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Jordan α-centralizers in rings and some applications

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ABSTRACT: Let R be a ring, and α be an endomorphism of R. An additive mapping $H\colon R \to R$ is called a left α-centralizer (resp. Jordan left α-centralizer) if $H(xy) = H(x)\alpha(y)$ for all $x, y \in R$ (resp. $H(x^2) = H(x)\alpha(x)$ for all $x \in R$). The purpose of this paper is to prove two results concerning Jordan α-centralizers and one result related to generalized Jordan (α, β)-derivations. The result which we refer state as follows: Let R be a 2-torsion-free semiprime ring, and α be an automorphism of R. If $H\colon R \to R$ is an additive mapping such that $H(x^2) = H(x)\alpha(x)$ for every $x \in R$ or $H(xyx) = H(x)\alpha(yx)$ for all $x, y \in R$, then H is a left α-centralizer on R. Secondly, this result is used to prove that every generalized Jordan (α, β)-derivation on a 2-torsion-free semiprime ring is a generalized (α, β)-derivation. Finally, some examples are given to demonstrate that the restrictions imposed on the hypothesis of the various theorems were not superfluous.

Key Words: Semiprime ring, 2-torsion-free ring, Jordan centralizer, Jordan α-centralizer, generalized derivations, generalized Jordan (α, β)-derivations.

Contents

1. Introduction

This research has been motivated by the works of E. Albas [1] and J. Vukman [13]. Throughout, the present paper R will denote an associative ring with center $Z(R)$, not necessarily with an identity element. For any $x, y \in R$, as usual $[x, y] = xy - yx$ and $x \circ y = xy + yx$ will denote the well-known Lie and Jordan products, respectively. We shall make extensive use of basic commutator identities: $[xy, z] = x[y, z] + [x, z]y$, $[x, yz] = y[x, z] + [x, y]z$. A ring R is *n-torsion-free*, where n is an integer in case $nx = 0$, for $x \in R$, implies $x = 0$. Recall that a ring R is *prime* if for any $a, b \in R$, $aRb = (0)$ implies that $a = 0$ or $b = 0$, and is called *semiprime* in case $aRa = (0)$ implies $a = 0$.

An additive mapping $d\colon R \to R$ is called a *derivation* (resp. *Jordan derivation*) if $d(ab) = d(a)b + ad(b)$ holds for all $a, b \in R$ (resp. $d(a^2) = d(a)a + ad(a)$ holds for all $a \in R$). For a fixed $a \in R$, define $d\colon R \to R$ by $d(x) = [x, a]$ for all $x \in R$, called an *inner derivation* (see [4] for a partial bibliography).

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Following B. Hvala [[9], page 1447], an additive mapping $F\colon R \to R$ is called a *generalized derivation* if there exists a derivation $d\colon R \to R$ such that $F(xy) = F(x)y + xd(y)$ holds for all $x, y \in R$. We call an additive mapping $F\colon R \to R$ a *generalized Jordan derivation* if there exists a derivation $d\colon R \to R$ such that $F(x^2) = F(x)x + xd(x)$ holds for all $x \in R$ [[5], page 7]. In [[5], Theorem], M. Ashraf and N. Rehman showed that in a 2-torsion-free ring R which has a commutator nonzero divisor, every generalized Jordan derivation on R is a generalized derivation. It is easy to see that $F\colon R \to R$ is a generalized derivation iff F is of the form $F = d + H$, where d is a derivation and H a left centralizer on R.

According B. Zalar [17], an additive mapping $H\colon R \to R$ is called a *left* (resp. *right*) *centralizer* of R if $H(xy) = H(x)y$ (resp. $H(xy) = xH(y)$) holds for all $x, y \in R$. If $a \in R$, then $L_a(x) = ax$ is a left centralizer and $R_a(x) = xa$ is a right centralizer. If H is both left as well right centralizer, then H is a *centralizer*. In case R has an identity element, $H\colon R \to R$ is a left (resp. right) centralizer iff H is of the form $L_a(x)$ (resp. $R_a x$) for some fixed element $a \in R$. An additive mapping $H\colon R \to R$ is called a *left* (resp. *right*) *Jordan centralizer* in case $H(x^2) = H(x)x$ (resp. $H(x^2) = xH(x)$) holds for $x \in R$.

