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On semi star generalized closed sets in bitopological spaces

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ABSTRACT: K. Chandrasekhara Rao and K. Joseph [5] introduced the concepts of semi star generalized open sets and semi star generalized closed sets in a topological space. The same concept was extended to bitopological spaces by K. Chandrasekhara Rao and K. Kannan [6,7]. In this paper, we continue the study of $\tau_1\tau_2$-s^*g closed sets in bitopology and we introduced the newly related concept of pairwise s^*g-continuous mappings. Also S^*GO-connectedness and S^*GO-compactness are introduced in bitopological spaces and some of their properties are established.

Key Words: pairwise s^*g-continuous functions, pairwise S^*GO-connected, pairwise S^*GO-compact space, $\tau_1\tau_2$-s^*g closed sets, pairwise gs-continuous functions.

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1. Introduction

In 1963, J.C. Kelly [15] initiated the study of bitopological spaces. Maheshwari and Prasad [19] introduced semi open sets in bitopological spaces in 1977 and further properties of this notion were studied by Bose [29] in 1981 and Fukutake [11] define one kind of semi open sets in bitopological spaces and studied their properties in 1989.

Mean while Fukutake introduced generalized closed sets and pairwise generalized closure operator [12] in bitopological spaces in 1986. He defined a set A of a topological space X to be $\tau_i\tau_j$-generalized closed set (briefly $\tau_i\tau_j$-g - closed) if τ_j-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_i-open in X. Also, he defined a new closure operator and strongly pairwise $T_{\frac{1}{2}}$-space.

Semi generalized closed sets and generalized semi closed sets are extended to bitopological settings by F. H. Khedr and H. S. Al-saadi [14]. K. Chandrasekhara

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Rao and K. Kannan [6,7] introduced the concepts of semi star generalized closed sets in bitopological spaces.

The connectedness and components were introduced by Pervin [25] in bitopological spaces while Reilly and Young [28] introduced the quasi components in bitopological spaces. We find more detailed study of connecteness in bitopological spaces in Birsan [2], Reilly [27] and Swart [30]. Das [9] initiated the study of semi connectedness in topological spaces and Dorsett [10] continued the study of the same further. M.N. Mukherjee [24] introduced pairwise semi connectedness in bitopological spaces. In the sequel pairwise S^*GO-connected space is introduced in fifth section.

In 1995, sg-compact spaces were introduced independently by Caldas [4]. According to him, a topological space (X, τ) is called sg-compact if every cover of X by sg-open sets has a finite subcover. Devi, Balachandran and Maki [20] defined the same concept and they used the term SGO-compactness. In the last section the concept of S^*GO-compact space, introduced by K. Chandrasekhara Rao and K. Joseph [5] is extended to bitopological spaces.

In the next section some prerequisites and abbreviations are established.

2. Preliminaries

Let (X, τ_1, τ_2) or simply X denote a bitopological space. For any subset $A \subseteq X$, τ_i-$int(A)$ and τ_i-$cl(A)$ denote the interior and closure of a set A with respect to the topology τ_i, respectively. A^C denotes the complement of A in X unless explicitly stated.

Definition 2.1 A set A of a bitopological space (X, τ_1, τ_2) is called

(a) $\tau_1\tau_2$-semi open if there exists an τ_1-open set U such that $U \subseteq A \subseteq \tau_2$-$cl(U)$.

(b) $\tau_1\tau_2$-semi closed if $X - A$ is $\tau_1\tau_2$-semi open.
Equivalently, a set A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$-semi closed if there exists an τ_1-closed set F such that τ_2-$int(F) \subseteq A \subseteq F$.

(c) $\tau_1\tau_2$-generalized closed ($\tau_1\tau_2$-g closed) if τ_2-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1-open in X.

(d) $\tau_1\tau_2$-generalized open ($\tau_1\tau_2$-g open) if $X - A$ is $\tau_1\tau_2$-g closed.

(e) $\tau_1\tau_2$-semi generalized closed ($\tau_1\tau_2$-sg closed) if τ_2-$scl(A) \subseteq U$ whenever $A \subseteq$U and U is τ_1-semi open in X.

(f) $\tau_1\tau_2$-semi generalized open ($\tau_1\tau_2$-sg open) if $X - A$ is $\tau_1\tau_2$-sg closed.

(g) $\tau_1\tau_2$-generalized semi open ($\tau_1\tau_2$-gs open) if $F \subseteq \tau_2$-$sint(A)$ whenever $F \subseteq A$ and F is τ_1-closed in X.

(h) $\tau_1\tau_2$-generalized semi closed ($\tau_1\tau_2$-gs closed) if $X - A$ is $\tau_1\tau_2$-gs open.

First we prove some results in topological spaces as prerequisites. Recall that arbitrary union of $cl(A_i), i \in I$ is contained in closure of arbitrary union of subsets A_i in any topological space. The equality holds if the collection $\{A_i, i \in I\}$ is locally finite. Hence we conclude the following.

Theorem 2.2 The arbitrary union of s^*g-closed sets $A_i, i \in I$ in a topological space (X, τ) is s^*g-closed if the family $\{A_i, i \in I\}$ is locally finite.

