m-e^*-closed.

(13) → (11) Let W be an m'-θ-semi open set in Y. $Y \setminus W$ is m'-θ-semiclosed. By hypothesis $f^{-1}(Y \setminus W)$ is m-e^*-closed. Follows that, $f^{-1}(W)$ is m-e^*-open.

(1) → (14) Let A be an m'-open set in Y. Since m'-$int(m'$-$cl(A))$ is m'-regular open, we obtain that $f^{-1}(m'$-$int(m'$-$cl((A)))$ is m-e^*-closed.

(14) → (1) Let A be an m'-regular open set in Y. By hypothesis, $f^{-1}(A)$ is m-e^*-closed in X. But $f^{-1}(A) = f^{-1}(m'$-$int(m'$-$cl((A)))$. Therefore $f^{-1}(m'$-$int(m'$-$cl((A)))$ is m-e^*-closed.

(2) → (15) Let F be m'-regular closed in Y. By hypothesis $f^{-1}(F)$ is m-e^*-open. Since $f^{-1}(F) = f^{-1}(m'$-$cl(m'$-$int((F)))$ then

$f^{-1}(m'\text{-}cl(m'\text{-}int((F)))$ is m-e^*-open.

(15) $\to$ (2) Let F be an m'-regular closed in Y. By hypothesis, we obtain $f^{-1}(m'\text{-}cl(m'\text{-}int((F)))$ is m-e^* -open. But $f^{-1}(F) = f^{-1}(m'\text{-}cl(m'\text{-}int((F)))$ that is m-e^*-open. Therefore, $f^{-1}(F)$ is m-e^*-open.

(2) $\to$ (16) Let $U \in m'\text{-}\beta O(Y)$. Always we obtain that $m'\text{-}int(m'\text{-}cl(U)) \subset m'\text{-}cl(U)$, follows that $m'\text{-}cl(m'\text{-}int(m'\text{-}cl(U))) \subset m'\text{-}cl(U)$. Since $U \in m'\text{-}\beta O(Y)$ then $U \subset m'\text{-}cl(m'\text{-}int(m'\text{-}cl(U)))$, and obtain that

$$m'\text{-}cl(U) \subset m'\text{-}cl(m'\text{-}cl(m'\text{-}int(m'\text{-}cl(U)))) = m'\text{-}cl(m'\text{-}int(m'\text{-}cl(U)))$$

Follows that $m'\text{-}cl(U) = m'\text{-}cl(m'\text{-}int(m'\text{-}cl(U)))$, from here $m'\text{-}cl(U)$ is m'-regular closed and therefore $f^{-1}(m'\text{-}cl(U))$ is m-e^*-open.

(16) $\to$ (17) Since $m'\text{-}SO(Y) \subset m'\text{-}\beta O(Y)$ then $f^{-1}(m'\text{-}cl(U))$ is m-e^*-open.

(17) $\to$ (18) Let $U \in m'\text{-}PO(Y)$. Since $Y \setminus m'\text{-}int(m'\text{-}cl(U))$ is an m'-regular closed set and therefore is m'-semi open. We obtain that $X \setminus f^{-1}(m'\text{-}int(m'\text{-}cl(U))) = f^{-1}(Y \setminus m'\text{-}int(m'\text{-}cl(U))) = f^{-1}(m'\text{-}cl(Y \setminus m'\text{-}int(m'\text{-}cl(U)))) \in m\text{-}e^*O(X)$. Follows that $f^{-1}(m'\text{-}int(m'\text{-}cl(U)))$ is m-e^*-closed.

(18) $\to$ (1) Let $U \in m'\text{-}RO(Y)$. Then $U \in m'\text{-}PO(Y)$ and therefore $f^{-1}(U) = f^{-1}(m'\text{-}int(m'\text{-}cl(U)))$ is m-e^*-closed set in X. □

Remark 3.22 *In the same form as we characterize the functions (m, m')-(e^*,s) -continuous, in the Theorem 3.21, we can obtain similar characterizations for functions (m, m')-*
(e,s)-continuous (respectively (m, m')-(a,s)-continuous) changing in the Theorem 3.21, e^* by e (respectively a).

