Bol. Soc. Paran. Mat.
(3s.) **v. 26** 1-2 (2008): 53–64.

Decomposition of continuity via b-open set

Ahmad Al-Omari and Mohd. Salmi Md. Noorani

ABSTRACT: We introduce the notions of locally b-closed, b-t-set, b-B-set, locally b-closed continuous, b-t-continuous, b-B-continuous functions and obtain decomposition of continuity and complete continuity.

Key Words: b-open set, t-set, B-set, locally closed, decomposition of continuity

Contents

1. introduction

Tong [17,18], Ganster-Reilly [5], Hatice [7], Hatir-Noiri [8], Przemski [20], Noiri-Sayed [13] and Erguang-Pengfei [4], gave some decompositions of continuity. Andrijevic [2] introduced a class of generalized open sets in a topological space, the so-called b-open sets. The class of b-open sets is contained in the class of semi-preopen sets and contains all semi-open sets and preopen sets. Tong [18] introduced the concept of t-set and B-set in topological space. In this paper, we introduce the notions of locally b-closed sets, b-t-set, b-B-set, b-closed continuous, b-t-continuous and b-B-continuous function, and obtain another decomposition of continuity. All through this paper (X, τ) and (Y, σ) stand for topological spaces with no separation axioms assumed, unless otherwise stated. Let $A \subseteq X$, the closure of A and the interior of A will be denoted by $Cl(A)$ and $Int(A)$, respectively. A is regular open if $A = Int(Cl(A))$ and A is regular closed if its complement is regular open; equivalently A is regular closed if $A = Cl(Int(A))$. The complement of a b-open set is said to be b-closed. The intersection of all b-closed sets of X containing A is called the b-closure of A and is denoted by $bCl(A)$ of A. The union of all b-open (resp. α-open, semi open, preopen) sets of X contain in A is called b-interior (resp. α-interior, semi-interior, pre-interior) of A and is denoted by $bInt(A)$ (resp. $\alpha Int(A)$, $sInt(A)$, $pInt(A)$). The family of all b-open (resp. α-open, semi-open, preopen, regular open, b-closed, preclosed) subsets of a space X is denoted by $bO(X)$ (resp. $\alpha O(X)$, $SO(X)$ and $PO(X)$, $bC(X)$, $PC(X)$ respectively).

2000 *Mathematics Subject Classification:* 54C05, 54C08, 54C10

Definition 1.1. A subset A of a space X is said to be:

1. α-open [12] if $A \subseteq Int(Cl(Int(A)))$;
2. Semi-open [9] if $A \subseteq Cl(Int(A))$;
3. preopen [15] if $A \subseteq Int(Cl(A))$;
4. b-open [1] if $A \subseteq Cl(Int(A)) \cup Int(Cl(A))$;
5. Semi-preopen [2] if $A \subseteq Cl(Int(Cl(A)))$.

The following result will be useful in the sequel.

Lemma 1.1. *[1] If A is a subset of a space (X, τ), then*

1. $sInt(A) = A \cap Cl(Int(A))$;
2. $pInt(A) = A \cap Int(Cl(A))$;
3. $\alpha Int(A) = A \cap Int(Cl(Int(A)))$;
4. $bInt(A) = sInt(A) \cup pInt(A)$.

2. locally b-closed sets

Definition 2.1. A subset A of a space X is called:

1. t-set [18] if $Int(A) = Int(Cl(A))$.
2. B-set [18] if $A = U \cap V$, where $U \in \tau$ and V is a t-set.
3. locally closed [3] if $A = U \cap V$, where $U \in \tau$ and V is a closed.
4. locally b-closed if $A = U \cap V$, where $U \in \tau$ and V is a b-closed.

We recall that a topological space (X, τ) is said to be extremally disconnected (briefly E.D.) if the closure of every open set of X is open in X. We note that a subset A of X is locally closed if and only if $A = U \cap Cl(A)$ for some open set U (see [3]). The following example shows that the two notions of b-open and locally closed are independent.