Proof. Let $\{A_i, i \in I\}$ be locally finite and A_i is s^*g-closed in a topological space (X, τ) for each $i \in I$. Let $\bigcup A_i \subseteq U$ and U is semi open in X. Then $A_i \subseteq U$ and U is semi open in X for each i. Since A_i is s^*g-closed in X for each $i \in I$, we have $cl(A_i) \subseteq U$. Consequently, $\bigcup[cl(A_i)] \subseteq U$. Since the family $\{A_i, i \in I\}$ is locally finite, $cl[\bigcup(A_i)] = \bigcup[cl(A_i)] \subseteq U$. Therefore, $\bigcup A_i$ is s^*g-closed in X. □

Theorem 2.3 The arbitrary intersection of s^*g-open sets $A_i, i \in I$ in a topological space (X, τ) is s^*g-open if the family $\{A_i^C, i \in I\}$ is locally finite.

Proof. Let $\{A_i^C, i \in I\}$ be locally finite and A_i is s^*g-open in X for each $i \in I$. Then A_i^C is s^*g-closed in X for each $i \in I$. Then by theorem , we have $\bigcup[A_i^C]$ is s^*g-closed in X. Consequently, Let $\{\bigcap(A_i)\}^C$ is s^*g-closed in X. Therefore, $\bigcap A_i$ is s^*g-open in X. □

3. $\tau_1\tau_2$-s^*g closed sets

Definition 3.1 A set A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$-semi star generalized closed ($\tau_1\tau_2$-s^*g closed) if τ_2-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1-semi open in X.

Example 3.2 Let X = {a, b, c}, τ_1 = {ϕ, X, {a}, {b, c}}, τ_2 = {ϕ, X, {a}, {b}, {a, b}}. Then {a, b} is $\tau_1\tau_2 - s^*g$ closed and {a} is not $\tau_1\tau_2 - s^*g$ closed.

Theorem 3.3 The arbitrary union of $\tau_1\tau_2$-s^*g closed sets $A_i, i \in I$ in a bitopological space (X, τ_1, τ_2) is $\tau_1\tau_2$-s^*g closed if the family $\{A_i, i \in I\}$ is τ_2-locally finite.

Proof. Let $\{A_i, i \in I\}$ be τ_2-locally finite and A_i is $\tau_1\tau_2$-s^*g closed in X for each $i \in I$. Let $\bigcup A_i \subseteq U$ and U is τ_1-semi open in X. Then, $A_i \subseteq U$ and U is τ_1-semi open in X for each i. Since A_i is $\tau_1\tau_2$-s^*g closed in X for each $i \in I$, we have τ_2-$cl(A_i) \subseteq U$. Consequently, $\bigcup[\tau_2$-$cl(A_i)] \subseteq U$. Since the family $\{A_i, i \in I\}$ is τ_2-locally finite, τ_2-$cl[\bigcup(A_i)] = \bigcup[\tau_2$-$cl(A_i)] \subseteq U$. Therefore, $\bigcup A_i$ is $\tau_1\tau_2$-s^*g closed in X. □

Theorem 3.4 The arbitrary intersection of $\tau_1\tau_2$-s^*g open sets $A_i, i \in I$ in a bitopological space (X, τ_1, τ_2) is $\tau_1\tau_2$-s^*g open if the family $\{A_i^C, i \in I\}$ is τ_2-locally finite.

Proof. Let $\{A_i^C, i \in I\}$ be τ_2-locally finite and A_i is $\tau_1\tau_2$-s^*g open in X for each $i \in I$. Then, A_i^C is $\tau_1\tau_2$-s^*g closed in X for each $i \in I$. Then by theorem, we have $\bigcup[A_i^C]$ is $\tau_1\tau_2$-s^*g closed in X. Consequently, $\{\bigcap(A_i)\}^C$ is $\tau_1\tau_2$-s^*g closed in X. Therefore, $\bigcap A_i$ is $\tau_1\tau_2$-s^*g open in X. □

4. Pairwise s^*g-continuous functions

First we recall the following known definitions.

Definition 4.1 A function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is

(a) pairwise g-continuous if $f^{-1}(U)$ is $\tau_i\tau_j$-g closed for each σ_j-closed set U in Y, $i\neq j$ and $i,j=1$, 2.

(b) pairwise sg-continuous if $f^{-1}(U)$ is $\tau_i\tau_j$-sg closed for each σ_j-closed set U in Y, $i\neq j$ and $i,j=1$, 2.

(c) pairwise gs-continuous if $f^{-1}(U)$ is $\tau_i\tau_j$-gs closed for each σ_j-closed set U in Y, $i\neq j$ and $i,j=1$, 2.

Definition 4.2 A function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is pairwise s^*g-continuous if $f^{-1}(U)$ is $\tau_i\tau_j$-s^*g closed for each σ_j-closed set U in Y, $i\neq j$ and $i,j=1$, 2.