Corollary 3.23 *Let $f : X \to Y$ be a function between m-spaces. The following statements are equivalent:*

1. *f is (m, m')-(e^*,s)-continuous.*
2. *$f^{-1}(m'\text{-}\alpha\text{-}cl(A))$ is an m-e^*-open set in X for all $A \in m'\text{-}\beta O(Y)$.*
3. *$f^{-1}(m'\text{-}p\text{-}cl(A))$ is an m-e^*-open set in X for all $A \in m'\text{-}SO(Y)$.*
4. *$f^{-1}(m'\text{-}s\text{-}cl(A))$ is an m-e^*-open set in X for all $A \in m'\text{-}PO(Y)$.*
5. *$m\text{-}e^*\text{-}cl(f^{-1}(R)) \subset f^{-1}(m'\text{-}\theta\text{-}s\text{-}cl(R))$ for all $R \subset m'\text{-}SO(Y)$.*
6. *$m\text{-}e^*\text{-}cl(f^{-1}(R)) \subset f^{-1}(m'\text{-}\theta\text{-}s\text{-}cl(R))$ for all $R \subset m'\text{-}PO(Y)$.*
7. *$m\text{-}e^*\text{-}cl(f^{-1}(R)) \subset f^{-1}(m'\text{-}\theta\text{-}s\text{-}cl(R))$ for all $R \subset m'\text{-}\beta O(Y)$.*

Proof: Is an immediate consequence of the Theorem 3.21. □

Remark 3.24 *There are an analogue to the Corollary 3.23 for functions (m, m') -(e, s)-continuous (respectively (m, m') -(a, s)-continuous), only change in Corollary 3.23, e^* by e (respectively by a).*

4. Compact sets in m-spaces

Definition 4.1 A subspace A of an m-space X is said to be m-compact relative to X if for all covering $\{P_i : i \in I\}$ of A by m-open sets in X, there exists a finite subset I_0 of I such that $A \subset \bigcup\{P_i : i \in I_0\}$. An m-space X is said to be m-compact if all m-open covering of X contains a finite subcollection that also covers X.

Remark 4.2

1. *If in Definition 4.1, the covering of A consist of m-e^*-open sets in X, then the notion of A m-compact relative to X is called m-e^*-compact relative to X*
2. *If in Definition 4.1, the covering of A consist of m-e-open sets in X, then the notion of A m-compact relative to X is called m-e-compact relative to X*
3. *If in Definition 4.1, the covering of A consist of m-a-open sets in X, then the notion of A m-compact relative to X is called m-a-compact relative to X*

Theorem 4.3 *Let (X, m) be an m-space and A a subset of X. Then*

1. *If A is m-e^*-compact then A is m-compact.*
2. *If A is m-e^*-compact then A is m-e-compact.*
3. *If A is m-e-compact then A is m-a-compact.*

Proof: (1) Suppose that A is m-e^*-compact and $\{U_\alpha : \alpha \in J\}$ is a covering of A by sets m-open in X. Since all m-open set is m-e^*-open, then $\{U_\alpha : \alpha \in J\}$ is a covering of A by sets m-e^*-open in X; hence a finite subcollection $\{U_{\alpha_1}, U_{\alpha_2}, ..., U_{\alpha_n}\}$ covers A. Therefore A is m-compact.

In analogue form follows (2) and (3) □

Theorem 4.4 *Every m-e^*-closed subset of an m-e^*-compact space is m-e^*-compact.*

Proof: Let A be an m-e^*-closed subset of an m-e^*-compact space. Given a covering $\{M_i : i \in I\}$ of A by sets m-e^*-open in X. Then $(X \setminus A) \cup (\bigcup_{i \in I} M_i)$ is a covering of X by sets m-e^*-open in X. Some finite subcollection of $(X \setminus A) \cup (\bigcup_{i \in I} M_i)$ covers X. If this subcollection contains the set $X \setminus A$ discard $X \setminus A$; otherwise leave the subcollection alone. The resulting collection is a finite subcollection of $\{M_i : i \in I\}$ that covers A. □

Theorem 4.5 *Every m-e-closed subset of an m-e-compact space is m-e-compact.*

Proof: The proof is similar to the proof of the Theorem 4.4. □

Theorem 4.6 *Every m-a-closed subset of an m-a-compact space is m-a-compact.*

Proof: The proof is similar to the proof of the Theorem 4.4. □

Definition 4.7 An m-space X is said to be *m-s-closed* if for all covering of X by sets m-regular closed in X contains a finite subcollection that also covers X.