Example 2.1. *Let $X = \{a, b, c, d\}$ and $\tau = \{X, \phi, \{b, c, d\}, \{b\}, \{a, b\}\}$ with $BO(X, \tau) = \{X, \phi, \{b\}, \{a, b\}, \{b, d\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}\}$, and the family of all locally closed is $LC(X, \tau) = \{X, \phi, \{b\}, \{a\}, \{a, b\}, \{c, d\}, \{b, c, d\}\}$ it is clear that $\{a\}$ is locally closed but not b-open and $\{b, d\}$ is b-open and not locally closed.*

Theorem 2.1. *For a subset A of an extremally disconnected space (X, τ), the following are equivalent:*

1. *A is open,*
2. *A is b-open and locally closed.*

Proof. (1)$\Rightarrow$ (2) This is obvious from definitions.
(2)$\Rightarrow$ (1) Let A be b-closed and locally closed so $A \subseteq (Int(Cl(A)) \cup Cl(Int(A)))$, $A = U \cap Cl(A)$. Then

$$\begin{aligned} A \quad &\subseteq U \cap (Int(Cl(A)) \cup Cl(Int(A))) \\ &\subseteq [Int(U \cap Cl(A))] \cup [U \cap Cl(Int(A))] \quad \text{(since } (X,\tau) \text{ is E.D. space we have)} \\ &\subseteq [Int(U \cap Cl(A))] \cup [U \cap Int(Cl(A))] \\ &\subseteq [Int(U \cap Cl(A))] \cup Int([U \cap Cl(A)]) \\ &= Int(A) \cup Int(A) = Int(A) \end{aligned}$$

Therefore A is open. □

Definition 2.2. A subset A of a topological space X is called $D(c,b)$-set if $Int(A) = bInt(A)$.

From the following examples one can deduce that b-open and $D(c,b)$-set are independent.

Example 2.2. *Let* $X = \{a,b,c,d\}$ *and* $\tau = \{X, \phi, \{c\}, \{d\}, \{c,d\}, \{a,c,d\}, \{b,c,d\}\}$. *Then* $BO(X,\tau) = \{\phi, X, \{b,c,d\}, \{a,c,d\}, \{a,b,d\}, \{a,b,c\}, \{c,d\}, \{b,c\}, \{b,d\}, \{a,b\}, \{a,c\}, \{d\}, \{c\}\}$, *it is clear that* $A = \{a\}$ *is* $D(c,b)$*-set but not* b*-open. Also* $B = \{a,b,d\}$ *is* b*-open but* B *is not* $D(c,b)$.

Theorem 2.2. *For a subset* A *of a space* (X,τ)*, the following are equivalent:*

1. *A is open,*

2. *A is b-open and $D(c,b)$-set.*

Proof. (1)$\Rightarrow$ (2) If A is open then A is b-open and $A = Int(A) = bInt(A)$ so A is $D(c,b)$-set.
(2) $\Rightarrow$ (1) The condition $A \in BO(X)$ and $A \in D(c,b)$ imply $A = bInt(A)$ and $Int(A) = bInt(A)$ and consequently A is open □

Proposition 2.1. *Let* H *be a subset of* (X,τ), H *is locally* b*-closed if and only if there exists an open set* $U \subseteq X$ *such that* $H = U \cap bCl(H)$

Proof. Since H is an locally b-closed, then $H = U \cap F$, where U is open and F is b-closed. So $H \subseteq U$ and $H \subseteq F$ then $H \subseteq bCl(H) \subseteq bCl(F) = F$. Therefore $H \subseteq U \cap bCl(H) \subseteq U \cap bCl(F) = U \cap F = H$. Hence $A = U \cap bCl(A)$. Conversely since $bCl(H)$ is b-closed and $H = U \cap bCl(H)$, then H is locally b-closed. □

Proposition 2.2. *[1].*

1. *The union of any family of b-open sets is b-open.*

2. *The intersection of an open set and a b-open set is a b-open set.*

Proposition 2.3. *Let A be a subset a topological space X if A is locally b-closed, then*

1. $bCl(A) - A$ *is b-closed set.*
2. $[A \cup (X - bCl(A))]$ *is b-open.*
3. $A \subseteq bInt(A \cup (X - bCl(A))$.