Example 4.3 Let $X=Y=\{a,b,c,d\}$, $\tau_1=\{\phi,X,\{a\},\{a,b\}\}$, $\tau_2=\{\phi,X,\{a,b\},\{a,b,c\},\{a,b,d\}\}$, $\sigma_1=\{\phi,Y,\{a\}\}$, $\sigma_2=\{\phi,Y,\{a,b\},\{a,b,c\}\}$. Let f: $(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be a function defined by $f(\phi)=\phi$, $f(X)=Y$, $f(a)=\{a,b,d\}$, $f(b)=\{c\}$, $f(c)=\{b\}$, $f(d)=\{d\}$, $f(a,b)=\{a,c\}$, $f(a,c)=\{a,b\}$, $f(a,d)=\{b,c\}$, $f(b,c)=\{a,d\}$, $f(b,d)=\{a,b,c\}$, $f(c,d)=\{c,d\}$, $f(a,b,c)=\{b,d\}$, $f(a,b,d)=\{a\}$, $f(a,c,d)=\{b,c,d\}$, $f(b,c,d)=\{a,c,d\}$. Then f is pairwise s^*g-continuous.

Theorem 4.4 Every pairwise continuous function is pairwise s^*g-continuous.

Proof. Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be pairwise continuous. Let U be a σ_j-closed set in Y. Then $f^{-1}(U)$ is τ_j-closed in X. Since every τ_j-closed set is $\tau_i\tau_j$-s^*g closed, $i\neq j$ and $i,j=1$, 2, we have f is pairwise s^*g-continuous. □

The converse of the above theorem need not be true in general. The next example supports our claim.

Example 4.5 In Example 4.3, $\{a\}$ is σ_1-open in Y. But $f^{-1}(a)=\{a,b,d\}$ is not τ_1-open in X. Therefore, f is pairwise s^*g-continuous but not pairwise continuous.

Since every $\tau_i\tau_j$-s^*g closed set is $\tau_i\tau_j$-g closed, $\tau_i\tau_j$-sg closed and $\tau_i\tau_j$-gs closed, $i\neq j$ and $i,j=1$,2, we have every pairwise s^*g-continuous function is pairwise g - continuous, pairwise sg-continuous and pairwise gs-continuous. But none of the above is reversible. The following examples support our claim.

Example 4.6 (a) Let $X=Y=\{a,b,c,d\}$,$\tau_1=\{\phi,X,\{a\},\{a,b\}\}$, $\tau_2=\{\phi,X,\{a,b\},\{a,b,c\},\{a,b,d\}\}$, $\sigma_1=\{\phi,Y,\{a\}\}$, $\sigma_2=\{\phi,Y,\{a,b\},\{a,b,c\}\}$. Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be a function defined by $f(\phi)=\phi$, $f(X)=Y$, $f(a)=\{b\}$, $f(b)=\{a\}$, $f(c)=\{a,b\}$, $f(d)=\{a,c,d\}$, $f(a,b)=\{c\}$, $f(a,c)=\{a,d\}$, $f(a,d)=\{a,c\}$, $f(b,c)=\{b,d\}$, $f(b,d)=\{b,c\}$, $f(c,d)=\{c,d\}$, $f(a,b,c)=\{a,b,d\}$, $f(a,b,d)=\{a,b,c\}$, $f(a,c,d)=\{d\}$, $f(b,c,d)=\{b,c,d\}$. Then f is pairwise g-continuous but not pairwise s^*g-continuous.

(b) Let $X = Y = \{a, b, c, d\}$,$\tau_1 = \{\phi, X, \{a\}, \{a, b\}\}$, $\tau_2 = \{\phi, X, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$, $\sigma_1 = \{\phi, Y, \{a\}\}$, $\sigma_2 = \{\phi, Y, \{a, b\}, \{a, b, c\}\}$. Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a function defined by $f(\phi) = \phi$, $f(X) = Y$, $f(a) = \{b\}$, $f(b) = \{a\}$, $f(c) = \{a, b\}$, $f(d) = \{d\}$, $f(a, b) = \{c\}$, $f(a, c) = \{a, d\}$, $f(a, d) = \{a, c\}$, $f(b, c) = \{a, b, c\}$, $f(b, d) = \{b, c, d\}$, $f(c, d) = \{c, d\}$, $f(a, b, c) = \{b, c\}$, $f(a, b, d) = \{a, c, d\}$, $f(a, c, d) = \{b, c, d\}$, $f(b, c, d) = \{a, b, d\}$. Then f is both pairwise gs-continuous and pairwise sg-continuous but not pairwise s^*g-continuous.

Theorem 4.7 The following are equivalent for a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$.

(a) f is pairwise s^*g-continuous.

(b) $f^{-1}(U)$ is $\tau_i\tau_j$-s^*g open for each σ_i-open set U in Y, $i \neq j$ and $i, j = 1, 2$.

Proof. $(a) \Rightarrow (b)$: Suppose that f is pairwise s^*g-continuous. Let A be σ_j-open in Y. Then A^C is σ_j-closed in Y. Since f is pairwise s^*g-continuous, we have $f^{-1}(A^C)$ is $\tau_i\tau_j$-s^*g closed in X, $i \neq j$ and $i, j = 1$,2. Consequently, $f^{-1}(A)$ is $\tau_i\tau_j$-s^*g open in X.

$(b) \Rightarrow (a)$ Suppose that $f^{-1}(U)$ is $\tau_i\tau_j$-s^*g open for each σ_i-open set U in Y, $i \neq j$ and $i, j = 1$,2. Let V be σ_j-closed in Y. Then V^C is σ_j-open in Y. Therefore, by our assumption, $f^{-1}(V^C)$ is $\tau_i\tau_j$-s^*g open in X, $i \neq j$ and $i, j = 1$,2. Hence $f^{-1}(V)$ is $\tau_i\tau_j$-s^*g closed in X. This completes the proof. □

Definition 4.8 A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is

(a) pairwise g-irresolute if $f^{-1}(U)$ is $\tau_i\tau_j$-g closed for each $\sigma_i\sigma_j$-g closed set U in Y, $i \neq j$ and $i, j = 1$,2.