It is well-known that Jordan derivations can be defined as $d(x \circ y) = d(x) \circ y + x \circ d(y)$, for all $x, y \in R$. Therefore, we can define a Jordan centralizer to be an additive mapping H which satisfies $H(x \circ y) = H(x) \circ y = x \circ H(y)$, for all $x, y \in R$. Since the product $\circ$ is commutative, there is no difference between the Jordan left and right centralizers. In [17], it was shown that a Jordan left centralizer of a semiprime ring is a left centralizer, and each Jordan centralizer is a centralizer.

Recently, E. Albaş [1] introduced the following definitions, which are generalizations of the definitions of centralizer and Jordan centralizer. Let R be a ring, and α be an endomorphism of R. A *Jordan α-centralizer* of R is an additive mapping $H\colon R \to R$ satisfying $H(xy+yx) = H(x)\alpha(y)+\alpha(y)H(x) = H(y)\alpha(x)+\alpha(x)H(y)$ for all $x, y \in R$. An additive mapping $H\colon R \to R$ is called a *left* (resp. *right*) *α-centralizer* of R if $H(xy) = H(x)\alpha(y)$ (resp. $H(xy) = \alpha(x)H(y)$) for all $x, y \in R$. If H is a left and right α-centralizer then it is natural to call H an *α-centralizer*. It is clear that for an additive mapping $H\colon R \to R$ associated with a homomorphism $\theta\colon R \to R$, if $L_a(x) = a\theta(x)$ and $R_a(x) = \theta(x)a$ for a fixed element $a \in R$ and for all $x \in R$, then L_a is a left $\theta-$centralizer and R_a is a right $\theta-$centralizer. Clearly every centralizer is a special case of a θ-centralizer with $\theta = id_R$.

Let $H\colon R \to R$ be an additive mapping and α be an endomorphism of R. We call H a *Jordan left* (resp. *right*) *α-centralizer* if $H(x^2) = H(x)\alpha(x)$ (resp. $H(x^2) = \alpha(x)H(x)$) holds for all $x \in R$. Note that for $\alpha = id_R$, identity map on R, then we have the usual well-known definitions of Jordan left and right centralizer mappings. Obviously every left (right) centralizer is a Jordan left (right) centralizer. The converse is in general not true (see [1], Example 1). In [17], B. Zalar proved that every Jordan left (right) centralizer on a 2-torsion-free semiprime ring is a left (right) centralizer. Considerable work has been done on Jordan left (right) centralizers in prime and semiprime rings during the last couple of decades (see for example: [3], [7], [8], [12], [14], [15], [16], where further references can be found).

If $H\colon R \to R$ is a centralizer, where R is an arbitrary ring, then H satisfies the relation

$$H(xyx) = xH(y)x, \text{for all } x, y \in R. \quad (1)$$

It seems natural to ask whether the converse is true. More precisely, asking for whether an additive mapping H on a ring R satisfying relation (1) is a centralizer. In [15], J. Vukman proved that the answer is affirmative in case R is a 2-torsion-free semiprime ring. In [1], Albaş proved, under some conditions, that in a 2-torsion-free semiprime ring R, every Jordan θ-centralizer is a θ-centralizer. In [7], W. Cortes and C. Haetinger proved this question changing the semiprimality condition on R by the existence of a commutator right (resp. left) nonzero divisor. And in [8], M.N. Daif, M.S. Tammam El-Sayiad and C. Haetinger proved that in a 2-torsion-free semiprime ring R, for an endomorphism θ of R and for an additive mapping $T{:}R \to R$ such that $T(xyx) = \theta(x)T(y)\theta(x)$ holds for all $x, y \in R$, then T is a θ-centralizer of R.

In the year 1995, L. Molnar [12] proved that if R is a 2-torsion-free prime ring and T: $R \to R$ is an additive function such that $T(xyx) = T(x)yx$ for all $x, y \in R$, then T is a left (right) centralizer.

In Section 2, we generalize the above mentioned result for semiprime rings. Further, some related result have been discussed.