Proof. 1. If A is an locally b-closed, there exist an U is open such that $A = U \cap bCl(A)$. Now

$$\begin{aligned} bCl(A) - A &= bCl(A) - [U \cap bCl(A)] \\ &= bCl(A) \cap [X - (U \cap bCl(A))] \\ &= bCl(A) \cap [(X - U) \cup (X - bCl(A))] \\ &= [bCl(A) \cap (X - U)] \cup [bCl(A) \cap (X - bCl(A))] \\ &= bCl(A) \cap (X - U) \end{aligned}$$

which is b-closed by Proposition 2.2

2. Since $bCl(A) - A$ is b-closed, then $[X - (bCl(A) - A)]$ is b-open and $[X - (bCl(A) - A)] = X - ((bCl(A) \cap (X - A)) = [A \cup (X - bCl(A))]$,
3. It is clear that $A \subseteq [A \cup (X - bCl(A))] = bInt[A \cup (X - bCl(A))]$.

□

As a consequence of Proposition 2.2, we have the following

Corollary 2.2A. *The intersection of a locally b-closed set and locally closed set is locally b-closed.*

Let $A, B \subseteq X$. Then A and B are said to be separated if $A \cap Cl(B) = \phi$ and $B \cap Cl(A) = \phi$.

Theorem 2.3. *Suppose (X, τ) is closed under finite unions of b-closed sets. Let A and B be locally b-closed. If A and B are separated, then $A \cup B$ is locally b-closed.*

Proof. Since A and B are locally b-closed, $A = G \cap bCl(A)$ and $B = H \cap bCl(B)$, where G and H are open in X. Put $U = G \cap (X \backslash Cl(B)$ and $V = H \cap (X \backslash Cl(A)$. Then $U \cap bCl(A) = (G \cap (X \backslash Cl(B))) \cap bCl(A) = A \cap (X \backslash Cl(B) = A$, since $A \subseteq X \backslash Cl(B)$. similarly, $V \cap bCl(B) = B$. And $U \cap bCl(B) \subseteq U \cap Cl(B) = \phi$ and $V \cap bCl(A) \subseteq V \cap Cl(A) = \phi$. Since, U and V are open.

$$\begin{aligned} (U \cup V) \cap bCl(A \cup B) &= (U \cup V) \cap (bCl(A) \cup bCl(B)) \\ &= (U \cap bCl(A)) \cup (U \cap bCl(B)) \cup (V \cap bCl(A)) \cup (V \cap bCl(B)) \\ &= A \cup B \end{aligned}$$

Hence $A \cup B$ is locally b-closed. □

Proposition 2.4. *[18] A subset A in a topological space X is open if and only it is pre-open set and a B-set.*

Proposition 2.5. *[10] A subset A in a topological space X is α-set if and only it is pre-open set and semi-open.*

Lemma 2.1. *[14] For a subset V of a topological space Y,we have $pCl(V) = Cl(V)$ for every $V \in SO(Y)$.*

In the topological space (X, τ) in [7] the author defined, $A_5 = B(X) = \{U \cap F | U \in \tau$ and $Int(Cl(F)) \subseteq F\}$. It is easy to see that every element in A_5 is B-set.

Proposition 2.6. *[7] Let H be a subset of (X, τ), $H \in A_5$ if and only if there exists an open set U such that $H = U \cap sCl(H)$.*

Lemma 2.2. *A subset A in a topological space X is B-set if it is locally b-closed and semi-open.*

Proof. Let A be locally b-closed and semi-open. Then by Proposition 2.1, there exists an open set U such that

$$\begin{aligned} A &= U \cap bCl(A) \\ &= U \cap [sCl(A) \cap pCl(A)] \\ &= U \cap [sCl(A) \cap Cl(A)] \qquad \text{by Lemma 2.1 we have} \\ &= U \cap sCl(A) \end{aligned}$$

Hence by Proposition 2.6 $A \in A_5$, so A is B-set. □

From the following examples one can deduce that α-sets and locally b-closed sets are independent.