(b) pairwise sg-irresolute if $f^{-1}(U)$ is $\tau_i\tau_j$-sg closed for each $\sigma_i\sigma_j$-sg closed set U in Y, $i \neq j$ and $i, j = 1$,2.

(c) pairwise gs-irresolute if $f^{-1}(U)$ is $\tau_i\tau_j$-gs closed for each $\sigma_i\sigma_j - gs$ closed set U in Y, $i \neq j$ and $i, j = 1$,2.

Definition 4.9 A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is pairwise s^*g-irresolute if $f^{-1}(U)$ is $\tau_i\tau_j$-s^*g closed for each $\sigma_i\sigma_j$-s^*g closed set U in Y, $i \neq j$ and $i, j = 1$,2.

Example 4.10 Let $X = Y = \{a, b, c, d\}$,$\tau_1 = \{\phi, X, \{a\}, \{a, b\}\}$, $\tau_2 = \{\phi, X, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$, $\sigma_1 = \{\phi, Y, \{a\}\}$, $\sigma_2 = \{\phi, Y, \{a, b\}, \{a, b, c\}\}$. Let f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a function defined by $f(\phi) = \phi$, $f(X) = Y$, $f(a) = \{b\}$, $f(b) = \{a\}$, $f(c) = \{c\}$, $f(d) = \{d\}$, $f(a, b) = \{a, c\}$, $f(a, c) = \{a, b\}$, $f(a, d) = \{b, c\}$, $f(b, c) = \{a, d\}$, $f(b, d) = \{b, d\}$, $f(c, d) = \{c, d\}$, $f(a, b, c) = \{a, b, d\}$, $f(a, b, d) = \{a, b, c\}$, $f(a, c, d) = \{a, c, d\}$, $f(b, c, d) = \{b, c, d\}$. Then f is pairwise s^*g-irresolute.

Concerning the composition of functions, we have the following.

Theorem 4.11 Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ and $g : (Y, \sigma_1, \sigma_2) \to (Z, \mu_1, \mu_2)$ be two functions. Then

(a) If f and g are pairwise s^*g-irresolute, then gof is pairwise s^*g-irresolute.

(b) If f is pairwise s^*g-irresolute and g is pairwise s^*g-continuous, then gof is pairwise s^*g-continuous.

(c) If f is pairwise g-irresolute and g is pairwise s^*g-continuous, then gof is pairwise g-continuous.

(d) If f is pairwise sg-irresolute and g is pairwise s^*g-continuous, then gof is pairwise sg-continuous.

(e) If f is pairwise gs-irresolute and g is pairwise s^*g-continuous, then gof is pairwise gs-continuous.

(f) If f is pairwise s^*g-continuous and g is pairwise continuous, then gof is pairwise s^*g-continuous.

Proof. Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ and $g : (Y, \sigma_1, \sigma_2) \to (Y, \mu_1, \mu_2)$ be pairwise s^*g-irresolute. Let U be $\mu_i\mu_j$-s^*g closed set in Z, , $i \neq j$ and i, j = 1,2. Since g is pairwise s^*g-irresolute, $g^{-1}(U)$ is $\sigma_i\sigma_j$-s^*g closed in Y. Since f is pairwise s^*g-irresolute, $(gof)^{-1} = f^{-1}[g^{-1}(U)]$ is $\tau_i\tau_j$-s^*g closed in X. Therefore, gof is pairwise s^*g-irresolute.

The proofs of (b)-(f) are similar. □

But the composition of two pairwise s^*g-continuous functions is not a pairwise s^*g-continuous function in general as shown in the following example.

Example 4.12 Let $X = Y = Z = \{a, b, c, d\}$,$\tau_1 = \{\phi, X, \{a\}, \{a, b\}\}$, $\tau_2 = \{\phi, X, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$, $\sigma_1 = \{\phi, Y, \{a\}\}$, $\sigma_2 = \{\phi, Y, \{a, b\}, \{a, b, c\}\}$, $\mu_1 = \{\phi, Z, \{a\}\}$, $\mu_2 = \{\phi, X, \{a\}, \{b, c\}\}$.

Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a function defined by $f(\phi) = \phi$, $f(X) = Y$, $f(a) = \{a, b, d\}$, $f(b) = \{c\}$, $f(c) = \{b\}$, $f(d) = \{d\}$, $f(a, b) = \{a, c\}$, $f(a, c) = \{a, b\}$, $f(a, d) = \{b, c\}$, $f(b, c) = \{a, d\}$, $f(b, d) = \{a, b, c\}$, $f(c, d) = \{c, d\}$, $f(a, b, c) = \{b, d\}$, $f(a, b, d) = \{a\}$, $f(a, c, d) = \{b, c, d\}$, $f(b, c, d) = \{a, c, d\}$. Then f is pairwise s^*g-continuous.