Let α, β be endomorphisms of a ring R. An additive mapping d: $R \to R$ is said to be an (α, β)-*derivation* (resp. *Jordan* (α, β)-*derivation*) if $d(xy) = d(x)\alpha(y) + \beta(x)d(y)$ holds for all $x, y \in R$ (resp. $d(x^2) = d(x)\alpha(x) + \beta(x)d(x)$ holds for all $x \in R$). Following M. Ashraf, A. Ali and S. Ali [2], an additive mapping F: $R \to R$ is called a *generalized* (α, β)-*derivation* (resp. *generalized Jordan* (α, β)-*derivation*) on R if there exists an (α, β)-derivation d: $R \to R$ such that $F(xy) = F(x)\alpha(y) + \beta(x)d(y)$ holds for all $x, y \in R$ (resp. $F(x^2) = F(x)\alpha(x) + \beta(x)d(x)$ holds for all $x \in R$). More general, we call an additive mapping F: $R \to R$ is a *generalized Jordan* (α, β)-*derivation*) on R if there exists a Jordan (α, β)-derivation d: $R \to R$ such that $F(x^2) = F(x)\alpha(x) + \beta(x)d(x)$ holds for all $x \in R$. Note that for id_R, the identity map on R, an generalized Jordan (id_R, id_R)-derivation is called simply a generalized Jordan derivation. Clearly, every generalized derivation on a ring is a generalized Jordan derivation. But the converse need not be true in general (see [6], Example). A number of authors have studied this problem in the setting of prime and semiprime rings (viz. [2], [6], [10] and [13], where further references can be found). In the year 2007, J. Vukman [13] proved that every generalized Jordan derivation on a 2-torsion-free semiprime ring is a generalized derivation.

In Section 3, we discuss the applications of the theory of α-centralizers (multipliers) and extend Vukman's result in the setting of generalized Jordan (α, β)-derivation.

We shall restrict our attention on left centralizers since all results presented in this paper are true also for right centralizers because of left and right symmetry.

2. Jordan α-centralizers

The main goal of this section is to prove the following theorem which generalizes Theorem 2 in [12]:

Theorem 2.1 *Let R be a 2-torsion-free semiprime ring and α be an automorphism of R. If $H\colon R \to R$ is an additive mapping such that $H(xyx) = H(x)\alpha(yx)$ (resp. $H(xyx) = \alpha(xy)H(x)$) for all $x, y \in R$, then H is a left (resp. right) α-centralizer on R.*

Now, we begin with the following known result:

Lemma 2.1 [[11], Theorem 2] *Let R be a 2-torsion-free semiprime ring and d a Jordan (θ, ϕ)-derivation of R with θ or ϕ an automorphism of R. Then d is a (θ, ϕ)-derivation of R.*

In [1], E. Albaş have proved the following result: Let R be a 2-torsion-free semiprime ring, and α be an automorphism of R. If $\alpha(Z(R)) = Z(R)$, then each Jordan left α-centralizer of R is a left α-centralizer. We now generalize Albas's result as follows:

Theorem 2.2 *Let R be a 2-torsion-free semiprime ring and α be an automorphism of R. If $H\colon R \to R$ is an additive mapping such that $H(x^2) = H(x)\alpha(x)$ for all $x \in R$, then H is a left α-centralizer.*

Proof: The proof runs on same parallel lines as of Theorem 2 of [11] under the case when ϕ is the trivial map, i.e., $\phi = 0$. □

As a corollary we obtain [[17], Proposition 1.4].

Corollary 2.2A *Let R be a 2-torsion-free semiprime ring. If $H\colon R \to R$ is an additive mapping such that $H(x^2) = H(x)x$ for all $x \in R$, then H is a left centralizer on R.*

Now we will prove our main theorem of this section:

Proof: [Proof of Theorem 2.1] By the hypothesis, we have

$$H(xyx) = H(x)\alpha(yx), \quad \forall\, x, y \in R. \tag{2}$$

Replacing x by $x + z$ in (2), we find, for every $x, y \in R$,

$$H((x+z)y(x+z)) = H(x)\alpha(yx) + H(x)\alpha(yz) + H(z)\alpha(yx) + H(z)\alpha(yz). \tag{3}$$