Example 2.3. *Let $X = \{a, b, c, d\}$ and $\tau = \{X, \phi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$. It is clearly that $\{a, d\}$ is locally b-closed but not α-open, since $\{a, d\} = \{a, c, d\} \cap \{a, b, d, \}$ and $\{a, d\} \nsubseteq Int(Cl(Int(\{a,d\}))) = \{d\}$.*

Example 2.4. *Let $X = \{a, b, c, d\}$, and $\tau = \{X, \phi, \{b, c, d\}, \{b\}, \{a, b\}\}$ with Then the family of all locally closed set is $LC(X, \tau) = \{X, \phi, \{b\}, \{a\}, \{a, b\}, \{c, d\}, \{b, c, d\}\}$, it is clearly $\{c, b\}$ is α-open but not locally b-closed since $\{c, b\} \neq \{open\} \cap \{b\text{-}closed\}$ and $\{c, b\} \subseteq Int(Cl(Int(\{c,b\}))) = X$.*

Theorem 2.4. *For a subset A of a space (X, τ), the following are equivalent:*

1. *A is open,*
2. *A is α-set and locally b-closed.*

Proof. It is immediate from Proposition 2.4, Proposition 2.5 and Lemma 2.2. □

Definition 2.3. Let X be a space. A subset A of X is called a generalized b-closed set (simply; gb-closed set) if $bCl(A) \subseteq U$, whenever $A \subseteq U$ and U is open. The complement of a generalized b-closed set is called generalized b-open (simply; gb-open).

Theorem 2.5. *For a subset A of a space (X, τ), the following are equivalent:*

1. *A is b-closed,*
2. *A is gb-closed and locally b-closed.*

Proof. Let A be b-closed so $A = A \cap X$, then A is locally b-closed. And if $A \subseteq U$ then $bCl(A) = A \subseteq U$ and A is gb-closed. Conversely if A is locally b-closed , then there exist open set U such that $A = U \cap bCl(A)$, since $A \subseteq U$ and A is gb-closed then $bCl(A) \subseteq U$. Therefore $bCl(A) \subseteq U \cap bCl(A) = A$. Hence A is b-closed. □

3. b-t-sets

In this section, we introduce the following notions.

Definition 3.1. A subset A of a space X is said to be:

1. b-t-set if $Int(A) = Int(bCl(A))$;
2. b-B-set if $A = U \cap V$, where $U \in \tau$ and V is a b-t-set;
3. b-semiopen if $A \subseteq Cl(bInt(A))$;
4. b-preopen if $A \subseteq Int(bCl(A))$.

Proposition 3.1. *For subsets A and B of a space (X, τ), the following properties hold:*

1. *A is a b-t-set if and only if it is b-semiclosed.*
2. *If A is b-closed, then it is a b-t-set.*
3. *If A and B are b-t-sets, then $A \cap B$ is a b-t-set.*

Proof. (1) Let A be b-t-set. Then $Int(A) = Int(bCl(A))$. Therefore $Int(bCl(A)) \subseteq Int(A) \subseteq A$ and A is b-semiclosed. Conversely if A is b-semiclosed , then $Int(bCl(A)) \subseteq A$ thus $Int(bCl(A)) \subseteq Int(A)$. Also $A \subseteq bCl(A)$ and $Int(A) \subseteq Int(bCl(A))$. Hence $Int(A) = Int(bCl(A))$.
(2) Let A be b-closed, then $A = bCl(A)$, and $Int(A) = Int(bCl(A))$ therefore A is b-t-set.
(3) Let A and B be b-t-set. Then we have

$$\begin{aligned}
Int(A \cap B) &\subseteq Int(bCl(A \cap B)) \\
&\subseteq (Int(bCl(A)) \cap (bCl(B))) \\
&= Int(bCl(A) \cap Int(bClB)) \\
&= Int(A) \cap Int(B) \qquad \text{(since } A \text{ and } B \text{ are } b\text{-}t\text{-set)} \\
&= Int(A \cap B)
\end{aligned}$$

Then $Int(A \cap B) = Int(bCl(A \cap B))$ hence $A \cap B$ is b-t-set. □

The converses of the statements in Proposition 3.1 (2) are false as the following example shows.