Let $g : (Y, \sigma_1, \sigma_2) \to (Z, \mu_1, \mu_2)$ be a function defined by $g(\phi) = \phi$, $g(Y) = Z$, $g(a) = \{b\}$, $g(b) = \{a\}$, $g(c) = \{d\}$, $g(d) = \{c\}$, $g(a, b) = \{a, c\}$, $g(a, c) = \{a, b\}$, $g(a, d) = \{a, d\}$, $g(b, c) = \{b, d\}$, $g(b, d) = \{b, c\}$, $g(c, d) = \{a, b, c\}$, $g(a, b, c) = \{c, d\}$, $g(a, b, d) = \{a, c, d\}$, $g(a, c, d) = \{a, b, d\}$, $g(b, c, d) = \{b, c, d\}$. Then g is pairwise s^*g-continuous.

But $(gof)^{-1}(\{b, c, d\}) = \{a, c, d\}$ is not $\tau_1\tau_2 - s^*g$ closed in X. Hence gof is not pairwise s^*g-continuous.

Definition 4.13 A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is pairwise pre s^*g-continuous if $f^{-1}(U)$ is $\tau_i\tau_j$-s^*g closed for each $\sigma_i\sigma_j$-semi closed set U in Y, $i \neq j$ and i, j = 1,2.

Example 4.14 Let $X = Y = \{a, b, c\}$,$\tau_1 = \{\phi, X, \{a\}, \{b, c\}\}$, $\tau_2 = \{\phi, X, \{a\}\}$, $\sigma_1 = \{\phi, Y, \{c\}, \{a, b\}\}$, $\sigma_2 = \{\phi, Y, \{c\}\}$. Let f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a function defined by $f(\phi) = \phi$, $f(X) = Y$, $f(a) = \{a, c\}$, $f(b) = \{b\}$, $f(c) = \{c\}$, $f(a, b) = \{a, b\}$, $f(a, c) = \{a\}$, $f(b, c) = \{a, b\}$. Then f is pairwise pre s^*g-continuous.

Obviously every pairwise pre s^*g-continuous function is pairwise s^*g-continuous. But it is not reversible. It is shown in the following example.

Example 4.15 In Example 4.3, f is pairwise s^*g-continuous but not pairwise pre s^*g-continuous.

Theorem 4.16 Let Y be a pairwise semi $T_{\frac{1}{2}}$-space. A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is pairwise s^*g-irresolute if it is pairwise pre s^*g-continuous.

Proof. Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is pairwise pre s^*g-continuous. Let A be $\sigma_i\sigma_j$-s^*g closed in Y, $i, j = 1, 2$ and $i \neq j$. Since every $\sigma_i\sigma_j$-s^*g closed set is $\sigma_i\sigma_j$-sg closed, we have A is $\sigma_i\sigma_j$-sg closed in Y. Since Y is pairwise semi $T_{\frac{1}{2}}$-space and every $\sigma_i\sigma_j$-sg closed set is σ_j-semi closed in a pairwise semi $T_{\frac{1}{2}}$-space, we have A is $\sigma_i\sigma_j$-semi closed. Since f is pairwise pre s^*g - continuous, we have $f^{-1}(A)$ is $\tau_i\tau_j$-s^*g closed in X. Hence f is pairwise s^*g-irresolute. □

Definition 4.17 A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is pairwise s^*g-closed if $f(U)$ is $\sigma_i\sigma_j$-s^*g closed for each τ_j-closed set U in X, $i \neq j$ and i, j = 1,2.

Example 4.18 Let $X = Y = \{a, b, c, d\}$,$\tau_1 = \{\phi, X, \{a\}, \{a, b\}\}$, $\tau_2 = \{\phi, X, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$, $\sigma_1 = \{\phi, Y, \{a\}\}$, $\sigma_2 = \{\phi, Y, \{a, b\}, \{a, b, c\}\}$. Let f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a function defined by $f(\phi) = \phi$, $f(X) = Y$, $f(a) = \{b\}$, $f(b) = \{a\}$, $f(c) = \{b, c, d\}$, $f(d) = \{d\}$, $f(a, b) = \{a, c\}$, $f(a, c) = \{a, b\}$, $f(a, d) = \{b, c\}$, $f(b, c) = \{a, d\}$, $f(b, d) = \{a, b, c\}$, $f(c, d) = \{c, d\}$, $f(a, b, c) = \{b, d\}$, $f(a, b, d) = \{a, b, d\}$, $f(a, c, d) = \{c\}$, $f(b, c, d) = \{a, c, d\}$. Then f is pairwise s^*g-closed.

Definition 4.19 A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is pairwise pre s^*g-closed if $f(U)$ is $\sigma_i\sigma_j$-s^*g closed for each $\tau_i\tau_j$-semi closed set U in X, $i \neq j$ and i, j = 1,2.

Example 4.20 Let $X = Y = \{a, b, c\}$,$\tau_1 = \{\phi, X, \{a\}, \{b, c\}\}$, $\tau_2 = \{\phi, X, \{a\}$, $\sigma_1 = \{\phi, Y, \{c\}, \{a, b\}\}$, $\sigma_2 = \{\phi, Y, \{c\}\}$. Let f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a function defined by $f(\phi) = \phi$, $f(X) = Y$, $f(a) = \{a\}$, $f(b) = \{b\}$, $f(c) = \{a, c\}$, $f(a, b) = \{b, c\}$, $f(a, c) = \{c\}$, $f(b, c) = \{a, b\}$. Then f is pairwise pre s^*g-closed.