On the other hand, we obtain, for all $x, y \in R$,

$$\begin{aligned} H((x+z)y(x+z)) &= H(xyz + zyx + xyx + zyz) \\ &= H(xyz + zyx) + H(x)\alpha(yx) + H(z)\alpha(yz). \end{aligned} \tag{4}$$

Combining (3) and (4), we get

$$H(xyz + zyx) = H(x)\alpha(yz) + H(z)\alpha(yx), \quad \forall\, x, y, z \in R. \tag{5}$$

Put $z = x^2$ in (5), to get

$$H(xyx^2 + x^2yx) = H(x)\alpha(yx^2) + H(x^2)\alpha(yx), \quad \forall\, x, y \in R. \tag{6}$$

Further, taking $y = xy + yx$ in (2) and using (2), we get

$$H(xyx^2 + x^2yx) = H(x)\alpha(xyx) + H(x)\alpha(yx^2), \quad \forall\, x, y \in R. \tag{7}$$

On combining last two equations, we obtain

$$H(x^2)\alpha(yx) - H(x)\alpha(x)\alpha(yx) = 0, \quad \forall\, x, y \in R. \tag{8}$$

Now, we set $A(x) = H(x^2) - H(x)\alpha(x)$ for every $x \in R$. Then, (8) reduces to

$$A(x)\alpha(yx) = 0, \quad \forall\, x, y \in R. \tag{9}$$

Since α is onto, (9) implies that

$$A(x)y_1\alpha(x) = 0, \quad \forall\, x, y_1 \in R. \tag{10}$$

Replacing y_1 by $\alpha(x)zA(x)$ in (10), then (10) gives that

$$A(x)\alpha(x)zA(x)\alpha(x) = 0, \ \textit{i.e.}, \quad A(x)\alpha(x)RA(x)\alpha(x) = (0), \quad \forall\, x \in R. \tag{11}$$

Semiprimeness of R yields that

$$A(x)\alpha(x) = 0, \quad \forall\, x \in R. \tag{12}$$

Linearizing (12), we get

$$A(x+y)\alpha(x) + A(x+y)\alpha(y) = 0, \quad \forall\, x, y \in R. \tag{13}$$

Now, we compute

$$\begin{aligned} A(x+y) &= (H(xy+yx) - H(x)\alpha(y) - H(y)\alpha(x)) \\ &\quad + \big(H(x^2) - H(x)\alpha(x)\big) + \big(H(y^2) - H(y)\alpha(y)\big) \\ &= B(x,y) + A(x) + A(y), \quad \forall\, x, y \in R, \end{aligned} \tag{14}$$

where $B(x,y) = H(xy+yx) - H(x)\alpha(y) - H(y)\alpha(x)$, for every $x, y \in R$.

Thus, in view of (14), expression (13) implies that

$$A(x)\alpha(y) + B(x,y)\alpha(x) + A(y)\alpha(x) + B(x,y)\alpha(y) = 0, \quad \forall\, x, y \in R. \tag{15}$$

Again, replace x by $-x$ in the last equation, to get

$$A(x)\alpha(y) + B(x,y)\alpha(x) - A(y)\alpha(x) - B(x,y)\alpha(y) = 0, \quad \forall\, x, y \in R. \tag{16}$$

Adding (15) with (16) and using the fact that R is a 2-torsion-free semiprime ring, we find that

$$A(x)\alpha(y) + B(x,y)\alpha(x) = 0, \quad \forall\, x, y \in R. \tag{17}$$

On right multiplication of equation (17) by $A(x)$, we obtain

$$A(x)\alpha(y)A(x) + B(x,y)\alpha(x)A(x) = 0, \quad \forall\, x, y \in R. \tag{18}$$

From equation (10), we find that

$$\alpha(x)A(x)y_1\alpha(x)A(x) = 0, \; i.e., \quad \alpha(x)A(x)R\alpha(x)A(x) = (0), \quad \forall\, x \in R. \tag{19}$$