Example 3.1. *Let $X = \{a, b, c, d\}$ and $\tau = \{X, \phi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$. Then $\{b, c\}$ is b-t-set but it is not b-closed.*

Proposition 3.2. *For a subset A of a space (X, τ), the following properties hold:*

1. *If A is t-set then it is b-t-set;*
2. *If A is b-t-set then it is b-B-set;*
3. *If A is B-set then it is b-B-set.*

Theorem 3.1. *For a subset A of a space (X, τ), the following are equivalent:*

1. *A is open,*
2. *A is b-preopen and a b-B-set.*

Proof. (1) ⇒ (2) Let A be open. Then $A \subseteq bCl(A)$, $A = Int(A) \subseteq Int(Cl_b(A))$ and A is b-preopens. Also $A = A \cap X$ hence A is b-B-set.
(2) ⇒ (1) Since A is b-B-set, we have $A = U \cap V$, where U is open set and $Int(V) = Int(bCl(V))$. By the hypothesis, A is also b-preopen, and we have

$$\begin{aligned} A &\subseteq Int(bCl(A) \\ &= Int(bCl(U \cap V) \\ &\subseteq Int(bCl(U) \cap bCl(V)) \\ &= Int(bCl(U)) \cap Int(bCl(V)) \\ &= Int(bCl(U)) \cap Int(V) \end{aligned}$$

Hence

$$\begin{aligned} A = U \cap V = & (U \cap V) \cap U \\ &\subseteq (Int(bCl(U)) \cap Int(V)) \cap U \\ &= (Int(bCl(U)) \cap U) \cap Int(V) \\ &= U \cap Int(V) \end{aligned}$$

Therefore $A = (U \cap V) = (U \cap Int(V))$, and A is open. □

From the following examples one can deduce that b-preopen sets and a b-B-sets are independent.

Example 3.2. *Let $X = \{a, b, c, d\}$ and $\tau = \{X, \phi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$. It is clearly that $\{c, b\}$ is b-B-set but it is not b-preopen, since $\{c, b\} = \{b, c, d\} \cap \{c, b\}$, $\{c, b\}$ is b-t-set and $\{c, b\} \nsubseteq Int(bCl(\{c, b\})) = \{c\}$*

Example 3.3. *Let $X = \{a, b, c, d\}$ and $\tau = \{X, \phi, \{b, c, d\}, \{b\}, \{a, b\}\}$. It is clearly that $\{c, b\}$ is b-preopen but it is not b-B-set , since $\{c, b\} \subseteq Int(bCl(\{c, b\})) = X$, since $\{c, b\}$ is not b-t-set and the open set containing $\{c, b\}$ is X or $\{b, c, d\}$, therefore $\{c, b\}$ is not b-B-set.*

Lemma 3.1. *Let A be an open subset of a space X. Then $bCl(A) = Int(Cl(A))$ and $Int(bCl(A)) = Int(Cl(A))$.*

Proof. Let A be open set, then

$$\begin{aligned} bCl(A) &= sCl(A) \cap pCl(A) \\ &= A \cup (Int(Cl(A)) \cap Cl(Int(A))) \\ &= A \cup (Int(Cl(A)) \cap Cl(A)) \\ &= A \cup (Int(Cl(A))) \\ &= Int(Cl(A)). \end{aligned}$$

□

Proposition 3.3. *For a subset A of space (X, τ), the following are equivalent:*

1. *A is regular open,*
2. *$A = Int(bCl(A))$,*
3. *A is b-preopen and a b-t-set.*

Proof. (1) ⇒ (2) Let A be regular open. Since $bCl \subseteq Cl(A)$. Therefore, we have $Int(bCl(A)) \subseteq Int(Cl(A)) = A$, since A is open, $A \subseteq Int(bCl(A))$ hence $A = Int(bCl(A))$.
(2) ⇒ (3) This is obvious
(3)⇒ (1) Let A be b-preopen and a b-t-set. Then $A \subseteq Int(bCl(A)) = Int(A) \subseteq A$ and A is open. by Lemma 3.1, $A = Int(bCl(A)) = Int(Cl(A))$, hence A is regular open. □

Definition 3.2. A subset A of a topological space X is called sb-generalized closed if $s(bCl(A)) \subseteq U$, whenever $A \subseteq U$ and U is b-preopen.