5. Pairwise S^*GO-connected space

Definition 5.1 A bitopological space (X, τ_1, τ_2) is pairwise S^*GO-connected if X can not be expressed as the union of two nonempty disjoint sets A and B such that $[A \cap \tau_1\text{-}s^*gcl(B)] \cup [\tau_2\text{-}s^*gcl(A) \cap B] = \phi$.

Suppose X can be so expressed then X is called pairwise S^*GO-disconnected and we write $X = A \backslash B$ and call this pairwise S^*GO-separation of X.

Example 5.2 (a) Let $X = \{a, b, c, d\}$,$\tau_1 = \{\phi, X, \{a\}, \{a, b\}\}$, $\tau_2 = \{\phi, X, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$. Then (X, τ_1, τ_2) is pairwise S^*GO-connected.

(b) $Y = \{a, b, c, d\}$, $\sigma_1 = \{\phi, Y, \{a\}\}$, $\sigma_2 = \{\phi, Y, \{a, b\}, \{a, b, c\}\}$. Then (Y, σ_1, σ_2) is pairwise S^*GO-connected.

Theorem 5.3 The following conditions are equivalent for any bitopological space.

(a) X is pairwise S^*GO-connected.

(b) X can not be expressed as the union of two nonempty disjoint sets A and B such that A is τ_1-s^*g open and B is τ_2-s^*g open.

(c) X contains no nonempty proper subset which is both τ_1-s^*g open and τ_2-s^*g closed.

Proof. $(a) \Rightarrow (b)$: Suppose that X is pairwise S^*GO-connected. Suppose that X can be expressed as the union of two nonempty disjoint sets A and B such that A is τ_1-s^*g open and B is τ_2-s^*g open. Then $A \cap B = \phi$. Consequently $A \subseteq B^C$. Then τ_2-$s^*gcl(A) \subseteq \tau_2$-$s^*gcl(B^C) = B^C$. Therefore, τ_2-$s^*gcl(A) \cap B = \phi$. Similarly we can prove $A \cap \tau_1$-$s^*gcl(B) = \phi$. Hence $[A \cap \tau_1\text{-}s^*gcl(B)] \cup [\tau_2\text{-}s^*gcl(A) \cap B] = \phi$. This is a contradiction to the fact that X is pairwise S^*GO-connected. Therefore, X can not be expressed as the union of two nonempty disjoint sets A and B such that A is τ_1-s^*g open and B is τ_2-s^*g open.

$(b) \Rightarrow (c)$: Suppose that X can not be expressed as the union of two nonempty disjoint sets A and B such that A is τ_1-s^*g open and B is τ_2-s^*g open. Suppose that X contains a nonempty proper subset A which is both τ_1-s^*g open and τ_2-s^*g closed. Then $X = A \cup A^C$ where A is τ_1-s^*g open, A^C is τ_2-s^*g open and A, A^C are disjoint. This is the contradiction to our assumption. Therefore, X contains no nonempty proper subset which is both τ_1-s^*g open and τ_2-s^*g closed.

$(c) \Rightarrow (a)$: Suppose that X contains no nonempty proper subset which is both τ_1-s^*g open and τ_2-s^*g closed. Suppose that X is pairwise S^*GO-disconnected. Then X can be expressed as the union of two nonempty disjoint sets A and B such that $[A \cap \tau_1\text{-}s^*gcl(B)] \cup [\tau_2\text{-}s^*gcl(A) \cap B] = \phi$. Since $A \cap B = \phi$, we have $A = B^C$ and $B = A^C$. Since τ_2-$s^*gcl(A) \cap B = \phi$, we have τ_2-$s^*gcl(A) \subseteq B^C$. Hence τ_2-$s^*gcl(A) \subseteq A$. Therefore, A is τ_2-s^*g closed. Similarly, B is τ_1-s^*g closed. Since $A = B^C$, A is τ_1-s^*g open. Therefore, there exists a nonempty proper set A which is both τ_1-s^*g open and τ_2-s^*g closed. This is the contradiction to our assumption. Therefore, X is pairwise S^*GO-connected. □

Theorem 5.4 If A is pairwise S^*GO-connected subset of a bitopological space (X, τ_1, τ_2) which has the pairwise S^*GO-separation $X = C \backslash D$, then $A \subseteq C$ or $A \subseteq D$.

Proof. Suppose that (X, τ_1, τ_2) has the pairwise S^*GO-separation $X = C \backslash D$. Then $X = C \cup D$ where C and D are two nonempty disjoint sets such that $[C \cap \tau_1\text{-}s^*gcl(D)] \cup [\tau_2\text{-}s^*gcl(C) \cap D] = \phi$. Since $C \cap D = \phi$, we have $C = D^C$ and $D = C^C$.

Now, $[(C\cap A)\cap\tau_1\text{-}s^*gcl(D\cap A)]\cup[\tau_2\text{-}s^*gcl(C\cap A)\cap(D\cap A)] \subseteq [C\cap\tau_1\text{-}s^*gcl(D)]\cup[\tau_2\text{-}s^*gcl(C)\cap D] = \phi$. Hence $A = (C\cap A)\backslash(D\cap A)$ is pairwise S^*GO-separation of A. Since A is pairwise S^*GO-connected, we have either $(C\cap A) = \phi$ or $(D\cap A) = \phi$. Consequently, $A \subseteq C^C$ or $A \subseteq D^C$. Therefore, $A \subseteq C$ or $A \subseteq D$. □

Theorem 5.5 If A is pairwise S^*GO-connected and $A \subseteq B \subseteq \tau_1\text{-}s^*gcl(A)\cap\tau_2\text{-}s^*gcl(A)$ then B is pairwise S^*GO-connected.