Theorem 4.8 *Let $f : X \to Y$ be surjective and (m, m')-(e^*,s)-continuous function. If X is an m-e^* compact space then Y is m'-s-closed.*

Proof: Let X be an m-e^*-compact space, $f : X \to Y$ an (m, m')-(e^*,s)-continuous and surjective. Given a covering $\{M_i : i \in I\}$ of Y by sets m'-regular closed. Since f is (m, m')-(e^*,s)-continuous, then $\{f^{-1}(M_i) : i \in I\}$ is a covering of X by sets m-e^*-open. Since X is m-e^*-compact, there exists a finite subset I_0 of I such that $X = \bigcup_{i \in I_0} f^{-1}(M_i)$. Now using the fact that f is surjective, Y is m'-s-closed. □

Theorem 4.9 *Let $f : X \to Y$ be surjective and (m, m')-(e,s)-continuous function. If X is an m-e-compact space then Y is m'-s-closed.*

Proof: The proof is similar to the proof of the Theorem 4.8 □

Theorem 4.10 *Let $f : X \to Y$ surjective and (m, m')-(a,s)-continuous function. If X is an m-a-compact space then Y is m'-s-closed.*

Proof: The proof is similar to the proof of the Theorem 4.8 □

The following Theorems are particular case of the Theorem 4.2 [21].

Theorem 4.11 *Let X, Y be m- spaces and f be an (m, m')-e^*-continuous function. If A is m-e^*-compact relative to X, then the image $f(A)$ is m'-compact relative to Y.*

Proof: Let $A \subset X$ be m-e^*-compact and $f : X \to Y$ be (m, m')-e^*-continuous. Let $\{U_\alpha : \alpha \in J\}$ be a covering of the set $f(A)$ by sets m'-open. The collection $\{f^{-1}(U_\alpha) : \alpha \in J\}$ is a covering of A by sets m-e^*-open in X. Hence finitely many of them, say $\{f^{-1}(U_{\alpha_1}), f^{-1}(U_{\alpha_2}), ..., f^{-1}(U_{\alpha_n})\}$ cover A. Then the sets $\{U_{\alpha_1}, U_{\alpha_2}, ..., U_{\alpha_n}\}$ cover $f(A)$. □

Theorem 4.12 *Let X, Y be m-spaces and f be an (m, m')-e-continuous. If A is m-e-compact relative to X, then the image $f(A)$ is m'-compact relative to Y.*

Proof: The proof is similar to the proof of the Theorem 4.11. □

Theorem 4.13 *Let X, Y be m-spaces and f be an (m, m')-a-continuous function. If A is m-a-compact relative to X, then the image $f(A)$ is m'-compact relative to Y.*

Proof: The proof is similar to the proof of the Theorem 4.11. □

Definition 4.14 An m-space X is said to be *weakly Hausdorff* if each element of X is the intersection of m-regular closed sets.

Theorem 4.15 *Let $f : X \to Y$ be a function. If f is injective, (m, m')-(e^*,s)-continuous and Y is weakly Hausdorff, then X is an m-e^*-T_1 space.*

Proof: Consider $x \neq y$ in X, since f is injective $f(x) \neq f(y)$ in Y, so there exist $P, R \in m'$-$RC(Y)$ such that $f(x) \in P$, $f(y) \notin P$, $f(x) \notin R$ and $f(y) \in R$. Since f is (m, m')-(e^*,s)-continuous, $f^{-1}(P)$ and $f^{-1}(R)$ are m-e^*-open subsets of X such that $x \in f^{-1}(P)$, $y \notin f^{-1}(P)$, $x \notin f^{-1}(R)$ y $y \in f^{-1}(R)$. Follows that X is an m-e^*-T_1 space. □

Theorem 4.16 *Let $f : X \to Y$ be a function. If f is injective, (m, m')-(e,s)-continuous and Y is weakly Hausdorff, then X is an m-e-T_1 space.*

Proof: The proof is similar to the proof of the Theorem 4.15. □

Theorem 4.17 *Let $f : X \to Y$ be a function. If f is injective, (m, m')-(a,s)-continuous and Y is weakly Hausdorff, then X is an m-a-T_1 space.*