Theorem 3.2. *For a subset A of topological space X, the following properties are equivalent:*

1. *A is regular open;*
2. *A is b-preopen and sb-generalized.*

Proof. (1)⇒ (2) Let A be regular open. Then A is b-open. $A \subseteq Int(bCl(A))$. Moreover,by Lemma 3.1 $s(bCl(A)) = A \cup Int(bCl(A)) = Int(bCl(A)) = Int(Cl(A)) = A$. Therefore, A is sb-generalized closed.
(2) ⇒ (1) Let A be b-preopen and sb-generalized closed. Then we have $s(bCl(A) \subseteq A$ and hence A is b-semiclosed. Therefore $Int(b(Cl(A))) \subseteq A$. Also A is b-preopen, $A \subseteq Int(b(Cl(A)))$ then $A = Int(bCl(A)))$. Therefore by Proposition 3.3 A is regular open. □

4. Decompositions of continuity

In this section, we provide some theorems concerning the decomposition of continuity via the notion of locally b-closed set.

Definition 4.1. A function $f : X \to Y$ is called b-continuous [19](resp. α-continuous [16], semi continuous [9], B-continuous [18] locally closed continuous [6], $D(c,b)$-continuous, locally b-closed continuous) if $f^{-1}(V)$ is b-open (resp. α-open, semi open, B-set, locally closed, $D(c,b)$-set, locally b-closed) in X for each open set V of Y.

Theorem 4.1. *Let $f : X \to Y$ be a function. Then*

1. *If X is extremally disconnected, f is continuous if and only if f is b-continuous and locally closed-continuous.*
2. *f is continuous if and only if f is b-continuous and $D(c,b)$-continuous.*
3. *f is continuous if and only if f is α-continuous and locally b-closed-continuous.*

Proof. 1. It follows from Theorem 2.1.

2. It follows from Theorem 2.2.

3. It follows from Theorem 2.4.

□

From the following example we can see that b-continuous functions and $D(c,b)$-continuous functions are independent.

Example 4.1. *Let $X = \{a,b,c,d\}$ and $\tau = \{X, \phi, \{c\}, \{d\}, \{c,d\}, \{a,c,d\}, \{b,c,d\}\}$.*
Define a function $f : X \longrightarrow X$ such that $f(a) = c$, $f(b) = a$ and $f(c) = d = f(d)$. Then f $D(c,d)$-continuous, but it is not b-continuous, since $f^{-1}(\{c\}) = \{a\}$ is $D(c,d)$-set, but it is not a b-open. And a function $f : X \longrightarrow X$ such that $f(a) = c$, $f(b) = c$, $f(c) = b, f(d) = d$. Then f is b-continuous, but it is not $D(c,d)$-continuous, since $f^{-1}(\{c,d\}) = \{a,b,d\}$ is b-open but it is not $D(c,d)$-set.

From the following example we can see that b-continuous functions and locally closed-continuous functions are independent.

Example 4.2. *Let $X = \{a,b,c,d\}$ and $\tau = \{X, \phi, \{b,c,d\}, \{b\}, \{a,b\}\}$*
Define a function $f : X \longrightarrow X$ such that $f(a) = b$, $f(b) = c$, $f(c) = c$ and $f(d) = d$. Then f is locally closed -continuous, but it is not b-continuous, since $f^{-1}(\{b\}) = \{a\}$ is locally closed but it is not b-open. And a function $f : X \longrightarrow X$ such that $f(a) = d$, $f(b) = b$, $f(c) = a, f(d) = c$, then f is b-continuous, but it is not locally closed-continuous, since $f^{-1}(\{b,c,d\}) = \{a,b,d\}$ which is b-open not locally closed

From the following examples we can see that α-continuous functions and locally b-closed-continuous functions are independent.