Proof. Suppose that B is not pairwise S^*GO-connected. Then $B = C\cup D$ where C and D are two nonempty disjoint sets such that $[C\cap\tau_1\text{-}s^*gcl(D)]\cup[\tau_2\text{-}s^*gcl(C)\cap D] = \phi$. Since A is pairwise S^*GO-connected, we have $A\subseteq C$ or $A \subseteq D$. Suppose $A \subseteq C$. Then $D \subseteq D\cap B \subseteq D\cap\tau_2\text{-}s^*gcl(A) \subseteq D\cap\tau_2\text{-}s^*gcl(C) = \phi$. Therefore, $\phi\subseteq D\subseteq\phi$. Consequently, $D = \phi$. Similarly, we can prove $C = \phi$ if $A \subseteq D$ {by Theorem 5.4}. This is the contradiction to the fact that C and D are nonempty. Therefore, B is pairwise S^*GO-connected. □

Theorem 5.6 The union of any family of pairwise S^*GO-connected sets having a nonempty intersection is pairwise S^*GO-connected.

Proof. Let I be an index set and $i \in I$. Let $A = \bigcup A_i$ where each A_i is pairwise S^*GO-connected with $\bigcap A_i \neq \phi$. Suppose that A is not pairwise S^*GO-connected. Then $A = C\cup D$, where C and D are two nonempty disjoint sets such that $[C\cap\tau_1\text{-}s^*gcl(D)]\cup[\tau_2\text{-}s^*gcl(C)\cap D] = \phi$. Since A_i is pairwise S^*GO-connected and $A_i \subseteq A$, we have $A_i \subseteq C$ or $A_i \subseteq D$. Therefore, $\bigcup(A_i) \subseteq C$ or $\bigcup(A_i) \subseteq D$. Hence, $A \subseteq C$ or $A \subseteq D$. Since $\bigcap A_i \neq \phi$, we have $x \in \bigcap A_i$. Therefore, $x \in A_i$ for all i. Consequently, $x \in A$. Therefore, $x \in C$ or $x \in D$. Suppose $x \in C$. Since $C\cap D = \phi$, we have $x \notin D$. Therefore, $A \nsubseteq D$. Therefore, $A \subseteq C$. Therefore, A is not pairwise S^*GO-connected. This shows that A is pairwise S^*GO-connected. □

Theorem 5.7 Let $f : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ be a pairwise continuous bijective and pairwise pre semi closed. Then inverse image of a $\sigma_i\text{-}s^*g$ closed set is $\tau_i\text{-}s^*g$ closed.

Theorem 5.8 Let $f : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ be a pairwise continuous bijective and pairwise pre semi closed function. Then the image of a pairwise S^*GO-connected space under f is pairwise S^*GO-connected.

Proof. Let $f : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ be pairwise continuous surjection and pairwise pre semi closed. Let X is pairwise S^*GO-connected. Suppose that Y is pairwise S^*GO-disconnected. Then $Y = A\cup B$ where A is $\sigma_1\text{-}s^*g$ open and B is $\sigma_2\text{-}s^*g$ open in Y. Since f is pairwise continuous and pairwise pre semi closed, we have $f^{-1}(A)$ is $\tau_1\text{-}s^*g$ open and $f^{-1}(B)$ is $\tau_2\text{-}s^*g$ open in X. Also $X = f^{-1}(A)\cup f^{-1}(B)$, $f^{-1}(A)$ and $f^{-1}(B)$ are two nonempty disjoint sets. Then X is pairwise S^*GO-disconnected. This is the contradiction to the fact that X is pairwise S^*GO-connected. Therefore, Y is pairwise S^*GO-connected. □

6. Pairwise S^*GO-compact space

Definition 6.1 A nonempty collection $\zeta = \{A_i, i \in I, \textit{an index set}\}$ is called a pairwise s^*g-open cover of a bitopological space (X, τ_1, τ_2) if $X = \bigcup A_i$ and $\zeta \subseteq \tau_1$-$S^*GO(X, \tau_1, \tau_2) \cup \tau_2$-$S^*GO(X, \tau_1, \tau_2)$ and ζ contains at least one member of τ_1-$S^*GO(X, \tau_1, \tau_2)$ and one member of τ_2-$S^*GO(X, \tau_1, \tau_2)$.

Definition 6.2 A bitopological space (X, τ_1, τ_2) is pairwise S^*GO-compact if every pairwise s^*g-open cover of X has a finite subcover.

Definition 6.3 A set A of a bitopological space (X, τ_1, τ_2) is pairwise S^*GO-compact relative to X if every pairwise s^*g-open cover of B by has a finite subcover as a subspace.

Example 6.4 Let $X = \{a, b, c, d\}$,$\tau_1 = \{\phi, X, \{a\}, \{a, b\}\}$, $\tau_2 = \{\phi, X, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$. Let $\zeta = \{\{a\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}$. Then (X, τ_1, τ_2) is pairwise S^*GO-compact.