The last expression forces that

$$\alpha(x)A(x) = 0, \quad \forall\, x \in R. \tag{20}$$

On combining (18) and (20), we have

$$A(x)\alpha(y)A(x) = 0, \; i.e., \quad A(x)RA(x) = (0), \quad \forall\, x \in R. \tag{21}$$

Thus our hypothesis yields that $A(x) = 0$, i.e., $H(x^2) - H(x)\alpha(x) = 0$, for each $x \in R$. Therefore, H is a Jordan left α-centralizer and hence, H is a left α-centralizer on R by Theorem 2.2. This completes the proof of our theorem. □

In Theorem 2.1, putting $y = x$, then we obtain the following:

Theorem 2.3 *Let R be a 2-torsion-free semiprime ring and α be an automorphism of R. If $H\colon R \to R$ is an additive mapping such that $H(x^3) = H(x)\alpha(x^2)$ (resp. $H(x^3) = \alpha(x^2)H(x)$) for all $x \in R$, then H is a left (resp. right) α-centralizer on R.*

Following are the immediate consequences of Theorem 2.1:

Corollary 2.3B [[12], Theorem 2] *Let R be a 2-torsion-free prime ring. If $H\colon R \to R$ is an additive mapping such that $H(xyx) = H(x)yx$ for all $x, y \in R$, then H is a left centralizer on R.*

Corollary 2.3C *Let R be a 2-torsion-free semiprime ring. If $H\colon R \to R$ is an additive mapping such that $H(xyx) = H(x)yx$ (resp. $H(xyx) = xyH(x)$) for all $x, y \in R$, then H is a left (resp. right) centralizer on R.*

The following example demonstrates that Theorems 2.1 and 2.2 are not true in the case of arbitrary rings:

Example 2.1 *Let R' be a ring such that the square of each element in R' is zero, but the product of some elements in R' is nonzero.*

Next, let $R = \left\{ r = \begin{pmatrix} x & y \\ 0 & 0 \end{pmatrix} \mid \forall\ x, y \in R' \right\}$*. We define the mappings* $H\colon R \to R$ *and* $\alpha\colon R \to R$ *as* $H(r) = \begin{pmatrix} 0 & x \\ 0 & 0 \end{pmatrix}$ *and* $\alpha(r) = \begin{pmatrix} x & -y \\ 0 & 0 \end{pmatrix}$*, for all $r \in R$. Then, H is a Jordan left α-centralizer but not a left α-centralizer on R. Also, it is easy to verify that $H(aba) \neq H(a)\alpha(ba)$ for some nonzero elements $a, b \in R$.*

We conclude this section with the following conjecture in view of Theorem 2.3:

Conjecture 1 *Let R be a semiprime ring with suitable torsion restrictions, and α be an automorphism of R. If $H : R \to R$ is an additive mapping such that $H(x^n) = H(x)\alpha(x^{n-1})$ holds for all $x \in R$ and $n \geq 1$, then H is a left α-centralizer on R.*

3. Applications

In this section, we present some applications of the theory of α-centralizes in rings. Very recently, J. Vukman in [13] proved a result concerning generalized Jordan derivations on semiprime rings. More, explicitly, he showed that every generalized Jordan derivation on a 2-torsion-free semiprime ring is a generalized derivation. Further in the year 2007, C. Lanski [11] obtained the following result: For $n \geq 2$, let R be a n-torsion-free semiprime ring with identity 1 and F, d: $R \to R$ be additive mappings. Let β be an endomorphism of R with $\beta(1) = 1$, and let α be an automorphism of R. Assume for all $x \in R$, $F(x^n) = F(x)\alpha(x^{n-1}) + \sum_{j=1}^{n-1} \beta(x^j)d(x)\alpha(x^{n-j-1})$, where $\alpha(x^0) = 1 = \beta(x^0)$. Then d is an (α, β)-derivation of R and F is a generalized (α, β)-derivation of R with respect to d. Further, if d is assumed to be an (α, β)-derivation of R, then one need assume only that α is an unital endomorphism of R. If F: $R \to R$ is a generalized Jordan (α, β)-derivation of R associated with Jordan (α, β)-derivation d: $R \to R$, then it is easy to see that F is a generalized Jordan (α, β)-derivation of R iff F is of the form $F = d + H$, where d is a Jordan (α, β)-derivation and H is a Jordan left α-centralizer of R. Thus, we can write $H = F - d$.