Proof: The proof is similar to the proof of the Theorem 4.15. □

Definition 4.18 *[23]An m-space X is said to be s-Urysohn if for each pair x, y of distinct points of X, there exist $M \in m$-$SO(X)$ and $N \in m$-$SO(X)$, $x \in M$, $y \in N$, such that m-$cl(M) \cap m$-$cl(N) = \emptyset$.*

Theorem 4.19 *Let $f : X \to Y$ be a function. If f is injective, (m, m')-(e^*,s)-continuous and Y is s-Urysohn, then X is an m-e^*-T_2 space.*

Proof: Let Y be *s-Urysohn*. For each pair x, y of distinct points of X, $f(x) \neq f(y)$. Hence there exist $P \in m'$-$SO(Y)$, $f(x) \in P$ and $R \in m'$-$SO(Y)$, $f(y) \in R$, such that m'-$cl(P) \cap m'$-$cl(R) = \emptyset$. Now using that f is (m, m')-(e^*,s)-continuous, then there exist m-e^*-open sets A, B in X such that $x \in A$, $y \in B$ and satisfy that $f(A) \subset m$-$cl(P)$ and $f(B) \subset m$-$cl(R)$. It follows $A \cap B = \emptyset$ and therefore, X is an m-e^*-T_2 space. □

Theorem 4.20 *Let $f : X \to Y$ be a function. If f is injective, (m, m')-(e,s)-continuous and Y is s-Urysohn, then X is an m-e-T_2 space.*

Proof: The proof is similar to the proof of the Theorem 4.19. □

Theorem 4.21 *Let $f : X \to Y$ be a function. If f is injective, (m, m')-(a,s)-continuous and Y is s-Urysohn, then X is m-a-T_2 space.*

Proof: The proof is similar to the proof of the Theorem 4.19. □

5. Generalized closed sets and $T_{1/2}$ spaces

In general topology, the notion of $T_{1/2}$ spaces is defined if every generalized closed set is closed. In 2007 Salas, M. Carpintero, C. and Rosas, E [24] studied and generalize these spaces using minimal structure. In 2007 Ekici [10] introduced the notion of $T_{1/2}$ spaces associated with the δ-closed sets. In this section, we compare the Definition of $m - T_{1/2}$ given by Salas, M. Carpintero, C. and Rosas, E [24] and the Definition 3.17.

Definition 5.1 *[18] Let (X, m) be an m-space, A a subset of X, A is said to be an m-generalized closed set, abbreviate m-g-closed, if m-cl$(A) \subset U$ whenever $A \subset U$ and $U \in m$.*

Definition 5.2 *[4] Let (X, m) be an m-space, X is said to be an m-$T_{1/2}$ space, if every m-g-closed set is m-closed.*

Theorem 5.3 *(Teorema 4.4 [24]) Let (X, m) be an m-space, X is an m-$T_{1/2}$space if and only if the following statements hold:*

1. *For each $x \in X$ we have that $\{x\}$ es m-open or m-closed.*
2. *The m structure satisfy the Maki condition.*

If m is an minimal structure on X consider the m space $(X, m\text{-}e^*O(X))$, denoted the generalized closed sets under this minimal structure by m-e^*-g-closed, and the m-$T_{1/2}$ space by m-(e^*)-$T_{1/2}$.
Our interest is to show that the Definition 5.2 of m-(e^*)-$T_{1/2}$ space is more general that Definition 3.17 of m-e^*-$T_{1/2}$ space.

Theorem 5.4 *Let (X, m) be an m-space, if X is m-e^*-$T_{1/2}$ space under Definition 3.17 then each $x \in X$, $\{x\}$ is m-δ-open or m-δ-closed.*

Proof: Let $x \in X$ and suppose that $\{x\}$ is not m-δ-open, then $X \setminus \{x\}$ is not m-δ-closed, follows that the only m-δ-closed set that contains $X \setminus \{x\}$ is X. Hence,

$$m\text{-}\delta\text{-}cl(X \setminus \{x\}) = X$$

$$m\text{-}cl(m\text{-}int(m\text{-}\delta\text{-}cl(X \setminus \{x\}))) = X$$

$$X \setminus \{x\} \subset m\text{-}cl(m\text{-}int(m\text{-}\delta\text{-}cl(X \setminus \{x\})))$$