Theorem 6.5 Every pairwise s^*g-compact space is pairwise compact.

Proof. Let (X, τ_1, τ_2) be pairwise S^*GO-compact. Let $\zeta = \{A_i, i \in I, \textit{an index set}\}$ be a pairwise open cover of X. Then $X = \bigcup A_i$ and $\zeta \subseteq \tau_1 \cup \tau_2$ and ζ contains at least one member of τ_1 and one member of τ_2. Since every τ_i-open set is τ_i-s^*g open, we have $X = \bigcup A_i$ and $\zeta \subseteq \tau_1$-$S^*GO(X, \tau_1, \tau_2) \cup \tau_2$-$S^*GO(X, \tau_1, \tau_2)$ and ζ contains at least one member of τ_1-$S^*GO(X, \tau_1, \tau_2)$ and one member of τ_2-$S^*GO(X, \tau_1, \tau_2)$. Therefore, ζ is the pairwise s^*g-open cover of X. Since X is pairwise S^*GO-compact, we have ζ has the finite subcover. Therefore, X is pairwise compact. □

But the converse of the above theorem need not be true in general.

Theorem 6.6 Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a pairwise continuous, bijective and pairwise pre semi closed. Then the image of a pairwise S^*GO-compact space under f is pairwise S^*GO-compact.

Proof. Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be pairwise continuous surjection and pairwise pre semi closed. Let X be pairwise S^*GO-compact. Let $\zeta = \{A_i, i \in I, \textit{an index set}\}$ be a pairwise s^*g-open cover of Y. Then $Y = \bigcup A_i$ and $\zeta \subseteq \sigma_1$-$S^*GO(Y, \sigma_1, \sigma_2) \cup \sigma_2$-$S^*GO(Y, \sigma_1, \sigma_2)$ and ζ contains at least one member of σ_1-$S^*GO(Y, \tau_1, \tau_2)$ and one member of σ_2-$S^*GO(Y, \sigma_1, \sigma_2)$. Therefore, $X = f^{-1}[\bigcup(A_i)]$ $= \bigcup f^{-1}(A_i)$ and $f^{-1}(\zeta) \subseteq \tau_1$-$S^*GO(X, \tau_1, \tau_2) \cup \tau_2$-$S^*GO(X, \tau_1, \tau_2)$ and $f^{-1}(\zeta)$ contains at least one member of τ_1-$S^*GO(X, \tau_1, \tau_2)$ and one member of τ_2-$S^*GO(X, \tau_1, \tau_2)$. Therefore, $f^{-1}(\zeta)$ is the pairwise s^*g-open cover of X. Since X is pairwise S^*GO-compact, we have $X = \bigcup f^{-1}(A_i)$, $i = 1$ to n. $\Rightarrow Y = f(X) = \bigcup(A_i)$, $i = 1$ to n. Hence, ζ has the finite subcover. Therefore, Y is pairwise S^*GO-compact. □

Theorem 6.7 If Y is τ_1-s^*g closed subset of a pairwise S^*GO-compact space (X, τ_1, τ_2), then Y is τ_2-S^*GO compact.

Proof. Let (X, τ_1, τ_2) be a pairwise S^*GO-compact space. Let $\zeta = \{A_i, i \in I, \textit{an index set}\}$ be a τ_2-s^*g open cover of Y. Since Y is τ_1-s^*g closed subset, Y^C is τ_1-s^*g open. Also $\zeta \cup Y^C = Y^C \cup \{A_i, i \in I, \textit{an index set}\}$ be a pairwise s^*g-open cover of X. Since X is pairwise S^*GO-compact, $X = Y^C \cup A_1 \cup .... \cup A_n$. Hence $Y = A_1 \cup .... \cup A_n$. Therefore, Y is τ_2-S^*GO compact. □

Since every τ_1-closed set is τ_1-s^*g closed, we have the following.

Theorem 6.8 If Y is τ_1-closed subset of a pairwise S^*GO-compact space (X, τ_1, τ_2), then Y is τ_2-S^*GO compact.

Theorem 6.9 If (X, τ_1) and (X, τ_2) are Hausdorff and (X, τ_1, τ_2) is pairwise S^*GO-compact, then $\tau_1 = \tau_2$.

Proof. Let (X, τ_1) and (X, τ_2) be Hausdorff and (X, τ_1, τ_2) is pairwise S^*GO-compact. Since every pairwise S^*GO - compact space is pairwise compact, we have (X, τ_1) and (X, τ_2) are Hausdorff and (X, τ_1, τ_2) is pairwise compact. Let F be τ_1-closed in X. Then F^C is τ_1 - open in X. Let $\zeta = \{A_i, i \in I, \textit{an index set}\}$ be the τ_2-open cover for X. Therefore, $\zeta \cup F^C$ is the pairwise open cover for X. Since X is pairwise compact, $X = F^C \cup A_1 \cup .... \cup A_n$. Hence $F = A_1 \cup .... \cup A_n$. Hence F is τ_2-compact. Since (X, τ_2) is Hausdorf, we have F is τ_2-closed. Similarly, every τ_2-closed set is τ_1-closed. Therefore, $\tau_1 = \tau_2$. □

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