In the proof of Theorem 3.1 below, we use this technique which can be regarded as a contribution to the theory of α-centralizers in rings. It is also to remark that our approach differs from those used by C. Lanski in [11]. Here, our main intention is to prove Lanski's result mentioned above for arbitrary semiprime rings (without assuming R has an identity element) in the case $n = 2$ which includes the result of J. Vukman [[13], Theorem 1]. In fact, we obtain the following result:

Theorem 3.1 *Let R be a 2-torsion-free semiprime ring. Suppose that α, β are endomorphisms of R such that α is an automorphism and d is a Jordan (α, β)-derivation of R. If $F\colon R \to R$ is a generalized Jordan (α, β)-derivation on R, then F is a generalized (α, β)-derivation on R.*

Proof: We are given that F is a generalized Jordan (α, β)-derivation. Therefore, we have

$$F(x^2) = F(x)\alpha(x) + \beta(x)d(x), \text{for all } x \in R. \tag{22}$$

In equation (22), we take d as a Jordan (α, β)-derivation on R. Since R is a 2-torsion-free semiprime ring, so in view of Lemma 2.1, d is an (α, β)-derivation on R. Now we write $H = F - d$. Then, find that

$$\begin{aligned} H(x^2) &= (F-d)(x^2) = F(x^2) - d(x^2) \\ &= F(x)\alpha(x) + \beta(x)d(x) - d(x)\alpha(x) - \beta(x)d(x) \\ &= F(x)\alpha(x) - d(x)\alpha(x) \\ &= (F(x) - d(x))\alpha(x) \\ &= H(x)\alpha(x), \text{ for all } x \in R. \end{aligned} \tag{23}$$

This implies that $H(x^2) = H(x)\alpha(x)$, for all $x \in R$. That is, H is a Jordan left α-centralizer. Thus, by Theorem 2.2, one can conclude that H is a left α-centralizer. Therefore, we prove that F can be written as $F = H + d$, where d is an (α, β)-derivation and H is a left α-centralizer on R. Hence, F is a generalized (α, β)-derivation on R. This completes the proof. □

As an immediate consequences of above theorem, we have the following:

Corollary 3.1A [[13], Theorem 1] *Let R be a 2-torsion-free semiprime ring and let $F\colon R \to R$ be a generalized Jordan derivation. Then F is a generalized derivation.*

Corollary 3.1B *Let R be a 2-torsion-free prime ring. Suppose that α, β are endomorphisms of R such that α is an automorphism and d is a Jordan (α, β)-derivation of R. If $F\colon R \to R$ is a generalized Jordan (α, β)-derivation on R, then F is a generalized (α, β)-derivation on R.*

Corollary 3.1C [[10], Theorem 2.5] *Let R be a 2-torsion-free prime ring and let $F\colon R \to R$ be a generalized Jordan derivation. Then F is a generalized derivation.*

We end our discussion with the following example which shows that the mentioned condition in the hypotheses of Theorem 3.1 is crucial:

Example 3.1 *Consider the ring R', as in Example 2.1.*

Next, let $R = \left\{ r = \begin{pmatrix} 0 & a & b \\ 0 & 0 & a \\ 0 & 0 & 0 \end{pmatrix} \mid \forall a, b \in R' \right\}$. *Define maps $F, d\colon R \to R$ and $\alpha, \beta\colon R \to R$, for every $r \in R$ as follows:*

$F(r) = \begin{pmatrix} 0 & 0 & b \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $\delta(r) = \begin{pmatrix} 0 & a & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $\alpha(r) = \begin{pmatrix} 0 & -a & b \\ 0 & 0 & -a \\ 0 & 0 & 0 \end{pmatrix}$, $\beta(r) = \begin{pmatrix} 0 & -a & -b \\ 0 & 0 & -a \\ 0 & 0 & 0 \end{pmatrix}$.

Then, it is straightforward to check that F is a generalized Jordan (α, β)-derivation but not a generalized (α, β)-derivation on R.

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