Example 4.3. *Let* $X = \{a, b, c, d\}$ *and* $\tau = \{X, \phi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$. *Define a function* $f : X \longrightarrow X$ *such that* $f(a) = c$, $f(b) = a$, $f(c) = b$ *and* $f(d) = d$ *is locally b-closed continuous but it is not* α*-continuous, since* $f^{-1}(\{c, d\}) = \{a, d\}$ *which is locally b-closed not* α*-open.*

Example 4.4. *Let* $X = \{a, b, c, d\}$ *and* $\tau = \{X, \phi, \{b, c, d\}, \{b\}, \{a, b\}\}$. *Define a function* $f : X \longrightarrow X$ *such that* $f(a) = c$, $f(b) = b$, $f(c) = a$ *and* $f(d) = d$ *then* f *is* α *-continuous but it is not locally b-closed-continuous, since* $f^{-1}(\{a, b\}) = \{c, b\}$ *which is* α*-open not locally b-closed.*

Definition 4.2. A function $f : X \to Y$ is called contra b-continuous [19](resp. contra gb-continuous) if $f^{-1}(V)$ is b-closed (resp. gb-closed) in X for each open set V of Y.

Theorem 4.2. *Let* $f : X \to Y$ *be a function . Then* f *is contra b-continuous if and only if it is locally b-closed-continuous and contra gb-continuous.*

Proof. It follows from Theorem 2.5 □

From the following examples we can see that locally b-closed-continuous and contra gb-continuous are independent.

Example 4.5. *Let* $X = \{a, b, c, d\}$ *and* $\tau = \{X, \phi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$. *Let* $Y = \{a, b, c, \}$, *with the topology* $\sigma = \{\phi, Y, \{a\}\}$. *Then the function* $f : (X, \tau) \to (Y, \sigma)$ *is defined by* $f(a) = b$, $f(b) = c$, $f(c) = a$ *and* $f(d) = a$. *Then* f *is contra gb-continuous, but it is not locally b-closed-continuous, since* $f^{-1}(\{a\}) = \{c, d\}$ *which is gb-closed not locally b-closed*

Example 4.6. *Let* $X = \{a, b, c, d\}$ *and* $\tau = \{X, \phi, \{b, c, d\}, \{b\}, \{a, b\}\}$. *Let* $Y = \{a, b, c, \}$, *with the topology* $\sigma = \{\phi, Y, \{a\}\}$. *Then the function* $f : (X, \tau) \to (Y, \sigma)$ *is defined by* $f(a) = b$, $f(b) = a$, $f(c) = a$ *and* $f(d) = c$. *Then* f *is locally b-closed-continuous, but it is not contra gb-continuous, since* $f^{-1}(\{a\}) = \{c, b\}$ *which is locally b-closed not gb-closed.*

Theorem 4.3. *Let* $f : X \to Y$ *be a function . Then* f *is* B*-continuous if and only if it is locally b-closed-continuous and semi-continuous.*

Proof. It follows from Lemma 2.2 □

Definition 4.3. A function $f : X \to Y$ is called b-pre continuous (resp. b-B-continuous, b-t-continuous contra sb-continuous) if $f^{-1}(V)$ is b-preopen (resp. b-B-set, b-t-set, sb-generalized closed) in X for each open set V of Y.

Theorem 4.4. *Let* $f : X \to Y$ *be a function. Then* f *is continuous if and only if it is b-pre-continuous and b-B-continuous.*

Proof. The proof is obvious from Theorem 3.1. □

From the following example we can see that b-pre continuous functions and b-B-continuous functions are independent.

Example 4.7. *Let* $X = \{a, b, c, d\}$ *and* $\tau = \{X, \phi, \{b, c, d\}, \{b\}, \{a, b\}\}$*. Let* $Y = \{a, b, c, \}$*, with the topology* $\sigma = \{\phi, Y, \{a\}\}$*. Then the function* $f : (X, \tau) \to (Y, \sigma)$ *is defined by* $f(a) = b$*,* $f(b) = a$*,* $f(c) = a$ *and* $f(d) = c$*. Then* f *is b-pre-continuous, but it is not b-*B*-continuous, since* $f^{-1}(\{a\}) = \{c, b\}$ *which is b-peropen not b-*B*-set.*

Example 4.8. *Let* $X = \{a, b, c, d\}$ *and* $\tau = \{X, \phi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$*. Let* $Y = \{a, b, c, \}$*, with the topology* $\sigma = \{\phi, Y, \{a\}\}$*. Then the function* $f : (X, \tau) \to (Y, \sigma)$ *is defined by* $f(a) = b$*,* $f(b) = a$*,* $f(c) = a$ *and* $f(d) = c$*. Then* f *is b-*B*-continuous, but it is not b-pre continuous, since* $f^{-1}(\{a\}) = \{c, b\}$ *which is b-*B*-set not b-peropen.*