Therefore, $X \setminus \{x\}$ is m-e^*-open and $\{x\}$ is m-e^*-closed. Since X is an m-e^*-$T_{1/2}$ space under Definition 3.17 then $\{x\}$ is m-δ-closed. □

Corollary 5.5 *Let (X, m) be an m-space where m satisfies the Maki condition, if X is an m-e^*-$T_{1/2}$space under Definition 3.17 then for each $x \in X$, $\{x\}$ is m-e^*-open or m-e^*-closed.*

Proof: By Theorem 5.4, each unitary set is m-δ-open or m-δ-closed. Now using the Maki condition each unitary set is m-e^*-open or m-e^*-closed. □

Theorem 5.6 *Let m be a minimal structure on X that satisfy Maki condition, if X is an m-e^*-$T_{1/2}$ space under Definition 3.17 then X is an m-(e^*)-$T_{1/2}$ space under Definition 5.2.*

Proof: Suppose that X is an m-e^*-$T_{1/2}$ space under Definition 3.17. By Theorem 5.4 each unitary set $\{x\}$ is m-δ-open or m-δ-closed, follows that each unitary set $\{x\}$ is m-e^*-open or m-e^*-closed. Now using Theorem 5.3 we obtain that X is an m-e^*-$T_{1/2}$ space under Definition 5.2. □

The converse of the above theorem is not true in general, as shown in the following examples.

Example 5.7 Let $X = \{a, b, c, d\}$ and $m = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$, then the m-space (X, m) is m-e^*-$T_{1/2}$ under the Definition 5.2 but not is m-e^*-$T_{1/2}$ under the Definition 3.17. In effect, the m-closed sets are: $\{\emptyset, X, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, \{c, d\}, \{b, d\}, \{a, d\}, \{d\}\}$.
m-$e^*O(X) = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, d\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, \{a, c, d\}\}$.
m-$e^*C(X) = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, d\}, \{b, c\}, \{c, d\}, \{d\}, \{a, b, d\}, \{b, c, d\}, \{a, c, d\}\}$.
m-e^*-*g-closed* $= \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, d\}, \{b, c\}, \{c, d\}, \{d\}, \{a, b, d\}, \{b, c, d\}, \{a, c, d\}\}$
m-$\delta C(X) = \{\emptyset, X, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{b, c, d\}, \{a, c, d\}\}$.

Observe that the m-e^*-generalized closed sets are all m-e^*-closed. Therefore, (X, m) is an m-(e^*)-$T_{1/2}$ space under Definition 5.2 but not is an m-e^*-$T_{1/2}$ space under Definition 3.17, because $\{a\}$ is an m-e^*-closed set that not is m-δ-closed. It said that the Definition 5.2 is more stronger that Definition 3.17.

Example 5.8 Consider the set of the real numbers $\mathbb{R}$ with the m-structure $m = \{\emptyset, \mathbb{R}, \{0\}\}$.
Let $x \in \mathbb{R}$, and consider $y \in \mathbb{R}$, if $y \neq 0$ then the only m-open set that contains y is $U = \mathbb{R}$, follows that

$$\{x\} \cap m\text{-}int(m\text{-}cl(U)) = \{x\}$$

If $y = 0$ then the m-open set that contains y are $V = \{0\}$ and $V = \mathbb{R}$, follows that

$$\{x\} \cap m\text{-}int(m\text{-}cl(V)) = \{x\}$$

.
In any case, we obtain that $y \in m\text{-}\delta\text{-}cl(\{x\})$, and hence $m\text{-}\delta\text{-}cl(\{x\}) = \mathbb{R}$. An follows that $\{x\} \subset m\text{-}cl(m\text{-}int(m\text{-}\delta\text{-}cl(\{x\})))$ and therefore $\{x\}$ is an $m\text{-}e^*$-open set.
Using the Theorem 5.3 follows that $(\mathbb{R}, m)$ is an $m\text{-}(e^*)\text{-}T_{1/2}$ space under Definition 5.2.
But $(\mathbb{R}, m)$ is not an $m\text{-}e^*\text{-}T_{1/2}$ space under the Definition 3.17, because $\mathbb{R}$-$\{0\}$ is $m\text{-}e^*$-closed but not is $m\text{-}\delta$-closed, since $m\text{-}\delta\text{-}cl(\mathbb{R}\text{-}\{0\}) = \mathbb{R}$.

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