Theorem 4.5. *Let* $f : X \to Y$ *be a function . Then* f *is completely continuous if and only if b-pre continuous and b-t-continuous.*

Proof. It follows from Theorem 3.3. □

Theorem 4.6. *Let* $f : X \to Y$ *be a function . Then* f *is completely continuous if and only if it is b-pre continuous and contra sb-continuous.*

Proof. It follows from Theorem 3.2. □

Acknowledgments

This work is financially supported by the Ministry of Higher Education, Malaysia under FRGS grant no: UKM-ST-06-FRGS0008-2008. We also would like to thank the referees for useful comments and suggestions.

References

1. D. Andrijević, "On b-open sets", *Mat. Vesnik* **48**(1996), 59-64.
2. D. Andrijević, "Semi-preopen sets" , *Mat. Vesnik* **38**(1)(1986), 24-32.
3. N. Bourbaki, "General Topology" , Part 1, Addison-Wesley (Reading, Mass, 1966).
4. Y. Erguang and T. Pengfei "On Decomposition Of A-continuity " *Acta Math. Hungar.* **110**(4)(2006), 309-313.
5. M. Ganster and I.L. Reilly, "A Decomposition of continuity" *Acta Math. Hungar.* **56**(1990), 229-301.
6. M. Ganster and I.L. Reilly, "Locally closed sets and LC-continuous functions" *International Journal of Mathematics and Mathematical Sciences*, vol.12, (1989), 239-249.
7. T. Hatice, "Decomposition of continuity " *Acta Math. Hungar.* **64**(3)(1994), 309-313.
8. T. Hatice, T. Noiri "Decomposition of continuity and complete Continuity" *Acta Math. Hungar.* **113**(4)(2006), 281-278.

9. N. Levine, "Semi-open sets and semi-continuity in topological spaces", *Amer Math. Monthly* **70**(1963), 36-41.

10. I.L. Reilly and M.K. Vamanamurthy, "On α-Continuity In Topological Spaces" *Acta Math. Hungar.* **45**(1-2)(1985), 27-32.

11. A.A. Nasef, "On b-locally closed sets and related topic", *Chaos Solitions* & *Fractals* **12**(2001), 1909-1915.

12. O. Njastad, "On some classes of nearly open sets" *Pacific J. Math.* **15**(1965), 961-970.

13. T. Noiri, R. Sayed "On Decomposition of continuity " *Acta Math. Hungar.* **111**(1-2)(2006), 1-8.

14. T. Noiri, On almost continuous functions, *Indian J. Pure Appl. Math.* **20**(1989), 571-576.

15. A.S. Mashhour , M.E. Abd El-Monsef and S.N. El-Deeb, "On precontinuous and weak precontinuous mappings", *Proc. Math. Phys. Soc. Egypt* **53**(1982), 47-53.

16. A.S. Mashhour ,I.A. Hasanein and S.N. El-Deeb,, "α-continuity and α-open mappings" *Acta Math. Hungar.* **41**(1983), 213-218.

17. J. Tong, "A Decomposition of continuity " *Acta Math. Hungar.* **48**(1986), 11-15.

18. J. Tong, "On Decomposition of continuity in topological spaces" *Acta Math. Hungar.* **54**(1-2)(1989), 51-55.

19. J.H. Park, "Strongly θ-b continuous functions" *Acta Math. Hungar.* **110**(4)(2006), 347-359.

20. M. Przemski, "A Decomposition of continuity and α-Continuity" *Acta Math. Hungar.* **61**(1-2)(1993), 93-98.

Ahmad Al-Omari and Mohd. Salmi Md. Noorani
School of mathematical Sciences, Faculty of Science and Technology
Universiti Kebangsaan Malaysia,
43600 UKM Bangi, Selangor,
Malaysia.
omarimutah1@yahoo.com, (Corresponding author)
msn@pkrisc.cc.ukm.my